Overview of latest results with Constituent Quark Models

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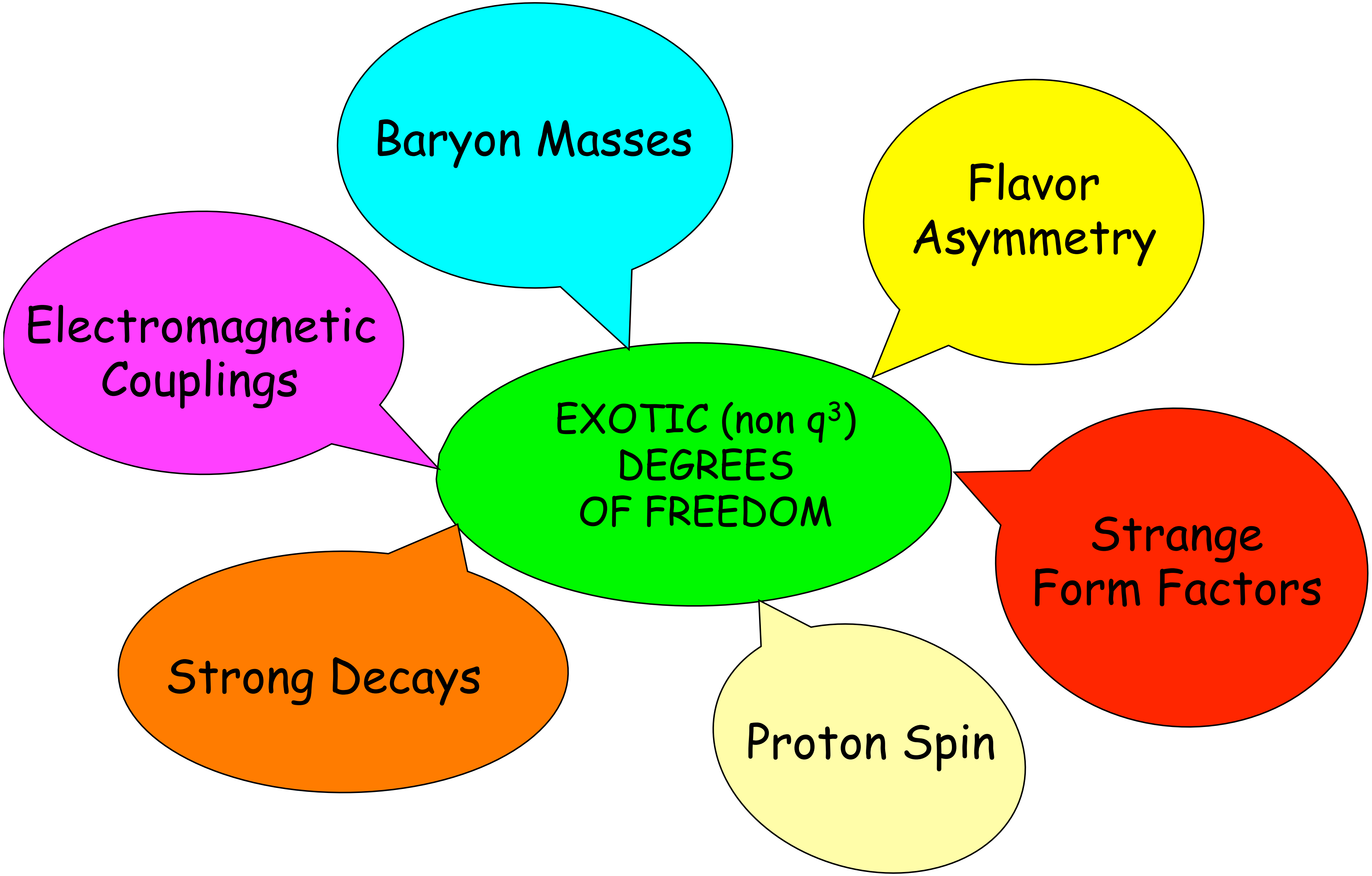

Baryon Masses
Flavor Asymmetry
Electromagnetic Couplings
EXOTIC (non q^3) DEGREES OF FREEDOM
Strange Form Factors
Strong Decays
Proton Spin

Unquenching the quark model for baryons
& Why Unquenching?

E. .Santopinto,. Bijker PRC 80, 065210 (2009),
PRC 82, 062202 (2010);J. Ferrettii,Santopinto, Bijker
Phys. Rev. C 85, 035204 (2012)

Many versions of CQMs have been developed

(IK, CI, GBE, U(7), hCQM,Bonn, etc.)

non relativistic and relativistic

While these models display peculiar features,

they share the following main features :

the effective degrees of freedom of 3q and a confining potential

the underling O(3) SU(3) symmetry

All of them are able to give a good description of the 3 and 4 stars spectrum

CQMs: S

Good description of the spectrum and magnetic moments

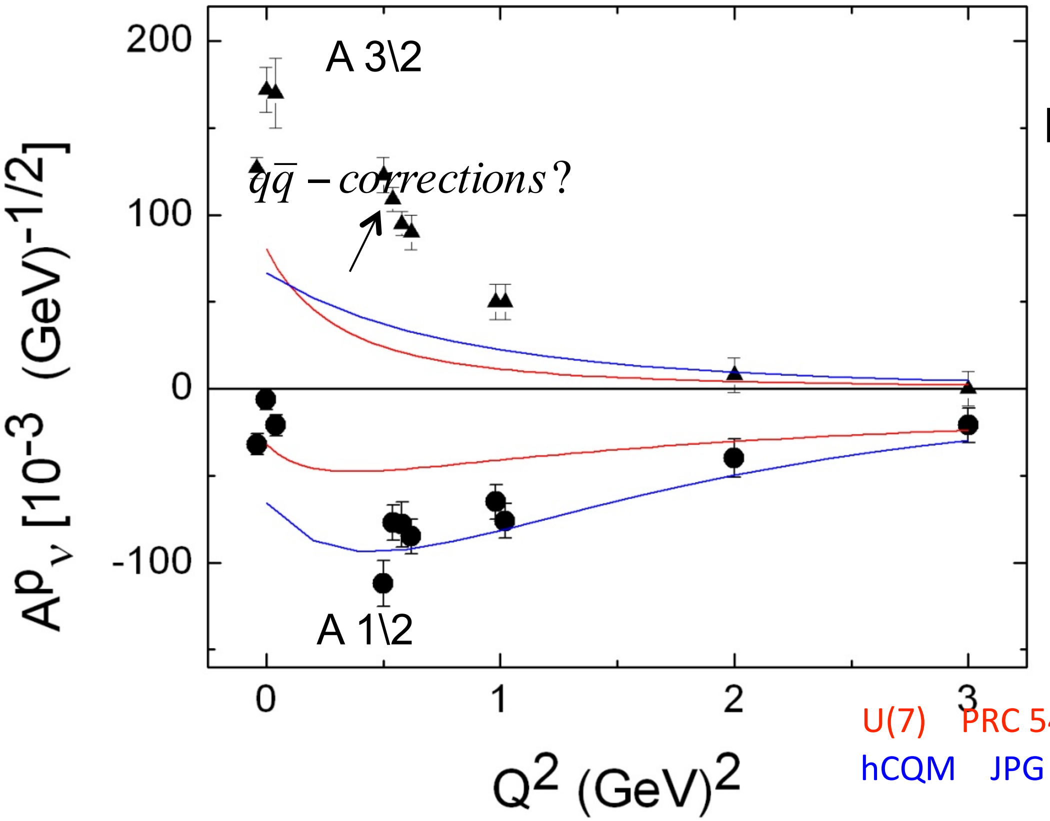
Predictions of many quantities:

- strong couplins
- photocouplings
- helicity amplitudes
- elastic form factors
- structure functions

Based on the effective degrees of freedom of 3 constituent quarks

Is it a degrees of freedom problem?

$q\bar{q}$ corrections ? important in the outer region



D13 transition amplitudes

U(7) PRC 54, 1935 (1996)

hCQM JPG 24, 753 (1998)

Considering also CQMs for mesons, CQMs able to reproduce the **overall trend of hundred of data**

- … but they show very similar deviations for observables such as

- photocouplings

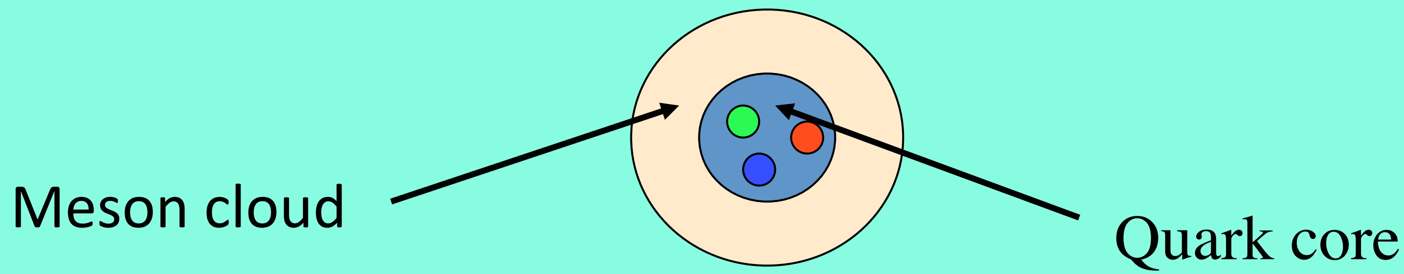
- helicity amplitudes,

please note

- the medium Q^2 behaviour is fairly well reproduced
- there is lack of strength at low Q^2 (outer region) in the e.m. transitions

- emerging picture:

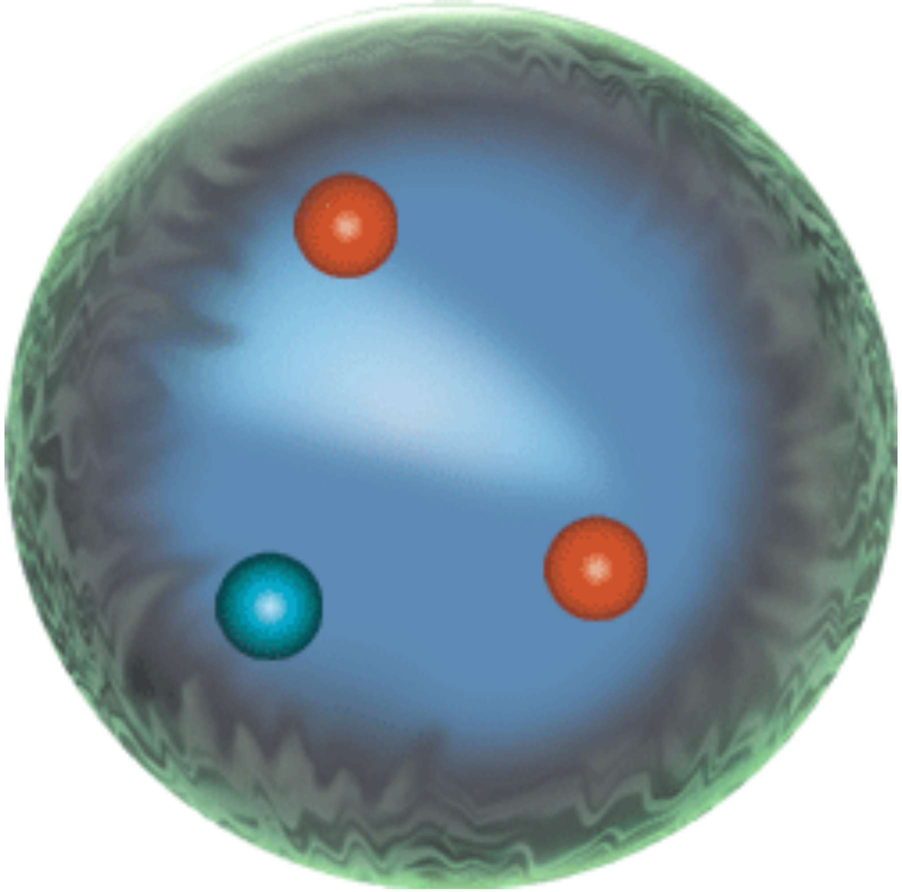
  quark core plus (meson or sea-quark) **cloud**

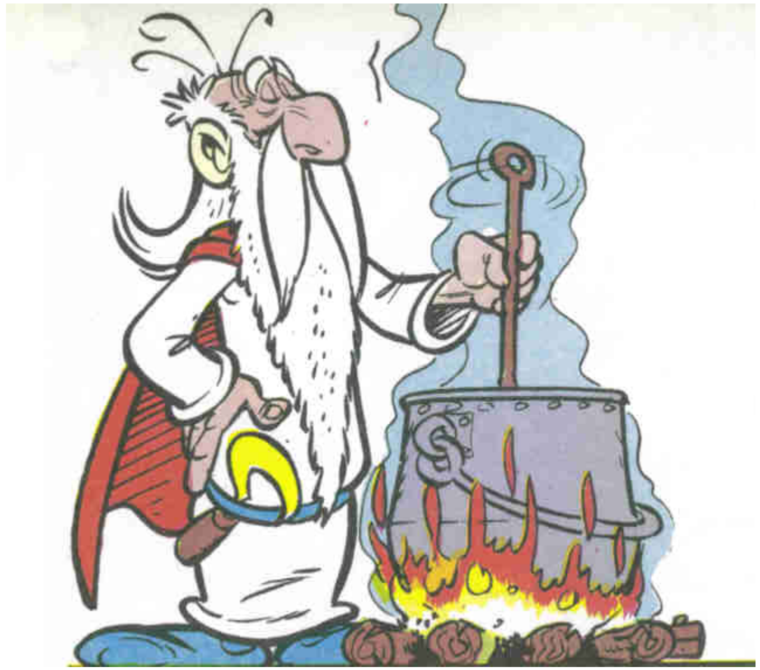


There are two possibilities:

phenomenological parametrization

microscopic explicit quark description





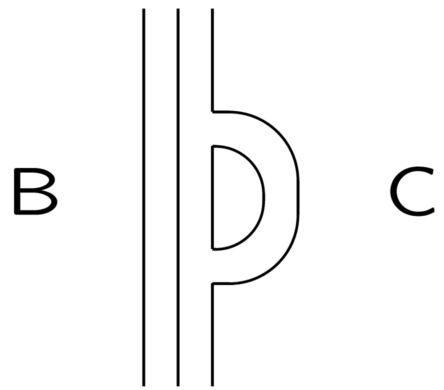
Problems

1) find a quark pair creation mechanism QCD inspired

2) implementation of this mechanism at the quark level but in such a way to

do not destroy the good CQMs results

Unquenched Quark Model



Strange quark-antiquark pairs in the proton with h.o. wave functions

Tornqvist & Zenczykowski (1984)
Geiger & Isgur, PRD 55, 299 (1997)
Isgur, NPA 623, 37 (1997)

- Pair-creation operator with $^3P_0$ quantum numbers of vacuum

Geiger & Isgur, PRD 55, 299 (1997)

It would be desirable to devise tests of the mechanisms underlying the delicate cancellations which conspire to hide the effects of the sea in the picture presented here. It also seems very worthwhile to extend this calculation to $\bar{u}u$ and $\bar{d}d$ loops. Such an extension could reveal the origin of the observed violations [38] of the Gottfried sum rule [39] and also complete our understanding of the origin of the spin crisis. From our previous calculations [4], the effects of ‘‘un-

Extensions

Bijker & Santopinto,
PRC 80, 065210 (2009)

- Any initial baryon or baryon resonance
- Any flavor of the quark-antiquark pair
- Any model of baryons and mesons

Formalism

$$| \psi_A \rangle = \mathcal{N} \left\{ | A \rangle + \sum_{BClJ} \int d\vec{K}\, k^2 dk \; | BC\vec{K}k\, lJ \rangle \frac{\langle BC\vec{K}k\, lJ \mid T^\dagger \mid A \rangle}{M_A - E_B - E_C} \right\}$$

Three-quark configuration
SU(3) flavor symmetry

Five-quark component
Isospin symmetry

Pair-creation operator: $T^\dagger = T^\dagger(^3P_0)$

L=S=1, J=0, color singlet, flavor singlet

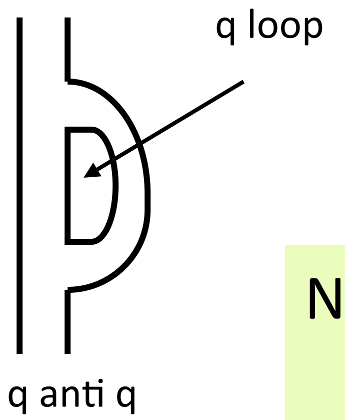
Unquenched Quark Model

- Harmonic oscillator quark model
- Sum over intermediate meson-baryon states includes for each oscillator shell all possible spin-flavor states
- Oscillator size parameters taken for baryons and mesons taken from literature (Capstick, Isgur, Karl)
- Smearing of the pair-creation vertex (Geiger, Isgur)
- Strength of $^3P_0$ coupling taken from literature on strong decays of baryons (Capstick, Roberts)

- No attempt to optimize the parameters

Unquenching the quark model

Mesons P. Geiger, N. Isgur, Phys. Rev. D41, 1595 (1990)
D44, 799 (1991)



Pair-creation operator with 3P0 quantum number

Note:

- sum over complete spin-flavor set of intermediate states necessary for preserving the OZI rule
- linear interaction is preserved after renormalization of the string constant

The good magnetic moment results of the CQM are preserved by the UCQM

Bijker, Santopinto,Phys.Rev.C80:065210,2009.

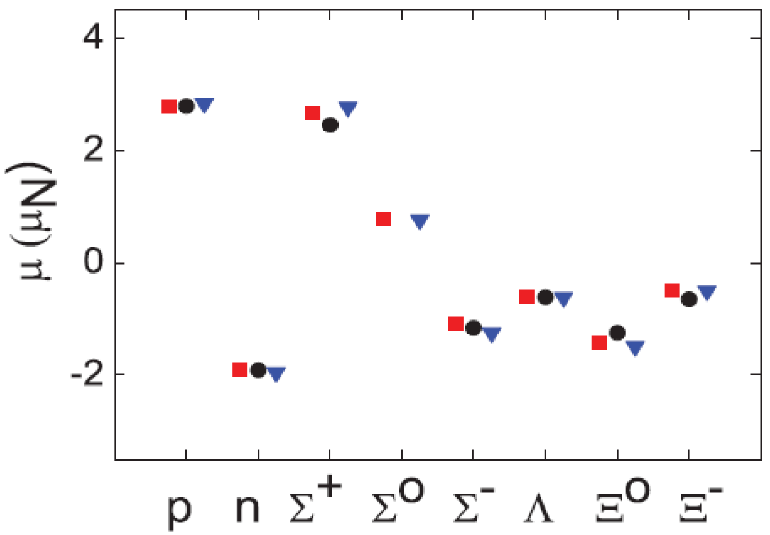


FIG. 3. (Color online) Magnetic moments of octet baryons: experimental values from the Particle Data Group [34] (circles), CQM (squares), and unquenched quark model (triangles).

Flavor Asymmetry

Gottfried sum rule

$$S_G = \int_0^1 dx \frac{F_{2p}(x) - F_{2n}(x)}{x} = \frac{1}{3} - \frac{2}{3}\int_0^1 dx \left[\bar{d}(x) - \bar{u}(x)\right]$$

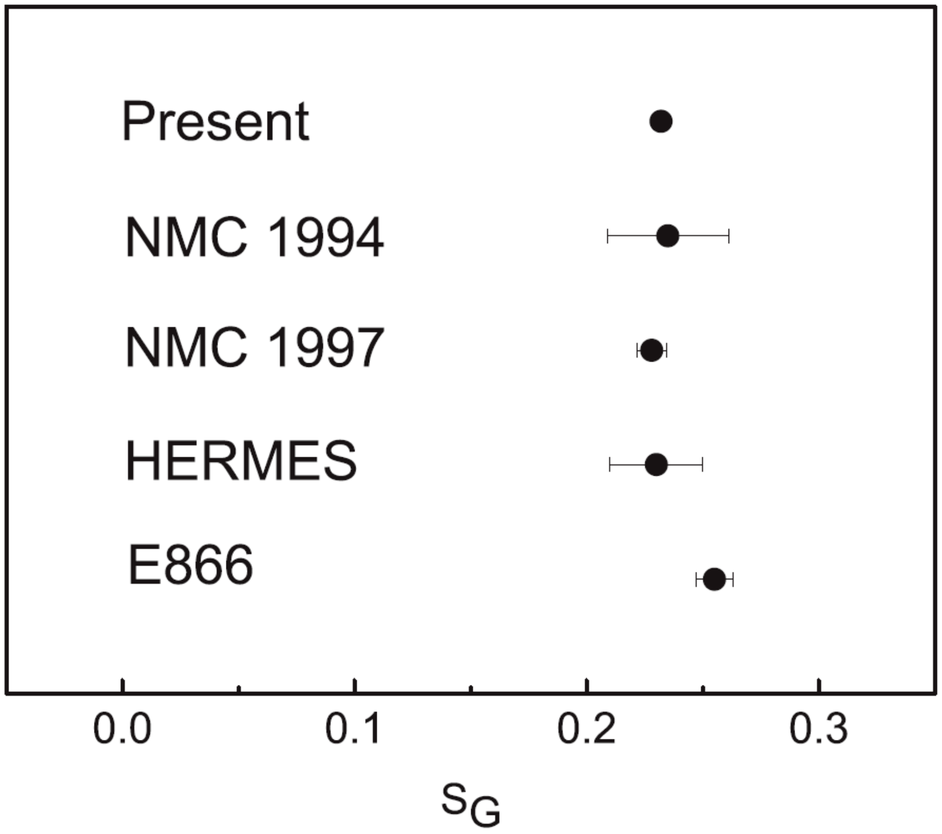
$$S_G \neq \frac{1}{3} \Rightarrow N_{\bar{d}} \neq N_{\bar{u}}$$

$$S_G = 0.2281 \pm 0.0065$$

$$\int_0^1 dx \left[\bar{d}(x) - \bar{u}(x)\right] = 0.16 \pm 0.01$$

Proton Flavor asymmetry

Santopinto, Bijker, PRC 82,062202(R) (2010)



Flavor asymmetry of the octect baryons in the UCQM

Santopinto, Bijker, PRC 82,062202(R) (2010)

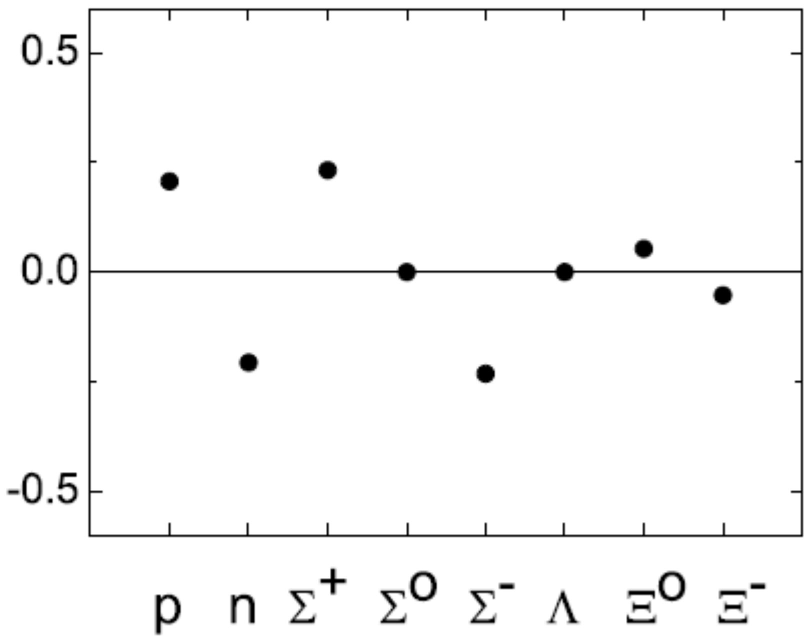


Figure 1. Flavor asymmetry of octet baryons

Flavor asymmetry
Nonperturbative QCD

Pauli blocking (Field & Feynman, 1977) too small
Pion dressing of the nucleon (Thomas et al., 1983)
Meson cloud models

Flavor asymmetries of octect baryons

Santopinto, Bijker, PRC 82,062202(R) (2010)

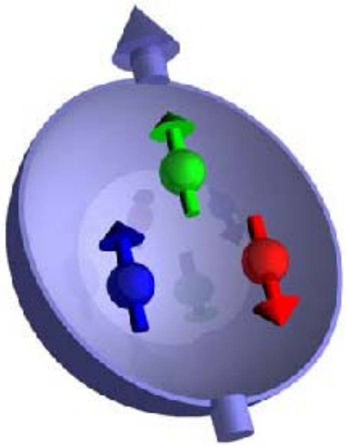
TABLE III. Relative flavor asymmetries of octet baryons.

Model	$\mathcal{A}(\Sigma^+)/\mathcal{A}(p)$	$\mathcal{A}(\Xi^0)/\mathcal{A}(p)$	Ref.
Unquenched CQM	0.833	−0.005	present
Chiral QM	2	1	Eichen
Balance model	3.083	2.075	Y.-J Zhang
Octet couplings	0.353	−0.647	Alberg

$$\Sigma^{\pm} p \to \ell^{+}\ell^{-} + X \text{ (e.g., at CERN).}$$

3. Proton Spin Crisis

1980's

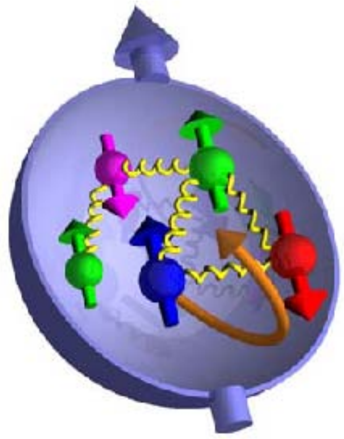


Naive parton model
3 valence quarks

1990's

QCD: contributions from
sea quarks and gluons

2000's



.. and orbital angular
momentum

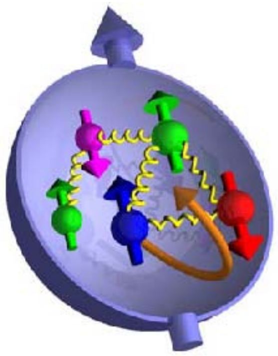
$$\frac{1}{2} = \frac{1}{2}(\Delta u + \Delta d)$$

$$\frac{1}{2} = \frac{1}{2}\underbrace{(\Delta u + \Delta d + \Delta s)}_{\Delta\Sigma} + \Delta G + \Delta L$$

$$\left.\begin{aligned} \Delta u &= 0.842 \\ \Delta d &= -0.427 \\ \Delta s &= -0.085 \end{aligned}\right\} \Delta\Sigma = 0.330 \pm 0.039$$

HERMES, PRD 75, 012007 (2007)
COMPASS, PLB 647, 8 (2007)

Proton Spin



- COMPASS@CERN: Gluon contribution is small (sign undetermined)
- Unquenched quark model

Ageev et al., PLB 633, 25 (2006)
Platchkov, NPA 790, 58 (2007)

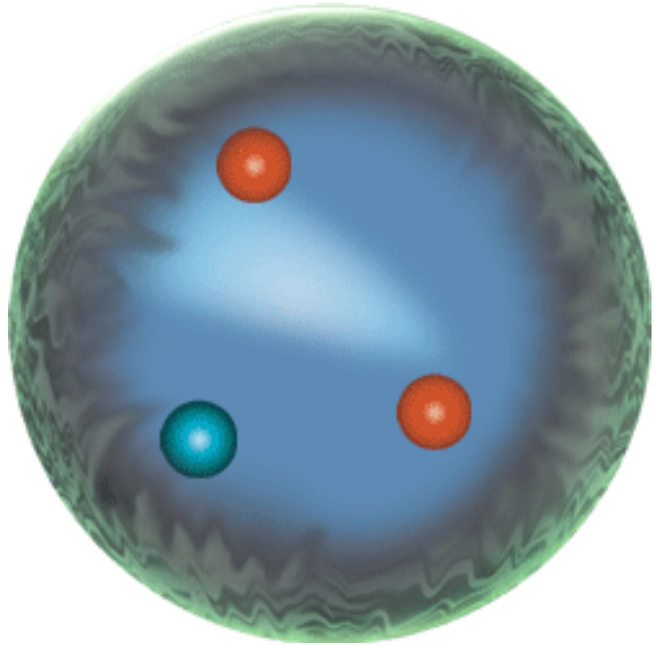
		CQM	Unquenched QM		
			Valence	Sea	Total
p	$\Delta\Sigma$	1	0.378	0.298	0.676
	$2\Delta L$	0	0.000	0.324	0.324
	$2\Delta J$	1	0.378	0.622	1.000

- More than half of the proton spin from the sea!
- Orbital angular momentum

Suggested by Myhrer & Thomas, 2008, but not explicitly calculated

4. Strangeness in the Proton

- The strange (anti)quarks come uniquely from the sea: there is no contamination from up or down valence quarks
- The strangeness distribution is a very sensitive probe of the nucleon's properties
- Flavor content of form factors
- New data from Parity Violating Electron Scattering experiments: SAMPLE, HAPPEX, PVA4 and G0 Collaborations



"There is no excellent beauty that hath not some strangeness in the proportion"
(Francis Bacon, 1561-1626)

Quark Form Factors

- Charge symmetry

$$G^{u,p} = G^{d,n} \equiv G^{u}$$
$$G^{d,p} = G^{u,n} \equiv G^{d}$$
$$G^{s,p} = G^{s,n} \equiv G^{s}$$

- Quark form factors

$$G^{u} = \left(3 - 4\sin^2\Theta_W\right) G^{\gamma,p} - G^{Z,p}$$
$$G^{d} = \left(2 - 4\sin^2\Theta_W\right) G^{\gamma,p} + G^{\gamma,n} - G^{Z,p}$$
$$G^{s} = \left(1 - 4\sin^2\Theta_W\right) G^{\gamma,p} - G^{\gamma,n} - G^{Z,p}$$

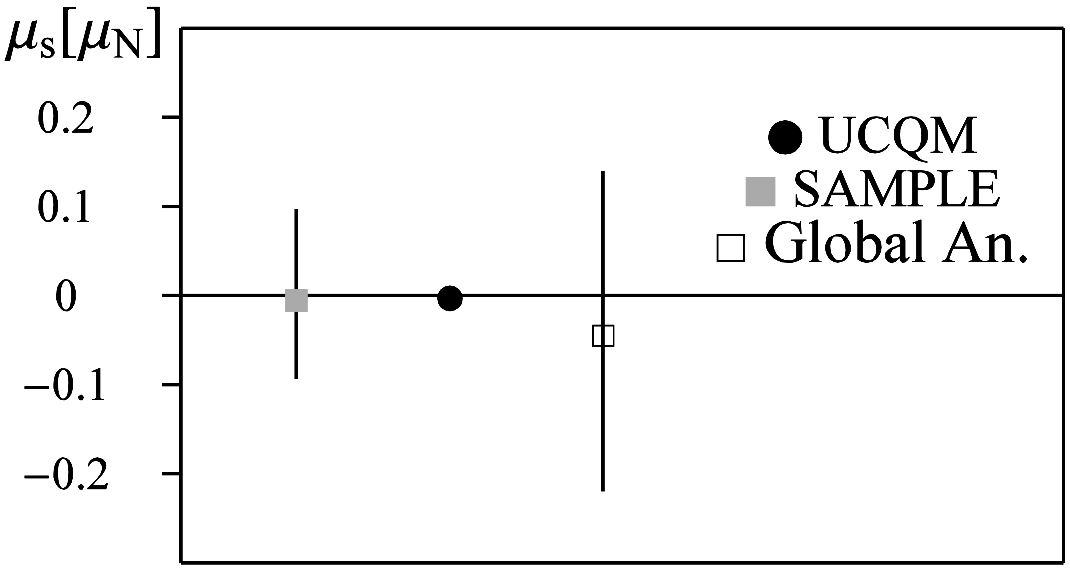
Kaplan & Manohar, NPB 310, 527 (1988)
Musolf et al, Phys. Rep. 239, 1 (1994)

Static Properties

$G_E(0) = e$	Electric charge
$G_M(0) = \mu$	Magnetic moment
$\left\langle r^2 \right\rangle_E = -6 \left. \frac{dG_E}{dQ^2} \right\|_{Q^2=0}$	Charge radius
$\left\langle r^2 \right\rangle_M = -\frac{6}{\mu} \left. \frac{dG_M}{dQ^2} \right\|_{Q^2=0}$	Magnetic radius

Strange Magnetic Moment

$$\vec{\mu}_s = \sum_i \mu_{i,s} \left[2\vec{s}(q_i) + \vec{\ell}(q_i) - 2\vec{s}(\bar{q}_i) - \vec{\ell}(\bar{q}_i) \right]$$

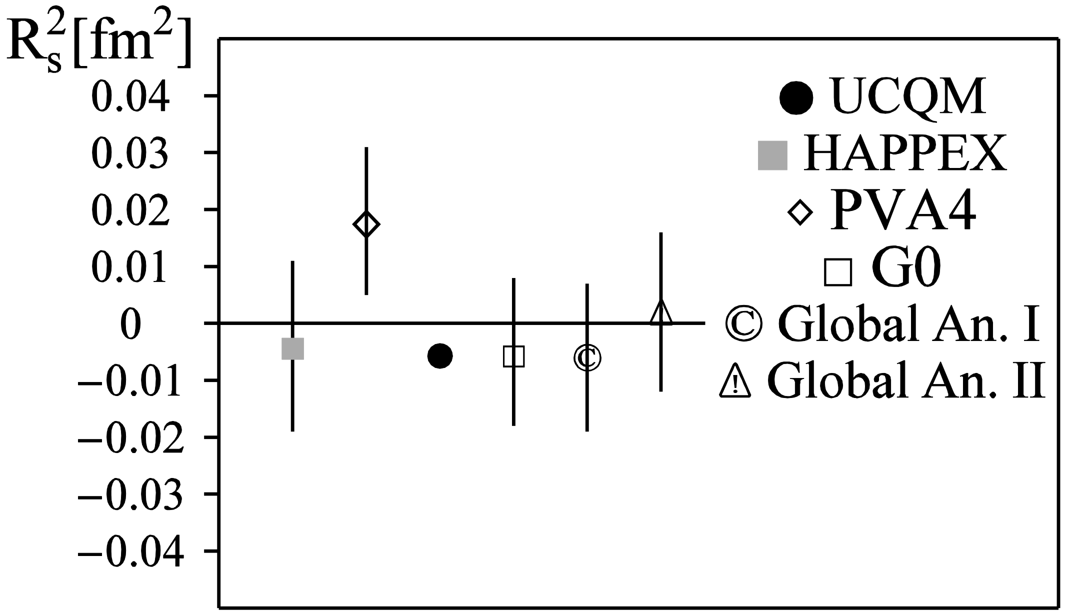


Jacopo Ferretti, Ph.D. Thesis, 2011

Bijker, Ferretti, Santopinto, Phys. Rev. C **85, 035204 (2012)**

Strange Radius

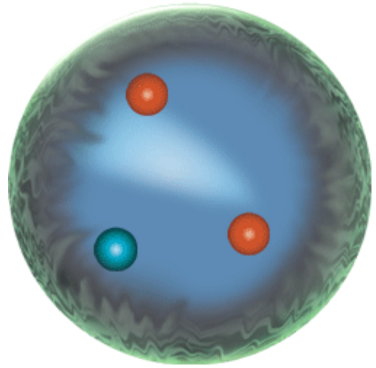
$$R_s^2 = \sum_{i=1}^{5} e_{i,s} \left(\vec{r}_i - \vec{R}_{CM}\right)^2$$



Jacopo Ferretti, Ph.D. Thesis, 2011

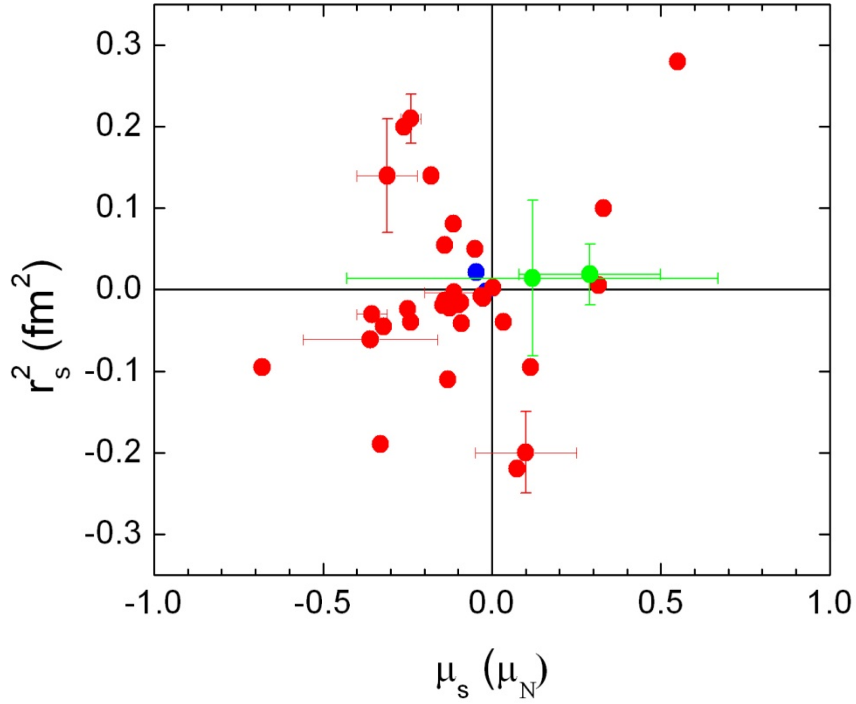
Bijker, Ferretti, Santopinto, Phys. Rev. C **85, 035204 (2012)**

Strange Proton



- Strange radius and magnetic moment of the proton
- Theory
- Lattice QCD
- Global analysis PVES
- Unquenched QM

$$\mu_s = -6 \cdot 10^{-4} \, (\mu_N)$$
$$\langle r^2 \rangle_s = -4 \cdot 10^{-3} \, (\text{fm}^2)$$



Jacopo Ferretti, Ph.D. Thesis, 2011
Bijker, Ferretti, Santopinto, Phys. Rev. C **85, 035204 (2012)**

Unquenching the quark model for the MESONS & Why Unquenching?

Santopinto, Galatà, Ferretti, Vassallo

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UQM: Meson Self Energies & couple channels

- Hamiltonian: $$H = H_0 + V$$
- H_0 act only in the bare meson space and it is chosen the Godfray and Isgur model
- V couples |A> to the continuum |BC>
- Dispersive equation $$\Sigma(E_a) = \sum_{BC} \int_0^\infty q^2 dq \, \frac{|V_{a,bc}(q)|^2}{E_a - E_{bc}}$$
- from non-relativistic Schrödinger equation
- Bare energy E_a (H_0 eigenvalue) satisfies: $$M_a = E_a + \Sigma(E_a)$$
- M_a = physical mass of meson A
- $\Sigma(E_a)$ = self energy of meson A

UQM: Meson Self Energies -- UQM I

- Coupling $V_{a,bc}(q)$ in $\Sigma(E_a)$ calculated as:

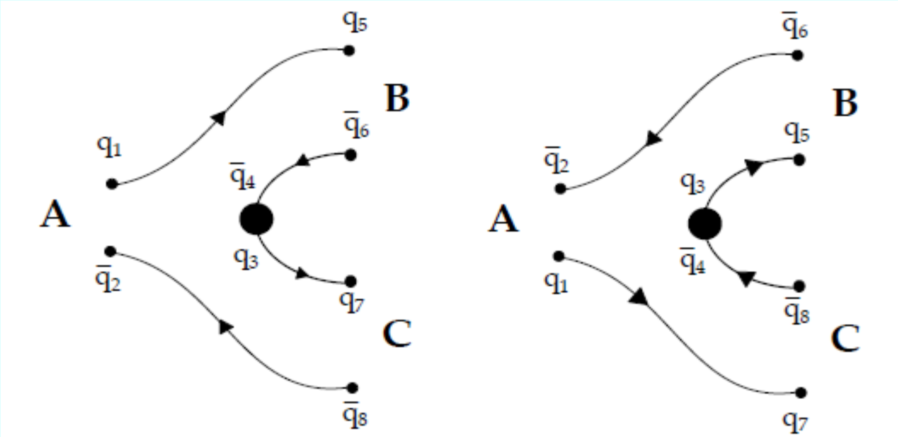
$$V_{a,bc}(q) = \sum_{\ell J} \langle BC\vec{q}\ell J | T^{\dagger} | A \rangle$$

Sum over a complete set of accesible

ground state (1S) mesons

Coupling calculated in the $^3P_0$ model

- Two possible diagrams contribute:



- Self energy in the UQM:

$$\Sigma(E_a) = \sum_{BC\ell J} \int_0^{\infty} q^2 dq \frac{\left| \langle BC\vec{q}\ell J | T^{\dagger} | A \rangle \right|^2}{E_a - E_b - E_c}$$

Godrey and Isgur model as bare mass

- Bare energies E_a calculated in the relativized G.I.Model for mesons
- Hamiltonian:

$$H = \sqrt{q^2+m_1^2}+\sqrt{q^2+m_2^2}+V_{\text{conf}}+V_{\text{hyp}}+V_{\text{so}}$$

- Confining potential:

$$V_{\text{conf}} = -\left(\frac{3}{4}c+\frac{3}{4}br-\frac{\alpha_s(r)}{r}\right)\vec{F}_1\cdot\vec{F}_2$$

- Hyperfine interaction:

$$V_{\text{hyp}} = -\frac{\alpha_s(r)}{m_1 m_2}\left[\frac{8\pi}{3}\vec{S}_1\cdot\vec{S}_2\,\delta^3(\vec{r}) + \frac{1}{r^3}\left(\frac{3\,\vec{S}_1\cdot\vec{r}\,\vec{S}_2\cdot\vec{r}}{r^2}-\vec{S}_1\cdot\vec{S}_2\right)\right]\vec{F}_i\cdot\vec{F}_j$$

- Spin-orb

$$V_{\text{so,cm}} = -\frac{\alpha_s(r)}{r^3}\left(\frac{1}{m_i}+\frac{1}{m_j}\right)\left(\frac{\vec{S}_i}{m_i}+\frac{\vec{S}_j}{m_j}\right)\cdot\vec{L}\;\;\vec{F}_i\cdot\vec{F}_j$$

$$V_{\text{so,tp}} = -\frac{1}{2r}\frac{\partial H_{ij}^{conf}}{\partial r}\left(\frac{\vec{S}_i}{m_i^2}+\frac{\vec{S}_j}{m_j^2}\right)$$

UQM or couple channel Quark Model

- Parameters of the relativized QM fitted to

$$M_a = E_a + \Sigma(E_a)$$

- Recursive fitting procedure

- M_a = calculated physical masses of q bar-q mesons → reproduce experimental spectrum [PDG]

- Intrinsic error of QM/UQM calculations: 30-50 MeV

UQM: charmonium with self-energy corr.

Parameters of the UQM ($^3P_0$ vertices)

Parameter	Value
γ_0	0.510
α	0.500 GeV
r_q	0.335 fm
m_n	0.330 GeV
m_s	0.550 GeV
m_c	1.50 GeV

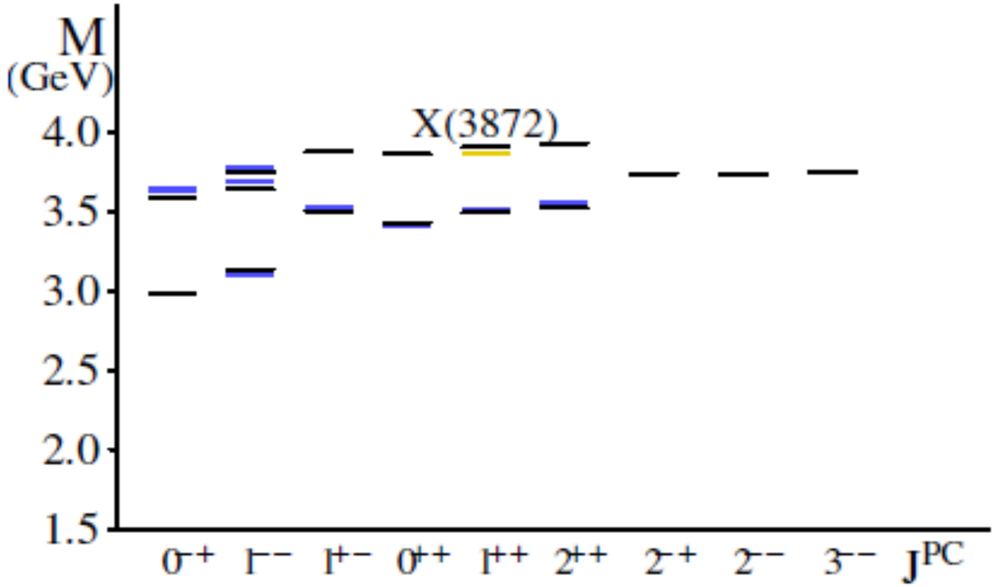
- fitted to:

State	DD	DD^*	D^*D^*	D_sD_s	$D_sD_s^*$	$D_s^*D_s^*$	Total	Exp.
$\eta_c(3^1S_0)$	–	38.8	52.3	–	–	–	91.1	–
$\Psi(4040)(3^3S_1)$	0.2	37.2	39.6	3.3	–	–	80.3	80 ± 10
$h_c(2^1P_1)$	–	64.6	–	–	–	–	64.6	–
$\chi_{c0}(2^3P_0)$	97.7	–	–	–	–	–	97.7	–
$\chi_{c2}(2^3P_2)$	27.2	9.8	–	–	–	–	37.0	–
$\Psi(3770)(1^3D_1)$	27.7	–	–	–	–	–	27.7	27.2 ± 1.0
$c\bar{c}(1^3D_3)$	1.7	–	–	–	–	–	1.7	–
$c\bar{c}(2^1D_2)$	–	62.7	46.4	–	8.8	–	117.9	–
$\Psi(4160)(2^3D_1)$	11.2	0.4	39.4	2.1	5.6	–	58.7	103 ± 8
$c\bar{c}(2^3D_2)$	–	43.5	49.3	–	11.3	–	104.1	–
$c\bar{c}(2^3D_3)$	17.2	58.3	48.1	3.6	2.6	–	129.8	–

UQM: charmonium spectrum with self-energy corr.

Ferretti, Galata' and Santopinto, Phys. Rev. C 88, 015207 (2013)

State	J^{PC}	$D\bar{D}$	$\bar{D}D^*$ $D\bar{D}^*$	$\bar{D}^*D^*$	$D_s\bar{D}_s$	$D_s\bar{D}_s^*$ $\bar{D}_sD_s^*$	$D_s^*\bar{D}_s^*$	$\eta_c\eta_c$	η_cJ/Ψ	$J/\Psi J/\Psi$	$\Sigma(E_a)$	E_a	M_a	$M_{exp.}$
$\eta_c(1^1S_0)$	0^{-+}	–	-34	-31	–	-8	-8	–	–	-2	-83	3062	2979	2980
$J/\Psi(1^3S_1)$	1^{--}	-8	-27	-41	-2	-6	-10	–	-2	–	-96	3233	3137	3097
$\eta_c(2^1S_0)$	0^{-+}	–	-52	-41	–	-9	-8	–	–	-1	-111	3699	3588	3637
$\Psi(2^3S_1)$	1^{--}	-18	-42	-54	-2	-7	-10	–	-1	–	-134	3774	3640	3686
$h_c(1^1P_1)$	1^{+-}	–	-59	-48	–	-11	-10	–	-2	–	-130	3631	3501	3525
$\chi_{c0}(1^3P_0)$	0^{++}	-31	–	-72	-4	–	-15	0	–	-3	-125	3555	3430	3415
$\chi_{c1}(1^3P_1)$	1^{++}	–	-54	-53	–	-9	-11	–	–	-2	-129	3623	3494	3511
$\chi_{c2}(1^3P_2)$	2^{++}	-17	-40	-57	-3	-8	-10	0	–	-2	-137	3664	3527	3556
$h_c(2^1P_1)$	1^{+-}	–	-55	-76	–	-12	-8	–	-1	–	-152	4029	3877	–
$\chi_{c0}(2^3P_0)$	0^{++}	-23	–	-86	-1	–	-13	0	–	-1	-124	3987	3863	–
$\chi_{c1}(2^3P_1)$	1^{++}	–	-30	-66	–	-11	-9	–	–	-1	-117	4025	3908	3872
$\chi_{c2}(2^3P_2)$	2^{++}	-2	-42	-54	-4	-8	-10	0	–	-1	-121	4053	3932	3927
$c\bar{c}(1^1D_2)$	2^{-+}	–	-99	-62	–	-12	-10	–						
$\Psi(3770)(1^3D_1)$	1^{--}	-11	-40	-84	-4	-2	-16	–						
$c\bar{c}(1^3D_2)$	2^{--}	–	-106	-61	–	-11	-11	–						
$c\bar{c}(1^3D_3)$	3^{--}	-25	-49	-88	-4	-8	-10	–						



M [X(3872); UQM] = 3908 MeV

UQM: charmonium with self-energy corr.

Ferretti, Galata' and Santopinto, Phys. Rev. C 88

- Experimental mass: 3871.68 ± 0.17 MeV [PDG]
- Several predictions for X(3872)'s mass. Here: c bar-c + continuum effects

$\chi_{c1}(2^3P_1)$'s mass (MeV)	Reference
3908	This paper
4007.5	[2]
3990	[3]
3920.5	[4]
3896	[5]

[1] Ferretti, Galata' and Santopinto, Phys. Rev. C **88**, 015207 (2013);

[2] Eichten et al., Phys. Rev. D 69,(2004)

[3] Kalashnikova, Phys. Rev. D 72, 034010 (2005)

[4] Eichten et al., Phys. Rev. D 73, 014014 (2008)

[5] Pennington and Wilson, Phys. Rev. D 76, 077502 (2007)

Interpretation of the X(3872) as a charmonium state plus an extra component due to the coupling to the meson-meson continuum

Ferretti, Galatà, Santopinto, **Phys.Rev. C88 (2013) 1, 015207**

- UCQM results used to study the problem of the **X(3872)** mass, meson with $J^{PC} = 1^{++}$, 2^3P_1 quantum numbers
- Experimental mass: 3871.68 ± 0.17 MeV [PDG]
- X(3872) very close to D bar-D* decay threshold
- Possible importance of continuum coupling effects?
- Several interpretations: pure c bar-c
- D bar-D* molecule
- tetraquark
- c bar-c + continuum effects

  - need to study strong and radiative decays to understand the situation

Radiative decays

Ferretti,Galatà,Santopinto,Phys.Rev. D90 (2014) 5, 054010

Transition	E_γ [MeV]	$\Gamma_{c\bar{c}}$ [KeV] present paper	$\Gamma_{D\bar{D}^*}$ [KeV] Ref. [7]	$\Gamma_{D\bar{D}^*}$ [KeV] Ref. [9]	$\Gamma_{D\bar{D}^*}$ [KeV] Ref. [59]	$\Gamma_{c\bar{c}+D\bar{D}^*}$ [KeV] Ref. [60]	$\Gamma_{exp.}$ [KeV] PDG [43]
$X(3872) \to J/\Psi\gamma$	697	11	8	$64-190$	$125-251$	$2-17$	≈ 7
$X(3872) \to \Psi(2S)\gamma$	181	70	0.03			$7-59$	≈ 36
$X(3872) \to \Psi(3770)\gamma$	101	4.0	0				
$X(3872) \to \Psi_2(1^3D_2)\gamma$	34	0.35	0				

[7] Swanson: molecular interpretation
[9] Oset: molecular interpretation
[59]-[60] Faessler : molecular ; ccbar +molecular

The Molecular model does not predict radiative decays into $\Psi(3770)$ and $\Psi_2(1^3D_2)$-→ Possible way to distinguish between the two interpretations

Quasi two-body decay $X(3872) \to D^0(\bar{D}^0\pi^0)_{\bar{D}^{0*}}$

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$$\Gamma_{\bar{D}^{0*}} < 2.1 \text{ MeV} \qquad \Gamma_{\bar{D}^{0*}} = 0.1 \text{ MeV}$$

$$\Gamma_{X(3872)\to D(\bar{D}\pi)_{\bar{D}^*}} = 0.50 - 0.62 \text{ MeV} , \quad M_{X(3872)} = 3871.85 \text{ MeV}$$

$$\Gamma_{X(3872)\to D(\bar{D}\pi)_{\bar{D}^*}} = 0.54 - 0.75 \text{ MeV} , \quad M_{X(3872)} = 3871.95 \text{ MeV}$$

Experimental results:

$\Gamma_{X(3872)\to D^0\bar{D}^{0*}} = 3.9^{+2.8+0.2}_{-1.4-1.1}$ MeV — PDG Aushev et al. [Belle Coll.], Phys. Rev. D **81**, 031103 (2010)

$\Gamma_{X(3872)\to D^0\bar{D}^{0*}} = 3.0^{+1.9}_{-1.4} \pm 0.9$ MeV — PDG Aubert et al. [BABAR Coll.], Phys. Rev. D 77011102(2008)

- **Prompt production from CDF collaboration in high-energy hadron collisions incompatible with a molecular interpretation**
- meson-meson molecule: large (a few fm) and fragile
- See: Bignamini et al., Phys. Rev. Lett. **103**, 162001 (2009); Bauer, Int. J. Mod. Phys. A **20**, 3765 (2005)

Bottomonium spectrum
(in a couple channel calculation)

Ferretti, Santopintio, Phys.Rev. D90, 094022 (2014)

- Parameters of the UQM ($^3P_0$ vertices)

Parameter	Value
γ_0	0.732
α	0.500 GeV
r_q	0.335 fm
m_n	0.330 GeV
m_s	0.550 GeV
m_c	1.50 GeV
m_b	4.70 GeV

- Pair-creation strength γ_0 fitted to:

$$\begin{aligned}\Gamma_{\Upsilon(4S)\to B\bar{B}} &= 2\Phi_{A\to BC}\left|\langle BC\vec{q}_0\,\ell J|T^\dagger|A\rangle\right|^2 \\ &= 2\Phi_{\Upsilon(4S)\to B\bar{B}} \\ &\quad \left|\langle B\bar{B}\vec{q}_0\,1^{--}|T^\dagger|\Upsilon(4S)\rangle\right|^2 \\ &= 21\text{ MeV}\,,\end{aligned}$$

Bottomonium Strong Decays

Ferretti, Santopinto, Phys.Rev. D90 094022 (2014)

- Two-body strong decays. Results:

State	Mass [MeV]	J^{PC}	BB	BB^* $\bar{B}B^*$	B^*B^*	B_sB_s	$B_sB_s^*$ $\bar{B}_sB_s^*$	$B_s^*B_s^*$
$\Upsilon(4^3S_1)$	10.595 $10579.4 \pm 1.2^\dagger$	1^{--}	21	–	–	–	–	–
$\chi_{b2}(2^3F_2)$	10585	2^{++}	34	–	–	–	–	–
$\Upsilon(3^3D_1)$	10661	1^{--}	23	4	15	–	–	–
$\Upsilon_2(3^3D_2)$	10667	2^{--}	–	37	30	–	–	–
$\Upsilon_2(3^1D_2)$	10668	2^{-+}	–	55	57	–	–	–
$\Upsilon_3(3^3D_3)$	10673	3^{--}	15	56	113	–	–	–
$\chi_{b0}(4^3P_0)$	10726	0^{++}	26	–	24	–	–	–
$\Upsilon_3(2^3G_3)$	10727	3^{--}	3	43	39	–	–	–
$\chi_{b1}(4^3P_1)$	10740	1^{++}	–	20	1	–	–	–
$h_b(4^1P_1)$	10744	1^{+-}	–	33	5	–	–	–
$\chi_{b2}(4^3P_2)$	10751	2^{++}	10	28	5	1	–	–
$\chi_{b2}(3^3F_2)$	10800	2^{++}	5	26	53	2	2	–
$\Upsilon_3(3^1F_3)$	10803	3^{+-}	–	28	46	–	3	–
$\Upsilon(10860)$	$10876 \pm 11^\dagger$	1^{--}	1	21	45	0	3	1
$\Upsilon_2(4^3D_2)$	10876	2^{--}	–	28	36	–	4	4
$\Upsilon_2(4^1D_2)$	10877	2^{-+}	–	22	37	–	4	3
$\Upsilon_3(4^3D_3)$	10881	3^{--}	1	4	49	0	1	2
$\Upsilon_3(3^3G_3)$	10926	3^{--}	7	0	13	2	0	5
$\Upsilon(11020)$	$11019 \pm 8^\dagger$	1^{--}	0	8	26	0	0	2

Bottomonium spectrum (in couple channel calculations)

Ferretti, Santopintio, Phys.Rev. D90, 094022 (2014)

State	J^{PC}	$B\bar{B}$	$B\bar{B}^*$ $\bar{B}B^*$	$B^*\bar{B}^*$	$B_s\bar{B}_s$	$B_s\bar{B}_s^*$ $\bar{B}_sB_s^*$	$B_s^*\bar{B}_s^*$	$B_c\bar{B}_c$	$B_c\bar{B}_c^*$ $\bar{B}_cB_c^*$	$B_c^*\bar{B}_c^*$	$\eta_b\eta_b$	$\eta_b\Upsilon$	$\Upsilon\Upsilon$	$\Sigma(E_a)$	E_a	M_a	$M_{exp.}$
$\eta_b(1^1S_0)$	0^{-+}	–	-26	-26	–	-5	-5	–	-1	-1	–	–	0	-64	9455	9391	9391
$\Upsilon(1^3S_1)$	1^{--}	-5	-19	-32	-1	-4	-7	0	0	-1	–	0	–	-69	9558	9489	9460
$\eta_b(2^1S_0)$	0^{-+}	–	-43	-41	–	-8	-7	–	-1	-1	–	–	0	-101	10081	9980	9999
$\Upsilon(2^3S_1)$	1^{--}	-8	-31	-51	-2	-6	-9	0	0	-1	–	0	–	-108	10130	10022	10023
$\eta_b(3^1S_0)$	0^{-+}	–	-59	-52	–	-8	-8	–	-1	-1	–	–	0	-129	10467	10338	–
$\Upsilon(3^3S_1)$	1^{--}	-14	-45	-68	-2	-6	-10	0	0	-1	–	0	–	-146	10504	10358	10355
$h_b(1^1P_1)$	1^{+-}	–	-49	-47	–	-9	-8	–	-1	-1	–	0	–	-115	10000	9885	9899
$\chi_{b0}(1^3P_0)$	0^{++}	-22	–	-69	-3	–	-13	0	–	-1	0	–	0	-108	9957	9849	9859
$\chi_{b1}(1^3P_1)$	1^{++}	–	-46	-49	–	-8	-9	–	-1	-1	–	–	0	-114	9993	9879	9893
$\chi_{b2}(1^3P_2)$	2^{++}	-11	-32	-55	-2	-6	-9	0	-1	-1	0	–	0	-117	10017	9900	9912
$h_b(2^1P_1)$	1^{+-}	–	-66	-59	–	-10	-9	–	-1	-1	–	0	–	-146	10393	10247	10260
$\chi_{b0}(2^3P_0)$	0^{++}	-33	–	-85	-4	–	-14	0	–	-1	0	–	0	-137	10363	10226	10233
$\chi_{b1}(2^3P_1)$	1^{++}	–	-63	-60	–	-9	-10	–	-1	-1	–	–	0	-144	10388	10244	10255
$\chi_{b2}(2^3P_2)$	2^{++}	-16	-42	-72	-2	-6	-10	0	0	-1	0	–	0	-149	10406	10257	10269
$h_b(3^1P_1)$	1^{+-}	–	-18	-73	–	-11	-10	–	-1	-1	–	0	–	-114	10705	10591	–
$\chi_{b0}(3^3P_0)$	0^{++}	-4	–	-160	-6	–	-15	0	–	-1	0	–	0	-186	10681	10495	–
$\chi_{b1}(3^3P_1)$	1^{++}	–	-25	-74	–	-11	-10	–	0	-1	–	–	0	-121	10701	10580	–
$\chi_{b2}(3^3P_2)$	2^{++}	-19	-16	-79	-3	-8	-12	0	0	-1	0	–	0	-138	10716	10578	–
$\Upsilon_2(1^1D_2)$	2^{-+}	–	-72	-66	–	-11	-10	–	-1	-1	–	–	0	-161	10283	10122	–
$\Upsilon(1^3D_1)$	1^{--}	-24	-22	-90	-3	-3	-16	0	0	-1	–	0	–	-159	10271	10112	–
$\Upsilon_2(1^3D_2)$	2^{--}	–	-70	-68	–	-10	-11	–	-1	-1	–	0	–	-161	10282	10121	10164
$\Upsilon_3(1^3D_3)$	3^{--}	-18	-43	-78	-3	-8	-11	0	-1	-1	–	0	–	-163	10290	10127	–

Bottomonium

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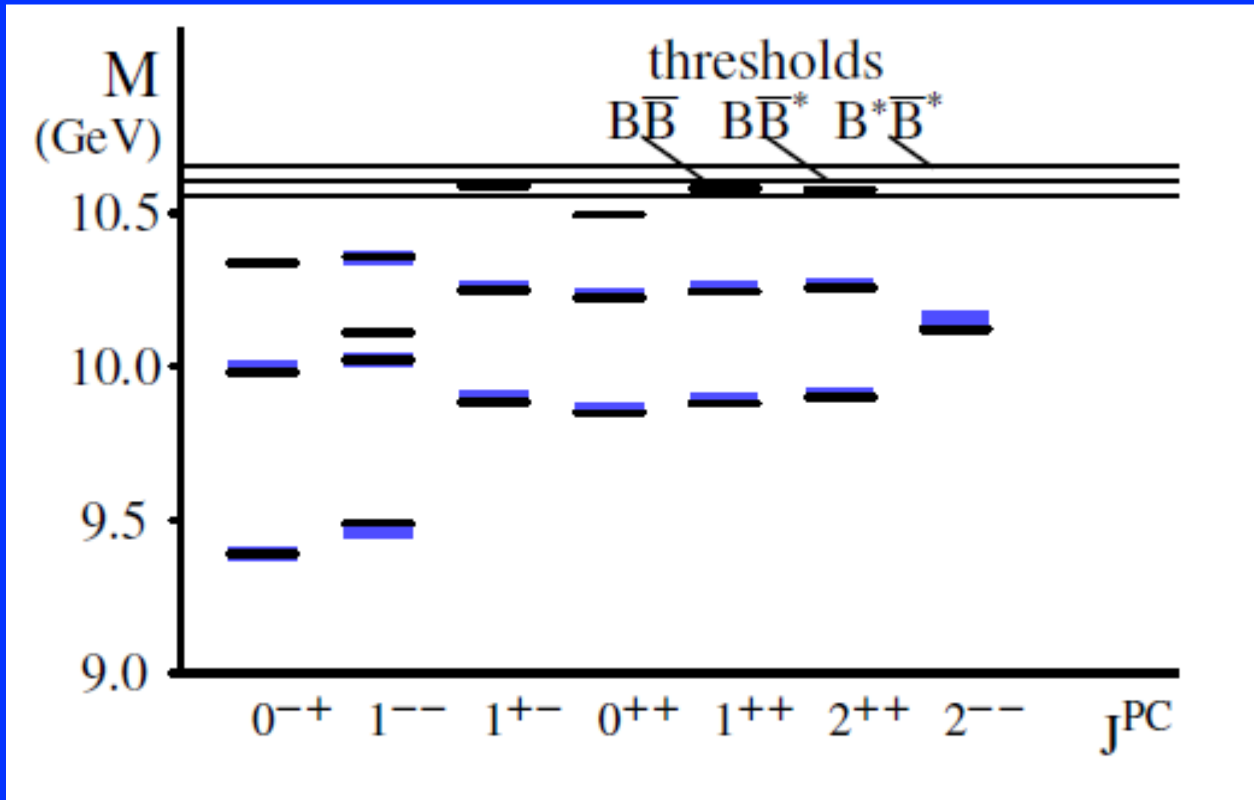
- Results:



Couple Channels corrections to Bottomonium , the $\chi_b(3P)$ system

Ferretti, Santopintio, Phys.Rev. D90 (2014) 9, 094022

- Results used to study some properties of the $\chi_b(3P)$ system, meson multiplet with N=3, L=1 quantum numbers
- $\chi_b(3P)$ states close to first open bottom decay thresholds
- Possible importance of continuum coupling effects?
- Pure c bar-c and c bar-c + continuum effects interpretations
- Necessary to study decays (strong, e.m., hadronic, ...) to confirm one interpretation

Couple Channels corrections to Bottomonium , the $\chi_b(3P)$ system

Ferretti, Santopintio, Phys.Rev. D90 (2014) 9, 094022

- Some experimental results for the mass barycenter of the system:
- $M[\chi_b(3P)] = 10.530 \pm 0.005$ (stat.) ± 0.009 (syst.) GeV
- Aad et al. [ATLAS Coll.], Phys. Rev. Lett. **108**, 152001 (2012)
- $M[\chi_b(3P)] = 10.551 \pm 0.014$ (stat.) ± 0.017 (syst.) GeV
- Abazov et al. [D0 Coll.], Phys. Rev. D 86, 031103 (2012)
- Mass barycenter in the UQM:

$$\Gamma_{A\to BC} = \Phi_{A\to BC}(q_0)\sum_{\ell,J}\left|\langle BC\vec{q}_0\,\ell J|\,T^\dagger\,|A\rangle\right|^2 \ . \tag{14}$$

Here, $\Phi_{A\to BC}(q_0)$ is the standard relativistic phase space factor [55, 73],

$$\Phi_{A\to BC} = 2\pi q_0 \frac{E_b(q_0)E_c(q_0)}{M_a} \ , \tag{15}$$

J. Ferretti E. Santopinto ,Phys.Rev. D90 (2014) 9, 094022, arXiv:1506.04415 [hep-ph]

State	Γ_{theor} ($^3P_0$) [MeV]	Γ_{exp} [MeV]
$\Upsilon(4^3S_1)$	21	20.5 ± 2.5
$\Upsilon(10860)$	71	42^{+29}_{-24}

Meson	Mass [MeV]	J^{PC}	$B\bar{B}$	$B\bar{B}^*$ $\bar{B}B^*$	$B^*\bar{B}^*$	$B_s\bar{B}_s$	$B_s\bar{B}_s^*$ $\bar{B}_sB_s^*$	$B_s^*\bar{B}_s^*$
$\Upsilon(10580)$ or $\Upsilon(4^3S_1)$	10.595 $10579.4 \pm 1.2^\dagger$	1^{--}	21	–	–	–	–	–
$\chi_{b2}(2^3F_2)$	10585	2^{++}	34	–	–	–	–	–
$\Upsilon(3^3D_1)$	10661	1^{--}	23	4	15	–	–	–
$\Upsilon_2(3^3D_2)$	10667	2^{--}	–	37	30	–	–	–
$\Upsilon_2(3^1D_2)$	10668	2^{-+}	–	55	57	–	–	–
$\Upsilon_3(3^3D_3)$	10673	3^{--}	15	56	113	–	–	–
$\chi_{b0}(4^3P_0)$	10726	0^{++}	26	–	24	–	–	–
$\Upsilon_3(2^3G_3)$	10727	3^{--}	3	43	39	–	–	–
$\chi_{b1}(4^3P_1)$	10740	1^{++}	–	20	1	–	–	–
$h_b(4^1P_1)$	10744	1^{+-}	–	33	5	–	–	–
$\chi_{b2}(4^3P_2)$	10751	2^{++}	10	28	5	1	–	–
$\chi_{b2}(3^3F_2)$	10800	2^{++}	5	26	53	2	2	–
$\Upsilon_3(3^1F_3)$	10803	3^{+-}	–	28	46	–	3	–
$\Upsilon(10860)$ or $\Upsilon(5^3S_1)$	$10876 \pm 11^\dagger$	1^{--}	1	21	45	0	3	1
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$\Upsilon_3(3^3G_3)$	10926	3^{--}	7	0	13	2	0	5
$\Upsilon(11020)$ or $\Upsilon(6^3S_1)$	$11019 \pm 8^\dagger$	1^{--}	0	8	26	0	0	2

State	J^P	Mass [MeV]	$D\pi$	$D^*\pi$	$D\rho$	$D^*\rho$	$D\eta$	$D^*\eta$	$D\omega$	$D^*\omega$	D_sK	D_s^*K	D_sK^*	$D_s^*K^*$
$D_1(1^3S_1)$	1^-	2038 $2009^\dagger$	0	–	–	–	–	–	–	–	–	–	–	–
$D_0^*(2400)$ or $D_0(1^3P_0)$	0^+	2398 $2318 \pm 29^\dagger$	66	–	–	–	–	–	–	–	–	–	–	–
$D_2^*(2460)$ or $D(1^3P_2)$	2^+	2501 $2463^\dagger$	6	2	–	–	0	–	–	–	–	–	–	–
$D_0(2550)$ or $D(2^1S_0)$	0^-	2582 $2539.4 \pm 4.5 \pm 6.8^\dagger$	–	42	–	–	–	–	–	–	–	–	–	–
$D_1(2^3S_1)$	1^-	2645	18	36	0	–	6	5	–	–	4	1	–	–
$D_1(1^3D_1)$	1^-	2816	20	13	13	1	10	5	4	0	6	2	–	–
$D_3(1^3D_3)$	3^-	2833	11	8	1	15	2	1	0	4	1	0	–	–
$D_0(2^3P_0)$	0^+	2931	18	–	–	38	2	–	–	12	0	–	–	–
$D_2(2^3P_2)$	2^+	2957	13	23	22	45	6	7	7	16	4	4	1	–
$D_0(3^1S_0)$	0^-	3067	–	1	4	38	–	1	1	13	–	3	8	8
$D_1(3^3S_1)$	1^-	3111	3	2	1	31	0	0	0	11	0	1	5	15
$D_4(1^3F_4)$	4^+	3113	11	8	4	36	2	1	1	12	1	0	0	1
$D_2(1^3F_2)$	2^+	3132	10	9	11	12	5	3	4	4	2	2	1	0
$D_3(2^3D_3)$	3^-	3226	8	14	16	21	4	5	5	7	3	3	2	9
$D_1(2^3D_1)$	1^-	3231	7	2	0	51	1	0	0	17	0	0	1	4
$D_0(3^3P_0)$	0^+	3343	1	–	–	13	0	–	–	4	1	–	–	11
$D_2(3^3P_2)$	2^+	3352	2	1	0	13	0	0	0	5	0	1	1	6
$D_3(1^3G_3)$	3^-	3398	5	2	7	15	2	2	2	5	1	1	1	1
$D_0(4^1S_0)$	0^-	3465	–	1	4	11	–	1	1	4	–	1	1	0
$D_4(2^3F_4)$	4^+	3466	5	8	10	12	2	3	3	4	2	2	2	4
$D_2(2^3F_2)$	2^+	3490	3	1	0	38	1	0	0	12	0	0	0	5
$D_3(3^3D_3)$	3^-	3578	2	1	0	6	0	0	0	2	0	0	1	2
$D_0(4^3P_0)$	0^+	3709	0	–	–	9	0	–	–	3	0	–	–	1
$D_3(2^3G_3)$	3^-	3721	2	1	0	24	0	0	0	8	0	0	0	3
$D_4(3^3F_4)$	4^+	3788	1	1	0	3	0	0	0	1	0	0	0	1
$D_4(4^3F_4)$	4^+	4085	0	0	0	2	0	0	0	1	0	0	0	0

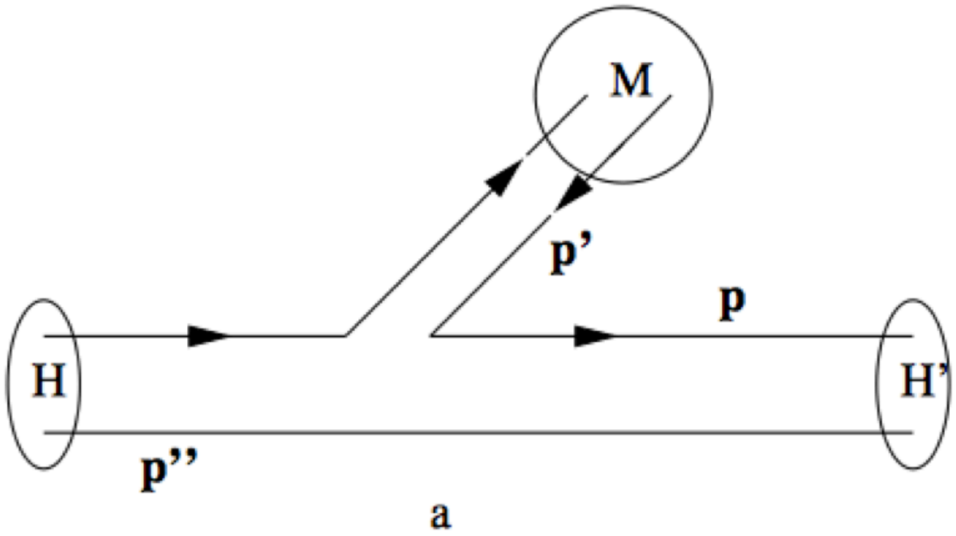
Main points

- Unquenching quark model:we have constructed the formalism in an explicit way, also thanks to group th. tecniques. Now, it can be applied to any quark model.
- We think we have maked up the problems of quark models adding the coupling with the continuum, thus opening the possibilty of many, many applications

- Future: application to open problems in hadron structure and spectroscopy : helicity amplitudes, strong decays,and so on.

3P0 pair created with vacuum qu. n.

Moreover, it takes into acount of the structure of the three mesons

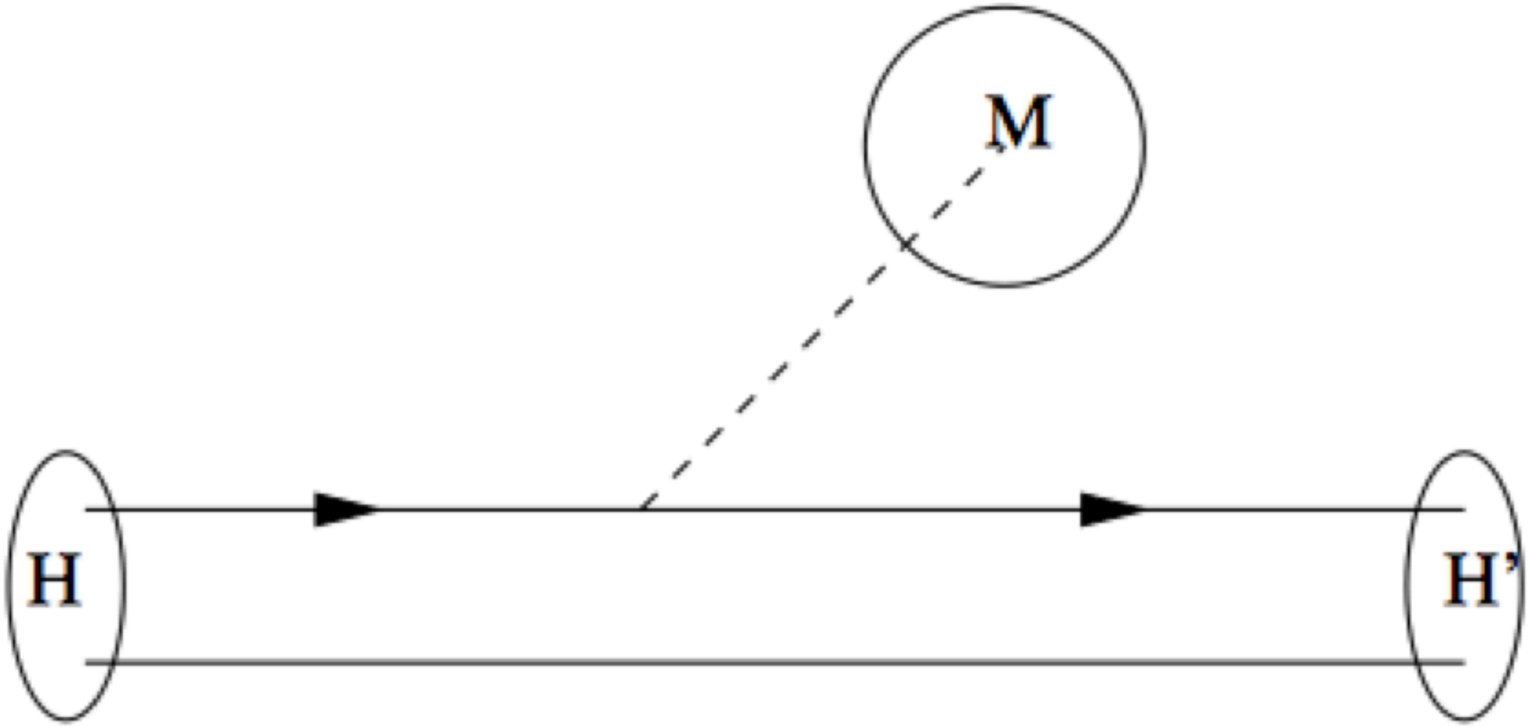


Elementary emission model

no wave fucuntion for the coupled meson

Ex. Send the dimension of one of the emitted meson to zero and

recoved the elementary e.model



b

Ex. Try to calculate the strong decays of the rho meson into two pions

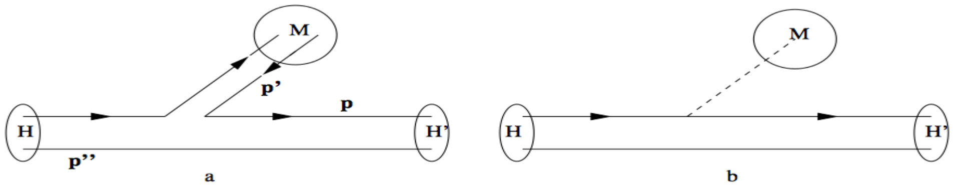


Figure : Diagrams representing a meson decay with a one-body interaction: within the $^3P_0$ decay model (a) and an elementary emission model (b).

The $^3P_0$ operator

$$T^\dagger = -3\gamma \sum_{i,j} \int d\vec{p}_i d\vec{p}_j \delta(\vec{p}_i + \vec{p}_j) C_{ij} F_{ij} \Gamma(\vec{p}_i - \vec{p}_j) \times \left[\chi_{ij} \times \mathcal{Y}_1(\vec{p}_i - \vec{p}_j)\right]_0^{(0)} b_i^\dagger(\vec{p}_i) d_j^\dagger(\vec{p}_j)$$

Transition amplitude for mesons

$$\langle BC, J_b, m_b, J_c, m_c \vec{K}_B, \vec{K}_C | T^\dagger | A, \vec{K}_A, J_a, m_a \rangle = 3\gamma \sum_m \langle 1,1; m, -m | 0,0 \rangle \times \langle \Phi_B^{m_b} \Phi_C^{m_c} | \Phi_A^{m_a} \Phi_{vac}^{-m} \rangle I_m(A,B,C)$$

$$I_m(A,B,C) = \frac{1}{8} \delta^3(\vec{p}_B + \vec{p}_C) \int d\vec{p} \mathcal{Y}_{1m}(\vec{p}_B - \vec{p}) \Psi_B^{*\mathrm{tot}}(-\vec{p}) \Psi_C^{*\mathrm{tot}}(\vec{p}) \Psi_A^{\mathrm{tot}}(\vec{p} + \vec{p}_B)$$

where

$$\Phi_{vac}^{-m} = \chi_1^{-m} \phi_0$$

wave functions

$$\psi(\vec{p}_1,\vec{p}_2)=\left(\frac{R_c^2}{\pi}\right)^{3/4}\exp\left[-\frac{1}{8}(\vec{p}_1-\vec{p}_2)^2\right]$$

$$I_m(A,B,C)=\left(\frac{3}{4\pi}\right)^{1/2}\pi^{-3/4}(\epsilon_m\cdot\vec{p}_c)\frac{4}{3}(2/3R_c)^{3/2}\exp\left(\frac{1}{12}R_c^2p_c^2\right)$$

where $\epsilon_m(m=\pm1,0)$ are the vectors $\epsilon_\pm=(0,\pm1/\sqrt{2},-i/\sqrt{2})$, and $\epsilon_0(0,0,1)$

Spin matrix elements: general expression

$$M_{spin}=\frac{1}{3}(-1)^{L_a+S_a+J_a}\hat{S}_a\hat{S}_b\hat{S}_c\hat{S}_{bc}\hat{\ell}\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & S_b \\ \frac{1}{2} & \frac{1}{2} & S_b \\ S_a & 1 & S_{bc} \end{Bmatrix}\begin{Bmatrix} S_a & L_a & J_a \\ \ell & S_{bc} & 1 \end{Bmatrix}$$

where $\hat{S} = \sqrt{2S+1}$

Spin matrix elements: $\rho \to \pi\pi$

$$M_{spin}=\frac{1}{3}(-1)^2\hat{1}\hat{0}\hat{0}\hat{0}\hat{1}\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 \\ 1 & 1 & 0 \end{Bmatrix}\begin{Bmatrix} 1 & 0 & 1 \\ 1 & 0 & 1 \end{Bmatrix}=\frac{1}{6\sqrt{3}}$$

Flavor matrix elements

$$M_{\rho\to\pi\pi} = 1$$

Complete matrix element

$$M_{tot} = I_{spatial} \times M_{spin} \times M_{flavor}$$

Decay width

$$\Gamma_{\rho \to \pi\pi} = 2\pi p_c \frac{E_\pi^2}{M_\rho} |M_{tot}|^2$$