From basic kinematics to Regge poles

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Introduction

- Donnachie and Landshoff in 1992 concluded their analysis on total cross sections based on Regge-type fits stating that *Regge theory remains one of the great truths of particle physics*.
- 15 years later this remains even more remarkably true than ever but,
- why?

  and,
- what was the birth of Regge poles?

- We shall briefly outline what motivated Regge poles
- why they are a theory more than a model
- how they were born
- how they made it to become an extremely useful instrument in high energy physics
- what made them slowly disappear from the scene
- what triggered their comeback
- why they are still with us (and presumably will remain for a long time to come if not forever)

- Regge poles were born in a brilliant attempt to learn about the properties of the full fledged S-Matrix theory starting from the only really fully manageable scheme, non relativistic potential model. The basic assumption is that we can use the Schrödinger equation to describe the elastic interaction of two (spinless, for simplicity)

$$a + b \rightarrow a + b$$

$$(-h^2/2\mu)\, \Delta\, \psi(r) + V(r)\, \psi(r) = E\, \psi(r)$$

- The conventional wisdom when using potential scattering to deal with particle physics problems is that the realistic underlying dynamics can be mimicked by Yukawa-like spherical (or central) potentials of the type:

$$V(r) = \int g(\alpha)\, e^{-\alpha r}\, d\alpha .$$

Using

$k = 2\mu/\hbar^2\ E$, $U(r) = 2\mu/\hbar^2\ V(r)$

the Schrödinger equation takes the form

$$[\Delta - U(r) - k^2]\ \psi(r) = 0.$$

- The assumption is that an impinging particle comes from (-)infinity as a plane wave to interact with a spherical symmetric potential so that at large (positive) distances, the asymptotic solution of (1.5) becomes the superposition of the incoming plane wave plus a distorted outgoing spherical wave of the form

$$\psi(\mathbf{r}) \sim_{r\to\infty} e^{i\mathbf{k}\cdot\mathbf{r}} + f(\mathbf{k}, \mathbf{k}')\, e^{ikr}/r$$

$\sum_{\ell=0}^{\infty}$

- f(**k, k'**) is known as the scattering amplitude whose squared modulus gives (in potential theory) the differential cross section

$$d\sigma/d\Omega = |f(\mathbf{k}, \mathbf{k'})|^2 = |f(k, \theta)|^2$$

As usual, for a spherical interaction, the scattering amplitude can be expanded in partial waves over all integer positive values of the angular momentum ℓ (from $\ell=0$ to $\ell=\infty$)

$$\mathbf{f(k, \theta) = \Sigma_\ell\, (2\ell+1)\, a_\ell(k)\, P_\ell(\cos\, \theta).}$$

- $a_\ell(k)$ are (complex) quantities known as partial waves related to the phase shifts $\delta_\ell(k)$ (and to the S-Matrix partial wave amplitudes) by

$$a_\ell(k) = (1/2ik)\,[\exp(2i\,\delta_\ell(k)) - 1] = (1/2ik)\,[S_\ell(k) - 1]$$

or

$$S_\ell(k) = \exp[2i\,\delta_\ell(k)]$$

- For elastic scattering (no inelastic channels open implies no absorption or, within the present scheme, real potentials), $\delta_\ell(k)$ are real quantities and the *elastic unitarity condition* reads $|\mathbf{S}_\ell(\mathbf{k})| = 1$ or

$$\mathbf{Im}\ \mathbf{a}_\ell(\mathbf{k}) = \mathbf{k}\ |\mathbf{a}_\ell(\mathbf{k})|^2$$

from the above results the *optical theorem*

$$\mathbf{Im}\ \mathbf{f}(\mathbf{k}, 0) = \mathbf{k}/\pi\ \ \sigma_{\mathbf{tot}}$$

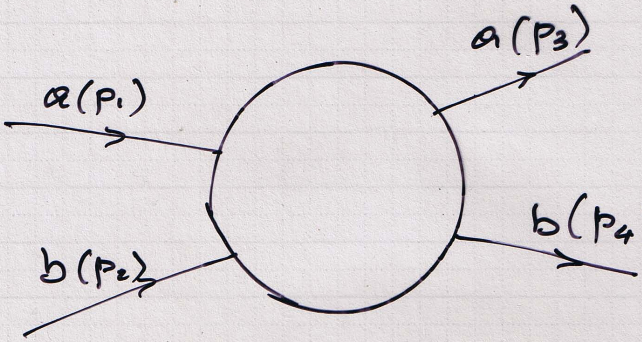
Moving to relativistic kinematics

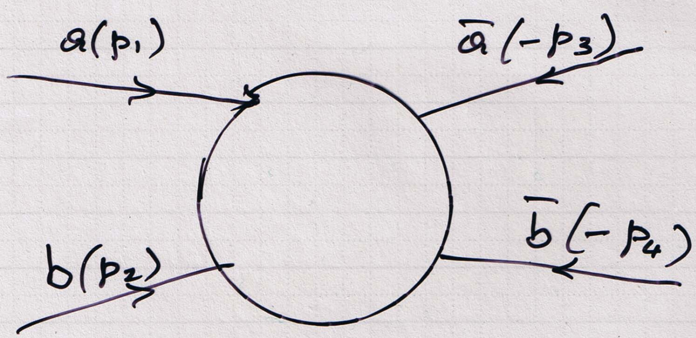
- In the relativistic scheme, the simple minded two body reaction $\mathbf{a+b \rightarrow a+b}$ is no longer the full story; due to crossing and the other properties in QFT, we have three independent but related reactions. More precisely, if we use the property that an outgoing particle can be viewed as an incoming antiparticle of reversed fourmomentum, for a two-body process, we have three channels in which related (but different) reactions occur. Schematically, if we underline a particle to denote the corresponding antiparticle we can relate $\mathbf{a+b \to a+b}$ to the annihilation process $\mathbf{a+b+\underline{a}+\underline{b}}$ and this leads to three channels which are called "s", "t" and "u" in reminiscence of the values taken by the corresponding Mandelstam variables

Crossing

$$a(p_1) + b(p_2) \longrightarrow a(p_3) + b(p_4)$$

$$p_1 + p_2 = p_3 + p_4$$





$$a(p_1) + b(p_2) + \bar{a}(-p_3) + \bar{b}(-p_4)$$

$$p_1 + p_2 - p_3 - p_4 = 0$$

(2.1) (s-channel)

$$a(p_1) + b(p_2) \rightarrow a(p_3) + b(p_4)$$

(2.2) (t-channel)

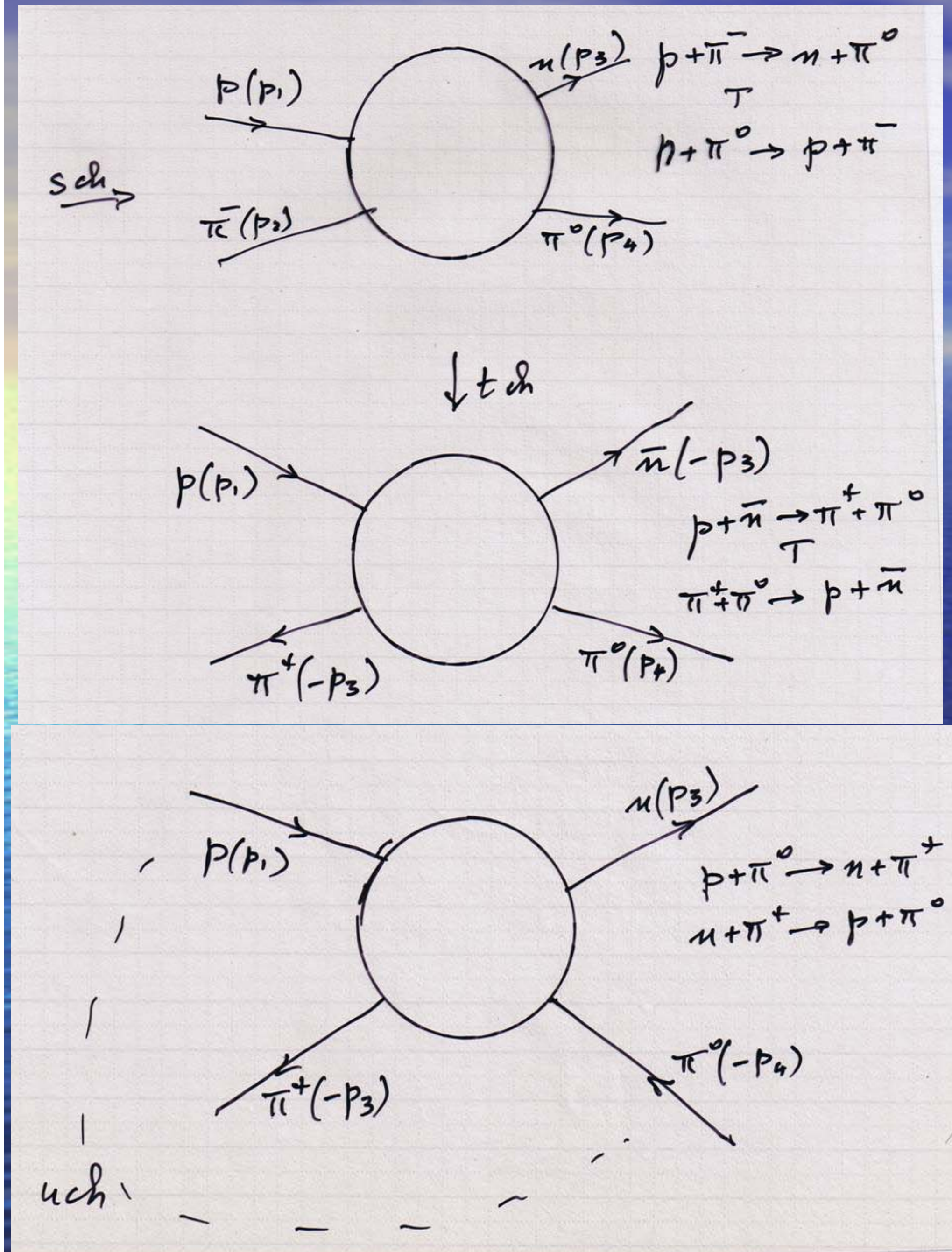
$$a(p_1) + \underline{a}(-p_3) \rightarrow \underline{b}(-p_2) + b(p_4)$$

(2.3) (u-channel)

$$a(p_1) + \underline{b}(-p_4) \rightarrow a(p_3) + \underline{b}(-p_2)$$

are, by analytic continuation one and thesame taken in different regions of the complex variables describing the above reactions.

- **In (2.1-3), incoming/outgoing particles are viewed as outgoing/ incoming antiparticles of reversed momentum (the other three possible reactions are simply the time reversed of the previous ones).**
- Just as an example, take the charge exchange reaction $\pi^- p \rightarrow \pi^\circ n$



Charge exchange reactions

- To exemplify, the s-channel (charge exchange) reaction

$$p(p_1) \;+\; \pi^-(p_2) \;\rightarrow\; n(p_3) \;+\; \pi^\circ(p_4)$$

becomes in the t-channel the annihilation reaction

$$p(p_1) \;+\; \underline{n}(-p_3) \;\rightarrow\; \pi^+(-p_2) \;+\; \pi^\circ(p_4)$$

and, in the u-channel the hitherto experimentally inaccessible charge exchange reaction

$$p(p_1) \;+\; \pi^\circ(-p_4) \;\rightarrow\; n(p_3) \;+\; \pi^+(-p_2)$$

(use has been made of the fact that π^+ is the antiparticle of π^- and that π° is its own antiparticle).

- **Formal complications arise in the case of realistic particles endowed with so far ignored quantum numbers (like spin etc.) but these need not to concern us here.**

- These three reactions (2.1-3) are labelled s-, t- and u-channel respectively since for each of them the corresponding invariant Mandelstam variable
- (2.4) $s = (p_1 + p_2)^2$
- (2.5) $t = (p_1 - p_3)^2$
- (2.6) $u = (p_1 - p_4)^2$

is positive definite while the other two are negative being, in essence, the four dimensional momentum transfer of the corresponding reaction.

For instance, in the s-channel (equal masses case), in the c. m. we have

$s = (p_1 + p_2)^2 = 4\,(k^2 + m^2) > 0$

$t = (p_1 - p_3)^2 = -\,2k^2\,(1 - \cos\theta_s) \leq 0$

$u = (p_1 - p_4)^2 = -\,2k^2\,(1 + \cos\theta_s) \leq 0$. **As a consequence,**

$$\cos\theta_s = 1 + 2t/(s - 4m^2) \qquad (2.7)$$

For on-shell particles only two of these variables are independent and, in fact

$$s + t + u = 4\,m^2 .$$

- This situation may change in case one of the particles is not on shell. Such is the case of an *inclusive* reaction such as

(2.8) $$a + b \rightarrow c + X,$$

(i.e. a reaction where X stays for an unresolved cluster of undetected particles in the final state) will be dealt with. In this case, a third variable, for instance the so-called *missing mass* (or any other independent variable)

(2.9) $$p_X^{\,2} = (p_1 + p_2 - p_3)^2$$

will have to be used to properly describe the process.

3. Problems with (high spin) meson exchange.

- According to the general wisdom going back to the old days of Yukawa (1935), the nuclear forces acting between hadrons are due to virtual particles (mesons) exchanged in the crossed t and/or u channels in strict analogy with e.m. interactions arising from the exchange of virtual photons between electrons.

- This picture becomes inapplicable at high energies (i.e. $s \rightarrow \infty$) for the following reason.
- Consider a generic two body reaction

(3.1) $1 + 2 \rightarrow 3 + 4$

mediated by single particle exchange in the t-channel. The scattering amplitude for the exchange of a particle of mass M and spin J goes as

(3.2) $A_{mes}(s, t) \sim A_J(t)\ P_J(\cos\theta_t) = P_J(\cos\theta_t) / (t - M^2)$

where m is the mass of the interacting particle and

(3.3) $\cos\theta_t = 1 + 2s / (t - 4m^2)$

is the t-channel scattering angle (compare with eq. (2.7)).

If in (3.2) we keep t fixed and let $s \to \infty$, using $P_\ell(z) \sim z^\ell$ as $z \to \infty$ we find

(3.4) $$A_{mes}(s, t) \sim s^J$$

which corresponds to a cross section growing like s^{2J-2} as $s \to \infty$.

This behaviour can be proved to violate (s-channel) unitarity since it violates the Froissart-Martin bound which, owing to unitarity requires σ_{tot} to be bounded by $\ln^2 s$).

- As we will see, Regge theory overcomes this difficulty while preserving the notion of crossed channel exchange. Also, according to Regge theory, the strong interaction will turn out to be due not to the exchange of particles of definite spin but to ***Regge trajectories*** i.e. to entire families of resonances.

4. Regge poles in non relativistic potential models.

The starting point is the partial wave s-channel expansion of the scattering amplitude (1.9) which we rewrite as

(4.1) $\mathbf{A(s, z) = \Sigma_{\ell} (2\ell+1) A_{\ell}(s) P_{\ell}(z)}$

where z is the cosine of the physical s-channel scattering angle

(4.2) $z \equiv \cos \theta_s = 1 + 2t/(s - 4m^2)$

- The representation (4.1) of the scattering amplitude is defined in the physical s-channel domain given by

(4.3) $s \geq 4\,m^2$ and $-1 \leq z \leq 1$.

- The question arises, therefore, whether (4.1) converges in a domain of the complex s, t and u variables larger than (4.3) and, more specifically, in a sufficiently large physical domains of the crossed t- and u-channels. As we shall see, this is not the case and the reason is simple to understand qualitatively.

- The s-channel singularities of A(s,t) are contained in the partial wave amplitudes $A_\ell(s)$ but the t-dependence is embodied in the Legendre polynomials and these are entire functions of their argument so that any singularity of the full amplitude must reveal itself from the divergence of the series (4.1) which becomes senseless.

- The problem of finding a representation of A(s,t) which can be used to connect the various channels and hence can be used to describe all physical reactions connected by crossing, is solved by introducing the seemingly unphysical concept of *complex angular momenta.* Before doing this, let us first investigate the region of convergence of the series (4.1) in the complex θ plane.

- The asymptotic expansion of $P_\ell(\cos\ \theta\ )$ for ℓ real and tending to ∞,

(4.4) $$P_\ell(\cos\ \theta\ ) = O(e^{\ell\,|\,\mathrm{Im}\ \theta\ |}\ )$$

implies its exponential growth (in ℓ) for complex θ .

- As a consequence, the only way the series (4.1) can converge is that, as $\ell \rightarrow \infty$

(4.5) $$A_\ell(s) \sim e^{-\ell\,\eta\,(s)}$$

- In this case convergence is guaranteed so long as

(4.6) $$\mathrm{Im}\ \theta \ \le\ \eta\,(s).$$

- Thus, convergence is insured in a horizontal strip in the complex θ plane

  symmetric with respect to the real axis of width η(s). Setting **$\chi = \text{ch}\ \eta(s)$** (which is always ≥ 1), the corresponding convergence domain of the partial wave expansion (4.1) in the complex cos θ plane (**$z = \cos\theta = x + iy$**) is
- (4.7) **$x^2/\chi^2 + y^2/(\chi^2 - 1) = 1$**
- which is an ellipse with foci ± 1 and semiaxes χ and $\sqrt{\chi^2 - 1}$ which is known as the ***Lehmann ellipse*** .

- The conclusion is that the representation (4.1) converges in a domain which, albeit greater than the physical domain $-1 \leq z \leq 1$ (to which it reduces if $\eta(s) \to 0$), is still finite i.e. never extends to arbitrarily large values of the complex variable $|z = \cos\theta|$. In the language of the s, t, u variables, the domain never extends to asymptotic values of $|t|$ (or $|u|$)[recall $\cos\theta_s = 1 + 2t/(s - 4m^2)$].
- **As expected, the representation (4.1) cannot be used to explore the large *crossed channels* energy domain .**

- **As already anticipated, the way to circumvent this difficulty is to continue the expansion (4.1) to complex values of the angular momentum ℓ.**
- To get a hint as to how to proceed, let us investigate the extreme case when ℓ is purely imaginary. In this case, repeating the previous procedure, provided
- (4.8) $$A_\ell(s) \sim e^{-|\ell|\,\delta(s)} \quad ,$$
- convergence will now be insured in a vertical strip parallel and symmetric to the imaginary θ axis i.e. in the strip

(4.9) $$|\,\mathrm{Re}\ \theta\,| \leq \delta(s)\ .$$

- Setting, accordingly $\xi = \cos \delta$ (which is always ≤ 1), the convergence domain is now given by

(4.19) $$x^2/\xi^2 - y^2/(1-\xi^2) = 1.$$

- Contrary to the previous case, (4.10) defines now an open domain, a hyperbola with foci ±1 and convergence is guaranteed in one of its halves.
- In addition, given that it overlaps in part with a portion of the Lehmann ellipse, once we have made an analytic continuation of the amplitude to complex angular momenta, the new representation will define exactly the very same scattering amplitude A(s,t) we started from so that we will be able to safely continue it to domain where the crossed channel energies $|t|$ and/or $|u|$ can become arbitrarily large.

5. Complex Angular Momenta

- We now have to find the condition to continue the partial wave scattering amplitude to complex angular momenta so as to find a representation suitable to make asymptotic expansions.
- First we have to assume that we can continue the partial wave amplitude $A_\ell(s)$ to complex values of ℓ and construct an interpolating function $A(\ell,s)$
- which reduces to $A_\ell(s)$ for real integer values of ℓ.
- For this, we suppose that

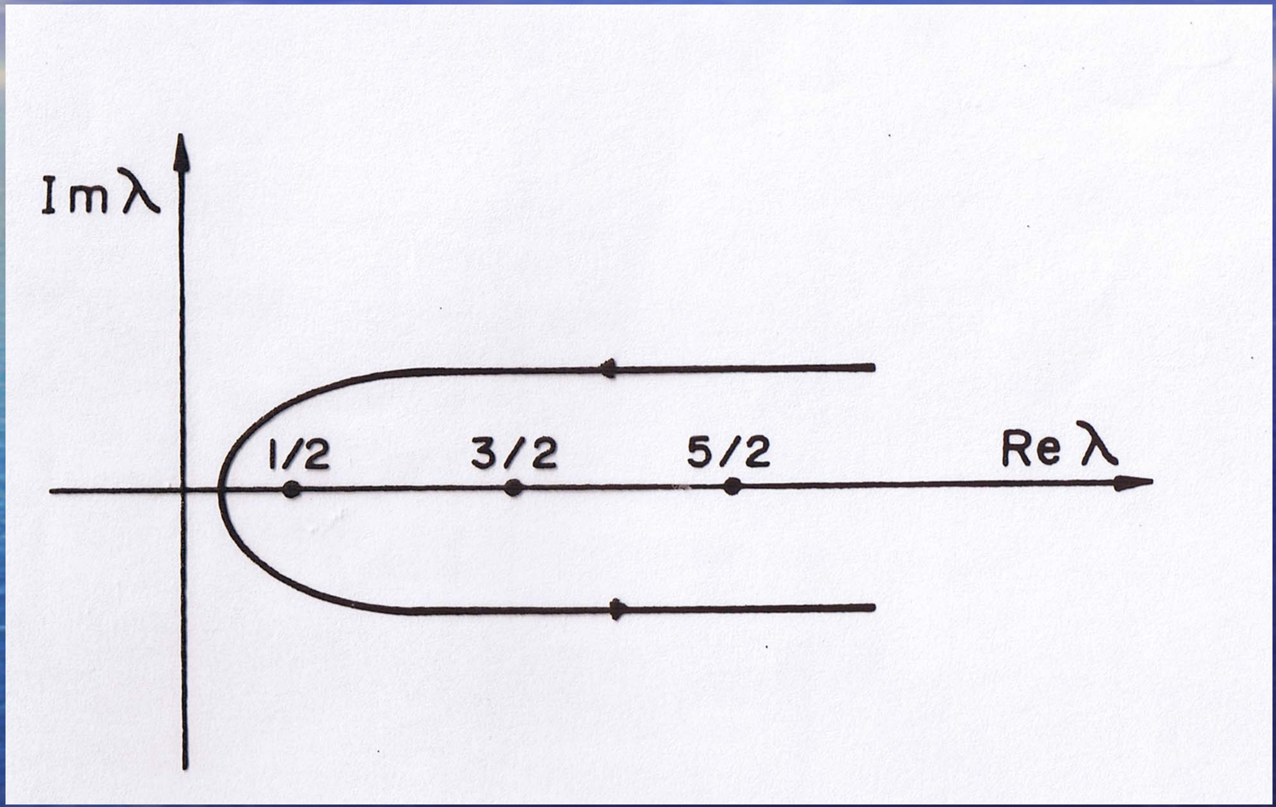
- $A(\ell,s)$ has only isolated singularities (poles) in the complex ℓ plane (and, for further simplicity, that they are simple poles);
- $A(\ell,s)$ is holomorphic for $\mathrm{Re}\,\ell \geq L$ (L positive arbitrary but finite)
- $A(\ell,s) \to 0$ as $|\ell| \to \infty$ for $\mathrm{Re}\,\ell > 0$.
- An amplitude $A(\ell,s)$ with the previous properties exists in at least two contexts:

- i) in non relativistic QM when the potential is a superposition of Yukawa potentials of the form

(1.3) $$V(r) = \int g(\alpha)\, e^{-\alpha r}\, d\alpha .$$

- This is supposed to mimic particle exchange in the t-channel of a general two-body reaction (3.1)
- ii) in the relativistic case when further requirements are satisfied (such as the validity of dispersion relations).

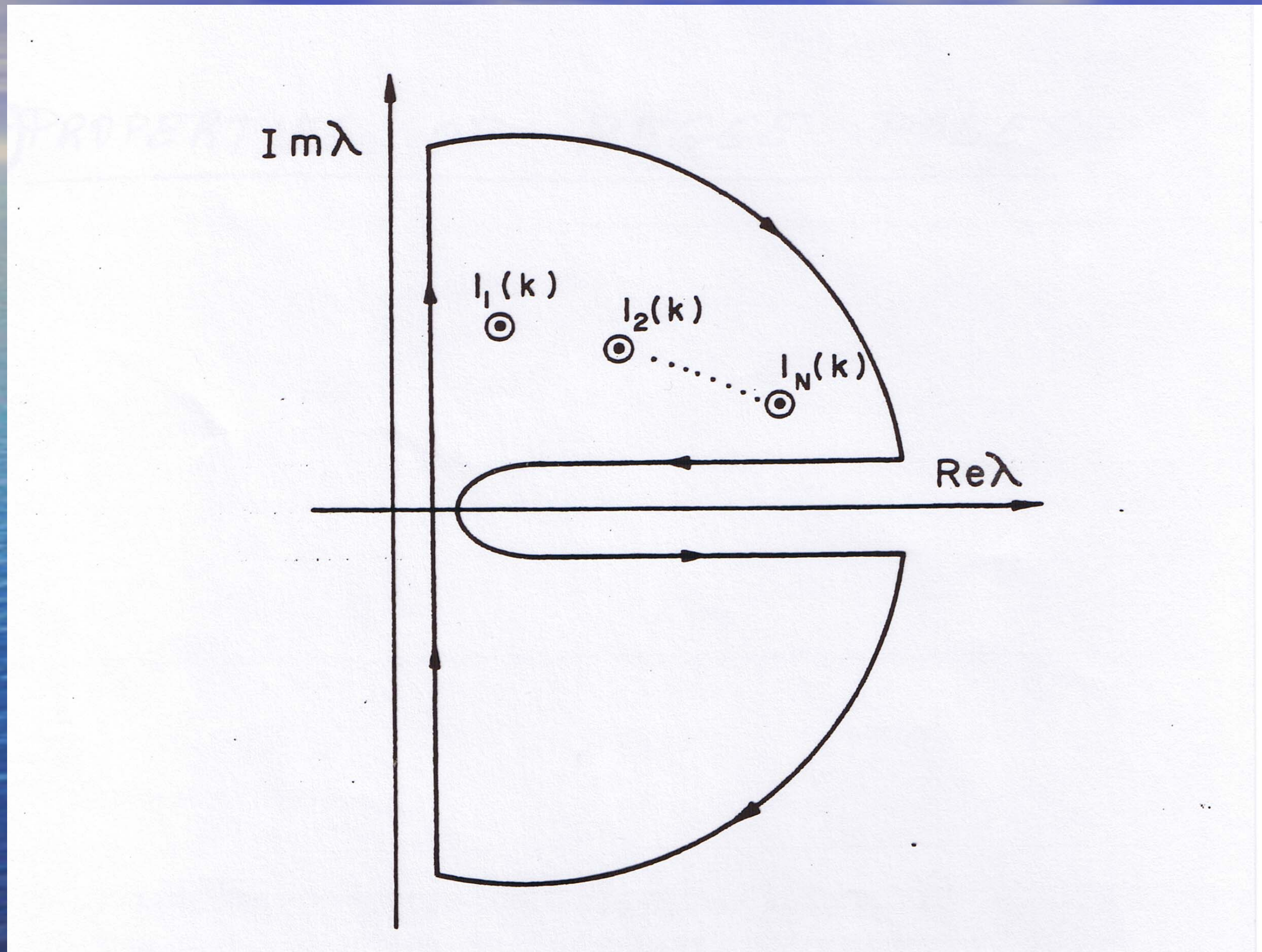
- If such an $A(\ell,s)$ exists, then it also unique thanks to a theorem by Carlson and the partial wave expansion (4.1) can be rewritten as
- (5.1) $A(s, z) = \sum_\ell (2\ell+1)\; A_\ell(s)\, P_\ell(z)$
  $- 1/2i \int_C (2\ell+1)\, A(\ell,s)\, P_\ell(-z)/\sin \pi\ell \;\; d\ell$
- where the discrete sum runs up to N-1 (N being the smallest integer larger than L) and C is the contour parallel to the real axis to the right of all singularities of $A(\ell,s)$ (see figure where $\lambda = \ell + ½$).


Im λ
1/2
3/2
5/2
Re λ

- The validity of (5.1) and its equivalence with (4.1) are a direct consequence of Cauchy residue's theorem (the integrand $f(\ell)$ in (5.1) has simple poles at all integers $\ell=n$ with residues $2i\,(2n+1)\,A_n(s)\,P_n(z)$).
- Given that there are no singularities of $A(\ell,s)$ at the right of $\ell=L$ and due to the asymptotic properties of $A(\ell,s)$ and of $P_\ell(-z)/\sin\pi\ell$, we can now deform the integration contour along the imaginary ℓ axis at the right of $\mathrm{Re}\,\ell>L$.

- Next, if the only singularities of A(ℓ,s) are simple poles, we can further move the integration contour to a parallel of the imaginary axis (see figure …) collecting the residues of A(ℓ,s) (at the same time, the residues of sin πℓ cancel the terms of the sum in (5.1)) so that, finally, we get the so called (Watson-Sommerfeld-Regge representation of the scattering amplitude whose asymptotically dominant contribution (we neglect the integral along a parallel to the left of the imaginary ℓ axis which vanishes as z goes to ∞) turns out to be

(5.2) $A(s, z) \sim -\sum_i [2\alpha_i(s)+1]\ \beta_i(s)\ P_{\alpha i}(-z)/\sin \pi\alpha_i(s)$.



Imλ
$l_1(k)$
$l_2(k)$
$l_N(k)$
Reλ

- The representation is now convergent in the half of the hyperbola (4.10) and we can now take the asymptotic z limit (i.e. the high energy limit in the crossed t/u) channel).
- If we call $\alpha(s)$ the pole with the largest real part, owing to $\mathbf{P_{\alpha(s)}(z) \sim z^{\alpha(s)}}$ and using (4.2) we get the asymptotic behaviour

(5.3) $\mathbf{A(s,t) \sim_{t\to\infty} - \beta(s)\, t^{\alpha(s)} / \sin \pi\alpha(s)}$

where nothing is known about the residue $\beta(s)$.

- Several complications can occur that muddle this simple result like, for instance higher order poles in the complex angular momentum plane, cuts etc. We shall not dig further on this matter in these notes and we refer the interested reader to the existing literature.
- One additional important point comes in the relativistic case due again to crossing.

- Invoking now the validity of crossing (....) we come, finally, to the amazingly simple *asymptotic Regge behaviour* as dominated by the singularity with the largest real part in the complex angular momentum plane **of the crossed t-channel** (beware, we are going to interchange s with t)

(5.4) $A_t(s,t) \sim_{s\to\infty} -\beta(t)\, s^{\alpha(t)}/\sin \pi\alpha(t)$

- When $s \to \infty$ at fixed t, also $u \to -\infty$ (recall that $s+t+u= 4m^2$). Consequently, if (5.4) is the asymptotic behaviour generated by the t-channel, we will have also a contribution from the u channel singularity $(-u)^{\alpha(t)}$ which,
- after some technical refinement, is written as

$$A_u(s, t) \sim_{s\to\infty} \beta(t)\ \xi\ e^{-i\pi\alpha(t)}/\sin \pi\alpha(t)$$

- where ξ is called *signature* and can take only the two possible values ±1.
- For future references, we shall introduce the *signature factor* $\eta(t)$

$$\eta(t) = -(1+\xi\ e^{-i\pi\alpha(t)})/\sin \pi\alpha(t)$$

- so that, in conclusion, the complete asymptotic behaviour of the scattering amplitude will be written as

(5.5) $$A(s,t) \sim_{s\to\infty} -\beta(t)\, s^{\alpha(t)} \left(1+\xi e^{-i\pi\alpha(t)}\right) / \sin \pi\alpha(t)$$

The first remarkable observations concerning Regge poles are:

i) The energy dependence of the asymptotic amplitude is predicted

ii) its phase is fixed

6. Regge trajectories.

- Near one of its poles in the complex angular momentum $\alpha(t)$, the partial wave amplitude $A(\ell, t)$ can be approximated by

  (6.1) $A(\ell, t) \sim_{\ell \to \alpha(t)} \beta(t) / (\ell - \alpha(t))$
- For t physical in the s-channel ($t \leq 0$), the ℓ plane singularities occur, in general at complex values of $\alpha(t)$; these can take on integer real values at unphysical ($t \geq 0$) t-values. In this case, Regge poles correspond to resonances or bound states.

- Suppose that for some real t0 value, we have

$$\alpha(t_0) = \ell + i\ \varepsilon$$

where ℓ is some integer and ε some real number (which we suppose much smaller than one).
Expanding $\alpha(t_0)$ around t_0,
we find

$$\alpha(t) = \ell + i\ \varepsilon\ +\ \alpha'(t_0)\,(t - t_0) + \ldots.$$

so that the denominator in (6.1) can be written as

- (6.4) $\quad 1/\,(\ell - \alpha(t)) \approx 1/(t - t_0 + i\,\Gamma\,)$

where $\Gamma\ = \mathrm{Im}\ \alpha(t_0)/\ \alpha'(t_0) =\ \varepsilon\,/\ \alpha'(t_0).$

- This is the typical structure of a Breit-Wigner resonance of mass $M=\sqrt{t_0}$ and width Γ which will be real iff

$$-\ d\ \mathrm{Im}\ \alpha(t)/dt\,\big|_{t0}\ <<\ d\ \mathrm{Re}\ \alpha(t)/dt\,\big|_{t0}$$

- Notice, however that while the vanishing of the denominator sin $\pi\alpha(t)$ signals the possible presence of a resonant state at every integer value of $\alpha(t)$, owing to the signature factor $(1+\xi\, e^{-i\pi\alpha(t)})$ induced by the crossed term in (5.5), **actual bound states will be interpolated by a Regge trajectory at *even values of the angular momentum (spin) if the signature ξ = +1 while negative signature if ξ = -1 will interpolate odd spin particle.***

- The crucial message is that the **asymptotic s-channel** behaviour is due to the **exchange of families of resonances in the crossed channels** which amplifies the message contained in the Yukawa message about the relevance of the exchange of particles extending its role to the determination of the asymptotic behaviour of the scattering amplitude.
- **Different processes will, in general, receive contribution from different trajectories according to their quantum numbers.**

- It is interesting to note that around t=0 one can expand $\alpha(t)$ in powers of t; for small enough t we can use the linear approximation

$$\alpha(t) = \alpha(0) + \alpha' t$$

- where $\alpha(0)$ and α' are known as the ***intercept*** and the ***slope*** respectively of the trajectory. According to (5.5) and all the previous discussion, it will be the trajectory with the highest intercept (whose **real part lies higher in the complex angular momentum plane**) which will **determine the asymptotic behaviour** of scattering amplitudes and cross sections.

- Quite unexpectedly, the linear approximation (6.6) turns out to provide a reasonably good account of the physical situation up to considerably higher values of t than one would a priori have guessed. The figure shows an example of the extent to which all dominant mesons lie on the same straight line up to |t| values of the order of 6 ÷ 7 $(GeV)^2$ irrespective of their spin being even or odd (exchange degeneracy). All leading meson trajectories, ρ , f_2 ,a_2 , and ω appear basically superimposed. Just as an example, we list the **quantum numbers of the leading mesonic trajectories** whose names come from the first resonance they interpolate.

- Quantum numbers of leading mesonic tr.
- f_2

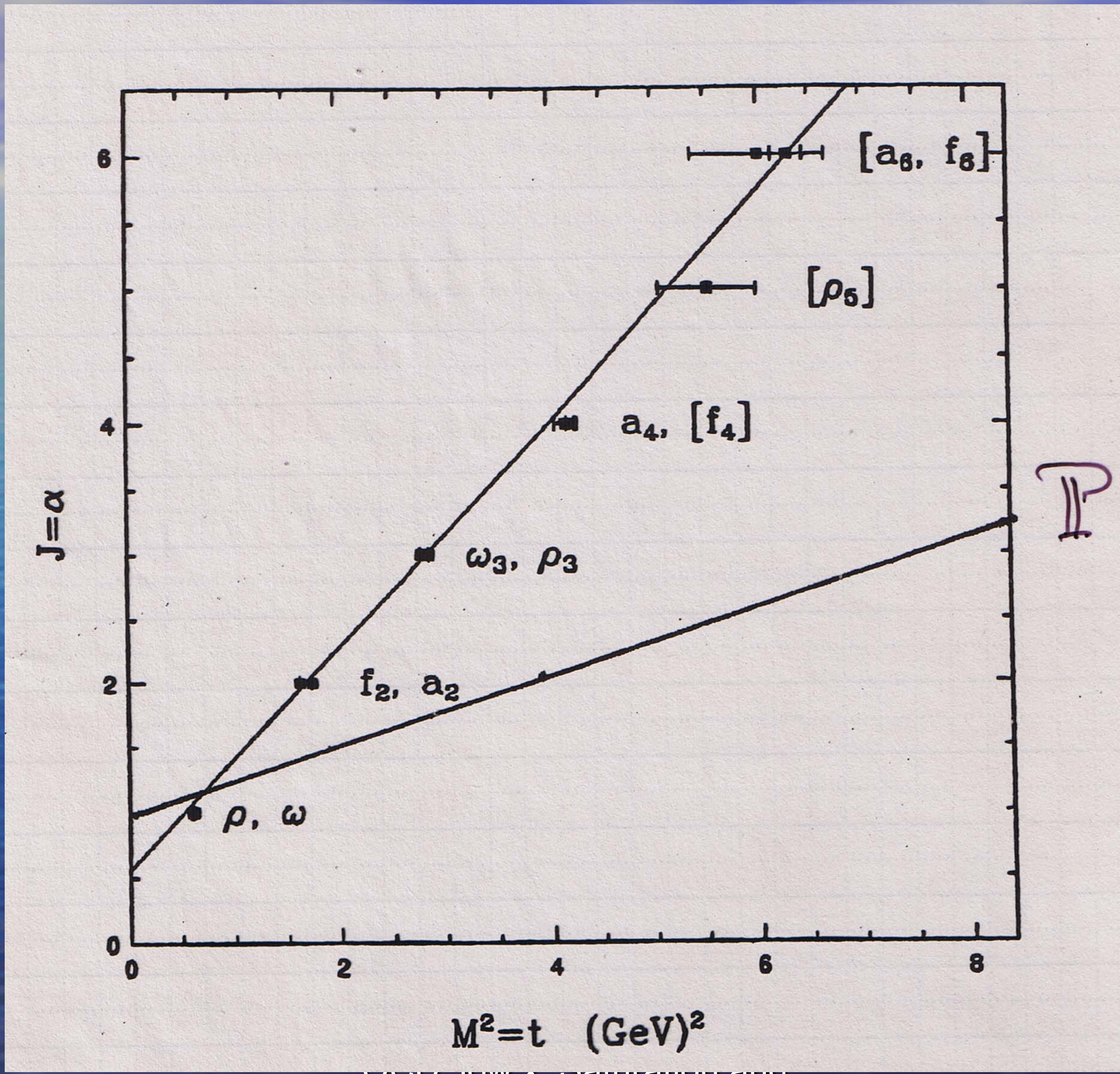
  $P = +1, C = +1, G = +1, I = 0, \xi = +1$
- ρ

  $P = -1, C = -1, G = +1, I = 1, \xi = -1$
- ω

  $P = -1, C = -1, G = -1, I = 0, \xi = -1$
- a_2

  $P = +1, C = +1, G = -1, I = 1, \xi = +1$

Mesonic Regge trajectory



- Note also that, among the above trajectories, f_2 has the quantum numbers of the vacuum. As it is well known, a special trajectory exists with the q. n. of the vacuum called *Pomeron*, whose existence determines the properties of the high energy cross sections (see Landshoff). In fact, **all mesonic trajectories appear basically *exchange degenerate*** and their **intercept** is very close to
- (6.7) $\alpha(0) \approx 1/2,$

- Needless to say, this *exchange degeneracy of the Regge trajectories* is slightly broken if one plots them more accurately. It is, in fact, impossible for a trajectory to rise linearly indefinitely. It can be proved that analyticity forces a trajectory to bend asymptotically to a lesser growth than linear (power or logarithmic as the case may be).
- Similar conclusions hold for fermionic trajectories. They, however, lie lower in intercept than the mesonic ones and their role is correspondingly less relevant to account for the gross features of the cross sections at high energies (but essential for subdominant features such as the backward behaviour or other).

- A most important exception in the above description is represented by the Pomeron trajectory will turn out to have an intercept very near 1 and will, therefore, provide the dominant behaviour of the cross sections

  (6.8) $\alpha_P(0) \approx 1.$

- We will not, however, spend much time on this subject which has been covered in great detail in another course.

- A last feature to be noticed is the almost universal slope of the mesonic (and also of the fermionic trajectories) which turns out to be very closely
- (6.9) $\alpha' \approx 1\ (\text{GeV})^{-2}$
- and is related to the so-called ***string tension*** in the realm of string theories. The noticeable exception to this almost universal property is, once again, the **Pomeron** whose slope turns out to be much smaller or close to zero so that many authors consider it on a different footing than the other Regge trajectories.

7. A brief account of Regge phenomenology.

- We will not spend much time on Regge phenomenology. We just wish to stress one feature which makes the Regge pole approach unique in the description of high energy physics phenomena. In fact, **Regge poles determine the asymptotic behaviour of the scattering amplitudes** (and, therefore, cross sections) in the s-channel (i.e. as $s \to \infty$) **when t is negative** and, at the same time, as we have just discussed, **they provide the basic information on resonant states when t is positive.**

- A further remarkable property is that **the phase of the dominant contribution is predicted and turns out to be imaginary in the forward direction** in perfect agreement with what high energy data demand.
- Rewriting the signature factor (5.6) for positive signature $\xi = +1$ as

  (7.1) $$\eta(t) = -e^{-i\frac{1}{2}\pi\alpha(t)}/\sin\tfrac{1}{2}\pi\alpha$$
- And using (6.6) with $\alpha(0) \approx 1$, near t=0

$$\eta(0) = i$$

- We also expand near $t \approx 0$ the residue function $\beta(t)$ which we assume to have an exponential form (this is unessential)

(7.2) $$\beta(t) \approx \beta(0)\exp[\tfrac{1}{2}B_0 t].$$

Putting everything together, the asymptotic ($s \to \infty$) near-forward scattering amplitude reads

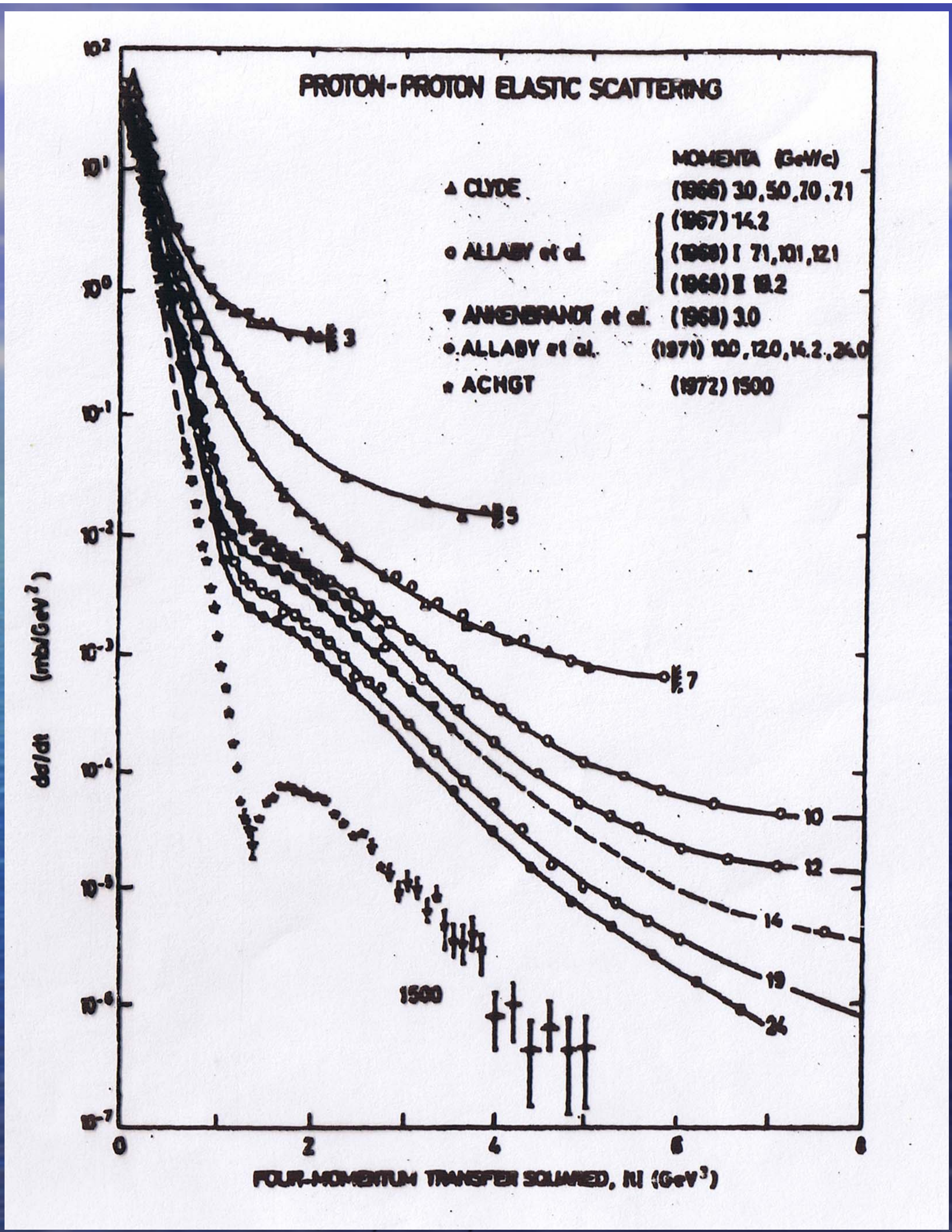
(7.3) $$A(s,t) \sim_{s\to\infty} i\,\beta(0)\, s^{\alpha(0)}\, e^{B(s)t}$$

- where the slope B(s) is thus predicted to have a ln s growth

(7.4) $$B(s) \approx \tfrac{1}{2}B_0 + \alpha'(\ell n\, s - i\,\pi/2).$$

- The result (7.3, 4) contains an unexpectedly simple and interesting set of properties. First of all, **at high energies *a diffraction peak is predicted* to appear in the near forward direction**. Furthermore, the ***slope is predicted grows logarithmically or the peak is predicted to shrink as the energy increases.***
- Notice that it is not the assumption on β (t) which accounts for these properties. The assumption on β (t) provides a constant term in the slope (as the data seem to require).

- The shrinkage of the forward peak (energy increase of the slope) is purely consequence of the exchange of a Regge trajectory with the vacuum quantum numbers and this seems indeed a general property supported by the data (see figure of angular pp distributions near t=0).
- In turn, the shrinkage of the diffraction peak is often interpreted **as the increase of the effective interaction radius** of the hadrons (we will not elaborate on this point here).

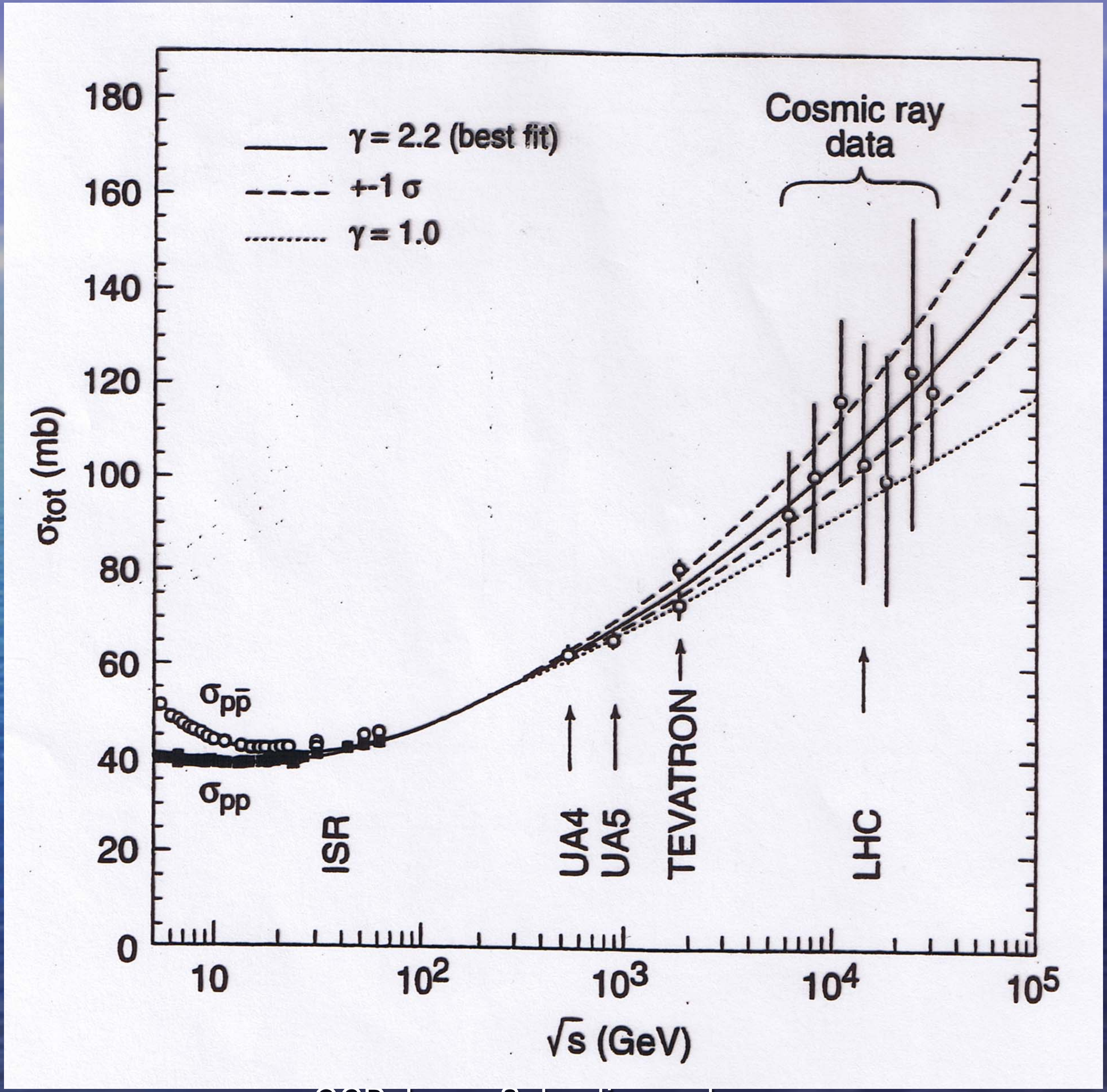

PROTON-PROTON ELASTIC SCATTERING
MOMENTA (GeV/c)
▲ CLYDE (1966) 3.0, 5.0, 7.0, 7.1
○ ALLABY et al. (1967) 14.2
(1968) I 7.1, 10.1, 12.1
(1968) II 19.2
▼ ANKENBRANDT et al. (1968) 3.0
⊙ ALLABY et al. (1971) 10.0, 12.0, 14.2, 24.0
★ ACHGT (1972) 1500
dσ/dt (mb/GeV²)
FOUR-MOMENTUM TRANSFER SQUARED, |t| (GeV²)
3
5
7
10
12
14
19
24
1500

- On the other hand, using the general form (5.5) or directly (7.3), we can now analyze the Regge pole prediction for what concerns high energy total cross sections. Due to the optical theorem which in the present case reads

(7.5) $\sigma_{tot} \sim_{s\to\infty} 1/s \; \mathrm{Im}\, A(s, t=0) \sim_{s\to\infty} s^{\alpha(0)-1}$

- we see that the total cross section σ_{tot} grows as a power if $\alpha(0) > 1$. This conflicts potentially with a bound on such a growth put by unitarity due to Froissart, Lukaszuk & Martin which restricts this growth to be at most

(7.6) $\sigma_{tot} \sim_{s\to\infty} O(\ell n^2 s)$.



180
160
140
120
100
80
60
40
20
0
σ_{tot} (mb)
γ = 2.2 (best fit)
+-1 σ
γ = 1.0
Cosmic ray data
$\sigma_{p\bar{p}}$
σ_{pp}
ISR
UA4
UA5
TEVATRON
LHC
10
10^2
10^3
10^4
10^5
$\sqrt{s}$ (GeV)

- In actual terms, the rate at which the Pomeron intercept $\alpha_P(0)$ would be required to exceed unity is so mild ($\alpha_P(0) \approx 1.08$) that the violation of the unitarity bound would occur at energies well beyond reach at any foreseeable future.
- This is true but:

i) the predicted growth of total c.s. is at risk of being exceeded at Tevatron energies

ii) at the same energies b-unitarity is near violation.

8. Conclusion

- Regge pole phenomenology has been extremely popular some decades ago when it was shown to describe successfully the gross features of a large class of reactions (basically all reactions – it was also extended to production amplitudes) with a rather limited number of adjustable parameters.

- At some point, however, people lost interest in the model when it became apparent that it was unable to reproduce a number of delicate points such as polarization data, charge exchange reactions and, in general, most subleading features of high energy processes.
- In addition, as soon as one tries to go beyond the simple notion of poles in the complex angular momentum plane, the complication and the arbitrariness grow fast and get rapidly out of control.

- A further reason of loss of interest in Regge poles was the difficulty of extending their notion and derivation to the realm of field theory beyond the original framework of non relativistic potential theory and beyond few manageable and simple exchange model.

- A last (psychological) reason for the diminution of interest in Regge models lies in the explosion of interest in Deep Inelastic Scattering (DIS) which in the Seventies gave rise to the new "Rutherford" experiment where it was shown that inside the proton exist seemingly point-like particles, the quarks (very much like Rutherford proved that inside the nuclei seemingly point-like particles, the nucleons, exist).