Amplitude analysis at BESIII

Jake Bennett
Carnegie Mellon University

2017 International Summer Workshop on Reaction Theory

Carnegie Mellon

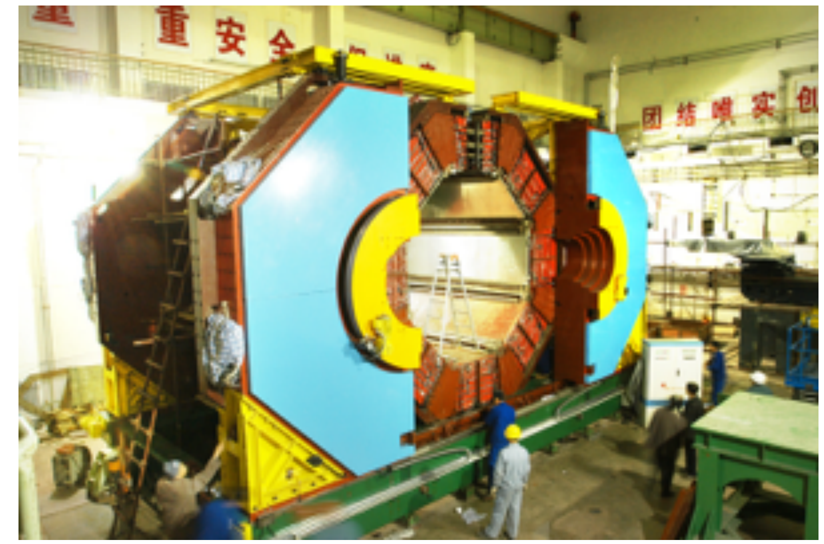
BESIII

Introduction to the experiment

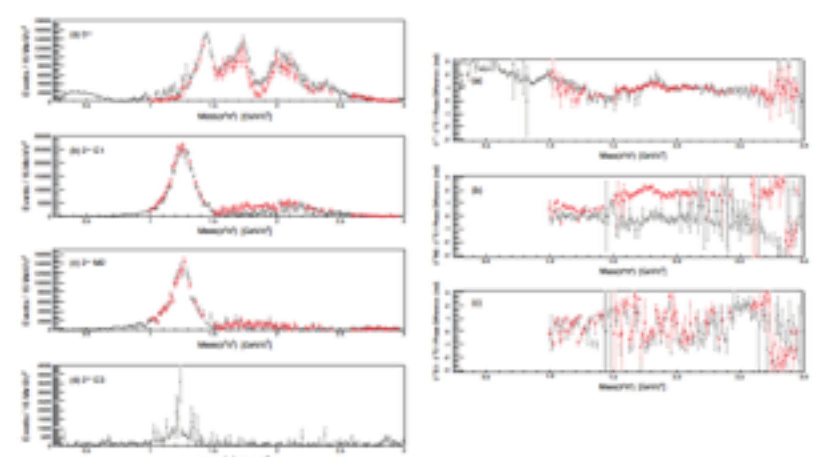
BEPCII accelerator

1. Produce e^+e^- collisions in the tau-charm region

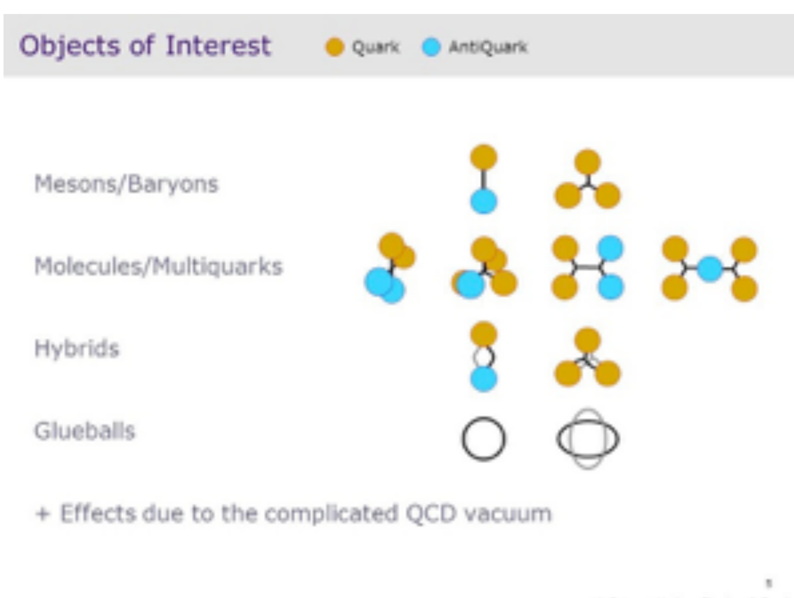
BESIII detector



2. Detect the decay products of charmonium states



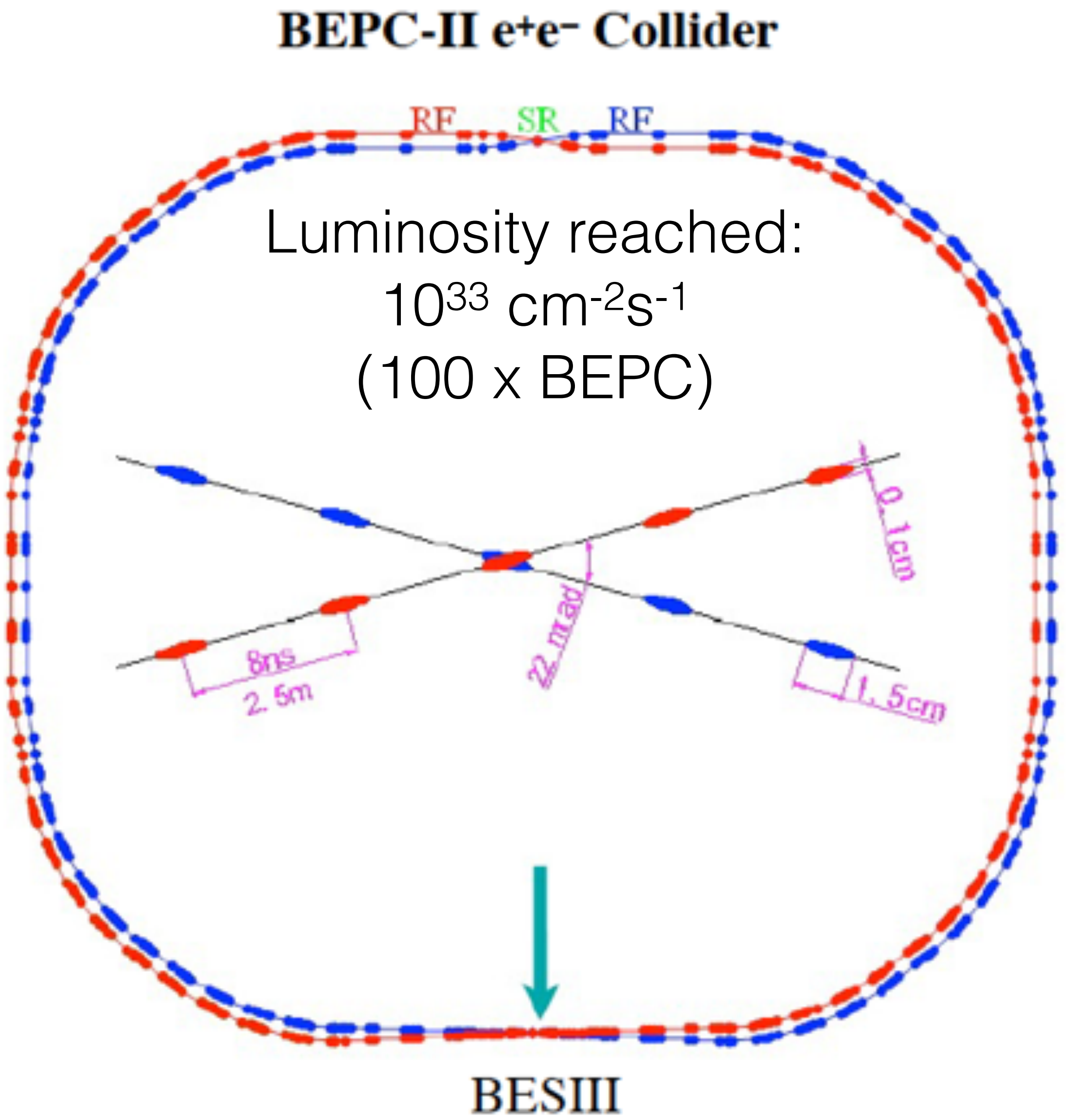
3. Study their properties and decays



4. Try to learn something about QCD

BESIII at BEPCII

- The physics goals of BESIII cover a diverse range:
  - Light hadron spectroscopy, charm physics, τ physics, charmonium physics
- e^+e^- collisions in the charmonium mass region
  - Use the properties and decays of charmonium states to study QCD



BESIII at BEPCII

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  - Light hadron spectroscopy,
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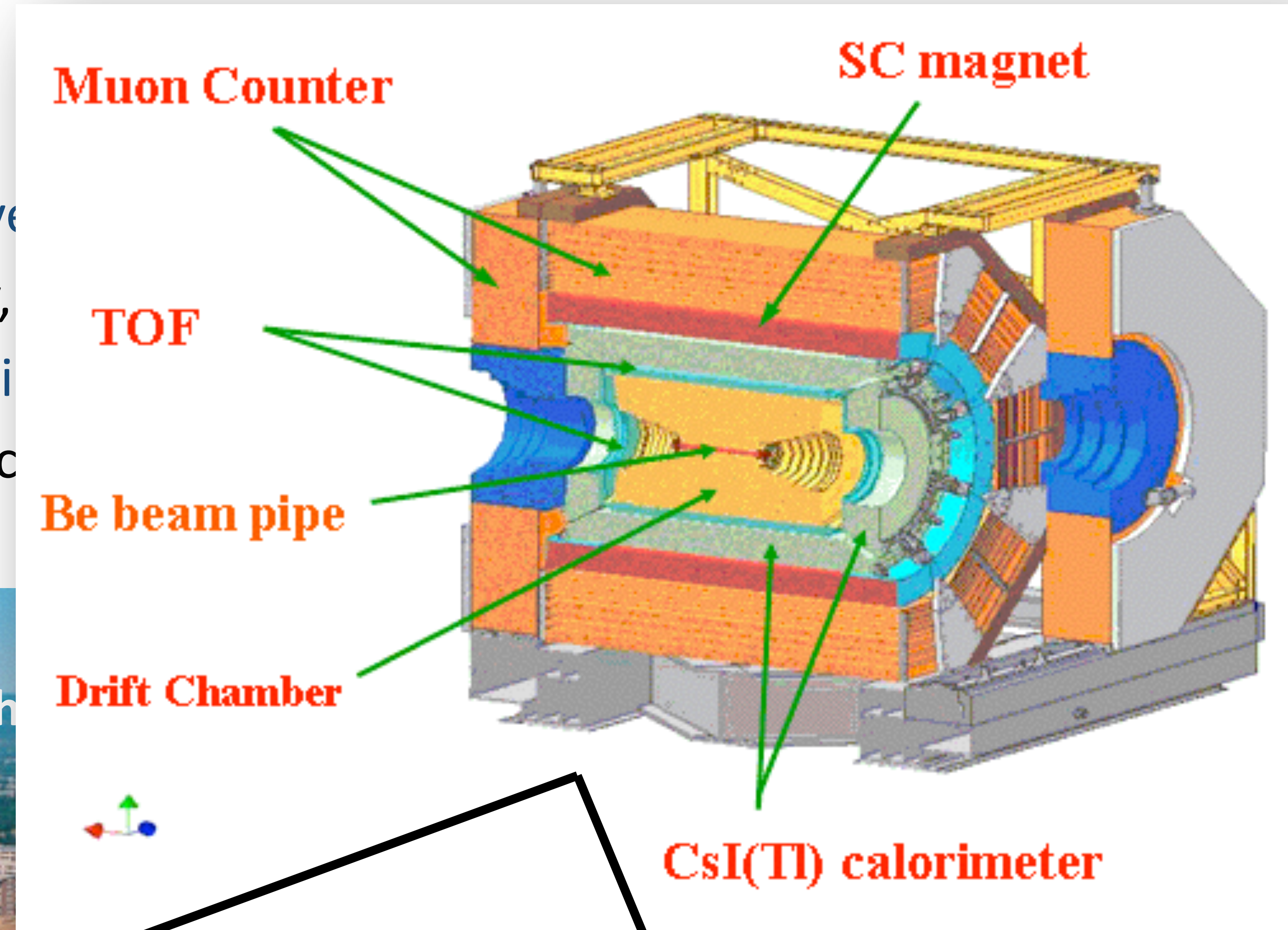

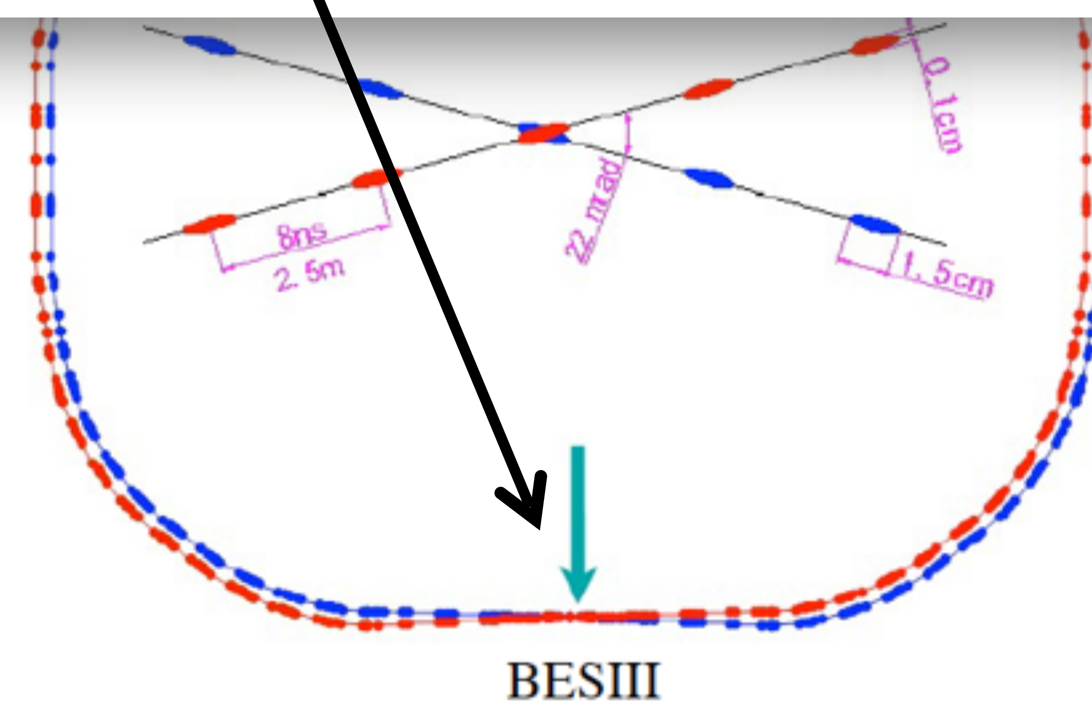

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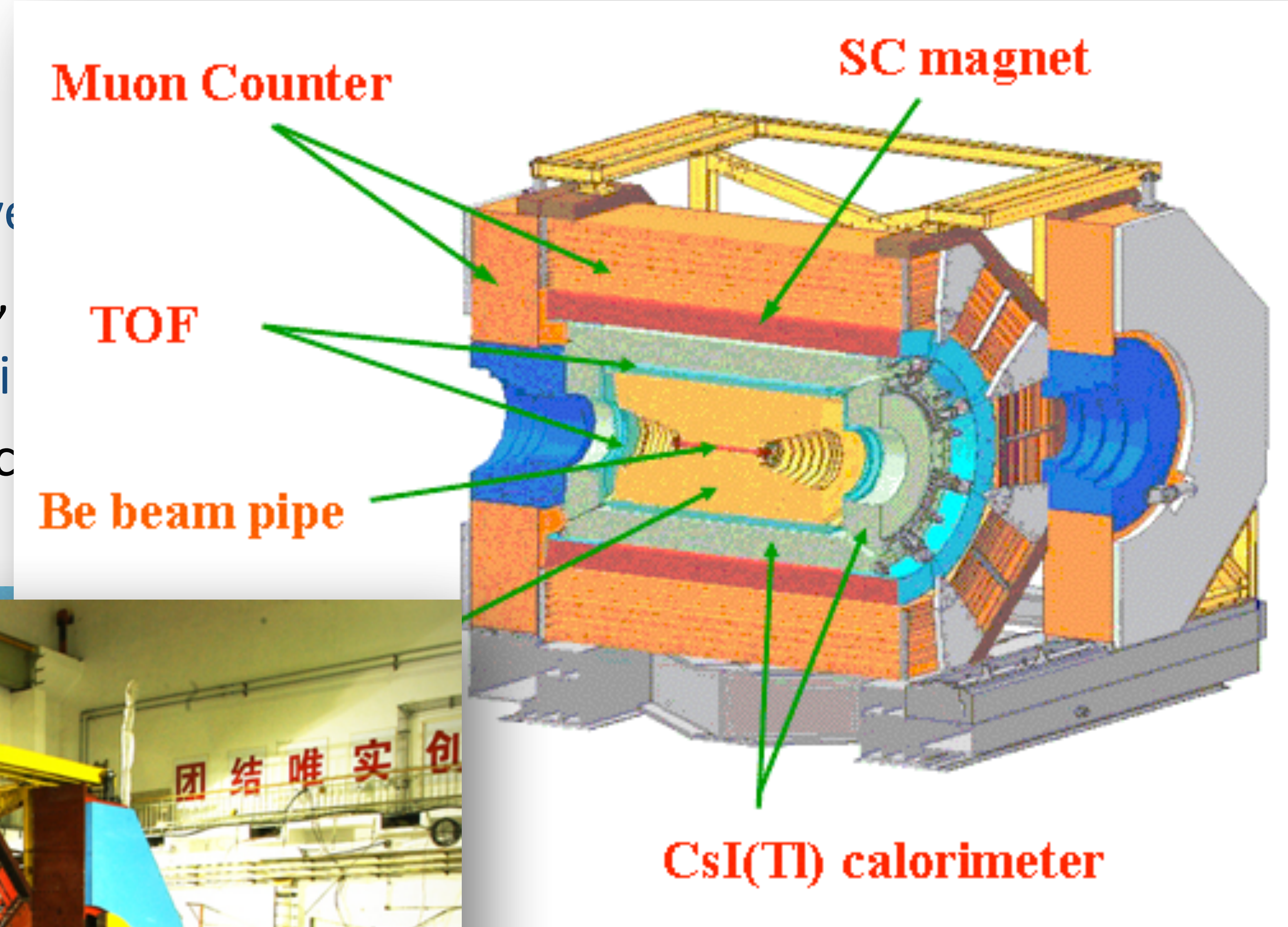

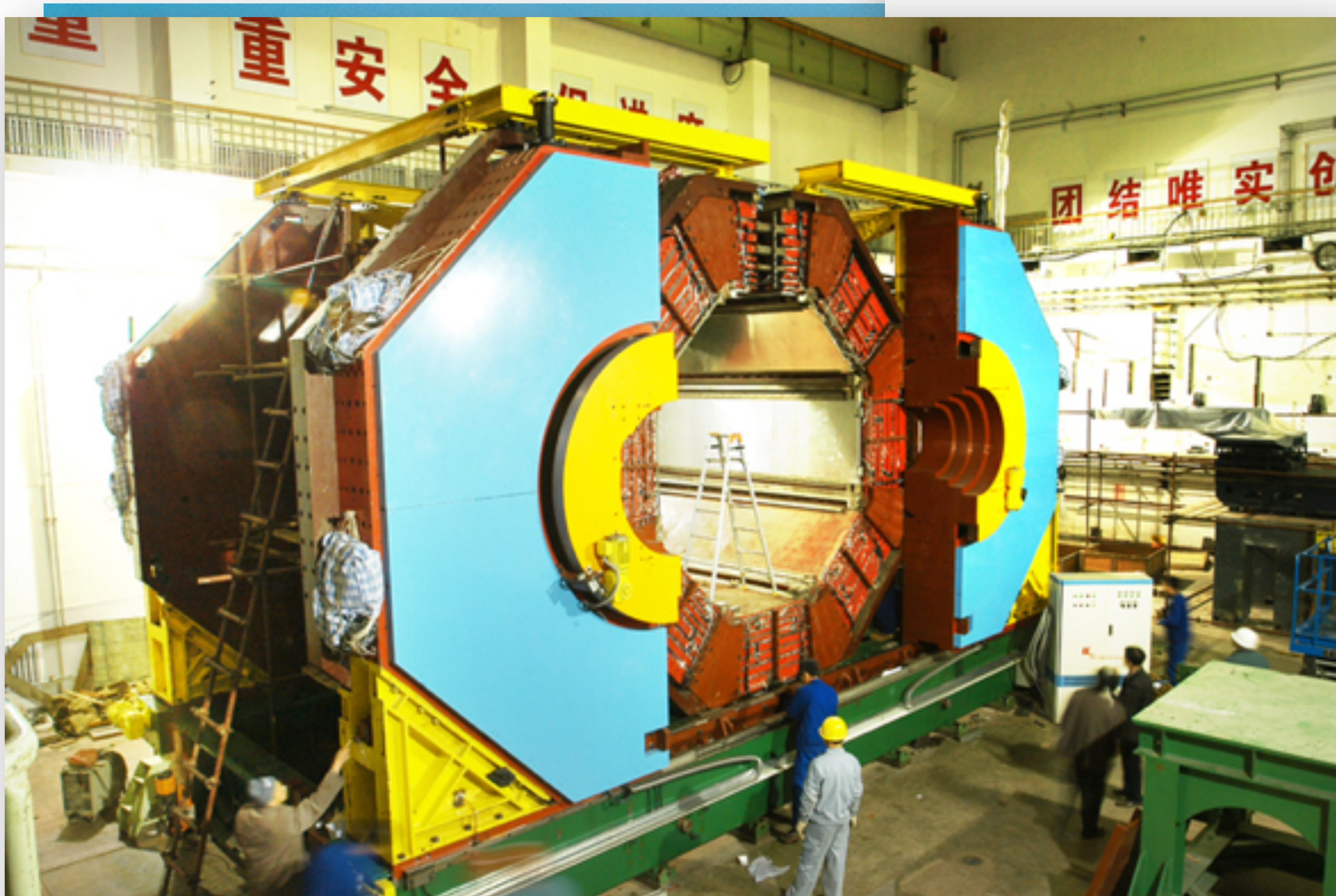

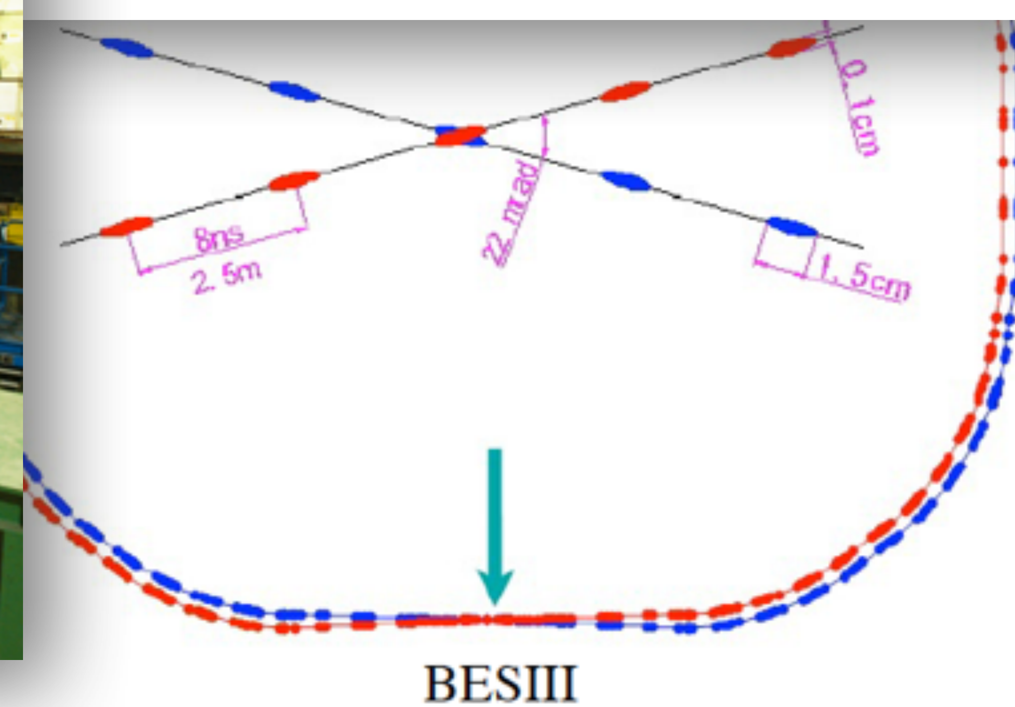

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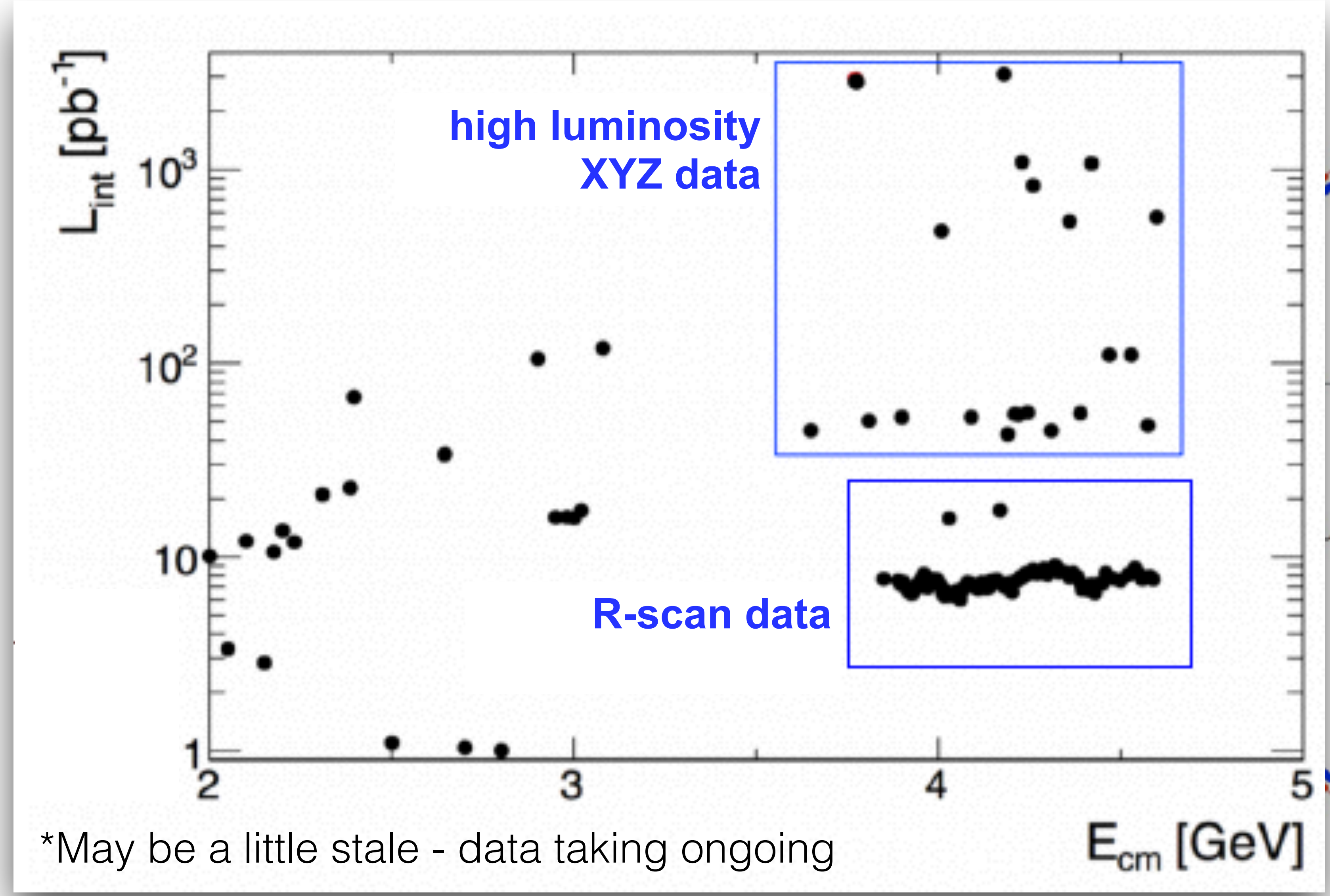

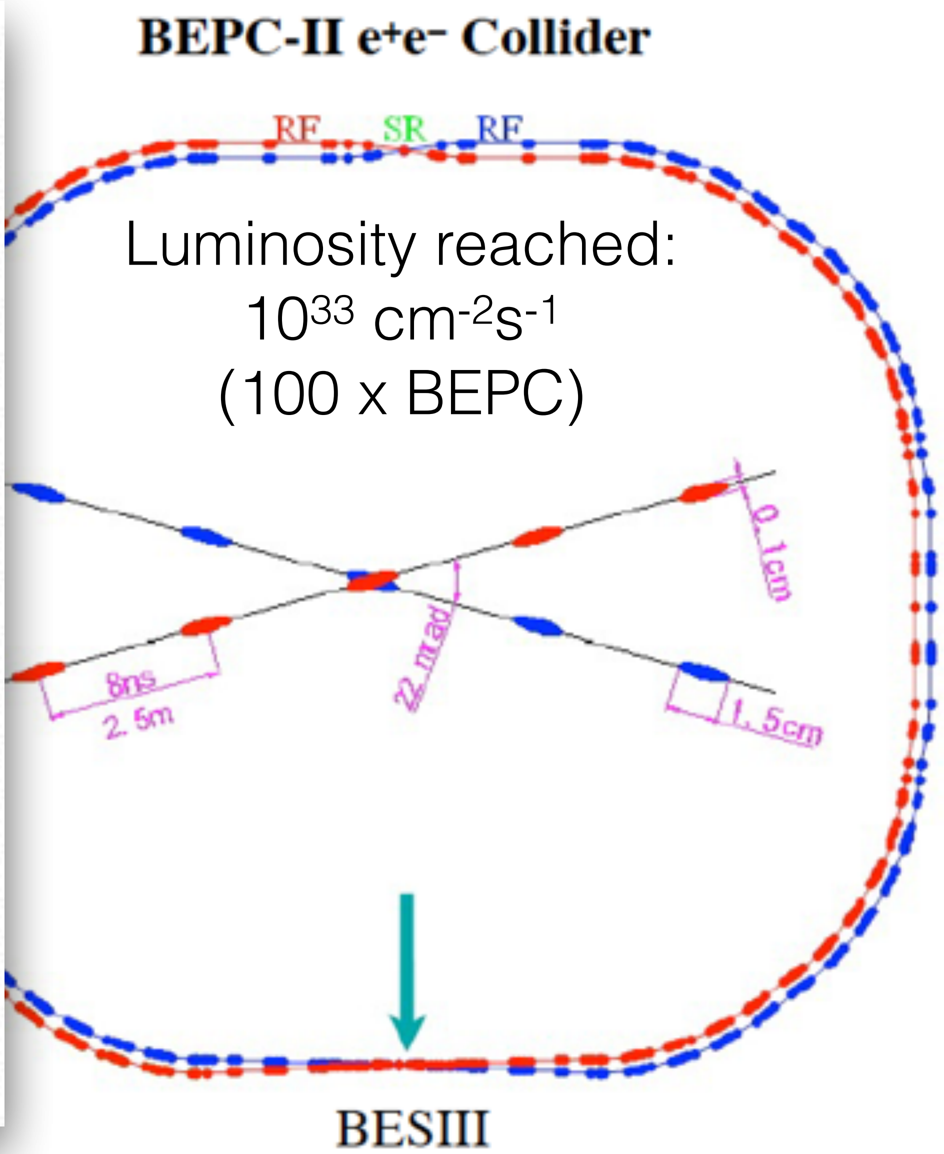

The challenges of the experimentalist

- Building and running the accelerator and detector are only the first step!

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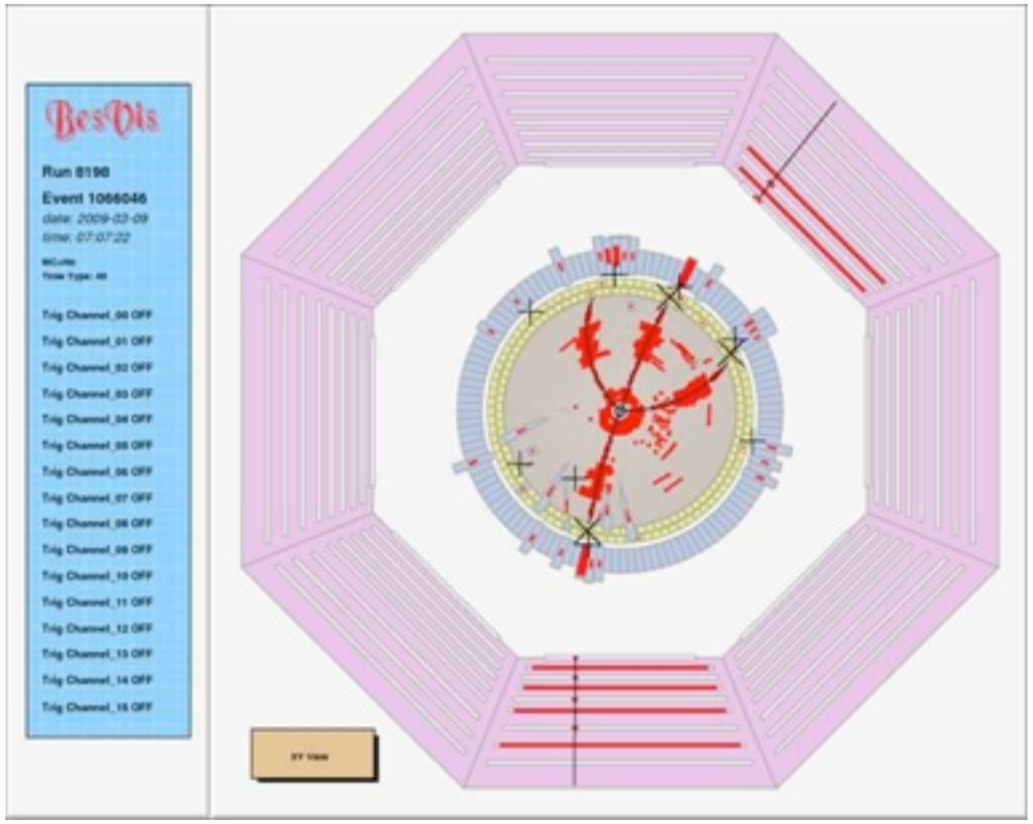
  - Translate detector-level hits and showers into analysis-level information



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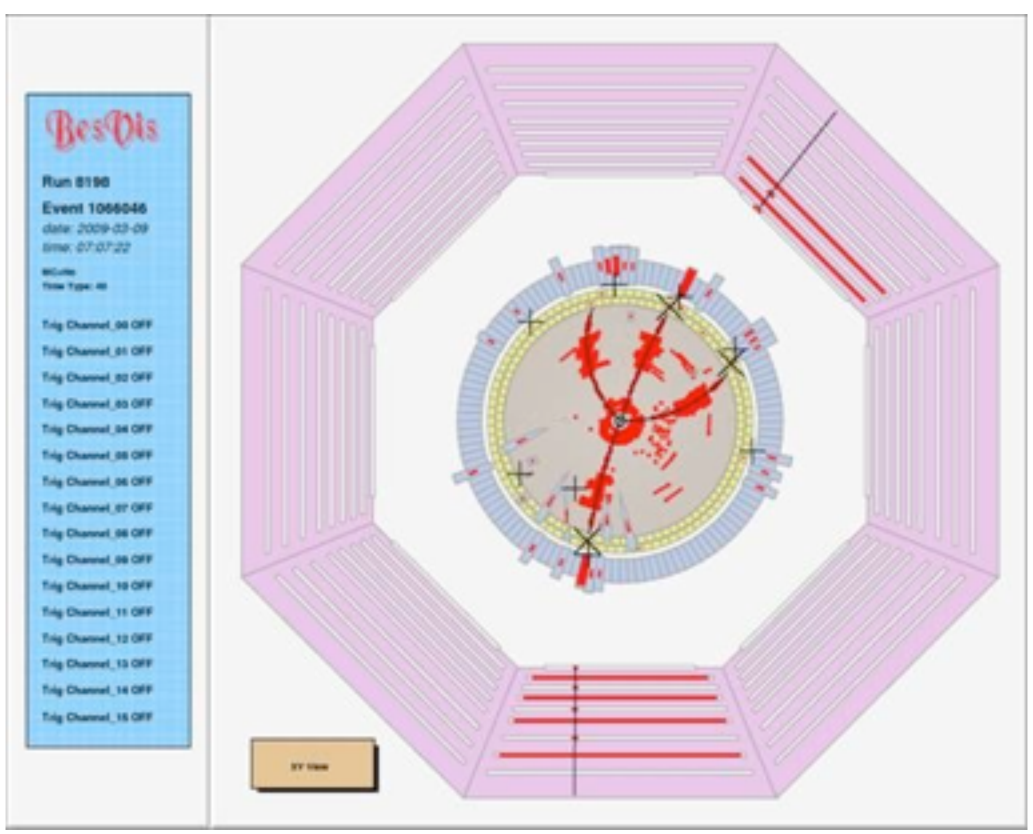
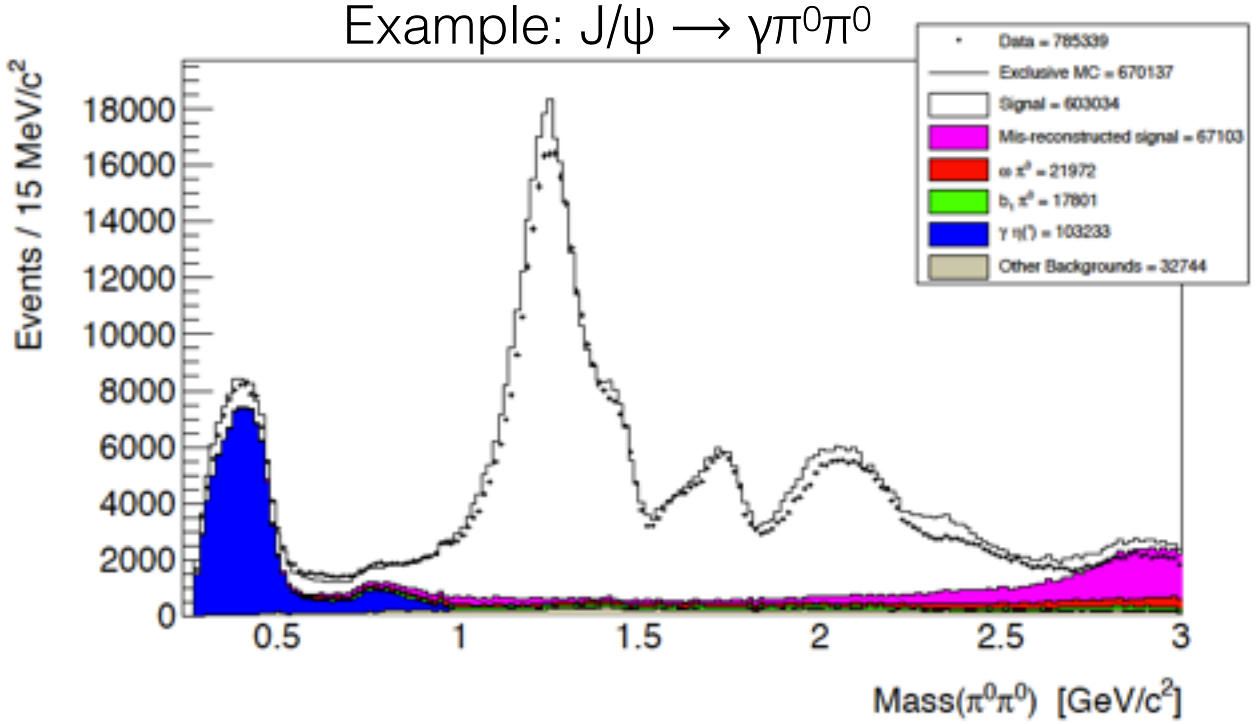


Example: $J/\psi \longrightarrow \gamma\pi^0\pi^0$



Total size of data sample: ~1.31 x 10^9 events

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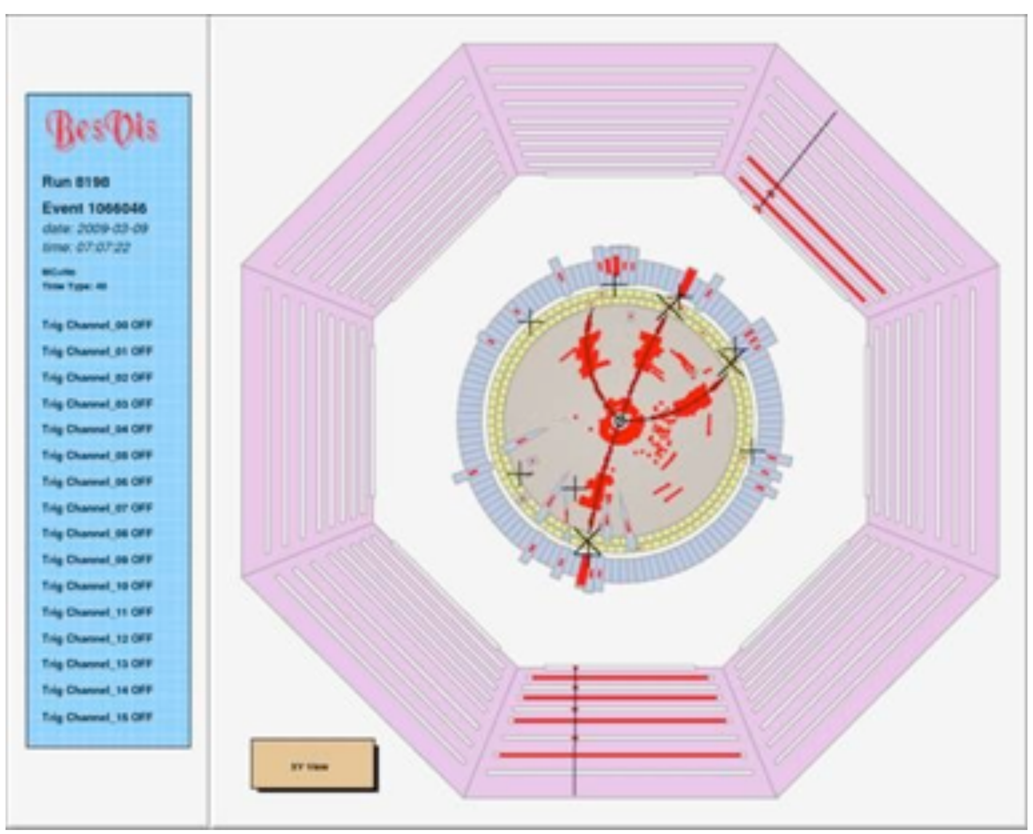
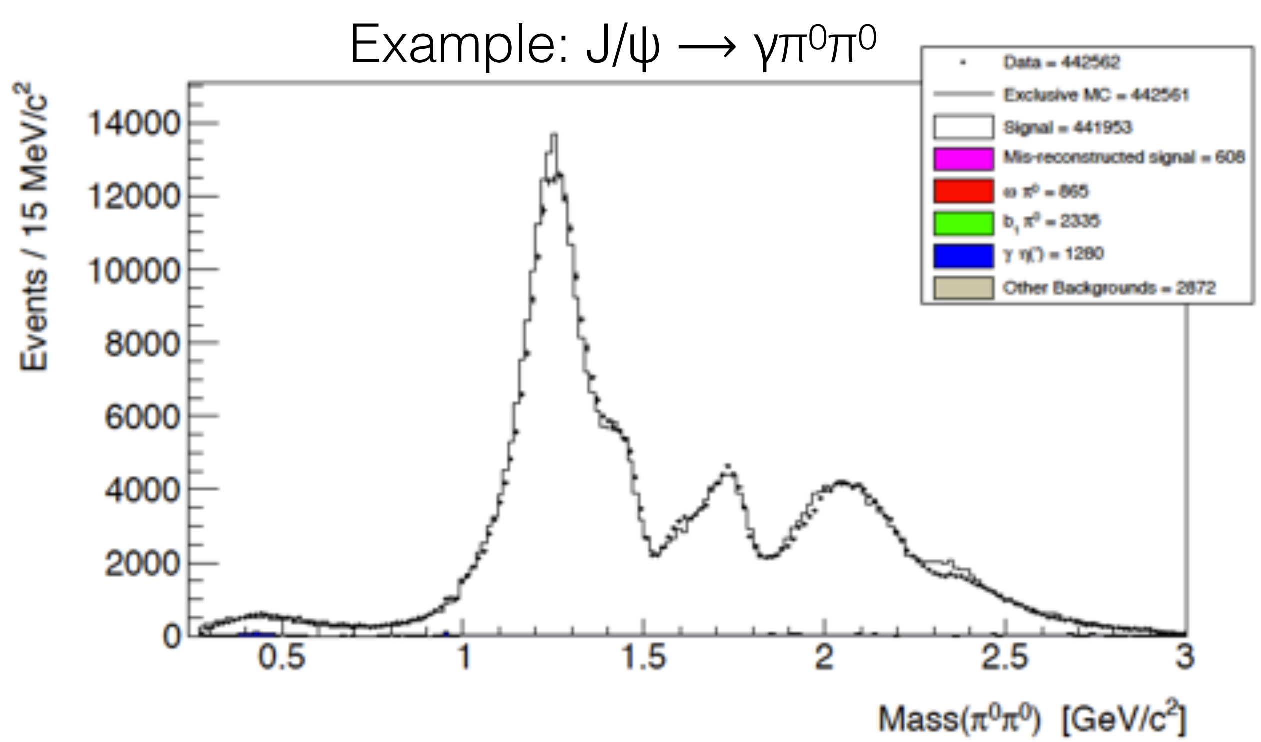




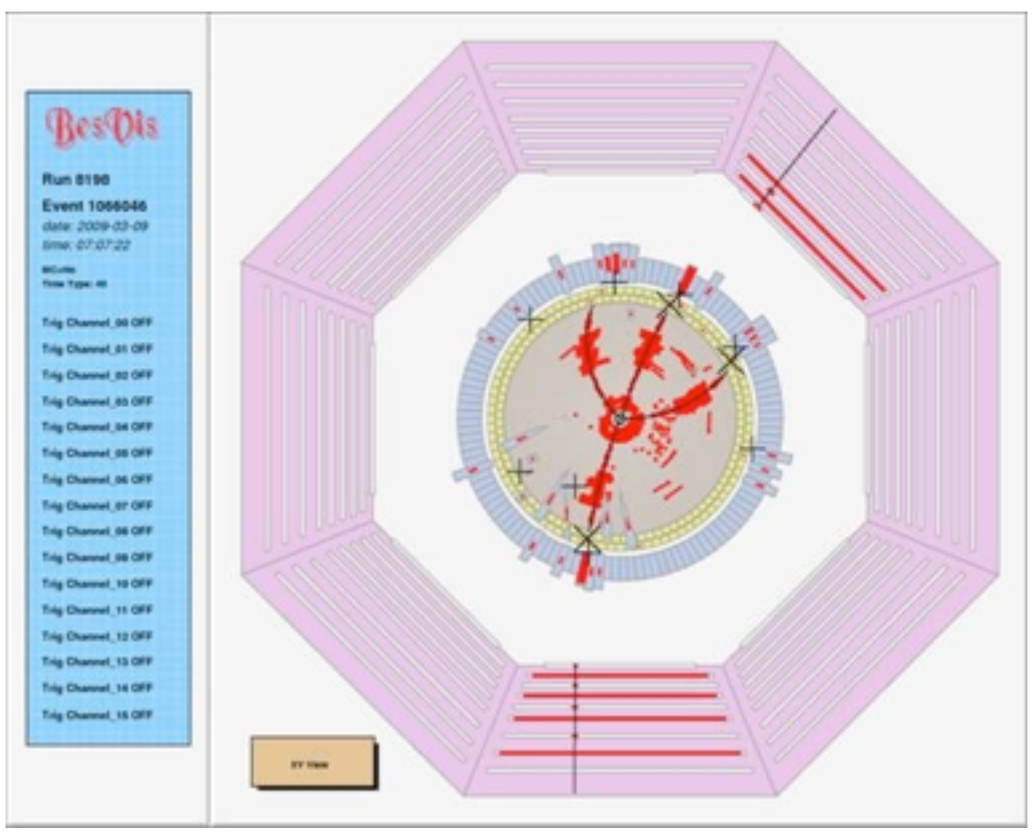
Total size of data sample: ~1.31 x 10^9 events

Size of sample after signal isolation: ~4.4 x 10^5 events

Background contamination remaining: < 2%

The challenges of the experimentalist

- Building and running the accelerator and detector are only the first step!
  - Carefully calibrate the detectors and reconstruction algorithms
  - Translate detector-level hits and showers into analysis-level information
  - Attempt to cleanly separate events of interest from backgrounds (intensive!)
  - Use the analysis-level information (basically four-vectors) in the analysis
  - Account for systematic uncertainties (difficult!)



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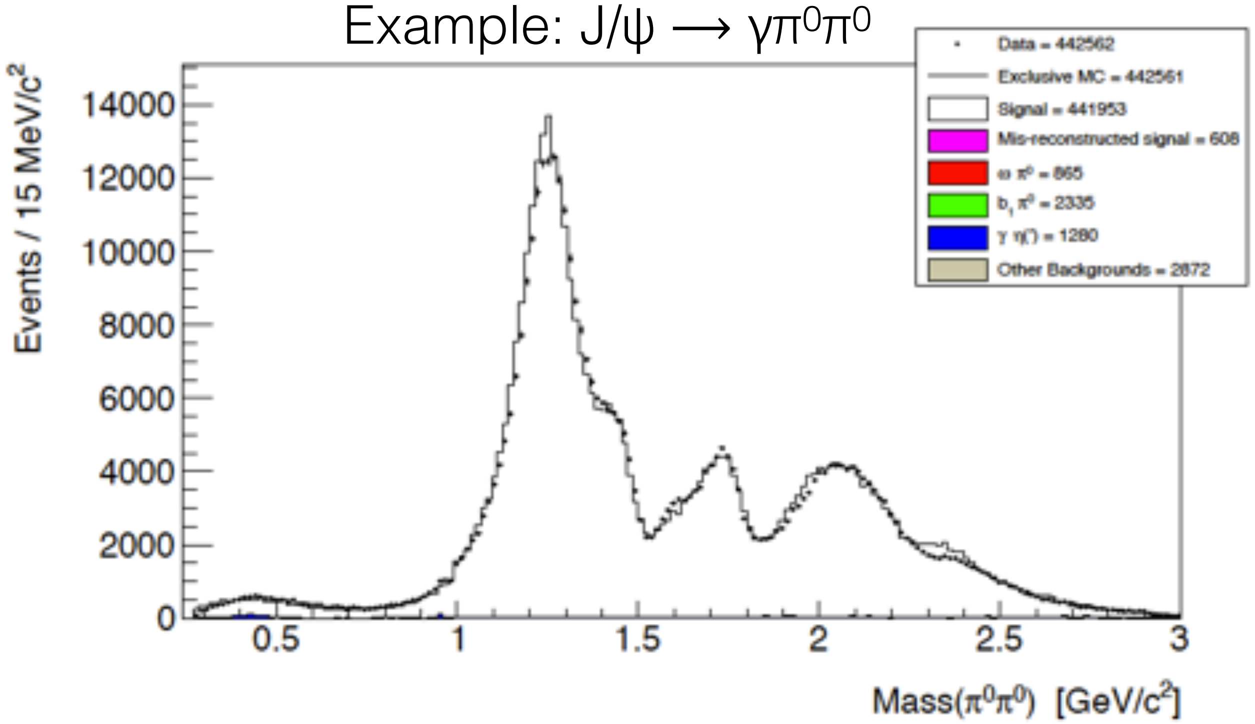

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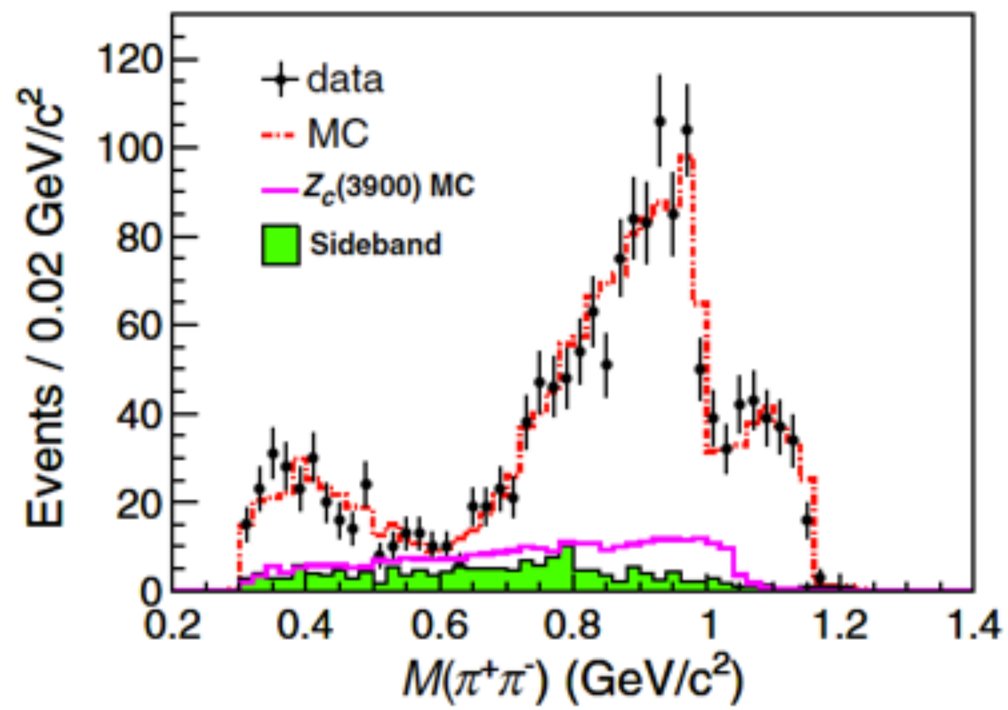
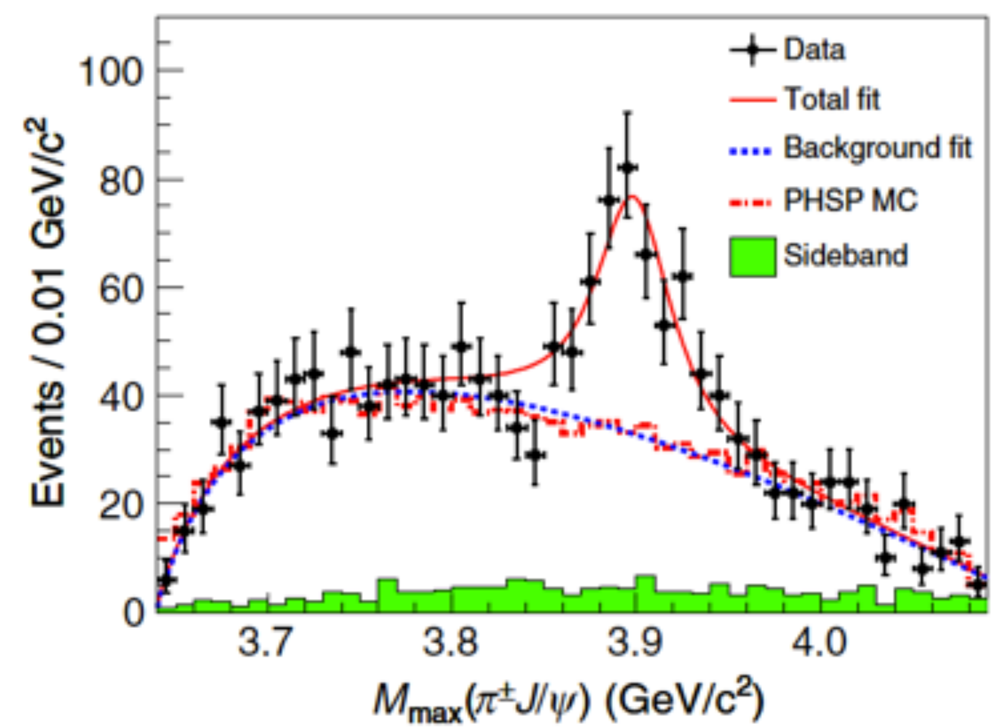
Amplitude analysis for constraining models

- Fairly recent puzzle: XYZ states
  - Appear to be at odds with standard quarkonium phenomenology
  - Interpretations abound: *multi-quark states, loosely bound hadron molecules, hybridized states, hadro-quarkonia, gluonic excitations, rescattering effects, virtual state poles, anomalous thresholds*
  - Is there a principle (or a few) that describe the new phenomena?





PhysRevLett.110.252001 (2013)

Amplitude analysis for constraining models

- Fairly recent puzzle: XYZ states

Viewpoint: New Particle Hints at Four-Quark Matter

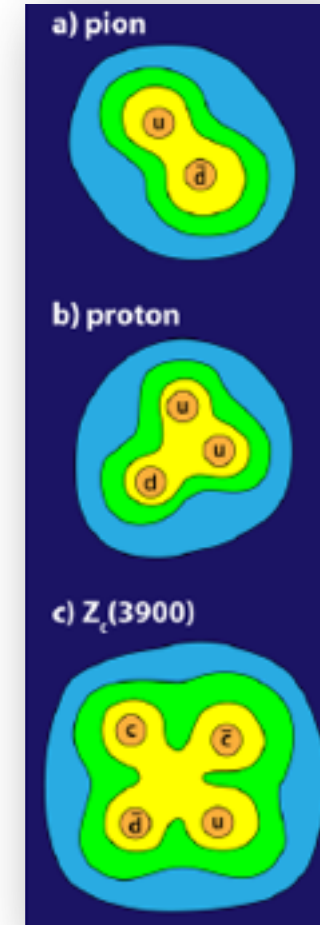
Eric Swanson, *University of Pittsburgh, Pittsburgh, PA 15260, USA*

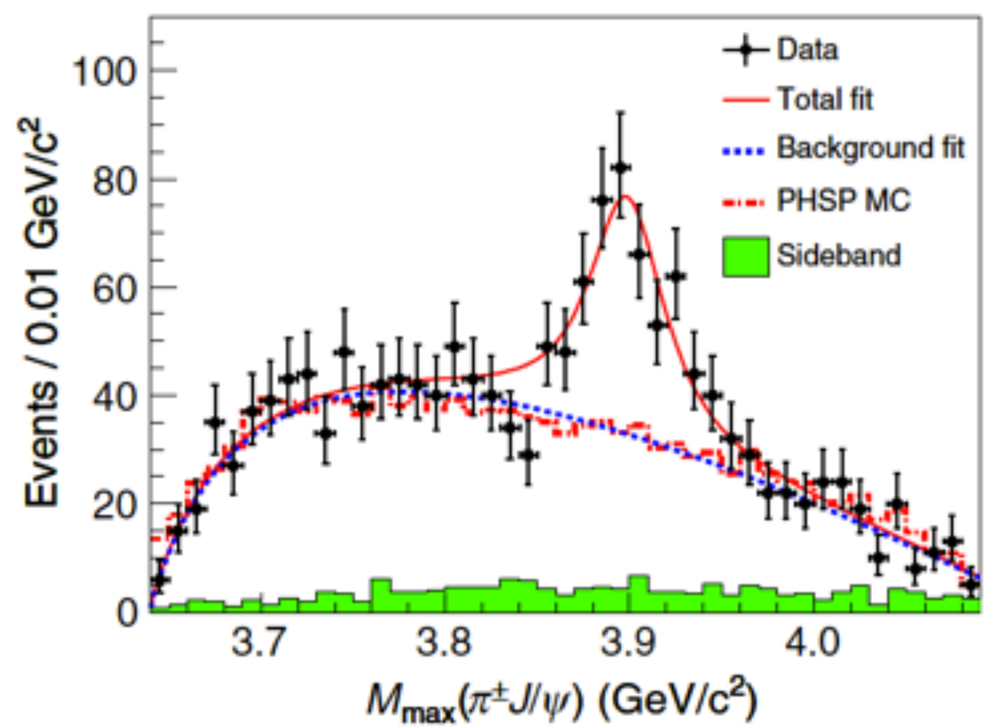
Published June 17, 2013 | Physics **6**, 69 (2013) | DOI: 10.1103/Physics.6.69

states, loosely bound hadron molecules, hybridized states, hadro-quarkonia, gluonic excitations, rescattering effects, virtual state poles, anomalous thresholds

- Is there a principle (or a few) that describe the new phenomena?

- At low statistics, simple fit with BW shape enough to stimulate interest
- In order to discriminate between models, need high statistics and must account for angular correlations/interference





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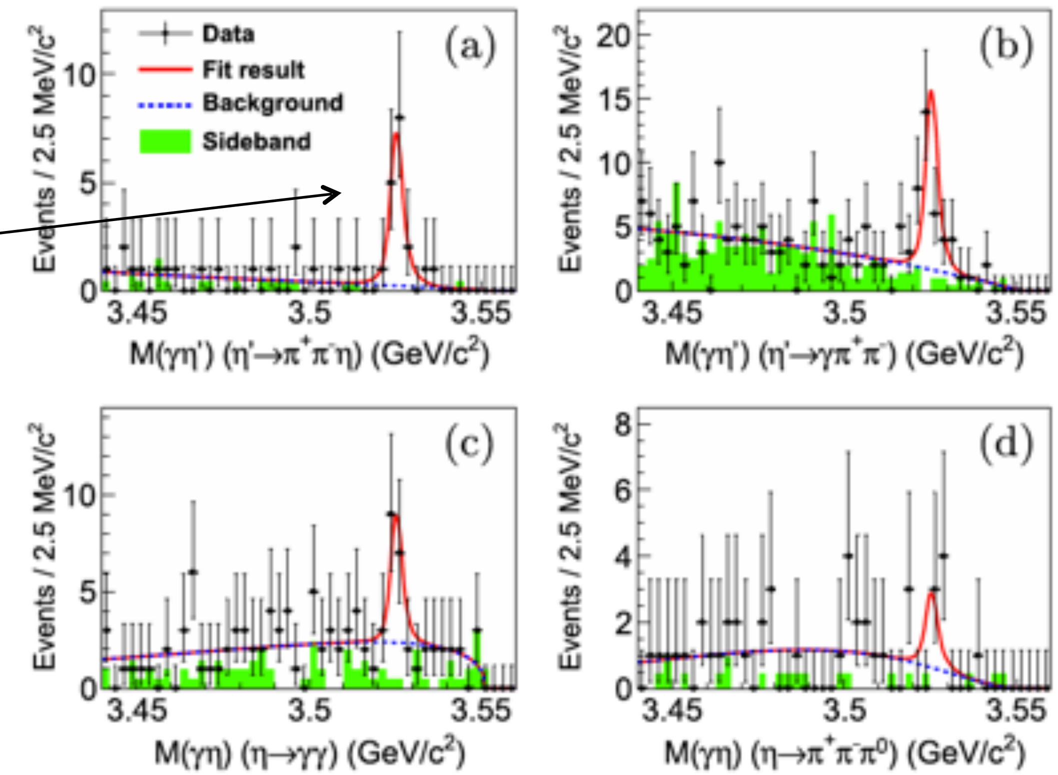
Amplitude analysis at BESIII

- Many HEP analyses involve searches for peaks in some invariant mass spectrum
    - Attempt to account for all backgrounds
    - Extract parameters of intermediate states

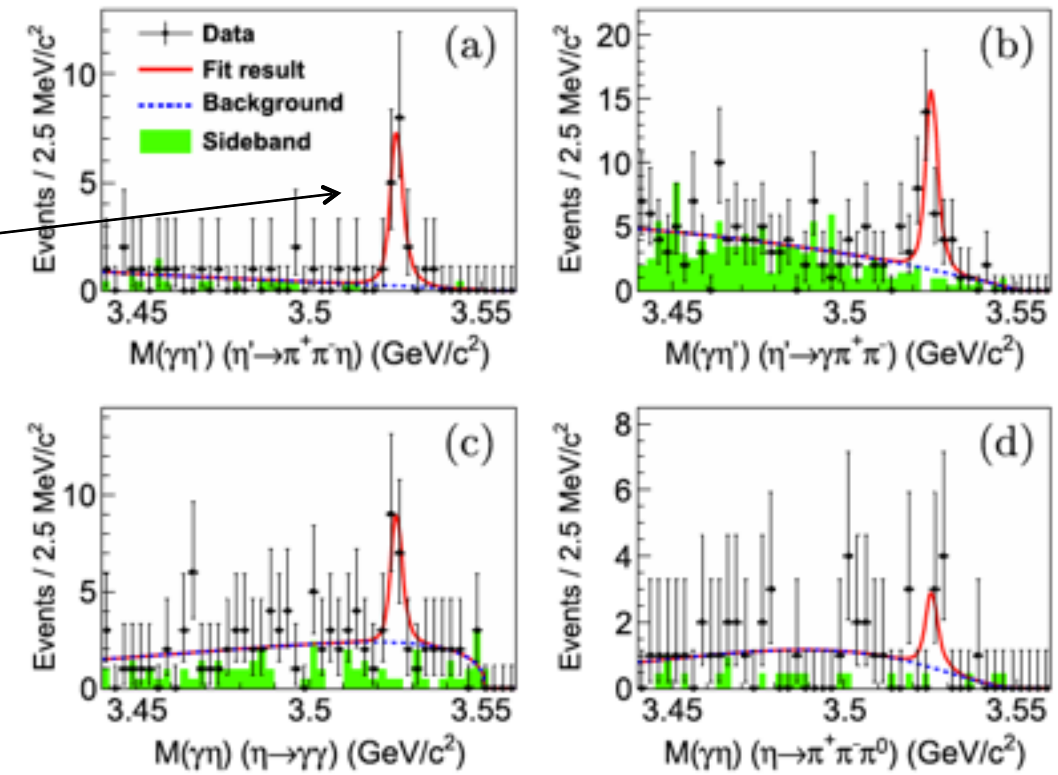
$h_c \to \gamma\eta(')$ PhysRevLett.116.251802 (2016)



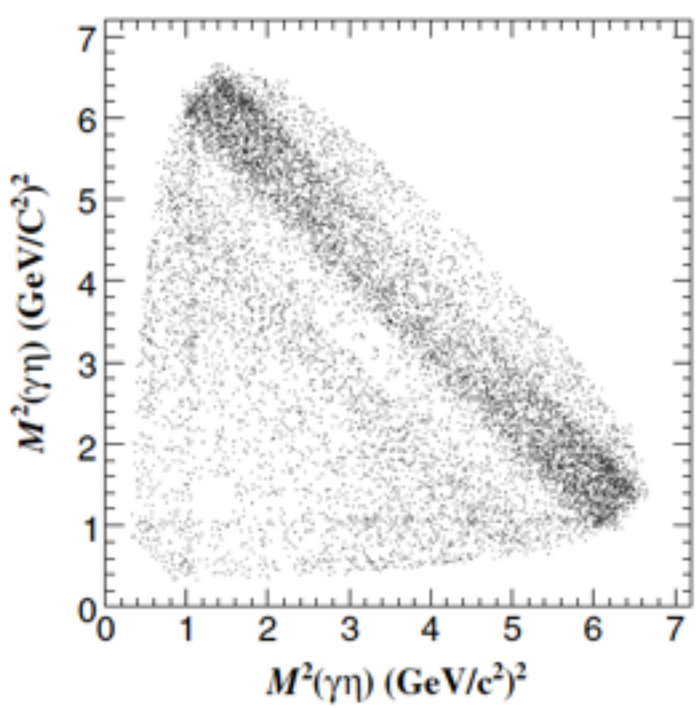
Amplitude analysis at BESIII

- Many HEP analyses involve searches for peaks in some invariant mass spectrum
  - Attempt to account for all backgrounds
  - Extract parameters of intermediate states
- Many systems are not so simple
  - Broad, overlapping, near thresholds

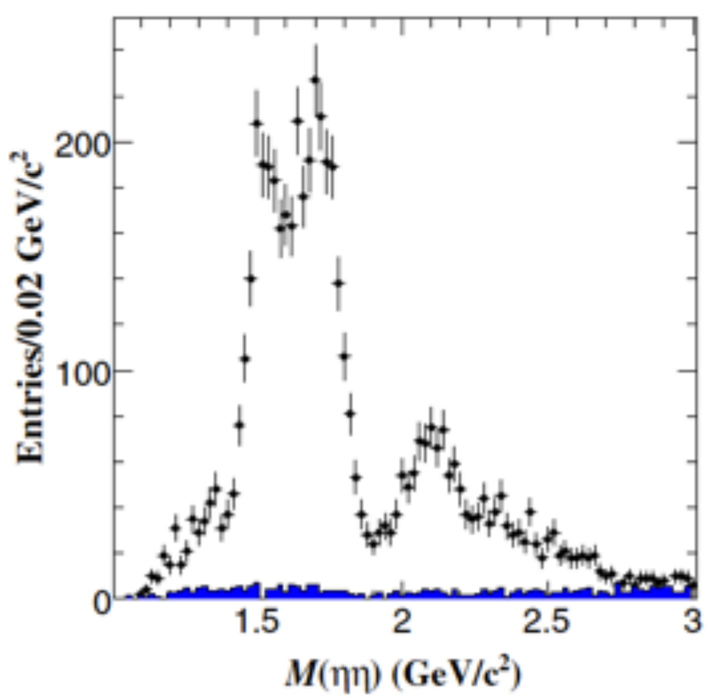
$h_c \to \gamma\eta(')$ PhysRevLett.116.251802 (2016)



$J/\psi \to \gamma\eta\eta$



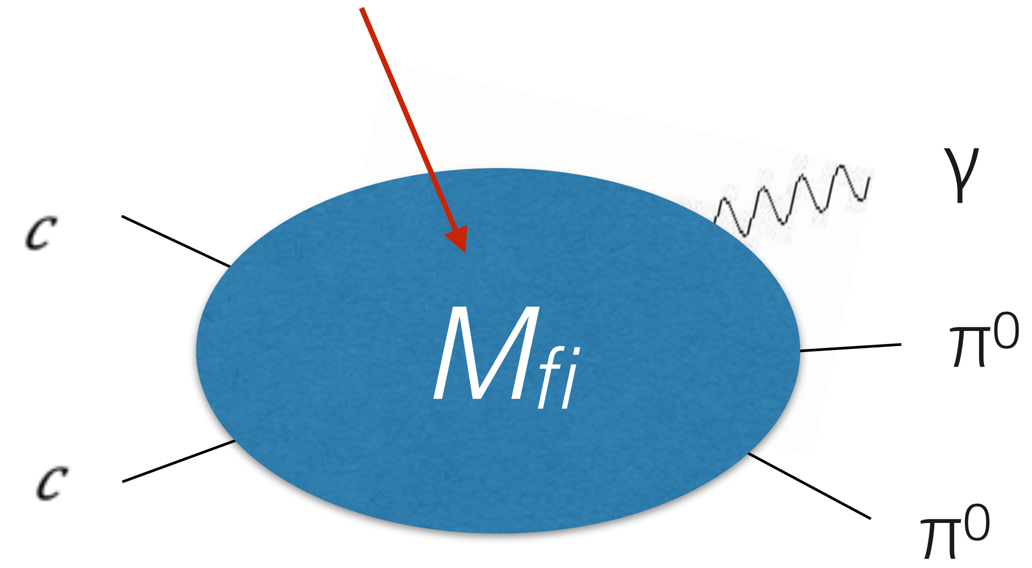
PhysRevD.87.092009 (2013)



- Rather than traditional “bump hunting” for states, requires more sophisticated techniques like amplitude analysis

Amplitude analysis

- Coupling of initial and final states given by invariant amplitudes
- Amplitude analysis: tool to extract the complex amplitudes from experimental data
  - Requires some model that contains free parameters
  - Consider all kinematics of final state particles
  - Vary the free parameters to maximize the likelihood that the model is a good description of the data sample



$$\frac{d\sigma_{fi}}{d\Omega} = \frac{1}{(8\pi)^2 s}\left(\frac{q_f}{q_i}\right)|\mathcal{M}_{fi}|^2 = |f_{fi}(\Omega)|^2$$

$$S_{fi} = \langle f|S|i\rangle$$

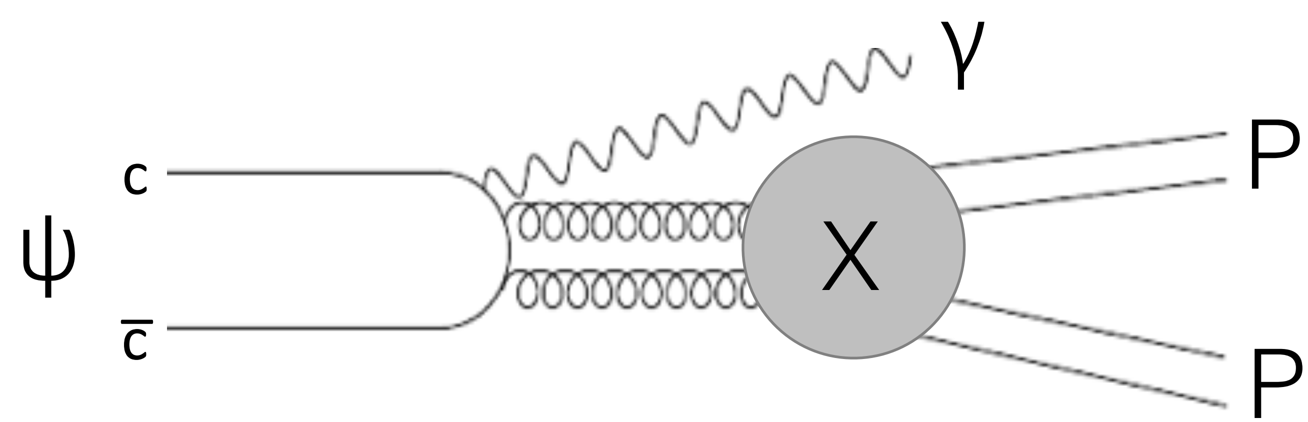
- Has its own challenges
  - How to construct amplitudes? How many amplitudes are needed? Are there ambiguities? How to deal with backgrounds?

Amplitude analysis

You will learn about this!

- Has its own challenges
  - **How to construct amplitudes?**

$$
\begin{aligned}
U^{M,\lambda_\gamma}(\vec{x},s) = &\sum_{j,J_\gamma,\mu} N_{J_\gamma} N_j D^J_{M,\mu-\lambda_\gamma}(\pi+\phi_\gamma,\pi-\theta_\gamma,0) \\
&\times D^j_{\mu,0}(\phi_\pi,\theta_\pi,0)\frac{1}{2}\frac{1+(-1)^j}{2} \\
&\times \langle J_\gamma-\lambda_\gamma; j\mu | J\mu-\lambda_\gamma \rangle \\
&\times \frac{1}{\sqrt{2}}[\delta_{\lambda_\gamma,1}+\delta_{\lambda_\gamma,-1}P(-1)^{J_\gamma-1}]V_{j,J_\gamma}(s),
\end{aligned}
$$



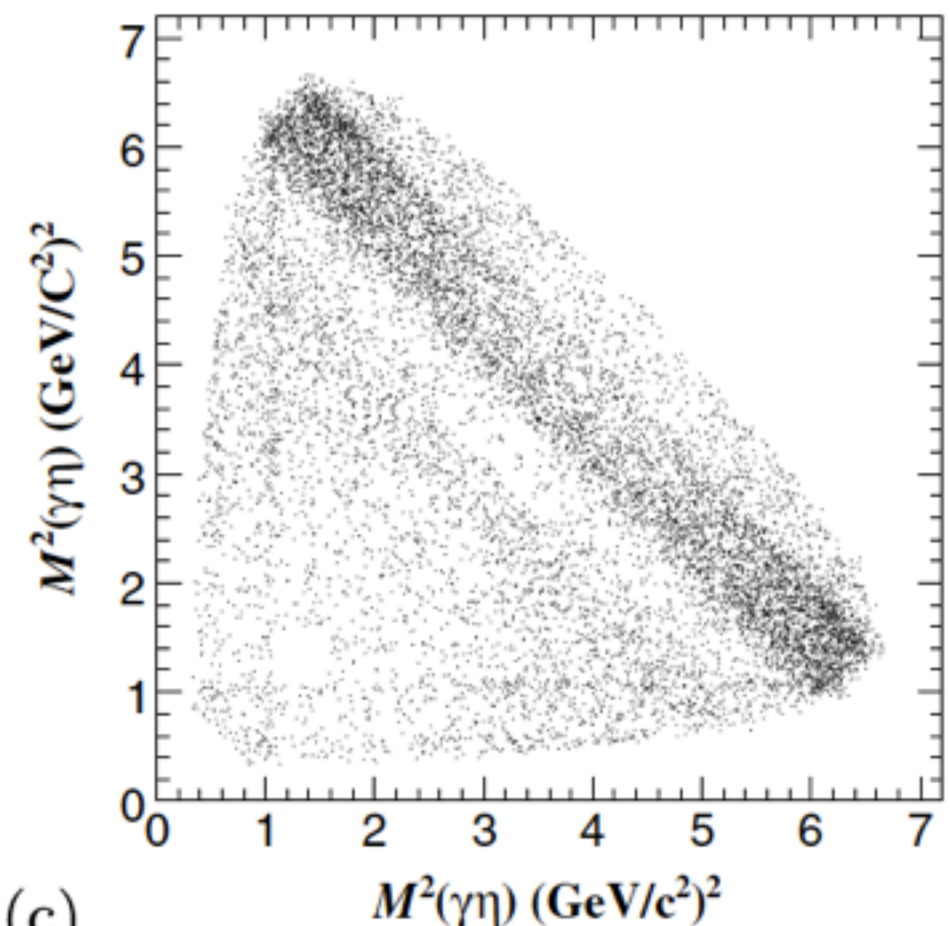
PhysRevD.92.052003 (2015)

Amplitude analysis

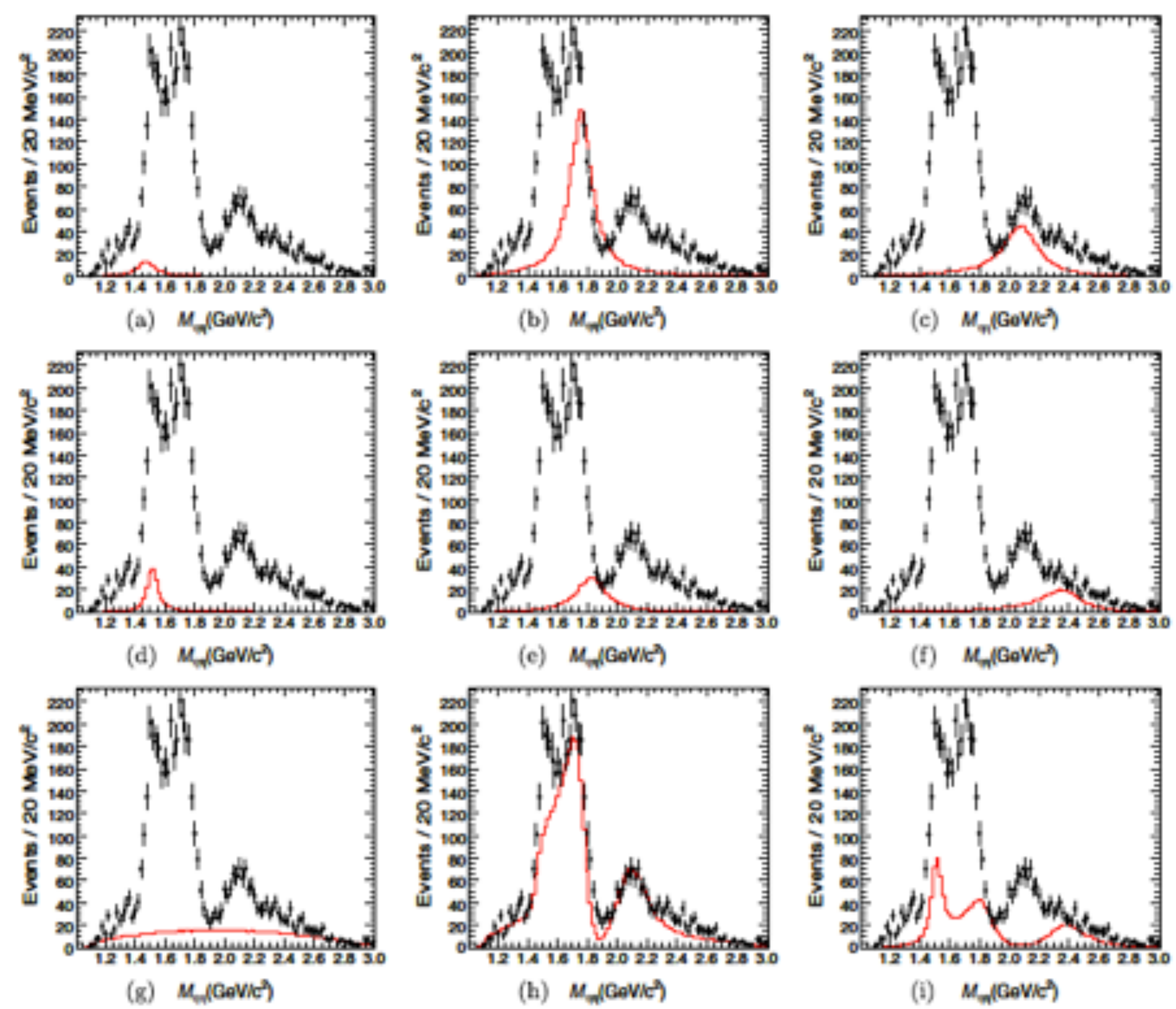
- Has its own challenges
  - How to construct amplitudes?
  - **How many amplitudes are needed?**

Many issues related to this!

J/ψ → γηη; a typical BESIII "PWA"



(c)



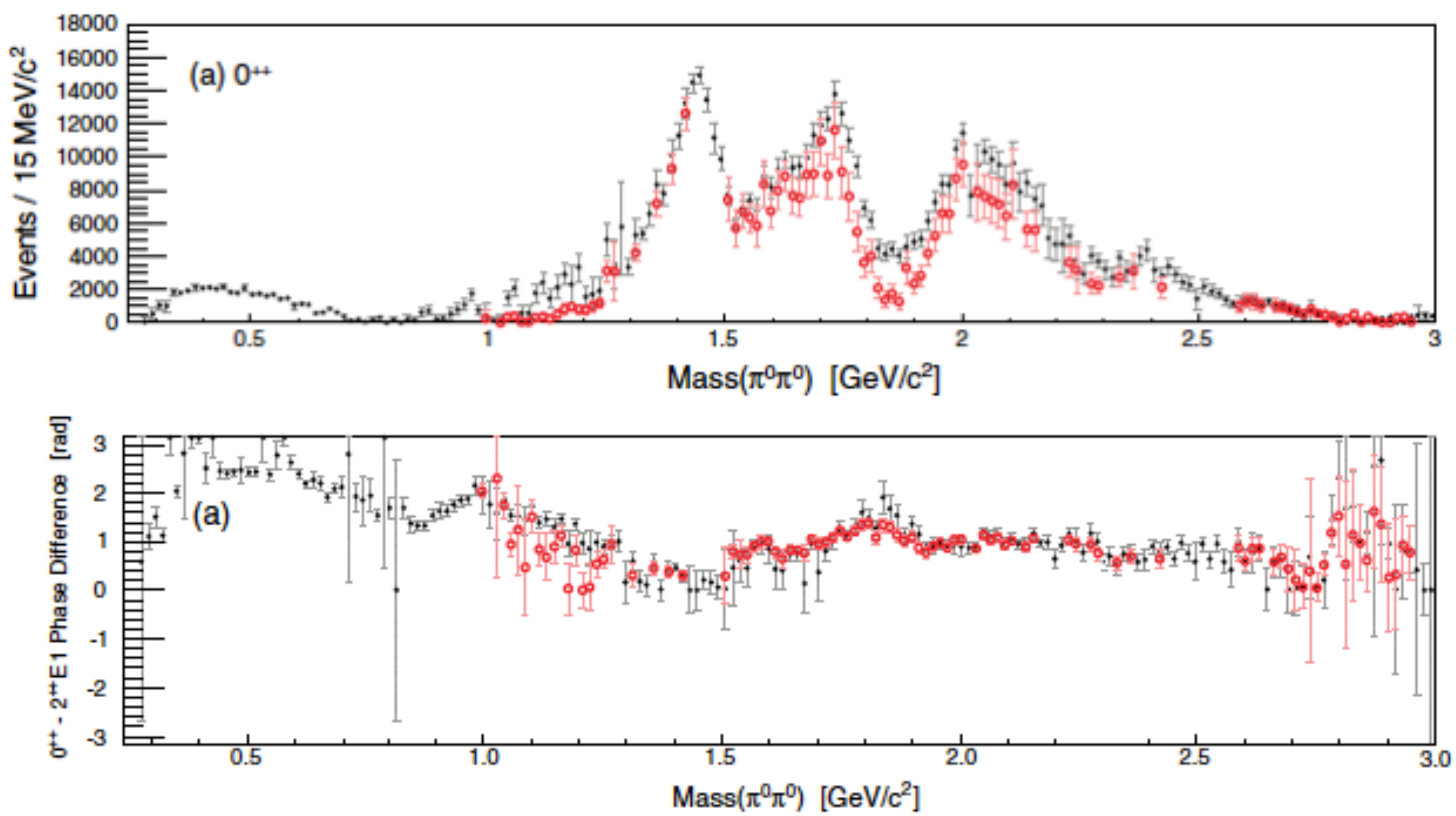
PhysRevD.87.092009 (2013)

Amplitude analysis

- Has its own challenges
  - How to construct amplitudes?
  - How many amplitudes are needed?
  - **Are there ambiguities?**

Somewhat inherent in observing only the amplitude squared

$J/\psi \to \gamma\pi^0\pi^0$; mass independent amplitude analysis



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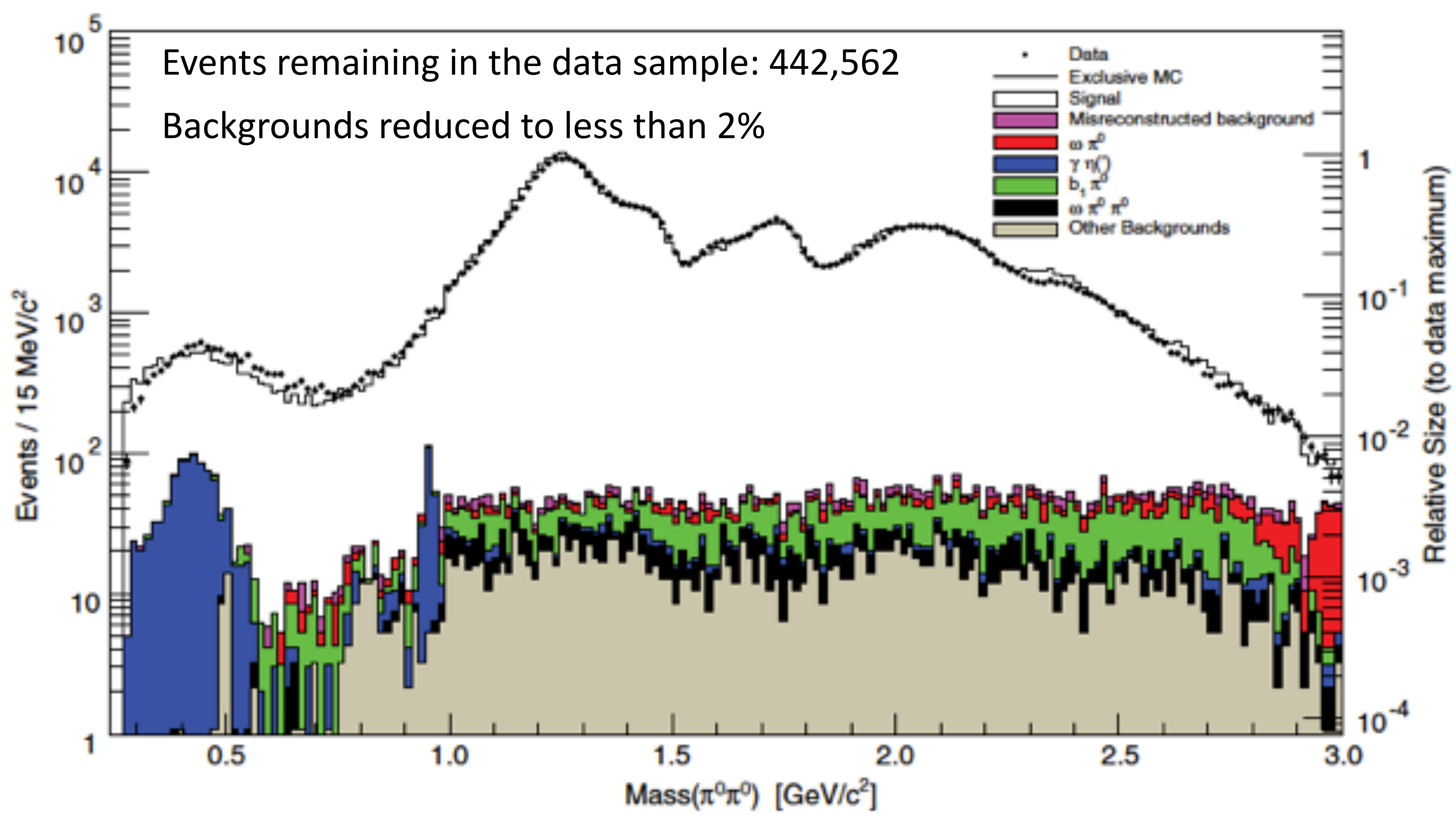

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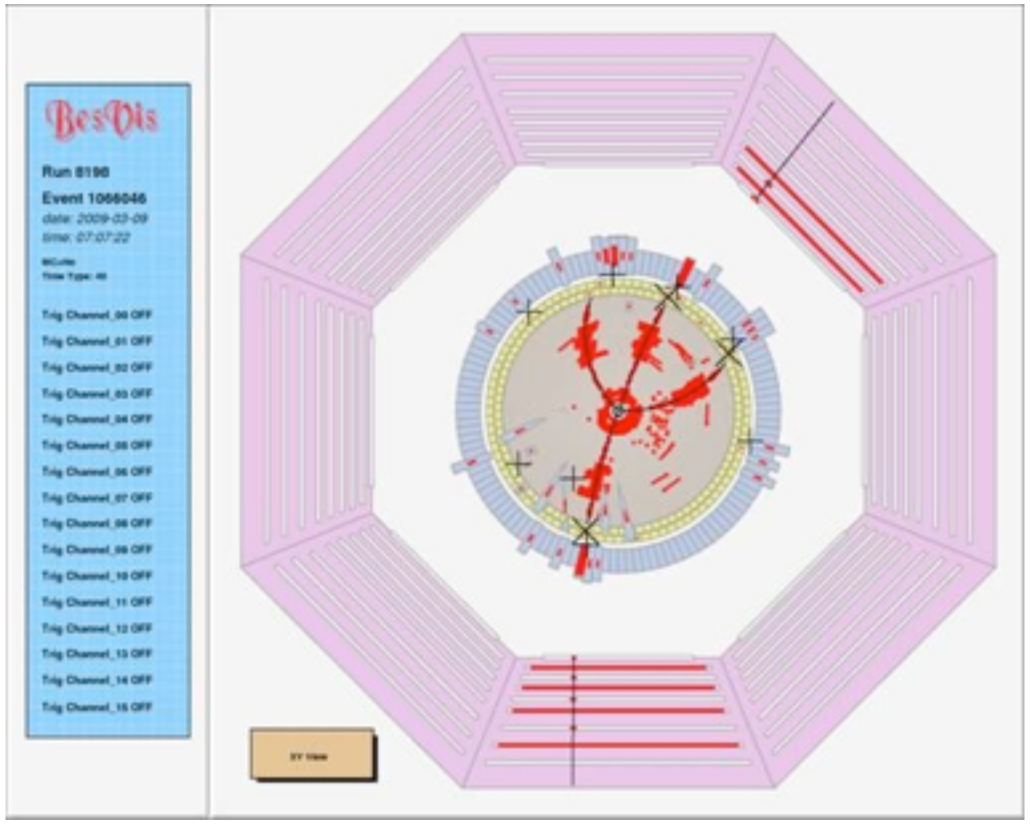
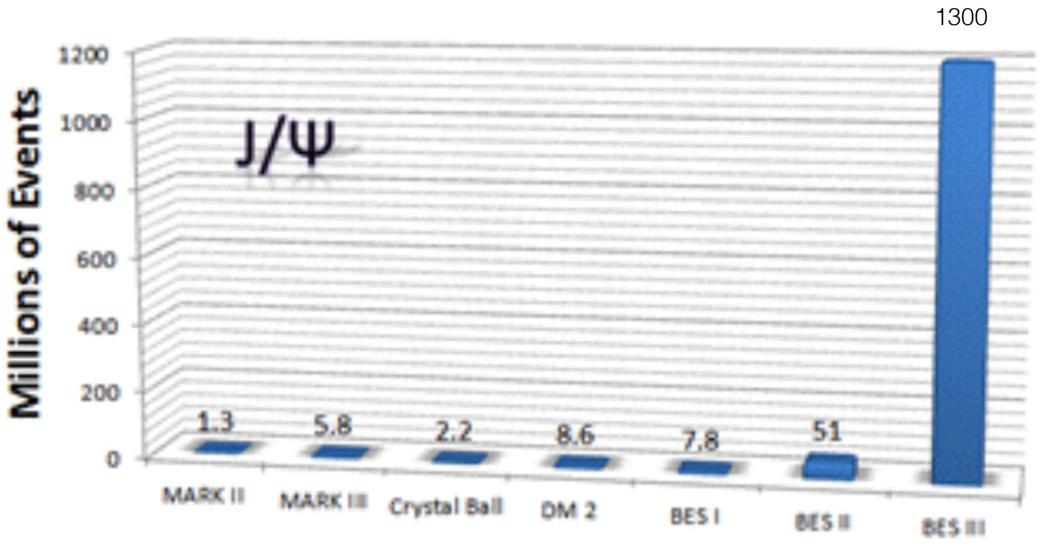
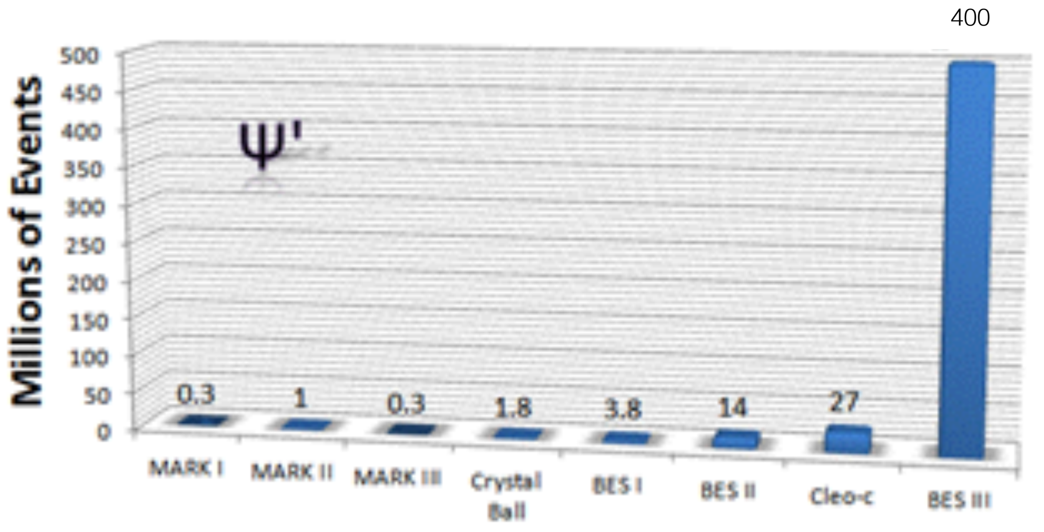
Hadron spectroscopy with charmonium decays

- BESIII has world leading samples of J/ψ and ψ' decays

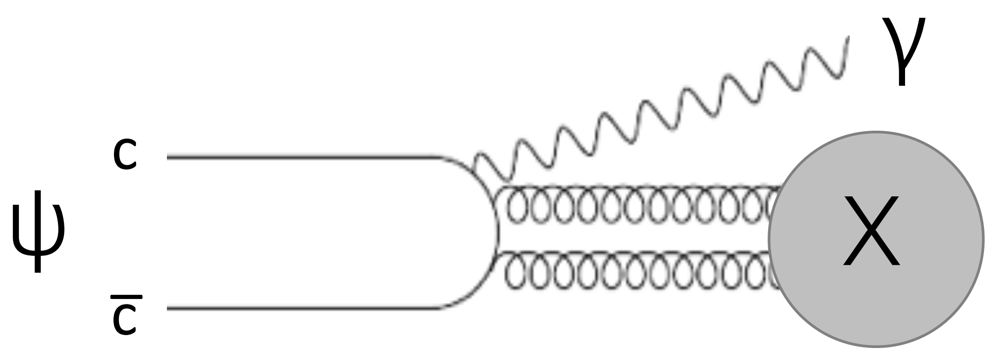






Hadron spectroscopy with charmonium decays

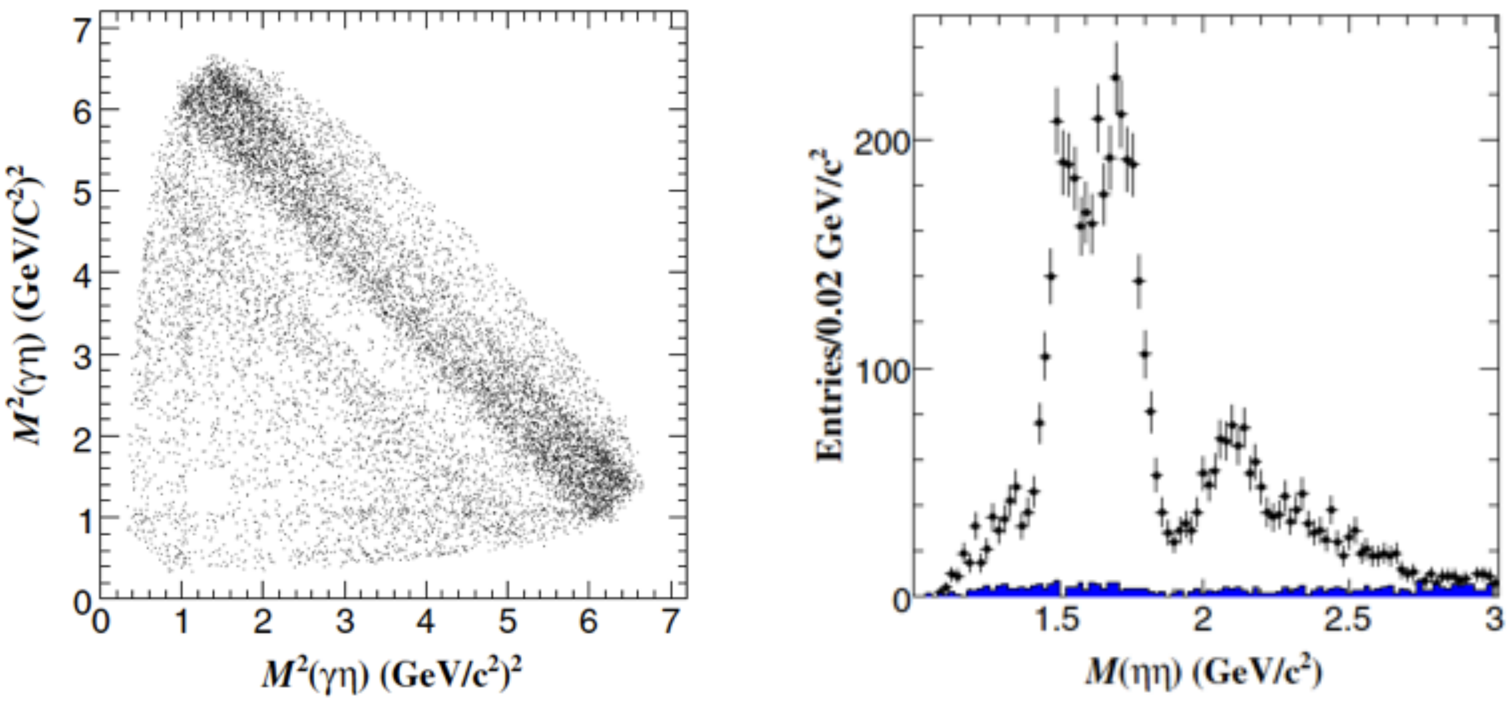
- BESIII has world leading samples of J/ψ and ψ' decays
- "Glue-rich" environment in which to search for glueballs
  - The J/ψ and ψ' masses are below open charm threshold, so OZI suppressed processes dominate
  - Suppression factor on radiative decays due to fine structure constant only about a factor of 10
  - Radiative decays account for about 8% of the total cross section



$J/\psi \rightarrow \gamma\eta\eta$; a typical BESIII "PWA"

- Amplitudes constructed in the covariant tensor formalism*
- Use a Breit-Wigner line shape to describe the decay dynamics
    - Easy, but mostly wrong… (more on this later)

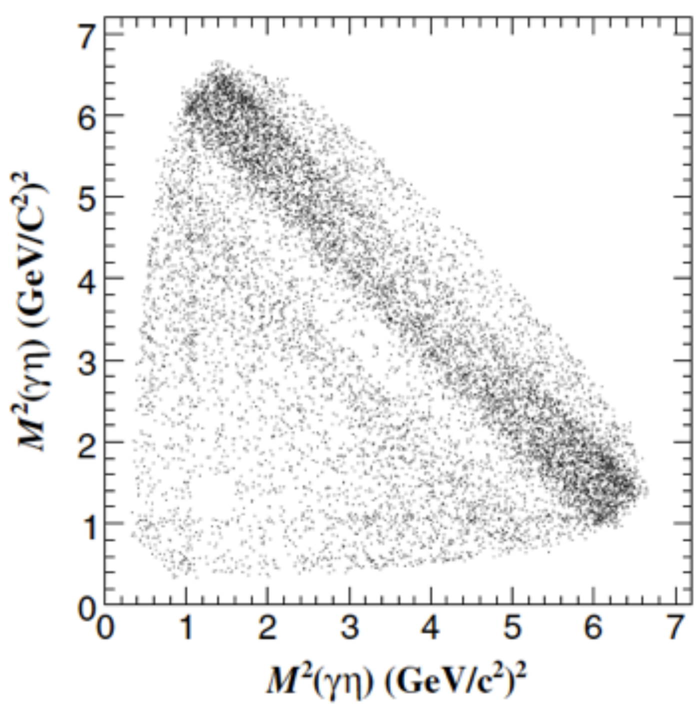
PhysRevD.87.092009 (2013)



* Zou & Bugg, Eur.Phys.J. A16 (2003) 537, Dulat & Zou, hep-ph/0403097, Dulat, Liu, Zou & Wu, hep-ph/0403136, Dulat & Zou Eur.Phys.J. A26 (2005) 125-134

$J/\psi \to \gamma\eta\eta$; a typical BESIII "PWA"

- Amplitudes constructed in the covariant tensor formalism*
- Use a Breit-Wigner line shape to describe the decay dynamics
- Add intermediate states according to some prescription
  1. *Choose some (not quite arbitrary) set of amplitudes as base model*
     - Consider previous studies, states in PDG, some educated guesses



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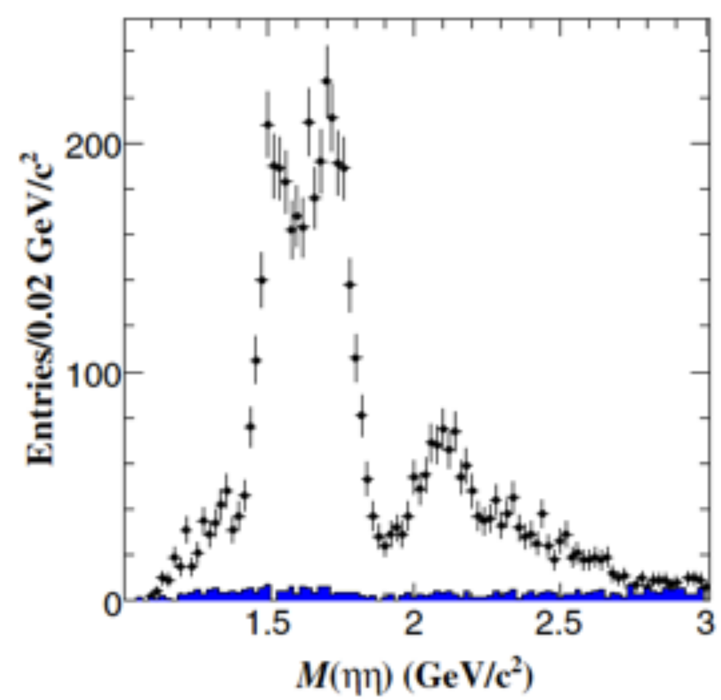

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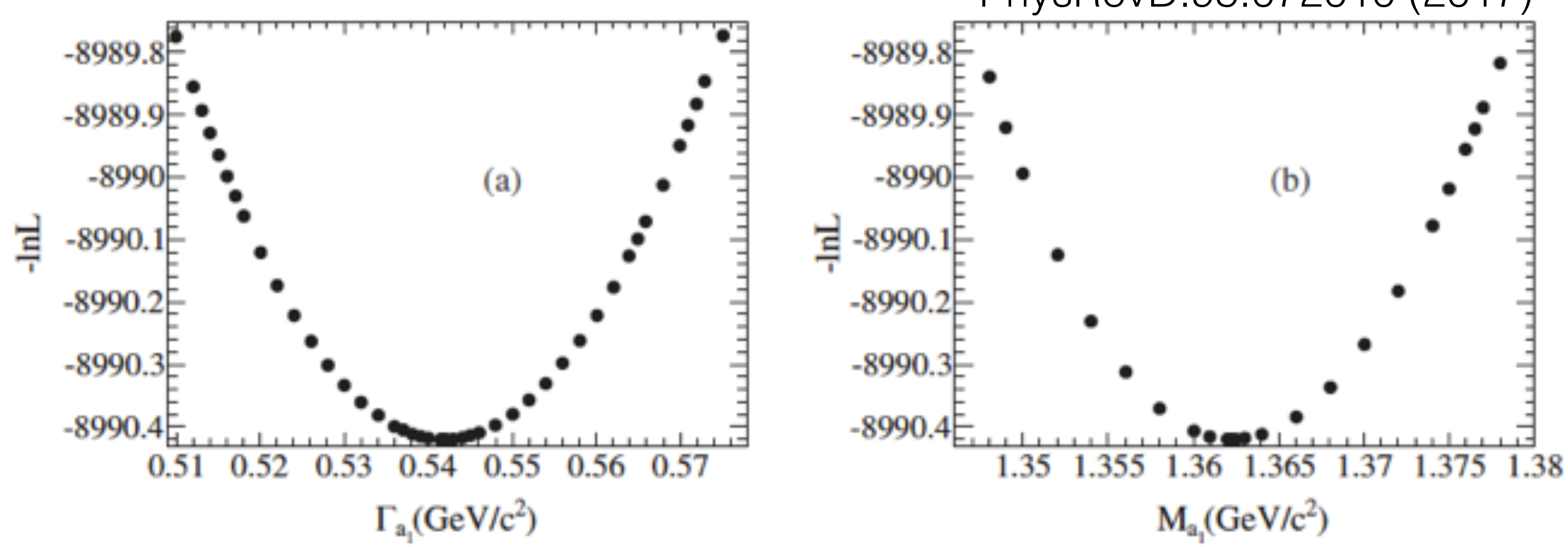
$J/\psi \to \gamma\eta\eta$; a typical BESIII "PWA"

- Amplitudes constructed in the covariant tensor formalism
- Use a Breit-Wigner line shape to describe the decay dynamics
- Add intermediate states according to some prescription
  1. *Choose some (not quite arbitrary) set of amplitudes as base model*
  2. *Add an additional amplitude*
     - Add one additional amplitude out of a pool of candidates*
  3. *Fit or scan likelihood to determine masses and widths of resonances*

PhysRevD.95.072010 (2017)



Not $J/\psi \to \gamma\eta\eta$, just an example

We tested the following mesons listed in PDG 2012: $f_2(1270)$, $f_0(1370)$, $f_2(1430)$, $f_0(1500)$, $f_2'(1525)$, $f_2(1565)$, $f_2(1640)$, $f_0(1710)$, $f_2(1810)$, $f_2(1910)$, $f_2(1950)$, $f_2(2010)$, $f_0(2020)$, $f_4(2050)$, $f_0(2100)$, $f_2(2150)$, $f_0(2200)$, $f_J(2220)$, $f_2(2300)$, $f_4(2300)$, $f_0(2330)$, $f_2(2340)$.

$J/\psi \rightarrow \gamma\eta\eta$; a typical BESIII "PWA"

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- Use a Breit-Wigner line shape to describe the decay dynamics
- Add intermediate states according to some prescription
    1. *Choose some (not quite arbitrary) set of amplitudes as base model*
    2. *Add an additional amplitude*
    3. *Fit or scan likelihood to determine masses and widths of resonances*
    4. *Take likelihood ratios to determine significance of amplitude*
    5. *Throw away amplitudes with less than 5σ significance*

$J/\psi \rightarrow \gamma\eta\eta$; a typical BESIII "PWA"

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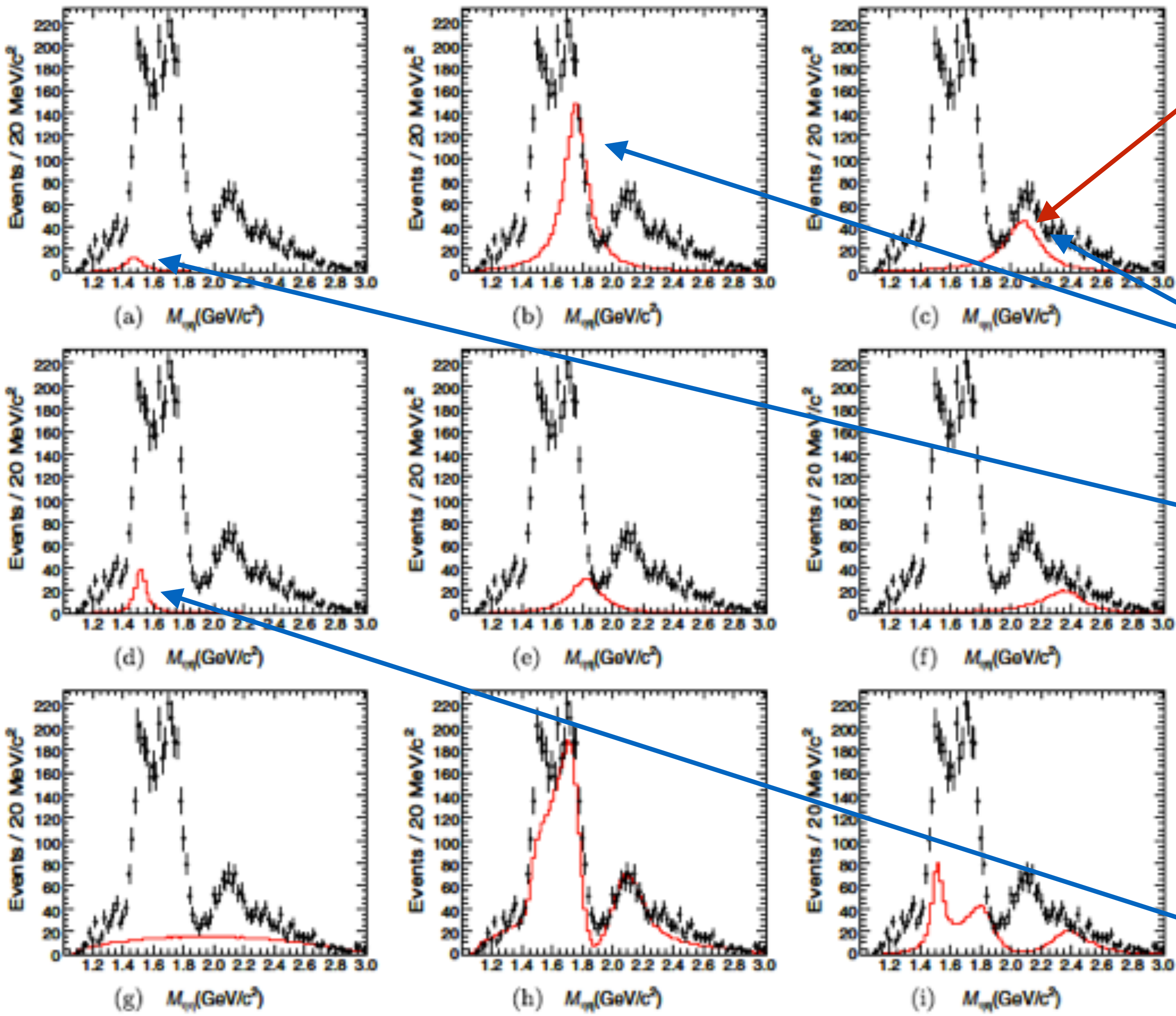
    2. *Add an additional amplitude*
    3. *Fit or scan likelihood to determine masses and widths of resonances*
    4. *Take likelihood ratios to determine significance of amplitude*
    5. *Throw away amplitudes with less than 5σ significance*
    6. *Iterate until solution converges*
        - Repeat and keep the most significant amplitude
        - Stop when no additional amplitudes are significant

Partial wave analysis of $J/\psi \rightarrow \gamma\eta\eta$



Red: extracted intensities for amplitudes

Resonance	Mass (MeV/c^2)	Width (MeV/c^2)	$\mathcal{B}(J/\psi \to \gamma X \to \gamma\eta\eta)$	Significance
$f_0(1500)$	1468^{+14+23}_{-15-74}	$136^{+41+28}_{-26-100}$	$(1.65^{+0.26+0.51}_{-0.31-1.40}) \times 10^{-5}$	8.2σ
$f_0(1710)$	$1759 \pm 6^{+14}_{-25}$	$172 \pm 10^{+32}_{-16}$	$(2.35^{+0.13+1.24}_{-0.11-0.74}) \times 10^{-4}$	25.0σ
$f_0(2100)$	$2081 \pm 13^{+24}_{-36}$	273^{+27+70}_{-24-23}	$(1.13^{+0.09+0.64}_{-0.10-0.28}) \times 10^{-4}$	13.9σ
$f_2'(1525)$	$1513 \pm 5^{+4}_{-10}$	75^{+12+16}_{-10-8}	$(3.42^{+0.43+1.37}_{-0.51-1.30}) \times 10^{-5}$	11.0σ
$f_2(1810)$	1822^{+29+66}_{-24-57}	$229^{+52+88}_{-42-155}$	$(5.40^{+0.60+3.42}_{-0.67-2.35}) \times 10^{-5}$	6.4σ
$f_2(2340)$	$2362^{+31+140}_{-30-63}$	$334^{+62+165}_{-54-100}$	$(5.60^{+0.62+2.37}_{-0.65-2.07}) \times 10^{-5}$	7.6σ

Results

- The $f_0(1710)$ and $f_0(2100)$ are the dominant scalars
- The $f_0(1500)$ exists (8.2σ).
- Branching fraction of the $f_0(1710)$ and $f_0(2100)$ are ~10x larger than that of the $f_0(1500)$
- The $f_2'(1525)$ is the dominant tensor

PhysRevD.87.092009 (2013)

*How many amplitudes are needed?

- A set of amplitudes can be "sufficient", how do we know it is "correct"?
- Common practice: Throw away amplitudes with less than 5σ significance
  - Somewhat arbitrary - why not 3σ, 4σ?
  - Often combined with other criteria - contributes >1% of events, size of interference with other amplitudes
  - Really only a valid statistical criterion if the background model (i.e. all other amplitudes) are correct
  - Get "fake" 5 sigmas more often than in truly Gaussian statistics

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- A set of amplitudes can be “sufficient”, how do we know it is “correct”?
- Common practice: Throw away amplitudes with less than 5σ significance
  - Somewhat arbitrary - why not 3σ, 4σ?
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More technical challenges

- Requires many many fits!
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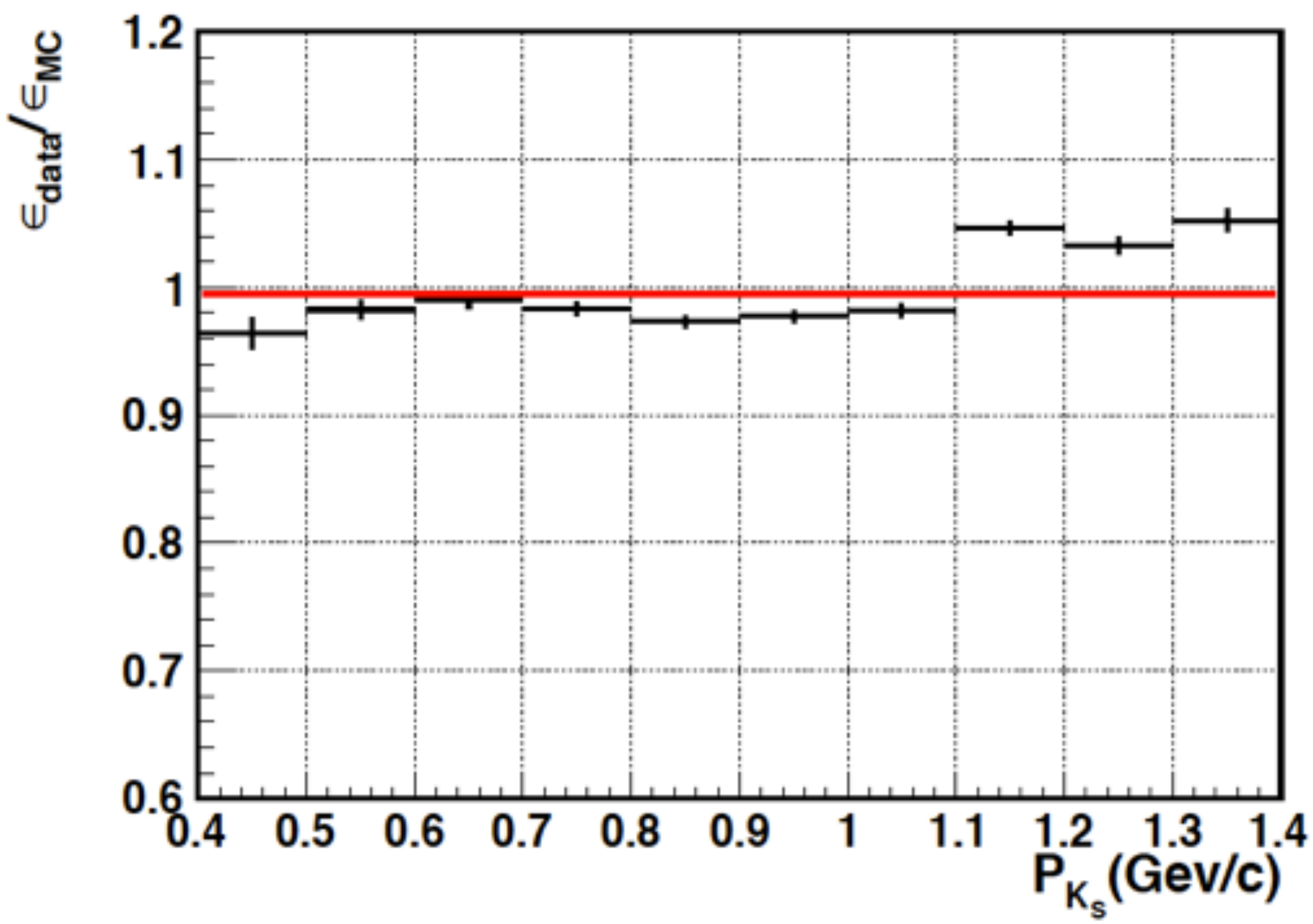
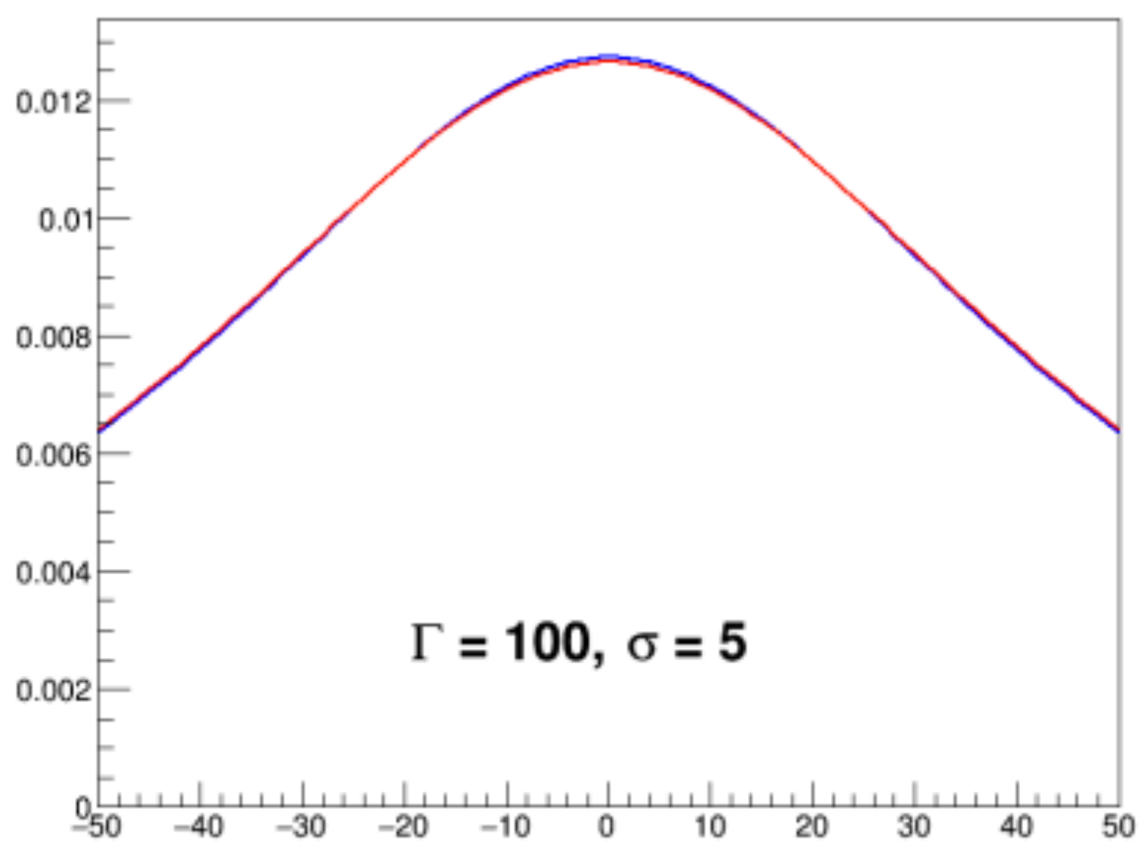
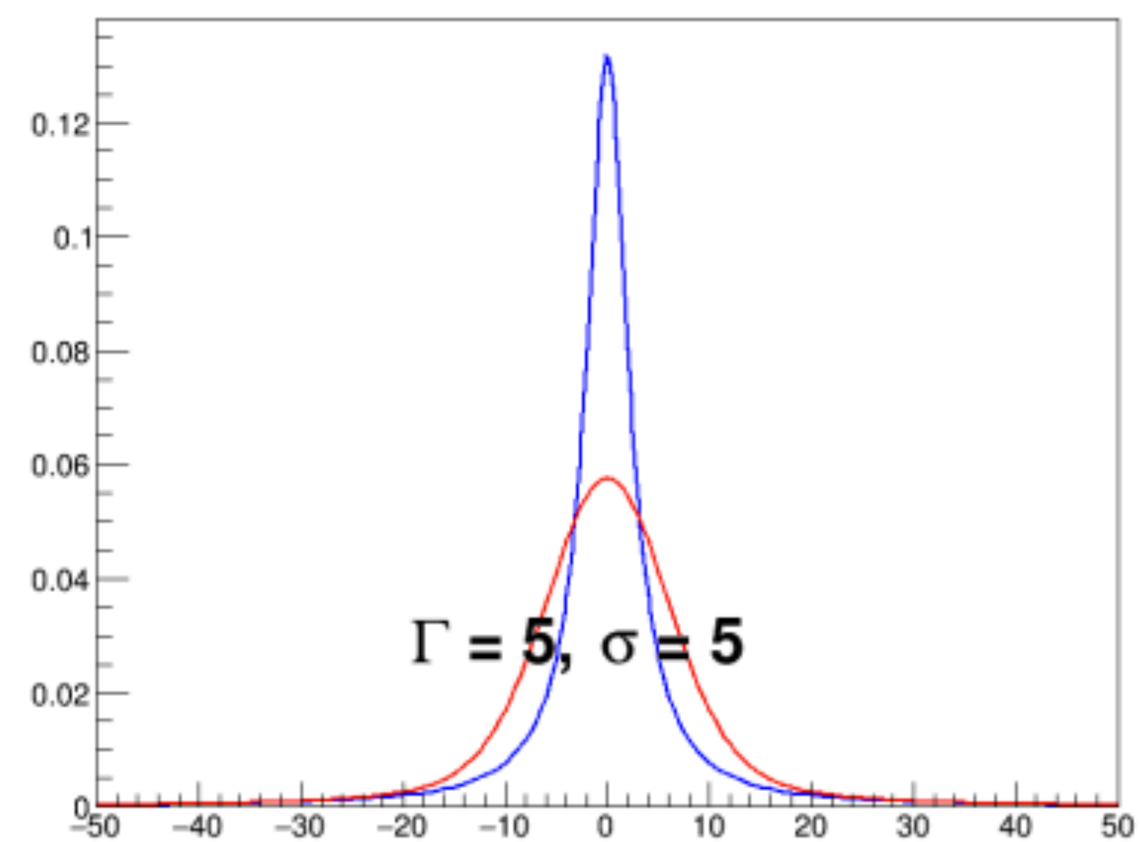
  - With ever increasing statistics, this becomes a computational problem
- Usually neglected: phase space dependent systematics
  (e.g. momentum dependent tracking efficiency)



How to treat detector resolution?

1. Very narrow states (e.g. K_S, J/ψ): typically force to nominal mass via a kinematic fit
2. Extremely broad states (e.g. ρ): resolution does not really matter
3. In between (e.g. ϕ): width and detector resolution are comparable (tricky!)





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3. In between (e.g. ϕ): width and detector resolution are comparable (tricky!)
   - Cannot convolute BW with a Gaussian because interference happens before resolution
   - Can cause significant deviations in model parameters
   - Some ideas to deal with this if the effect (e.g. KK mass for the ϕ): computationally expensive Gaussian sampling near measured phase space point
   - No obvious extension to a high-dimensional phase-space

Breit-Wigner Parametrization of a Resonance

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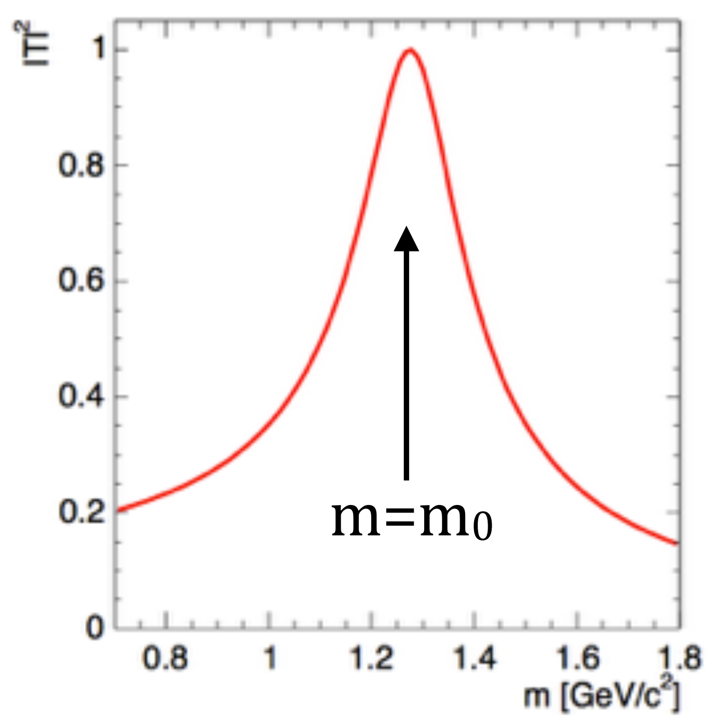
$$T(m) = \frac{\Gamma/2}{m_0 - m - i\Gamma/2}$$

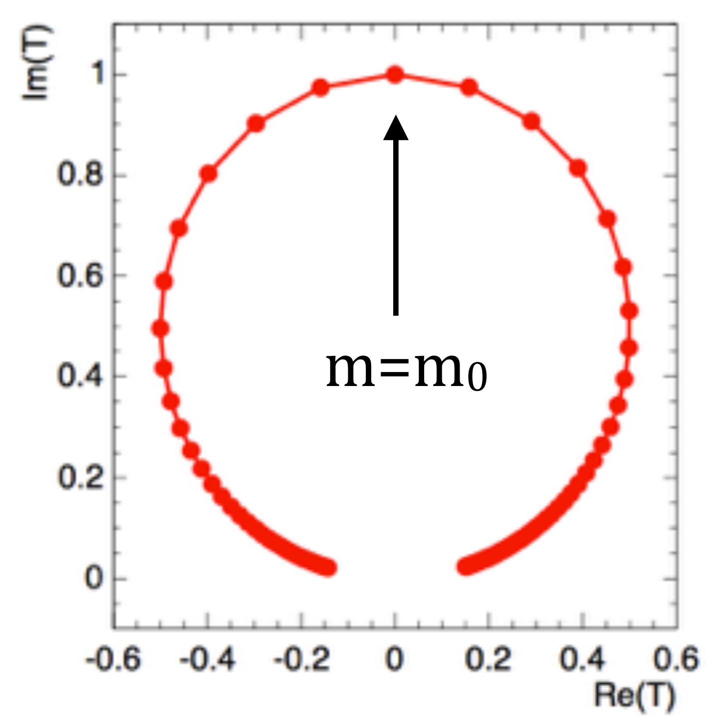
[simple Breit-Wigner (non-relativistic, constant width)]

$$I(m) = |T(m)|^2 = \frac{(\Gamma/2)^2}{(m_0 - m)^2 + (\Gamma/2)^2}$$

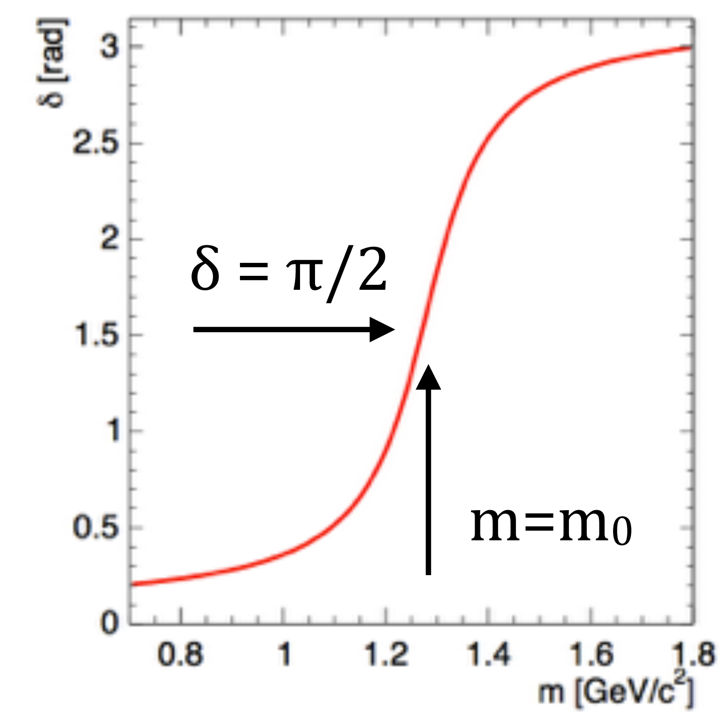
Intensity



Argand plot



Phase



K. Peters, arXiv:hep-ph/0412069v1

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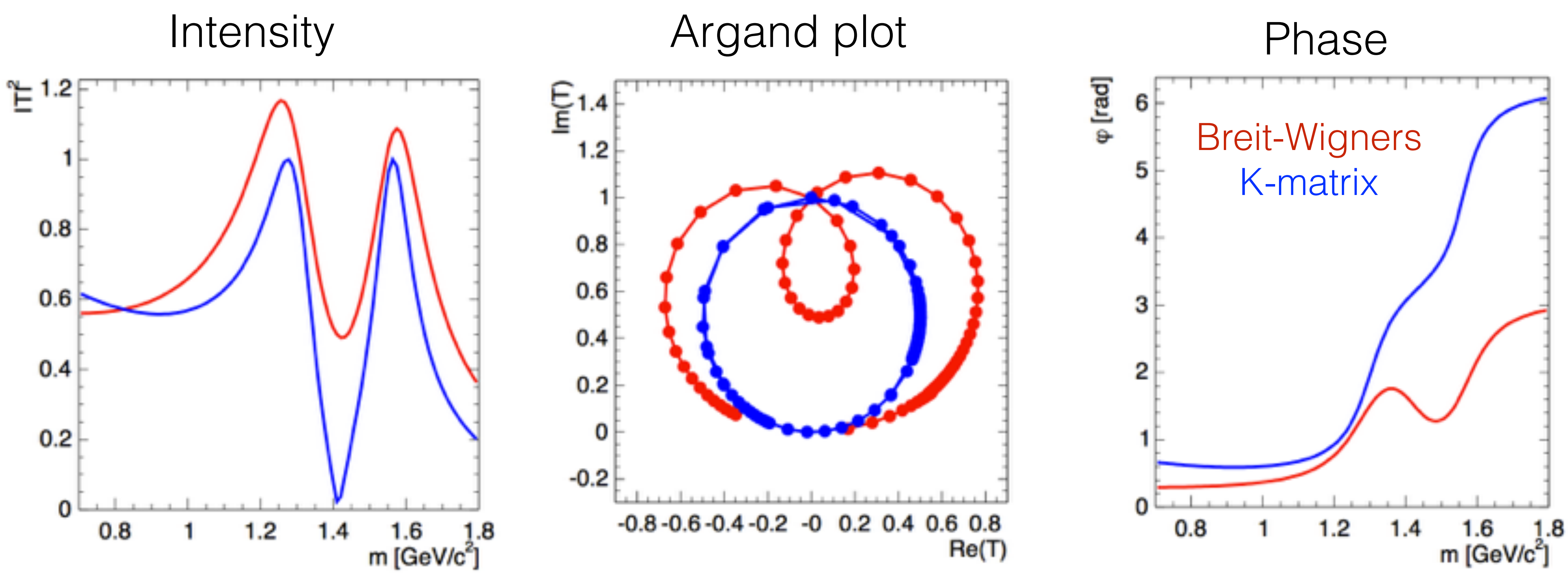
Example in scattering:

two hypothetical overlapping resonances decaying to $\pi\pi$

m_A = 1275 MeV/c^2; Γ_A = 185 MeV

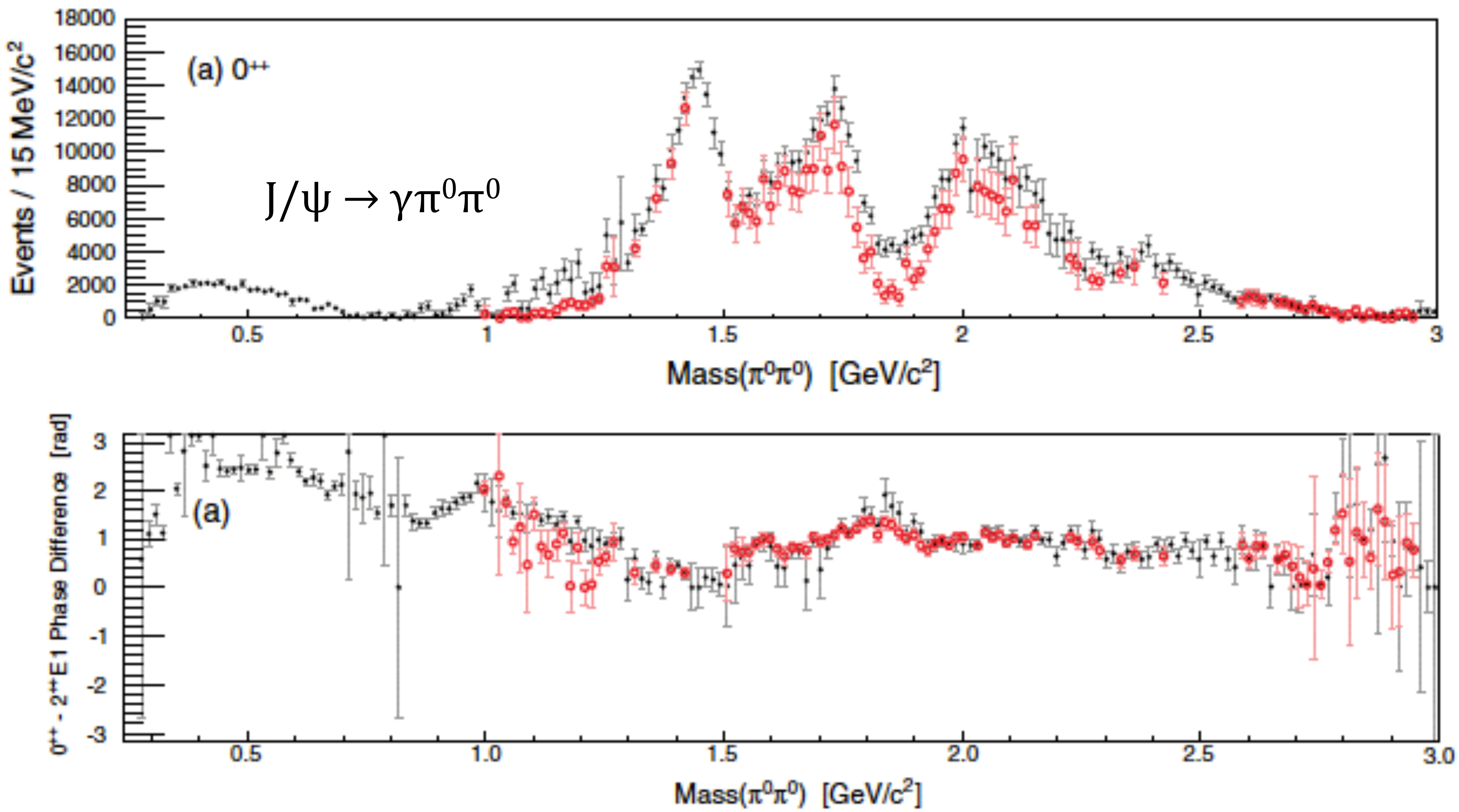
m_B = 1565 MeV/c^2; Γ_B = 150 MeV



K. Peters, arXiv:hep-ph/0412069v1

Mass independent approach

- Instead of modeling the s-dependence (eg. with a Breit-Wigner), make minimal model assumptions and measure the amplitudes independently in small bins of s
    - Construct a piecewise complex function that describes the s-dependence of the hadron dynamics
    - Provide useful results for model development



PhysRevD.92.052003 (2015)

Mass independent amplitude analysis

- The decay $J/\psi \longrightarrow \gamma\pi^0\pi^0$ factorizes into

radiative transition ππ interaction

$$\Sigma_{X=\pi\pi,KK,\ldots} \langle J/\psi | H_{EM} | \gamma_{J\gamma} X_{J12} \rangle \langle X_{J12} | H_{QCD} | \pi\pi \rangle A_{J\gamma,\, J12}$$

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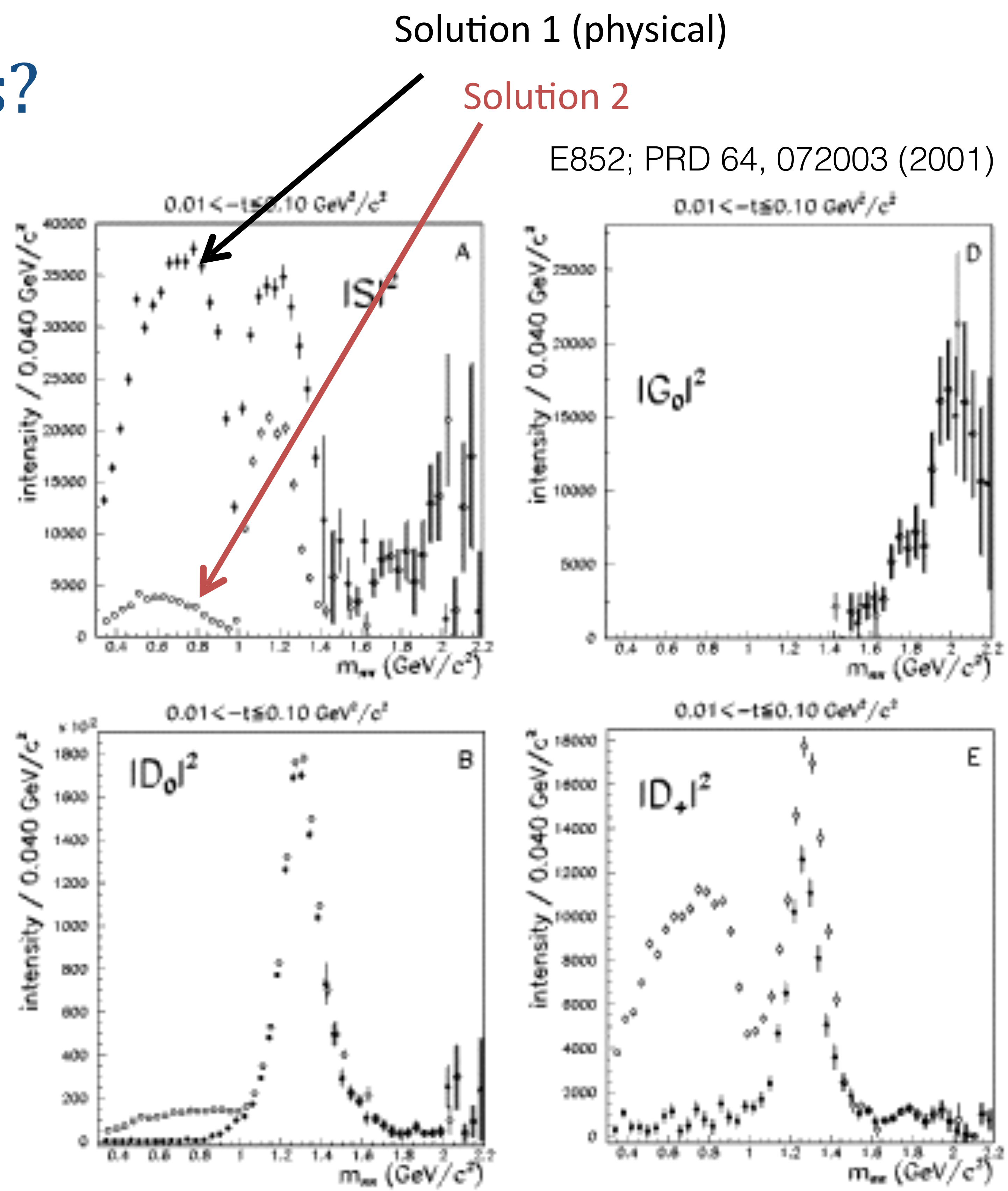
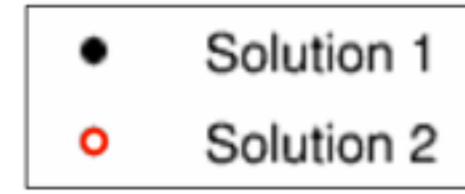
  - The function describing the ππ interaction is constant over a small range (15 MeV) of center of mass energy ($\sqrt{s}$)
- Rescattering effects, KK → ππ for example, have the potential to produce *phase differences* between the different components of the 2^{++} amplitude
  - Below KK threshold, the phases of the 2^{++} amplitudes may be constrained to be the same
  - Above KK threshold, rescattering effects introduce ambiguities

*Are there ambiguities?

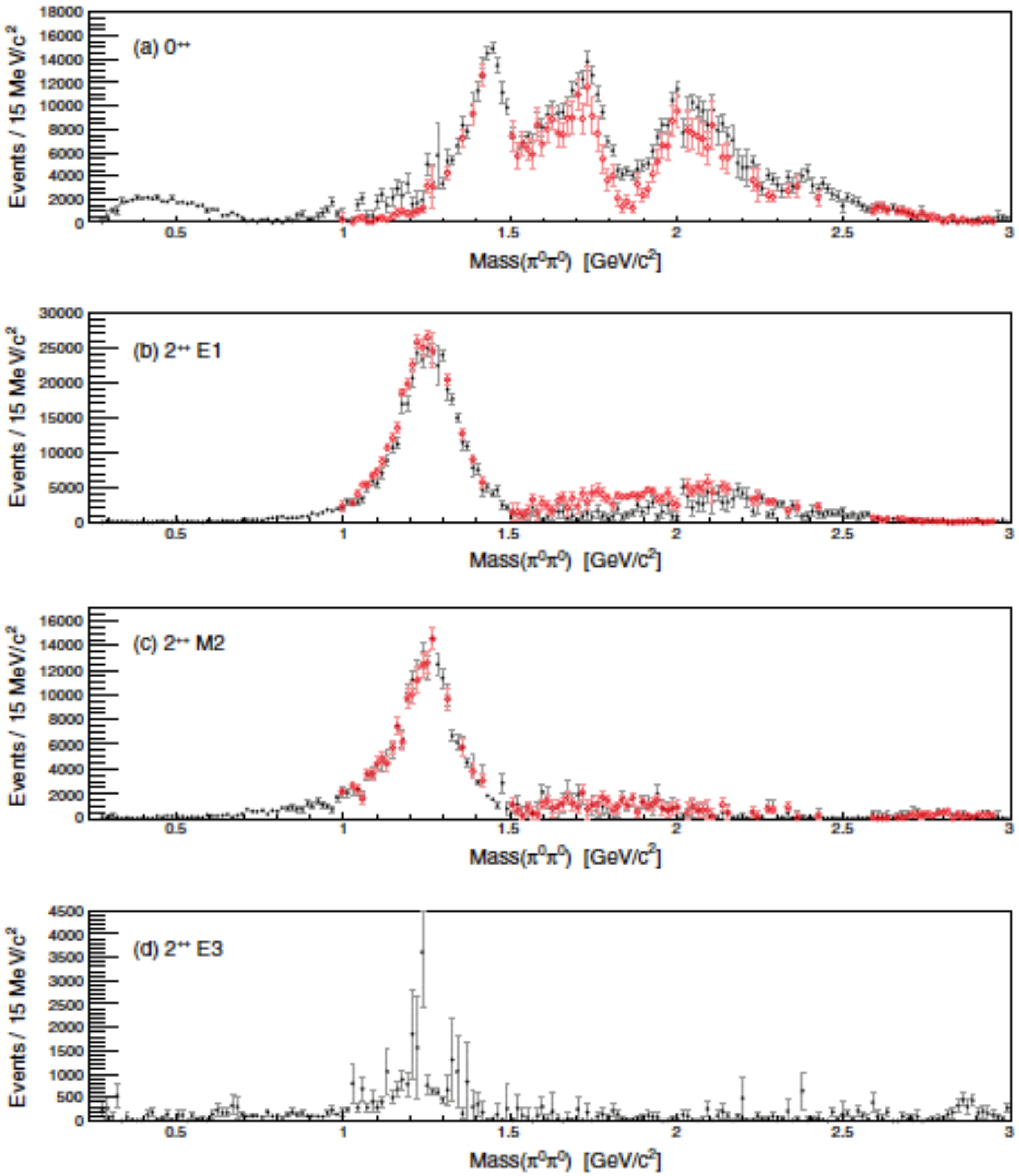
- An ambiguity arises when multiple sets of parameters yield the same overall value for a function (in this case the intensity)
- Ambiguities are present in many amplitude analyses
  - $\pi^- p \to \pi^0 \pi^0 n$ (E852)
  - *Barrelet ambiguities*
- General idea:
  - Publish both solutions
  - Alternate interpretations may also be used to fit data



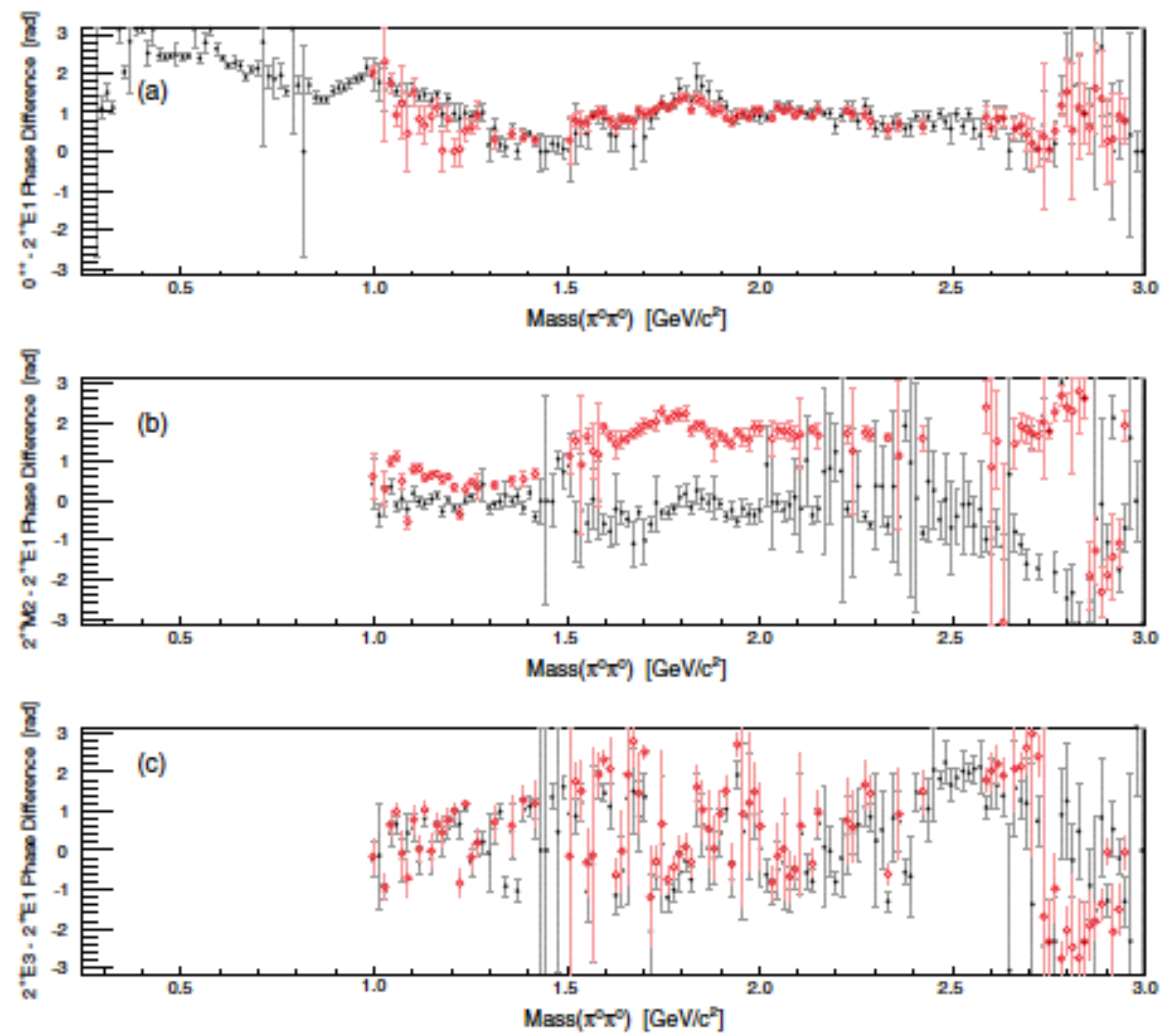
$J/\psi \to \gamma\pi^0\pi^0$: Nominal Results



Intensities



Phase differences

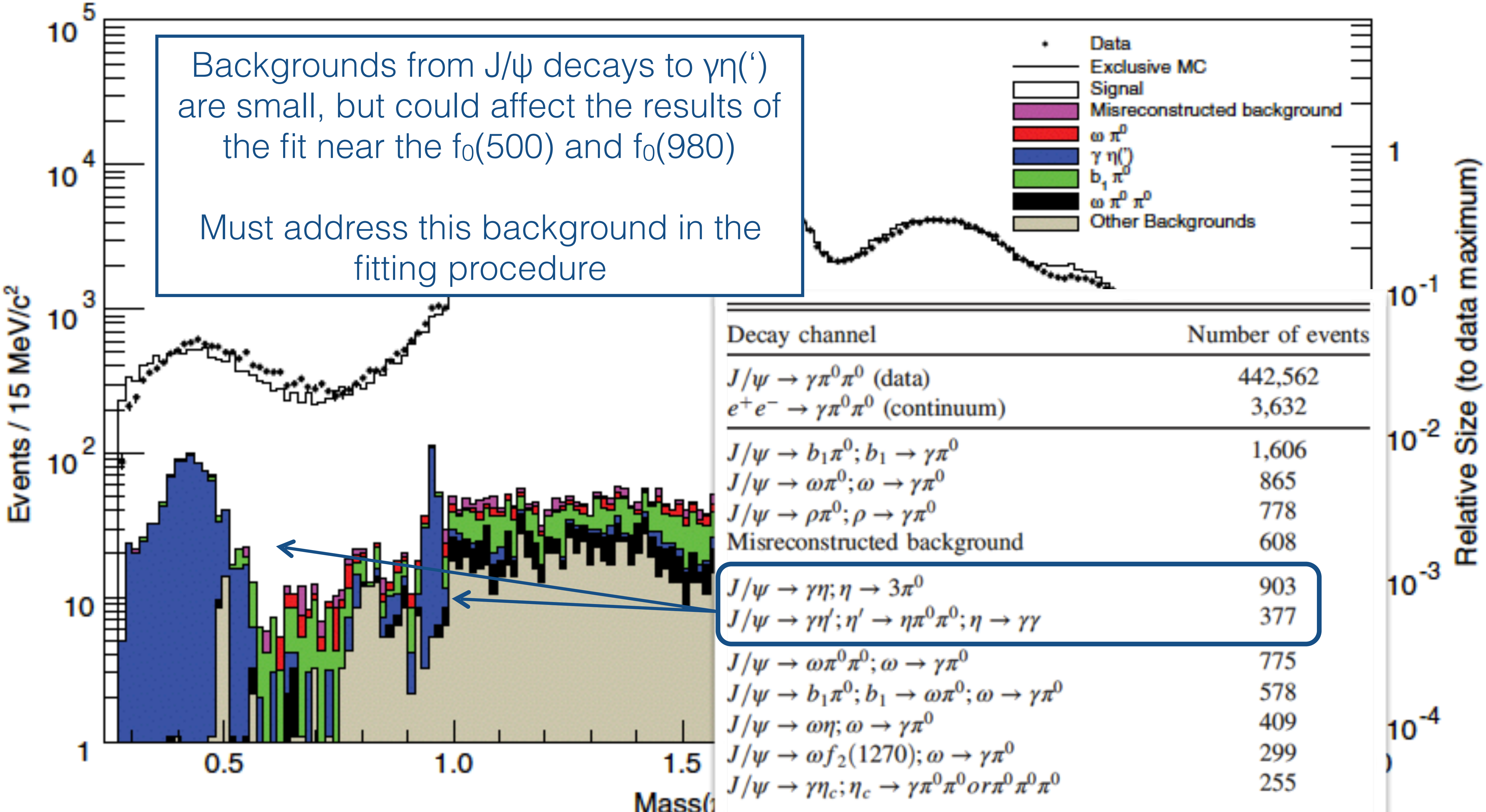


PhysRevD.92.052003 (2015)

*How to deal with backgrounds?

Backgrounds from J/ψ decays to γη(') are small, but could affect the results of the fit near the $f_0(500)$ and $f_0(980)$

Must address this background in the fitting procedure



Decay channel	Number of events
$J/\psi \to \gamma\pi^0\pi^0$ (data)	442,562
$e^+e^- \to \gamma\pi^0\pi^0$ (continuum)	3,632
$J/\psi \to b_1\pi^0; b_1 \to \gamma\pi^0$	1,606
$J/\psi \to \omega\pi^0; \omega \to \gamma\pi^0$	865
$J/\psi \to \rho\pi^0; \rho \to \gamma\pi^0$	778
Misreconstructed background	608
$J/\psi \to \gamma\eta; \eta \to 3\pi^0$	903
$J/\psi \to \gamma\eta'; \eta' \to \eta\pi^0\pi^0; \eta \to \gamma\gamma$	377
$J/\psi \to \omega\pi^0\pi^0; \omega \to \gamma\pi^0$	775
$J/\psi \to b_1\pi^0; b_1 \to \omega\pi^0; \omega \to \gamma\pi^0$	578
$J/\psi \to \omega\eta; \omega \to \gamma\pi^0$	409
$J/\psi \to \omega f_2(1270); \omega \to \gamma\pi^0$	299
$J/\psi \to \gamma\eta_c; \eta_c \to \gamma\pi^0\pi^0 or \pi^0\pi^0\pi^0$	255
Other backgrounds	507
Total background (MC)	7,960

PhysRevD.92.052003 (2015)

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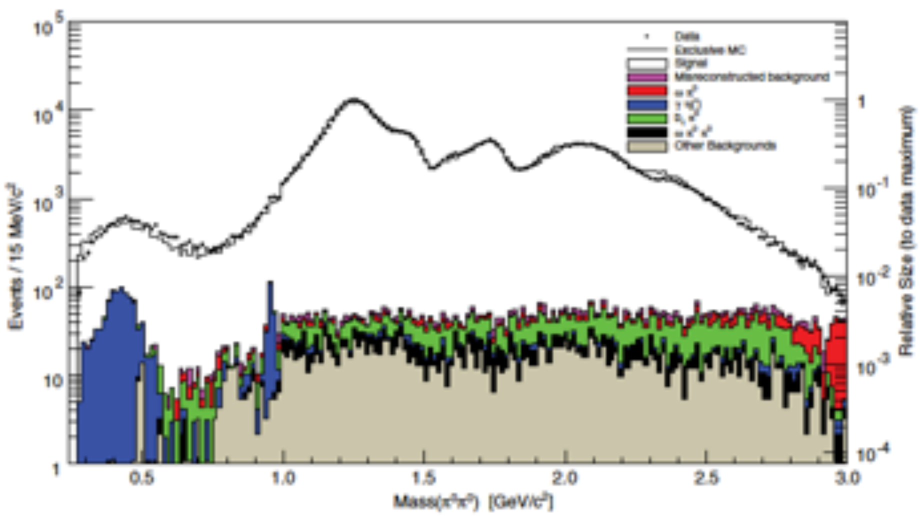

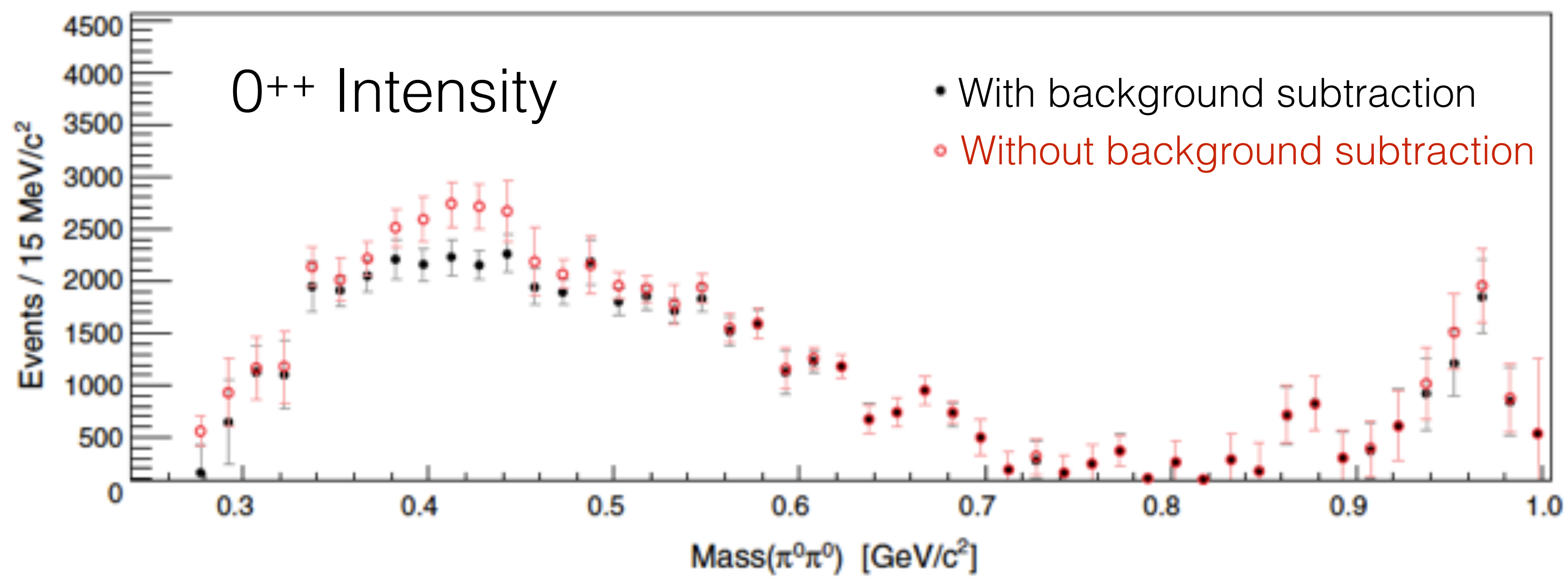

- Add background events with a negative weight (MC or data sidebands)

$$L(\vec{a}) = \prod_{i=1}^{N_{\mathrm{data}}^{\mathrm{sig}}} f(\vec{a}, \vec{x}_i) \prod_{j=1}^{N_{\mathrm{data}}^{\mathrm{bkg}}} f(\vec{a}, \vec{x}_j)$$

PhysRevD.92.052003 (2015)

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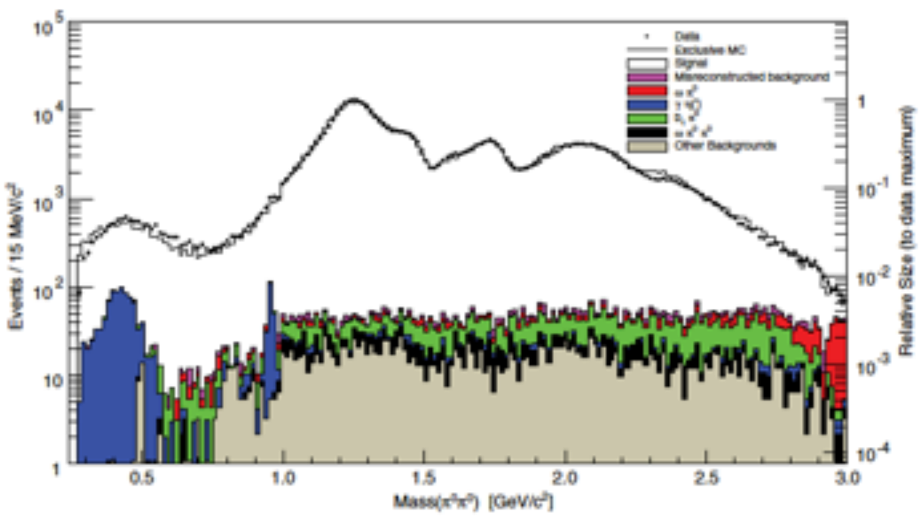

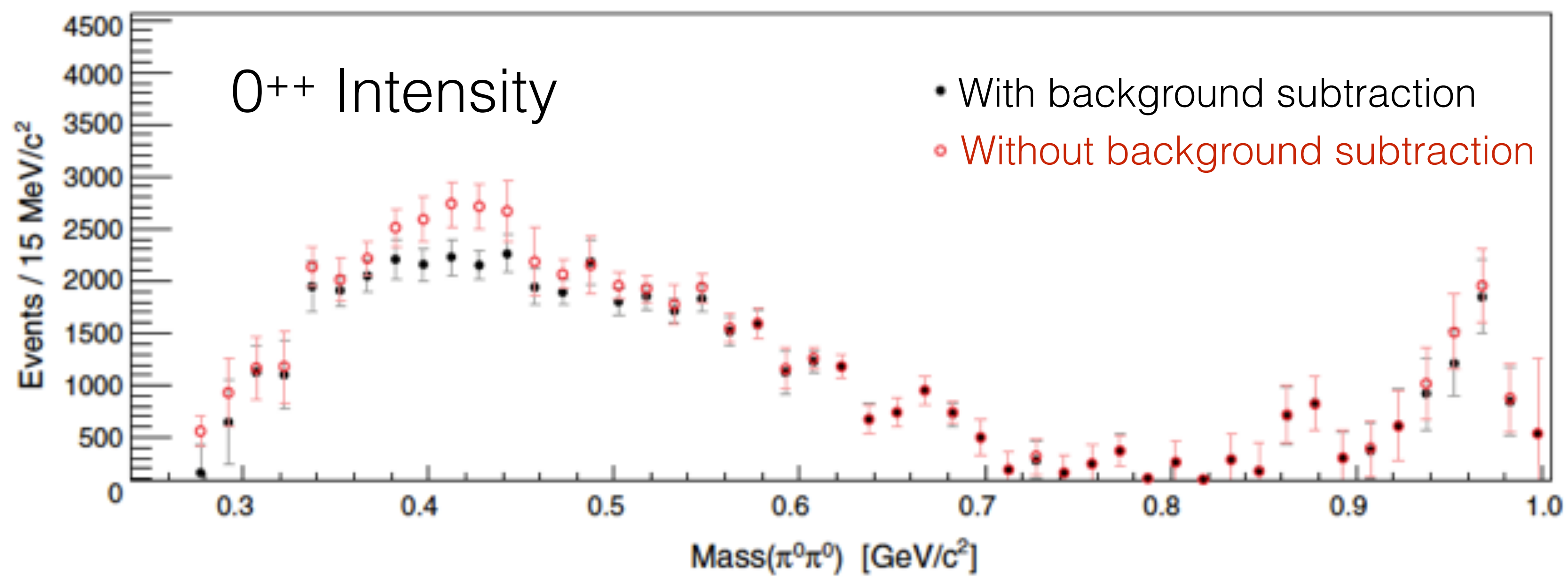

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PhysRevD.92.052003 (2015)

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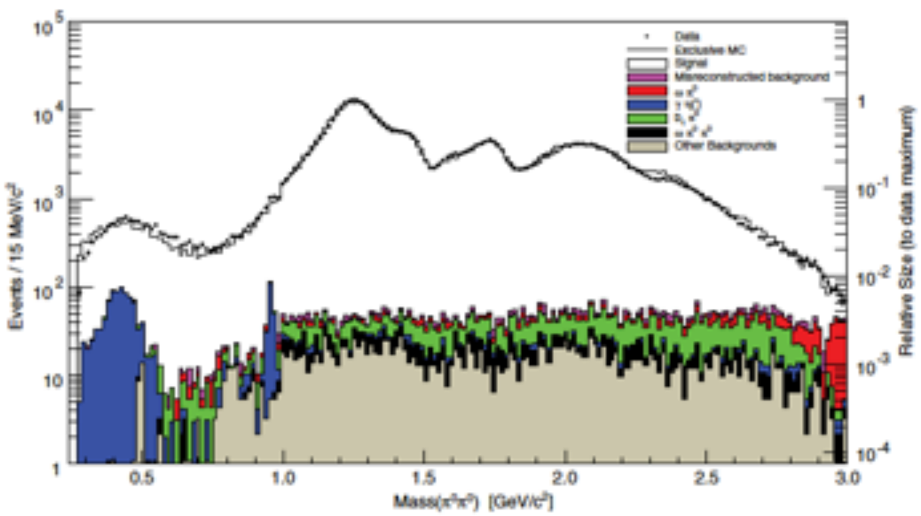

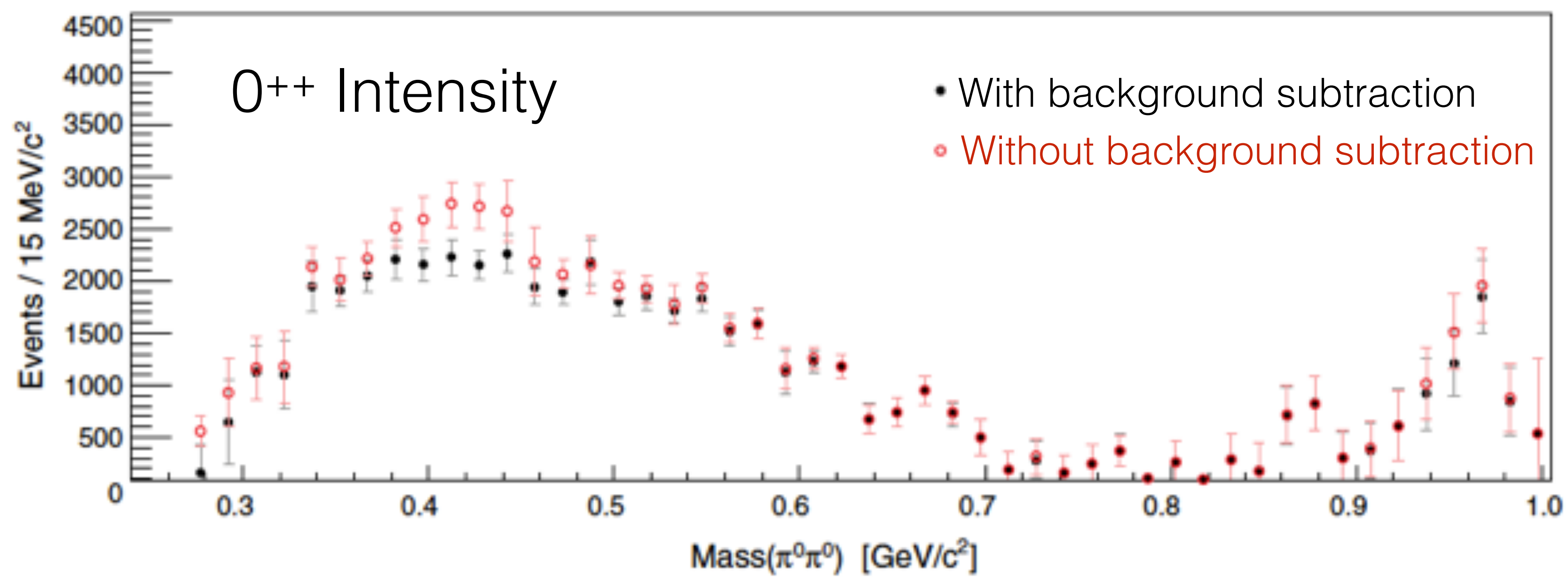

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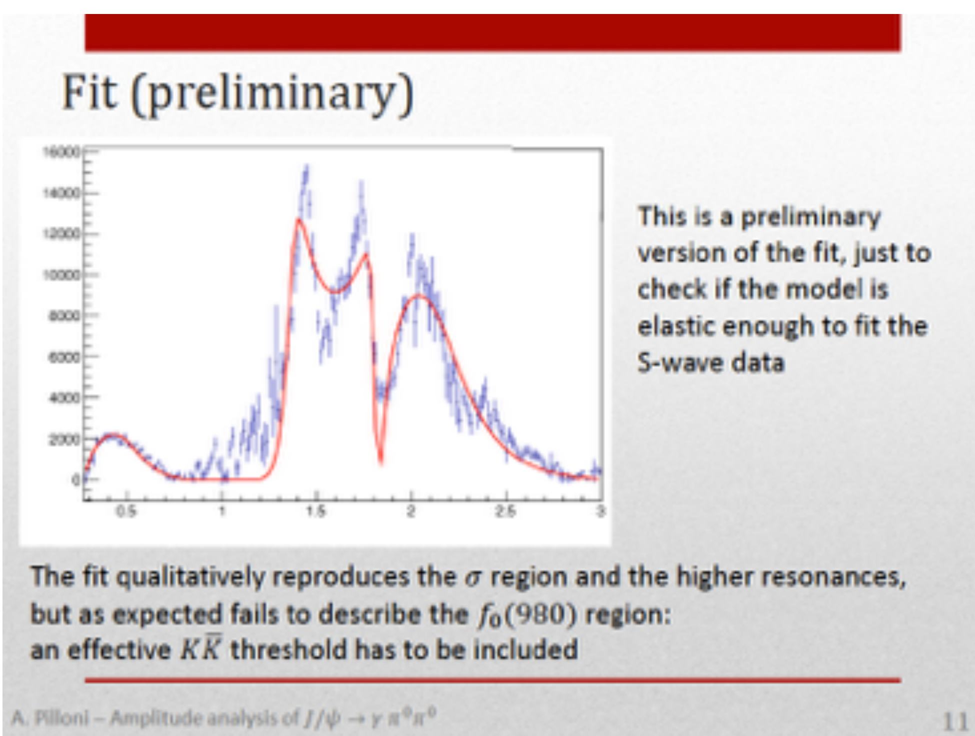
$$\mathcal{L}(\vec{a}) = \frac{e^{-\mu}\mu^{N_{\mathrm{data}}}}{N_{\mathrm{data}}!} \left(\prod_{i=1}^{N_{\mathrm{data}}} f(\vec{a},\vec{x}_i) \right) \left(\prod_{j=1}^{N_{\mathrm{MC}}^{\mathrm{big}}} f(\vec{a},\vec{x}_j)^{-w_j} \right)$$

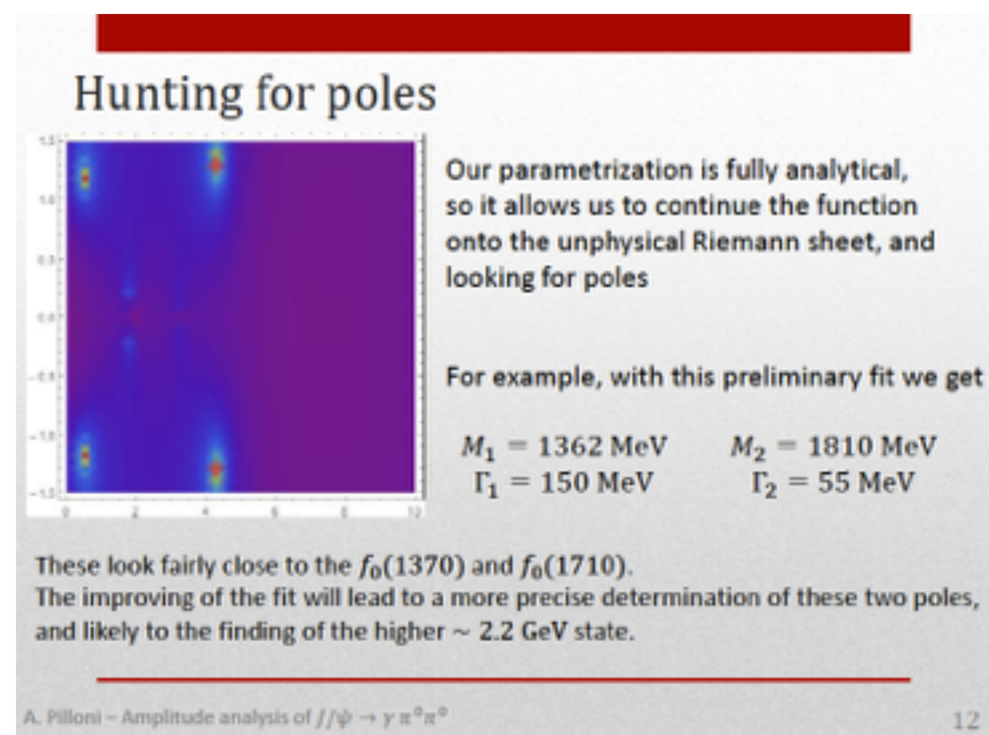
- Result is signal only likelihood

PhysRevD.92.052003 (2015)

What have we gained?

- Experiment independent information about scattering amplitude
  - Minimizes systematic bias arising from assumptions about $\pi\pi$ dynamics
  - Permits the development of dynamical models or parametrizations (no experimental knowledge is needed)
  - Combine results with data from other experiments in a common fit
  - Controlled study of coupled channel effects

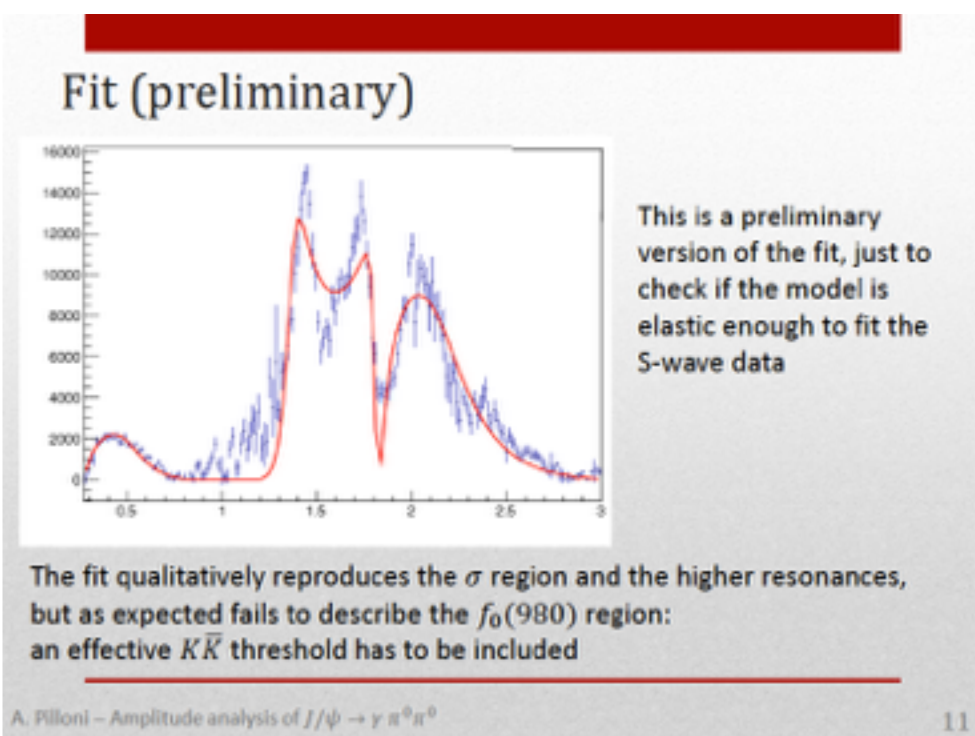


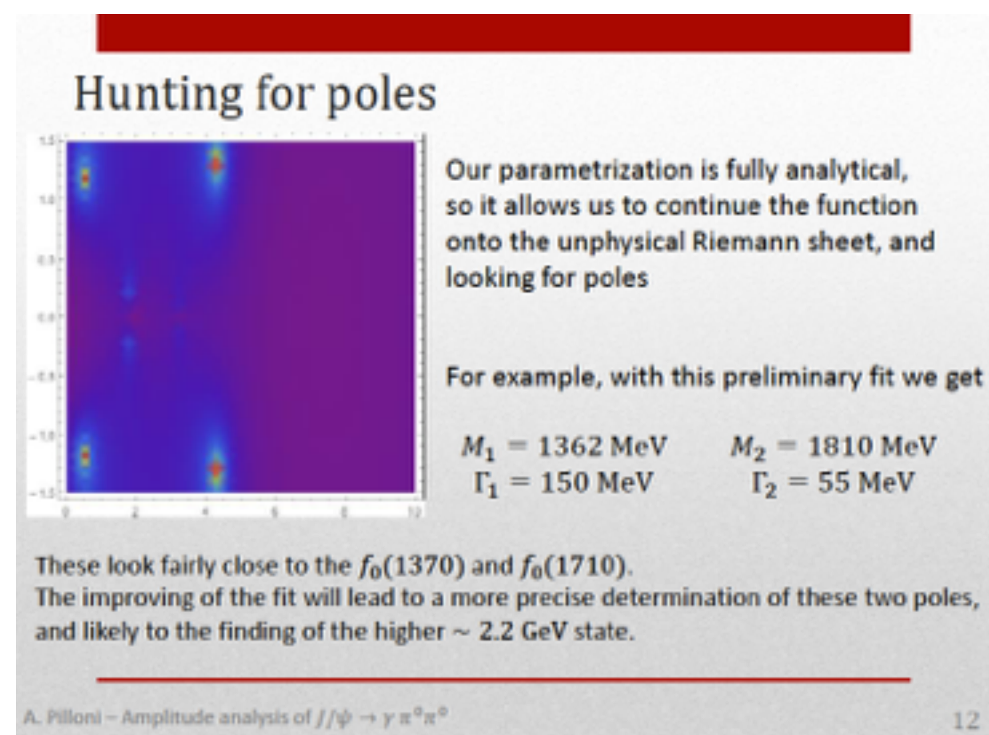


JPAC results from MESON 2016

What have we lost?

- Still has drawbacks
  - Ambiguous solutions
  - Large number of parameters
  - Potential bias in subsequent analyses from non-Gaussian effects
- Validity and precision at a level sufficient for model development, but extraction of rigorous values for model parameters only reliably obtained by fitting directly to the data





JPAC results from MESON 2016

Try both? PWA of J/ψ → γϕϕ

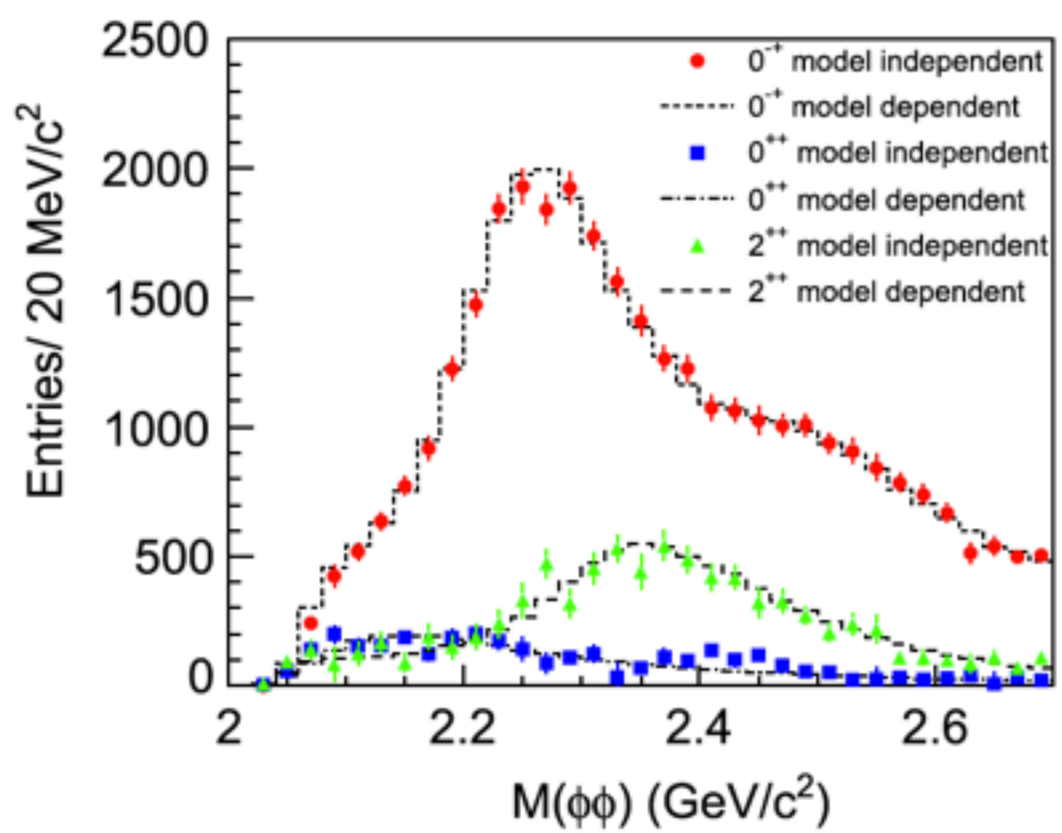
- Similar mass-dependent procedure as in J/ψ → γηη
  - Amplitudes in covariant tensor formalism
  - Data-driven background subtraction

$$-\ln \mathcal{L}_{\mathrm{sig}} = -(\ln \mathcal{L}_{\mathrm{data}} - \ln \mathcal{L}_{\mathrm{bkg}})$$

  - Resonances parametrized with relativistic Breit-Wigner with constant width

- Also perform mass-independent analysis
  - Fit for each amplitude (J^{PC}) in bins of ϕϕ invariant mass
  - Results of the two methods are consistent

Resonance	M (MeV/c^2)	Γ (MeV/c^2)	B.F. (×10^{-4})	Sig.
$\eta(2225)$	2216^{+4+21}_{-5-11}	185^{+12+43}_{-14-17}	$(2.40 \pm 0.10^{+2.47}_{-0.18})$	28σ
$\eta(2100)$	2050^{+30+75}_{-24-26}	$250^{+36+181}_{-30-164}$	$(3.30 \pm 0.09^{+0.18}_{-3.04})$	22σ
$X(2500)$	$2470^{+15+101}_{-19-23}$	230^{+64+56}_{-35-33}	$(0.17 \pm 0.02^{+0.02}_{-0.08})$	8.8σ
$f_0(2100)$	2101	224	$(0.43 \pm 0.04^{+0.24}_{-0.03})$	24σ
$f_2(2010)$	2011	202	$(0.35 \pm 0.05^{+0.28}_{-0.15})$	9.5σ
$f_2(2300)$	2297	149	$(0.44 \pm 0.07^{+0.09}_{-0.15})$	6.4σ
$f_2(2340)$	2339	319	$(1.91 \pm 0.14^{+0.72}_{-0.73})$	11σ
0^{-+} PHSP			$(2.74 \pm 0.15^{+0.16}_{-1.48})$	6.8σ

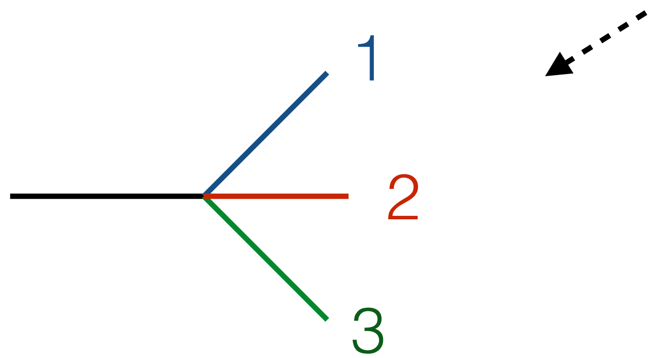


PhysRevD.93.112011 (2016)

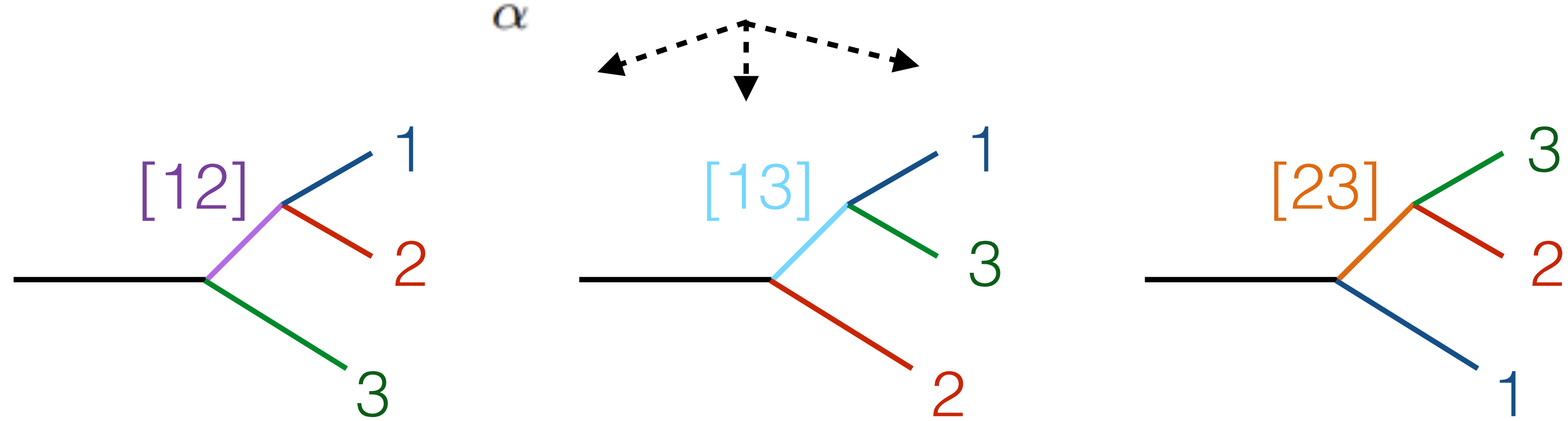
Amplitude analysis with three-body decays

- Most commonly performed using the isobar model (and extensions)
  - Express the total amplitude as a coherent sum of quasi-two-body contributions

$$\mathcal{A}_{D^0}(\vec{a},\vec{x}) = a_0\mathcal{A}_0 + \sum_{\alpha} a_\alpha \mathcal{A}_\alpha(\vec{x})$$



Often include (complex) non-resonant term



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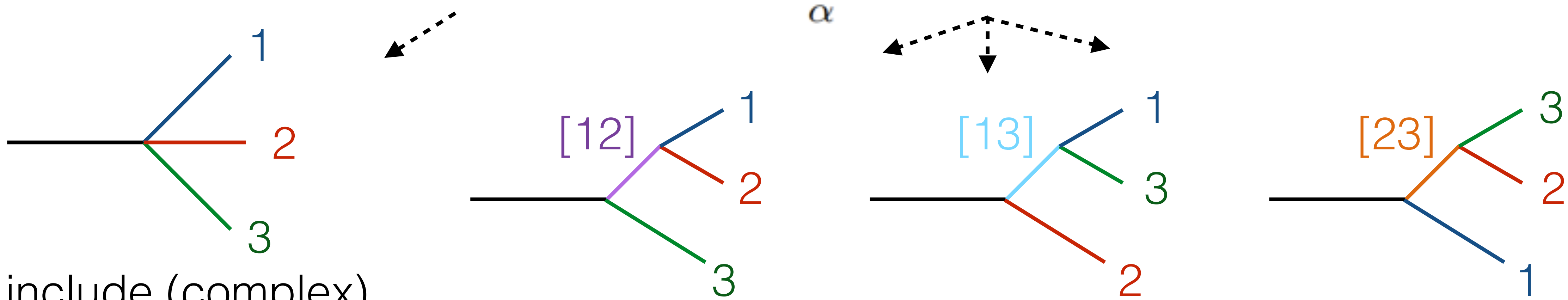

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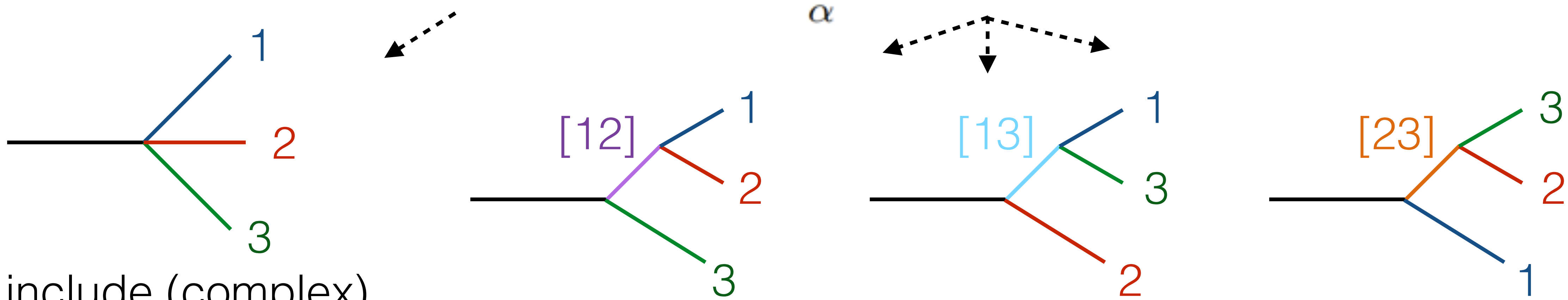

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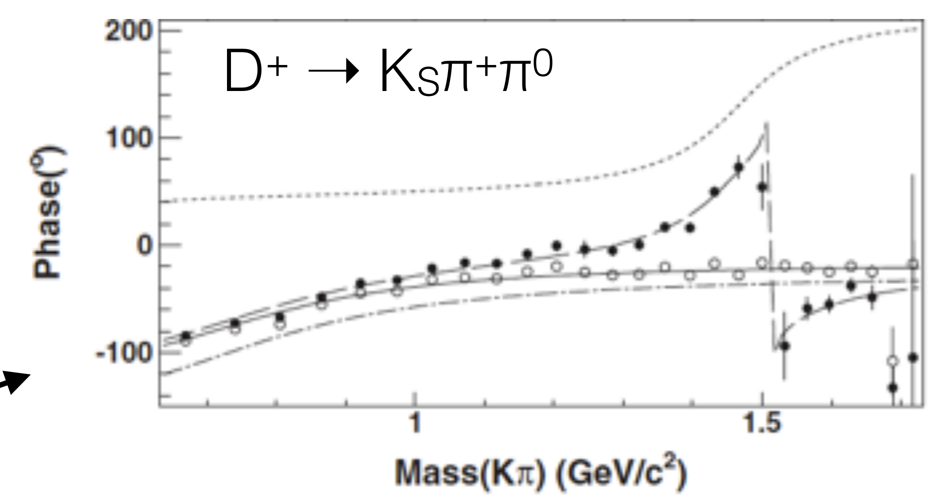
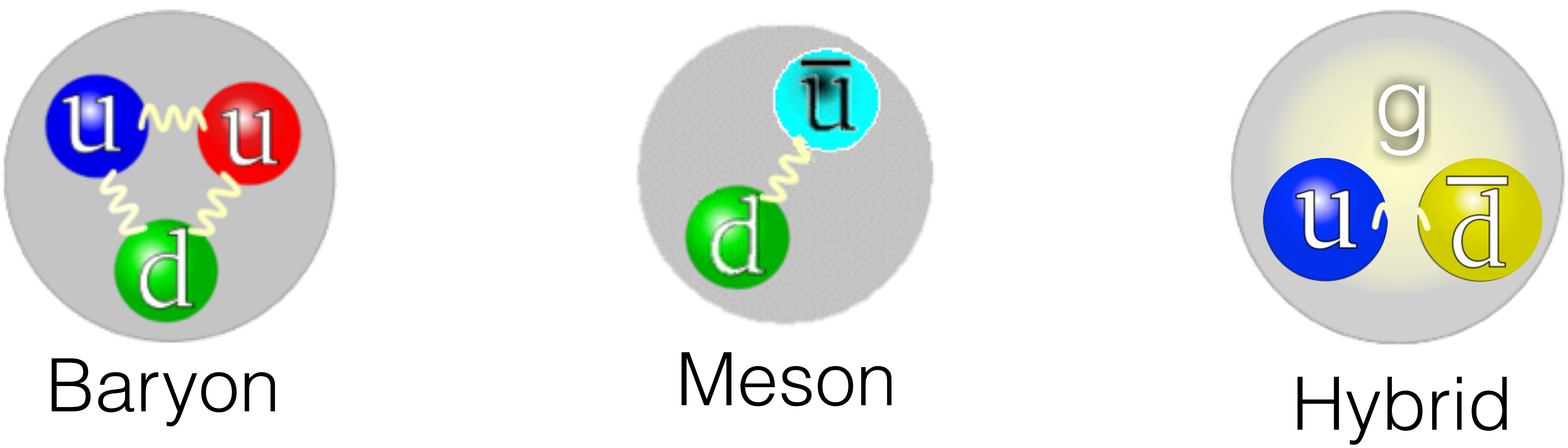
- Need to input strong interaction dynamics (line shapes, barrier factors, etc.)
- Alternative approaches to avoid model dependence usually involve binning



PhysRevD.89.052001 (2014)

$\chi_{c1} \rightarrow \eta\pi^+\pi^-$

- Amplitude analysis of $\chi_{c1} \rightarrow \eta\pi^+\pi^-$ decays
  - Potential exotic amplitude ($J^{PC} = 1^{-+}$) - lowest orbital excitation of a two-body combination in χ_{c1} decays to three pseudoscalars
  - Several candidate exotic states decaying into different final states, such as $\eta\pi$, $\eta'\pi$, $f_1(1270)\pi$, $b_1(1235)\pi$ and $\rho\pi$ have been reported by various experiments



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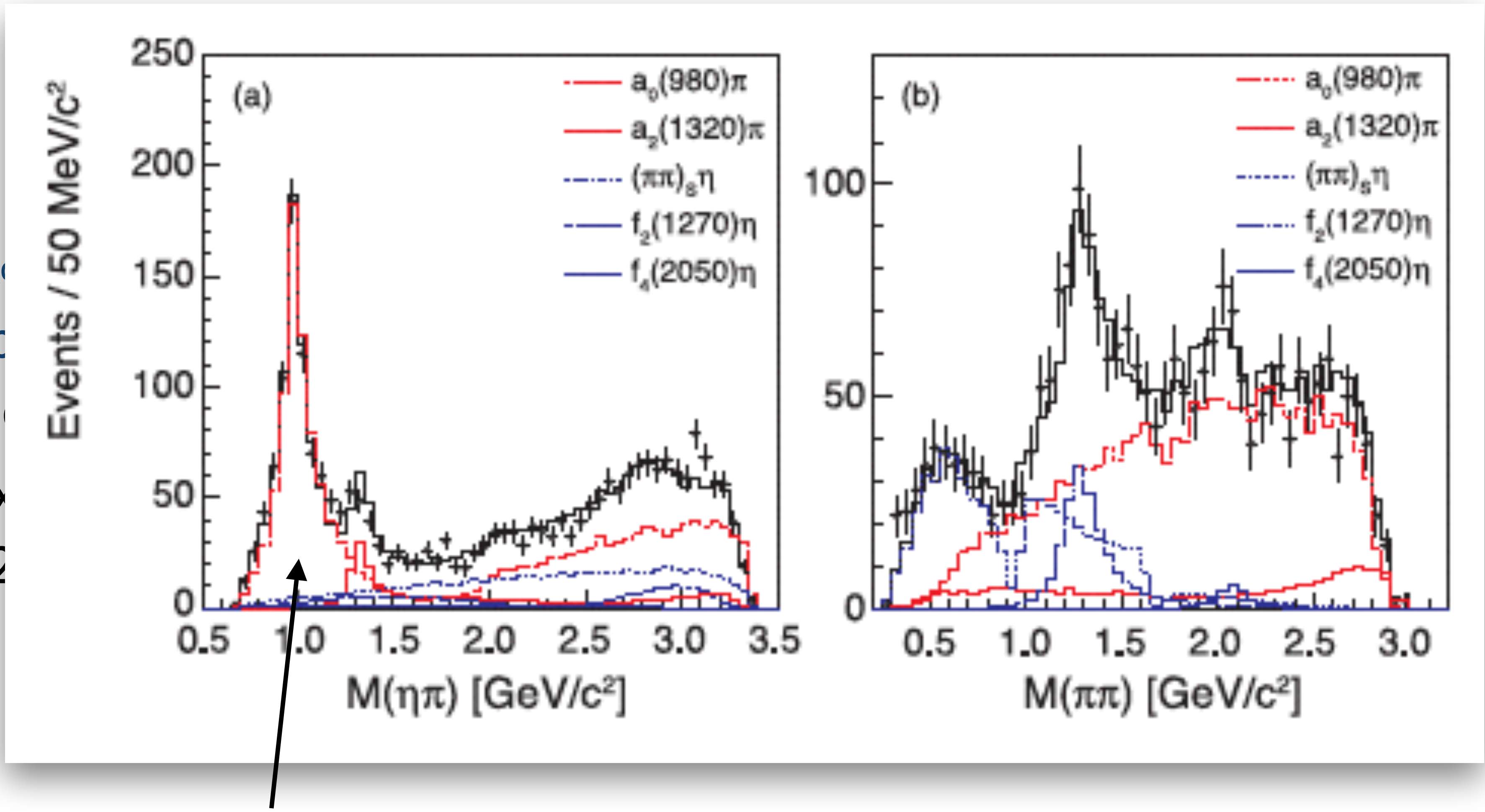
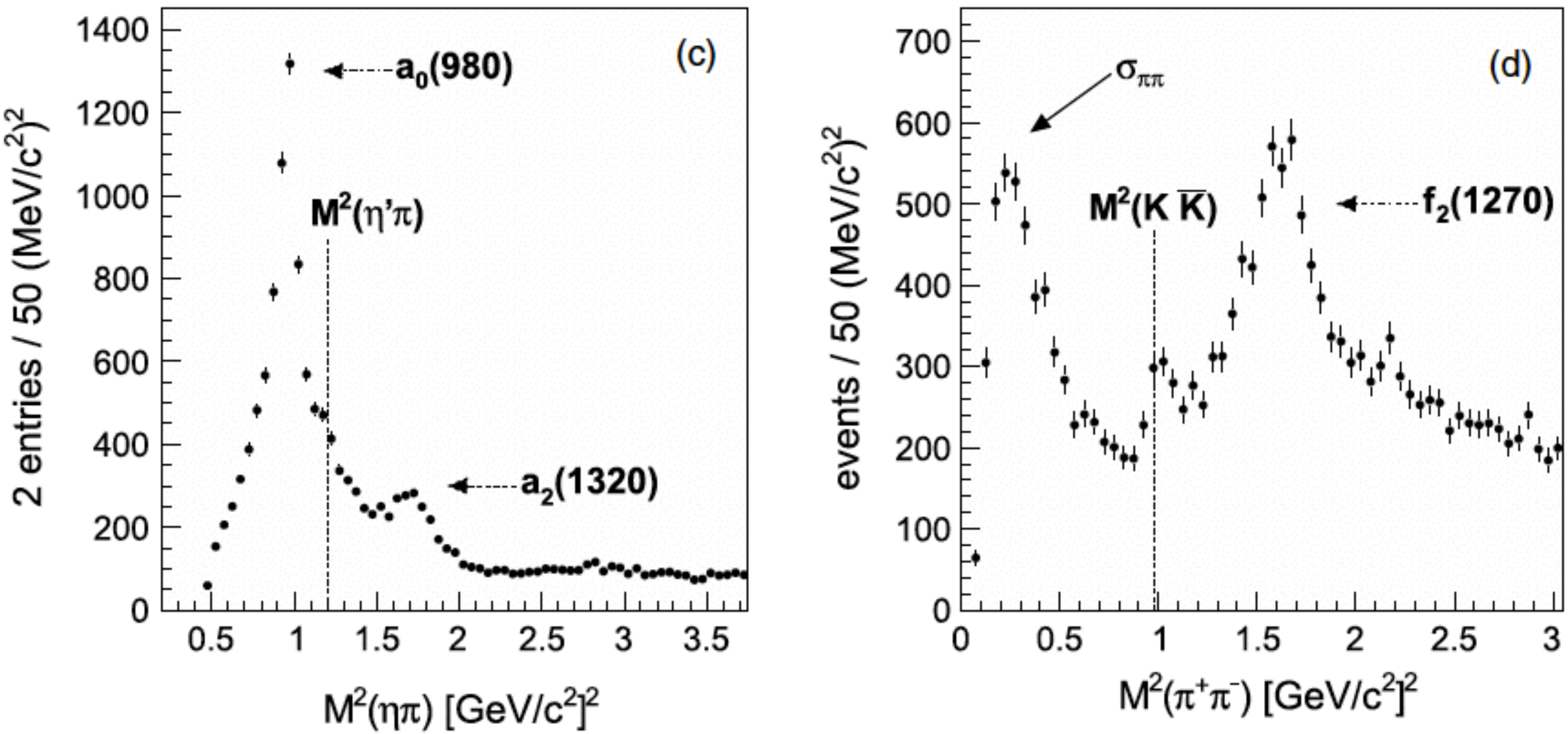


- Amplitude analysis of χ
  - Potential exotic amp combination in χ_{c1} d
  - Several candidate ex η'π, $f_1(1270)\pi$, $b_1(12$
- Another interesting state, the $a_0(980)$
  - Four-quark state? ordinary qq state? dynamically generated through meson-meson interactions?
  - Just below KK threshold: strong coupling generates cusp-like behavior in resonant amplitudes → line shape distorted so mass and width parameters do not correspond to the pole parameters
  - Use dispersion integral to describe line shape and extract information useful to determine the quark structure

PhysRevD.93.112011 (2016)

$\chi_{c1} \rightarrow \eta\pi^+\pi^-$

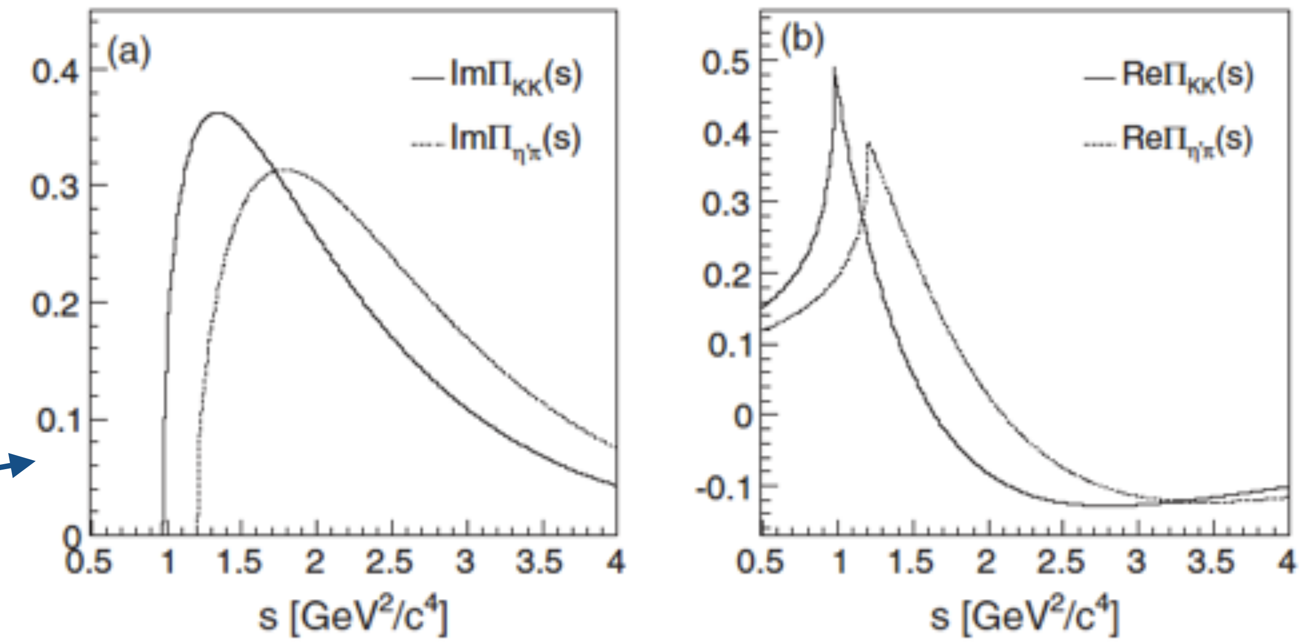
- Influence of thresholds is apparent (virtual channel influences distribution)

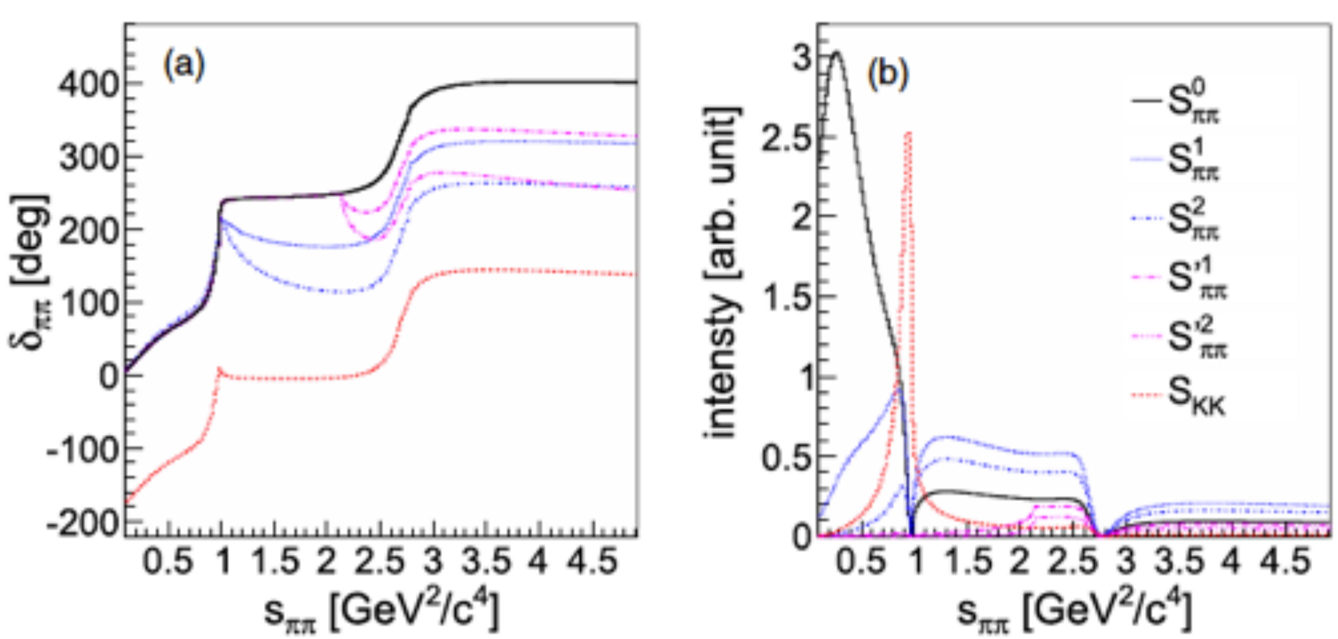


PhysRevD.93.112011 (2016)

Describing the dynamics

- Isobar model, helicity formalism
  - Background subtraction using sidebands from data
  - BW line shape for most resonances
- Dispersion integrals for $a_0(980)$ and $\pi\pi$ S-wave
  - Amplitudes respect unitarity
  - Accounts for differences between the $\pi\pi$ production in scattering and decay processes
- Also add a phase-space amplitude taking into account all possible helicity amplitudes





PhysRevD.93.112011 (2016)

$\chi_{c1} \rightarrow \eta\pi^+\pi^-$

- First nonzero coupling of $a_0(980)$ to $\eta'\pi$ (8.9σ)

Data	m_0 [GeV/c^2]	$g^2_{\eta\pi}$ [GeV/c^2]^2	$g^2_{K\bar{K}}/g^2_{\eta\pi}$	$g^2_{\eta'\pi}/g^2_{\eta\pi}$
CLEO-c [10]	0.998 ± 0.016	0.36 ± 0.04	0.872 ± 0.148	0.00 ± 0.17
C.Barrel [20]	0.987 ± 0.004	0.164 ± 0.011	1.05 ± 0.09	**0.772**
BESIII	$0.996 \pm 0.002 \pm 0.007$	$0.368 \pm 0.003 \pm 0.013$	$0.931 \pm 0.028 \pm 0.090$	$0.489 \pm 0.046 \pm 0.103$
BESIII ($R^2_{31} \equiv 0$)	0.990 ± 0.001	0.341 ± 0.004	0.892 ± 0.022	**0.0**

- First evidence for $a_2(1700)$ in this channel
- Only weak evidence (non-observation) for the $\pi_1(1400)$
  - $\pi_1(1600)$ and $\pi_1(2015)$ not significant

Decay	$\mathcal{F}$ [%]	Significance [σ]	$\mathcal{B}(\chi_{c1} \to \eta\pi^+\pi^-)$ [10^{-3}]
$\eta\pi^+\pi^-$	...	...	$4.67 \pm 0.03 \pm 0.23 \pm 0.16$
$a_0(980)^+\pi^-$	$72.8 \pm 0.6 \pm 2.3$	>100	$3.40 \pm 0.03 \pm 0.19 \pm 0.11$
$a_2(1320)^+\pi^-$	$3.8 \pm 0.2 \pm 0.3$	32	$0.18 \pm 0.01 \pm 0.02 \pm 0.01$
$a_2(1700)^+\pi^-$	$1.0 \pm 0.1 \pm 0.1$	20	$0.047 \pm 0.004 \pm 0.006 \pm 0.002$
$S_{K\bar{K}}\eta$	$2.5 \pm 0.2 \pm 0.3$	22	$0.119 \pm 0.007 \pm 0.015 \pm 0.004$
$S_{\pi\pi}\eta$	$16.4 \pm 0.5 \pm 0.7$	>100	$0.76 \pm 0.02 \pm 0.05 \pm 0.03$
$(\pi^+\pi^-)_S\eta$	$17.8 \pm 0.5 \pm 0.6$	...	$0.83 \pm 0.02 \pm 0.05 \pm 0.03$
$f_2(1270)\eta$	$7.8 \pm 0.3 \pm 1.1$	>100	$0.36 \pm 0.01 \pm 0.06 \pm 0.01$
$f_4(2050)\eta$	$0.6 \pm 0.1 \pm 0.2$	9.8	$0.026 \pm 0.004 \pm 0.008 \pm 0.001$
Exotic candidates			U.L. [90% C.L.]
$\pi_1(1400)^+\pi^-$	0.58 ± 0.20	3.5	<0.046
$\pi_1(1600)^+\pi^-$	0.11 ± 0.10	1.3	<0.015
$\pi_1(2015)^+\pi^-$	0.06 ± 0.03	2.6	<0.008

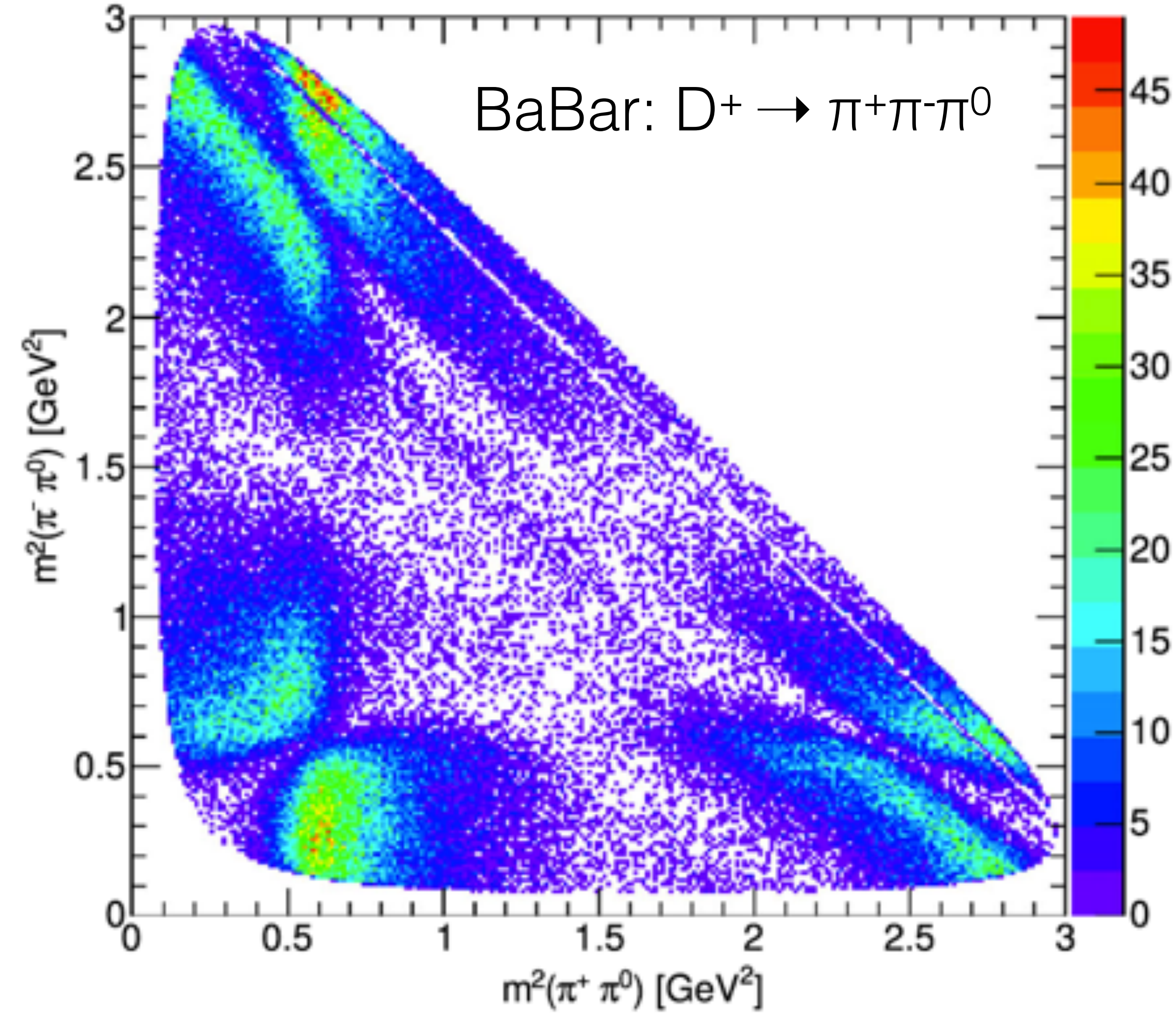
PhysRevD.93.112011 (2016)

Amplitude analysis in charm decays

- Decays of a heavy meson into three or more light mesons is ideal for CP studies
  - Large number of light meson resonances → lots of phase motion in a non-trivial distribution over Dalitz plot

Amplitude analysis in charm decays

- Decays of a heavy meson into three or more light mesons is ideal for CP studies
  - Large number of light meson resonances → lots of phase motion in a non-trivial distribution over Dalitz plot
- For the special case of decays to three pseudoscalars, the phase space density is uniform across the Dalitz plot
  - Visible structure is a direct consequence of the dynamics of the accessible amplitudes



BaBar; PhysRevD.93.112014 (2016)

Amplitude analysis in charm decays

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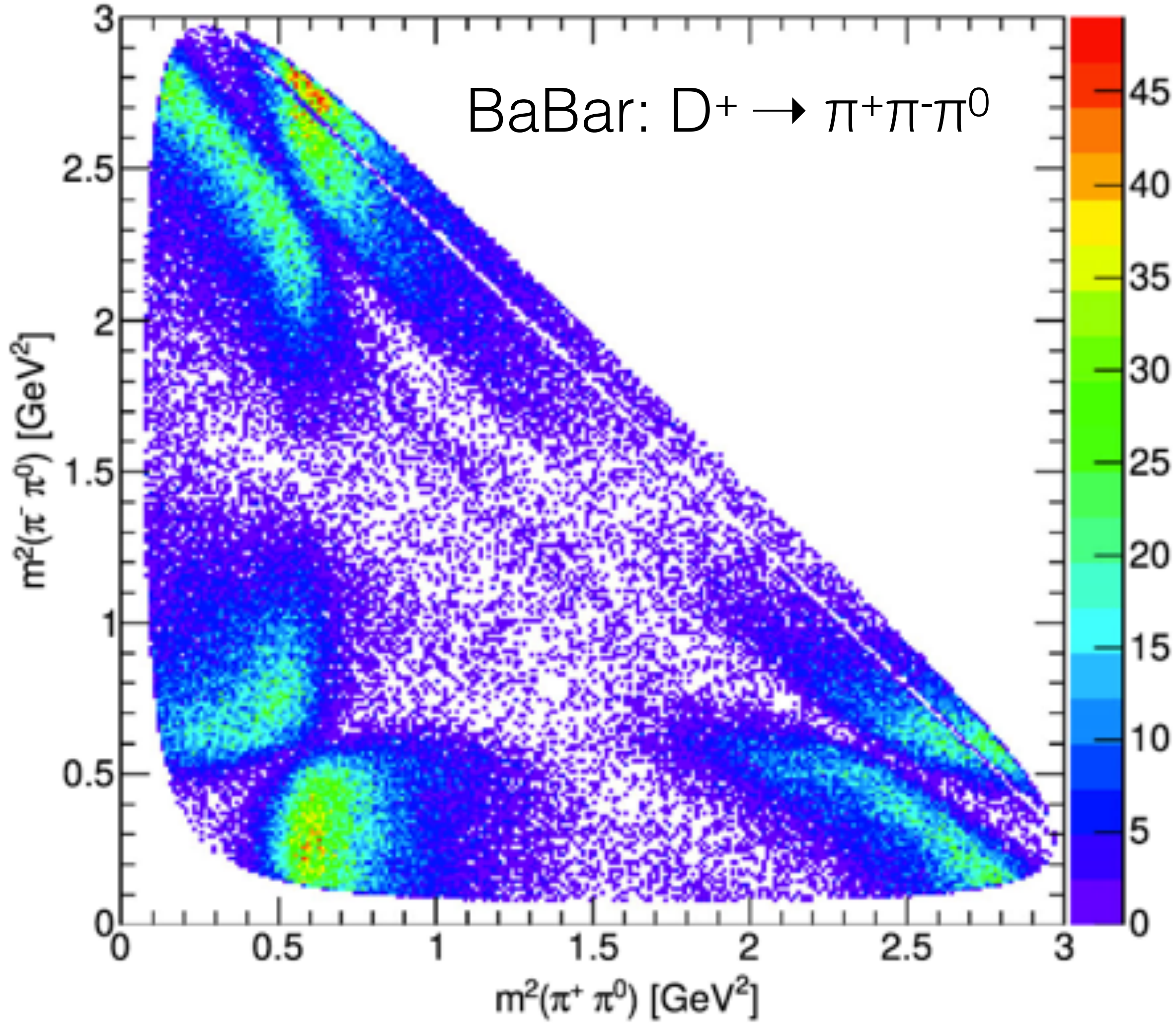
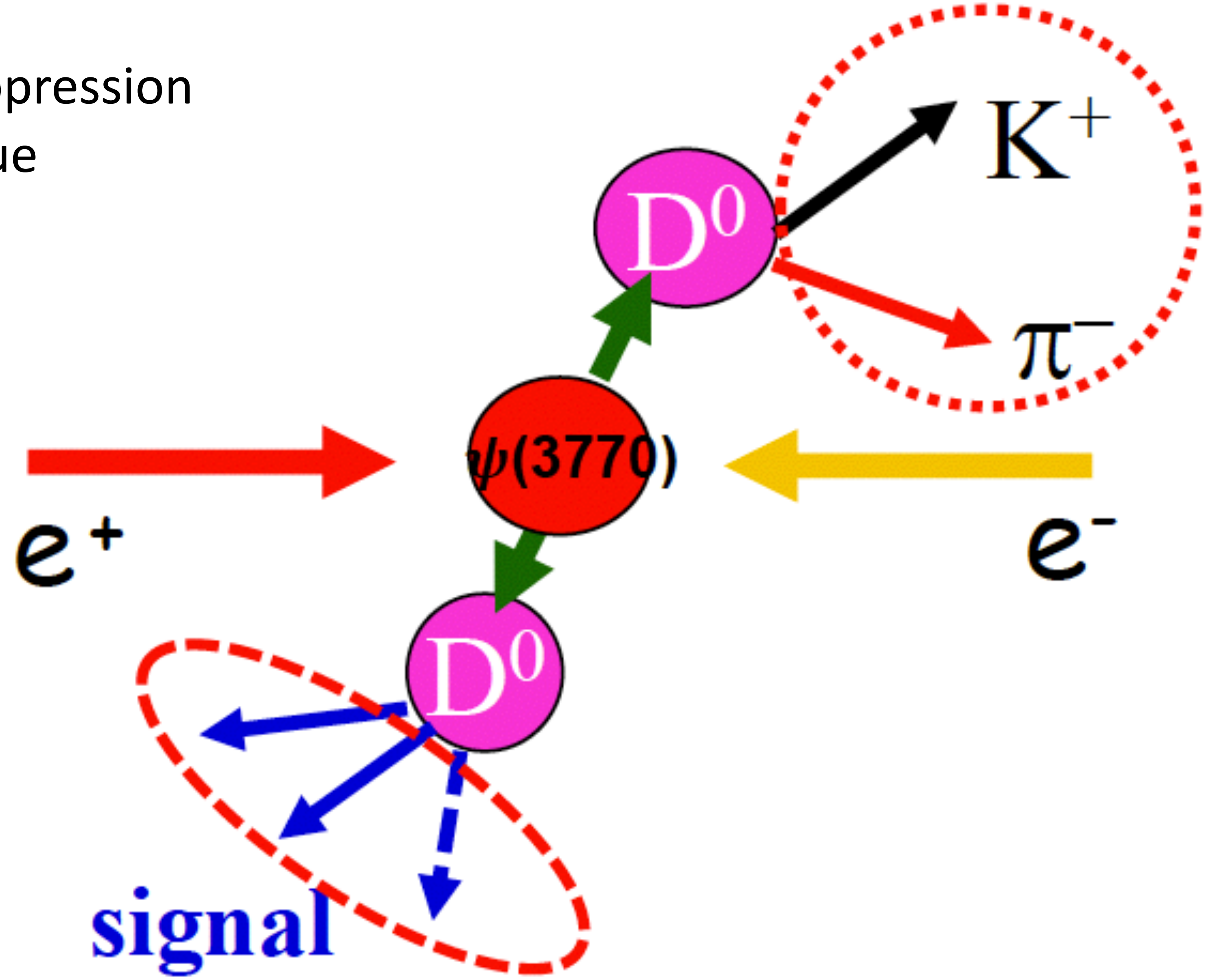


- For the special case of decays to three pseudoscalars, the phase space density is uniform across the Dalitz plot
  - Visible structure is a direct consequence of the dynamics of the accessible amplitudes
- Amplitude analysis provides complete description of data
  - Measure decay amplitudes and phases
  - Enables accurate measurements of branching fractions
  - Environment to study the effects of final state interactions

BaBar; PhysRevD.93.112014 (2016)

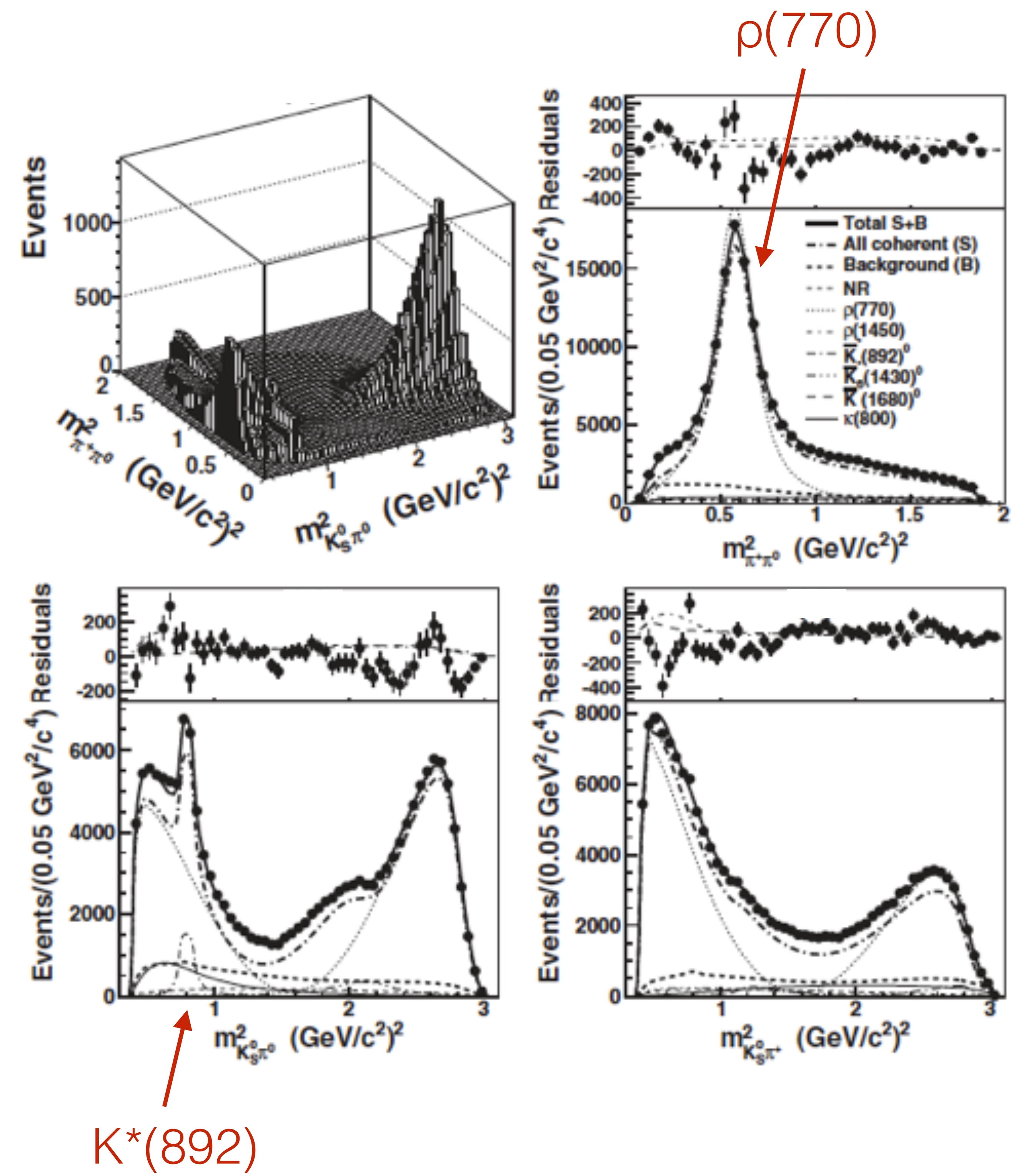
Running at threshold

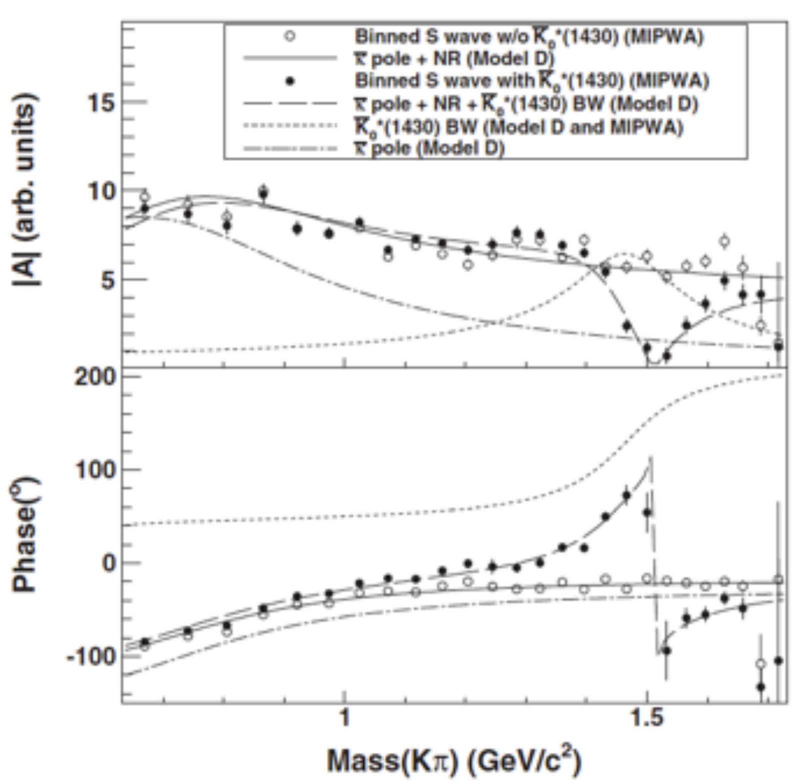
- Quantum correlated D mesons
- No additional hadrons
- Effective background suppression with double-tag technique



Amplitude analysis of $D^+ \to K_S\pi^+\pi^0$

- Isobar model of six quasi-two-body CF amplitudes plus a non resonant term
- Golden mode to study $K\pi$ S-wave in D decays
- Cross check with a "quasi-model-independent" analysis to test the $K_S\pi^0$ S-wave
  - Still use BW form for K*(1430)
  - Assumes no interaction with π^+



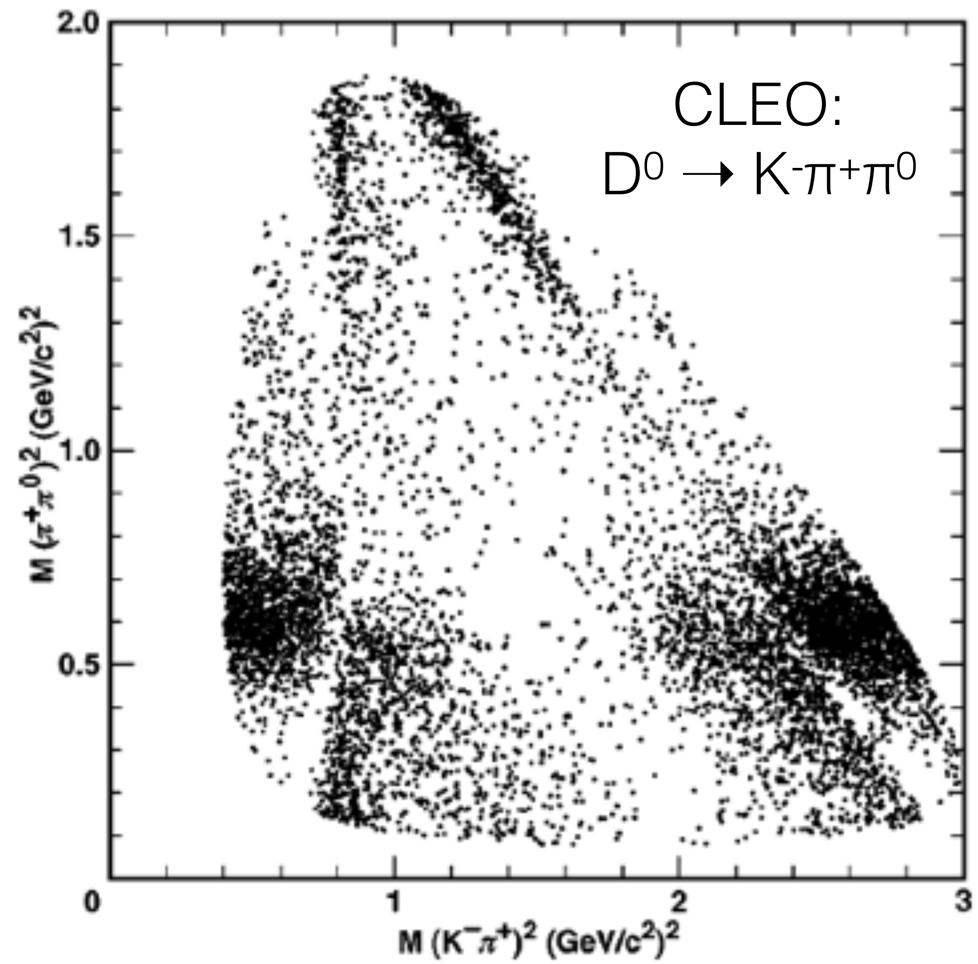


PhysRevD.89.052001 (2014)

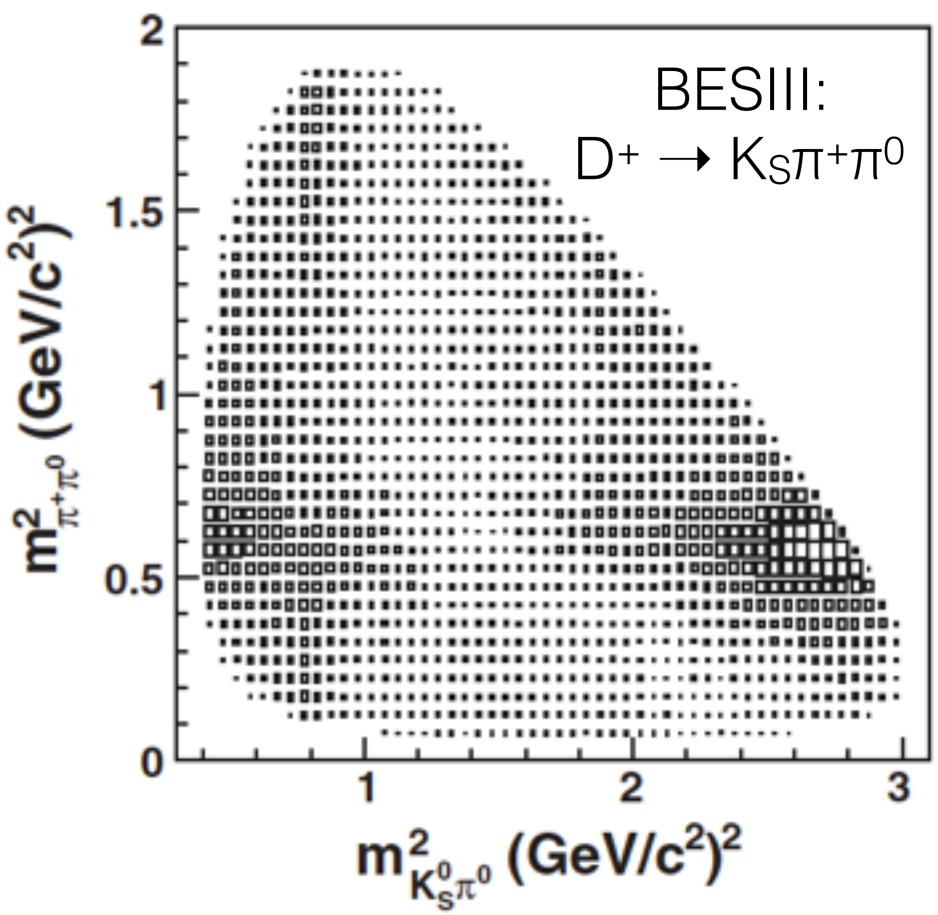
Fake interference?

- Heavy ρ mesons, ρ(1450) and ρ(1700) both lie outside Dalitz-plot, but are wide
  - Tails extend into region of interest
- Both have large fit fractions
  - … but have a small net contribution (9 ± 2) %
  - … and have nearly 180° difference in phase

PhysRevD.63.092001 (2001)



	Phase	Fit fraction
$\rho(1700)^+$	149 ± 8	75 ± 18
$\rho(1450)^+$	-45 ± 10	34 ± 11



PhysRevD.89.052001 (2014)

- Probably a misrepresentation of the contents of the Dalitz plot
  - Choose one (best goodness-of-fit)
  - Consider other as systematic uncertainty

Fake interference?

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  outside Dalitz-plot, but are v
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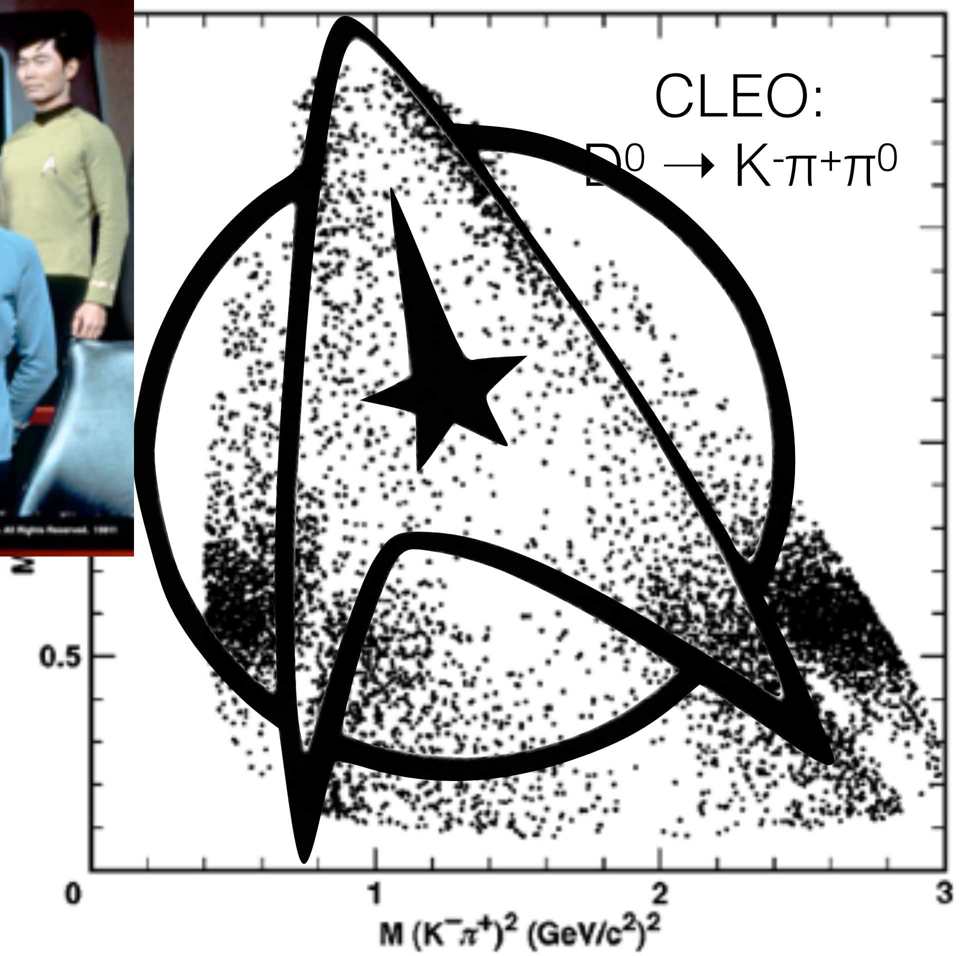
PhysRevD.63.092001 (2001)



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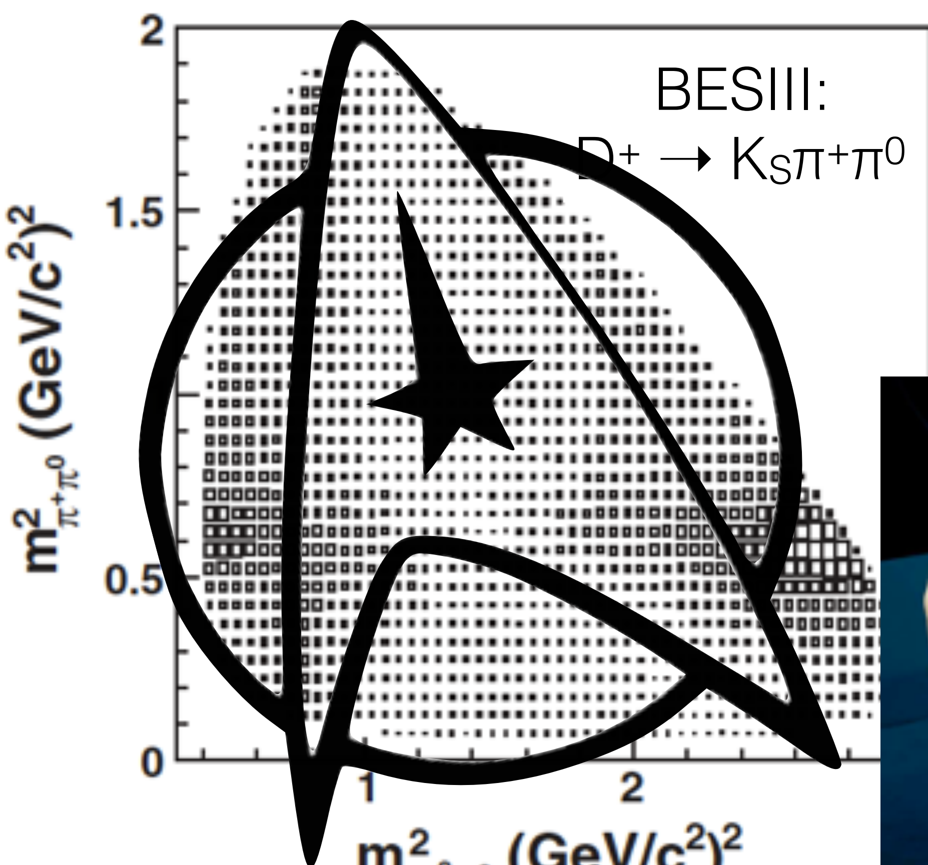

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goodness-of-fit)

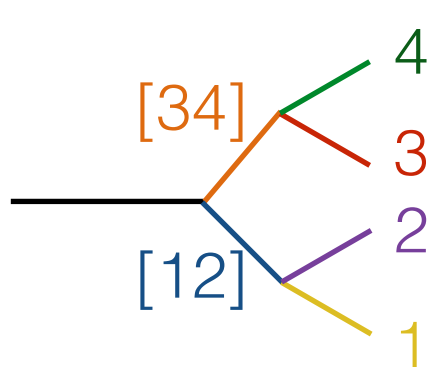
s systematic uncertainty

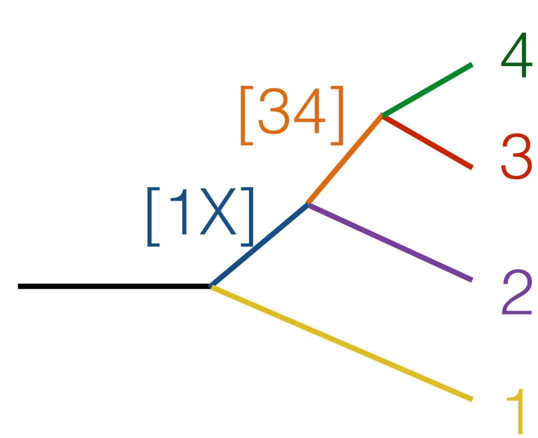
PhysRevD.89.052001 (2014)

Amplitude analysis of $D^0 \to K^-\pi^+\pi^+\pi^-$

- One of the three neutral D golden modes (large BF and low background)
- Accurate knowledge of substructure is important to reduce systematic uncertainties for analyses that use this mode as a reference
  - Absolute BF measurements of D hadronic modes
  - Along with strong phase measurement can help improve precision of γ
  - Theoretical studies of D^0-$\bar{D}^0$ mixing

- Complicated due to nonuniform phase space of four-body decay and possibility to have two separate intermediate resonances contributing





Decay mode
$D[S] \to V_1V_2,\ V_1 \to P_1P_2,\ V_2 \to P_3P_4$
$D[P] \to V_1V_2,\ V_1 \to P_1P_2,\ V_2 \to P_3P_4$
$D[D] \to V_1V_2,\ V_1 \to P_1P_2,\ V_2 \to P_3P_4$
$D \to AP_1, A[S] \to VP_2,\ V \to P_3P_4$
$D \to AP_1, A[D] \to VP_2,\ V \to P_3P_4$
$D \to AP_1, A \to SP_2,\ S \to P_3P_4$
$D \to VS,\ V \to P_1P_2,\ S \to P_3P_4$
$D \to V_1P_1,\ V_1 \to V_2P_2,\ V_2 \to P_3P_4$
$D \to PP_1, P \to VP_2,\ V \to P_3P_4$
$D \to TS,\ T \to P_1P_2,\ S \to P_3P_4$

PhysRevD.95.072010 (2017)

Limitations on amplitude models

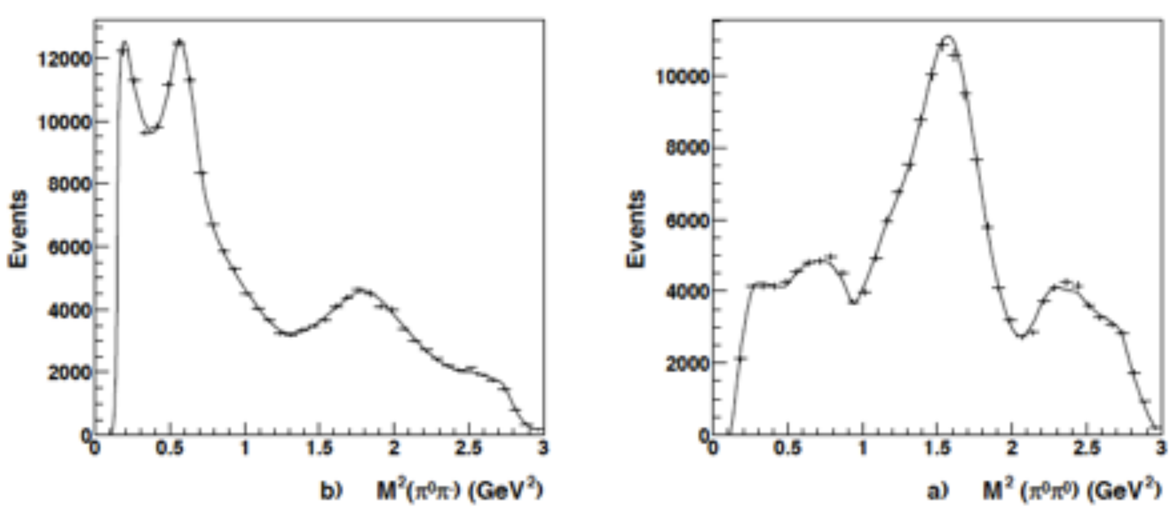
- Model dependence (again…)
  - Lineshapes (coupled channels, threshold effects, etc.)
  - "Sum of Breit-Wigners" model violates unitarity (especially for broad, overlapping resonances)
  - Difficult to differentiate S-wave amplitudes and non-resonant terms (can lead to unphysical phase variations)

- More robust methods
  - K-matrix (e.g. for S-wave): elegant way to consider unitarity
  - Scattering data to constrain phase variations
  - Input from theory (chiral symmetry, dispersion relations)

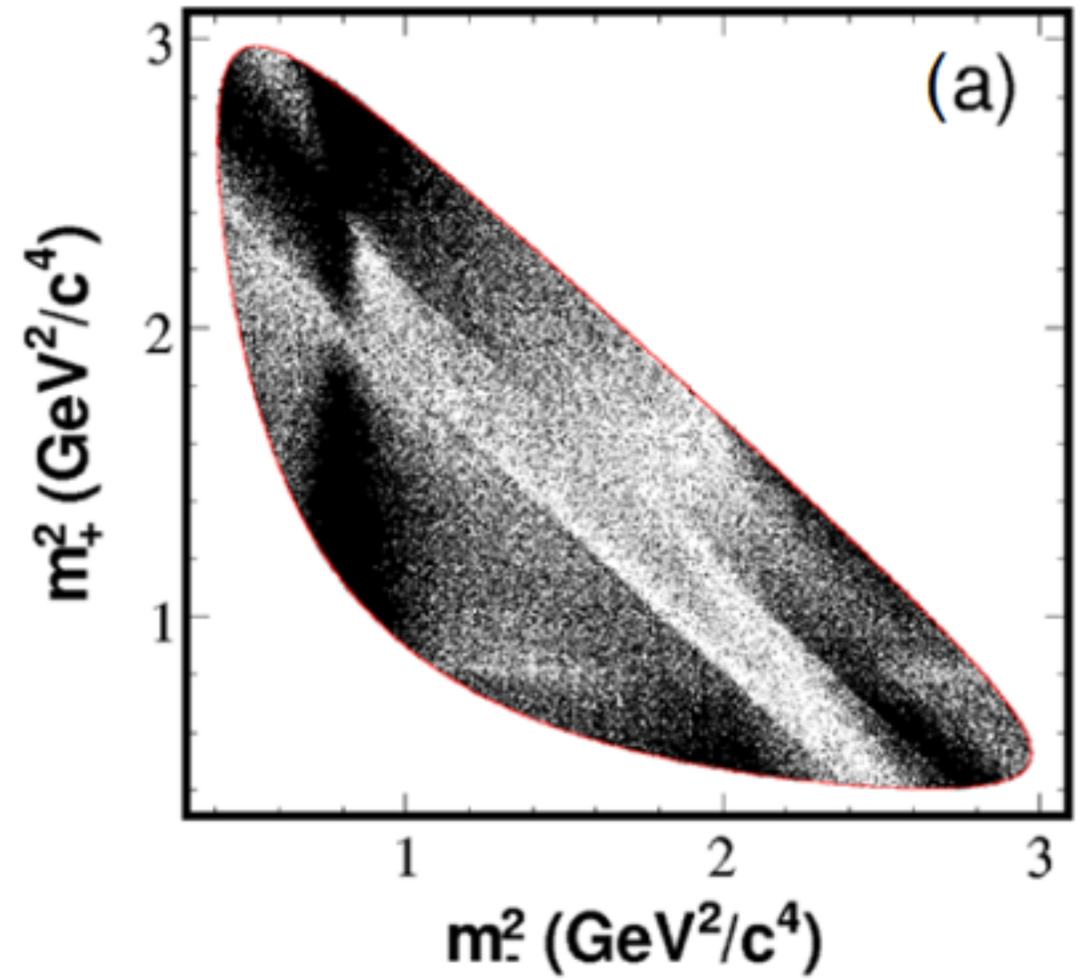
Kπ S-wave parametrization

- BW line shape for the K*(1430) plus a parametrization for non-resonant component from scattering data

e.g. Crystal Barrel data: $p\bar{p}$ annihilation into $\pi^0\pi^0\pi^-$ in liquid D_2



BaBar: B → $D^{(*)}K^{(*)}$, D → $K_S\pi^+\pi^-$



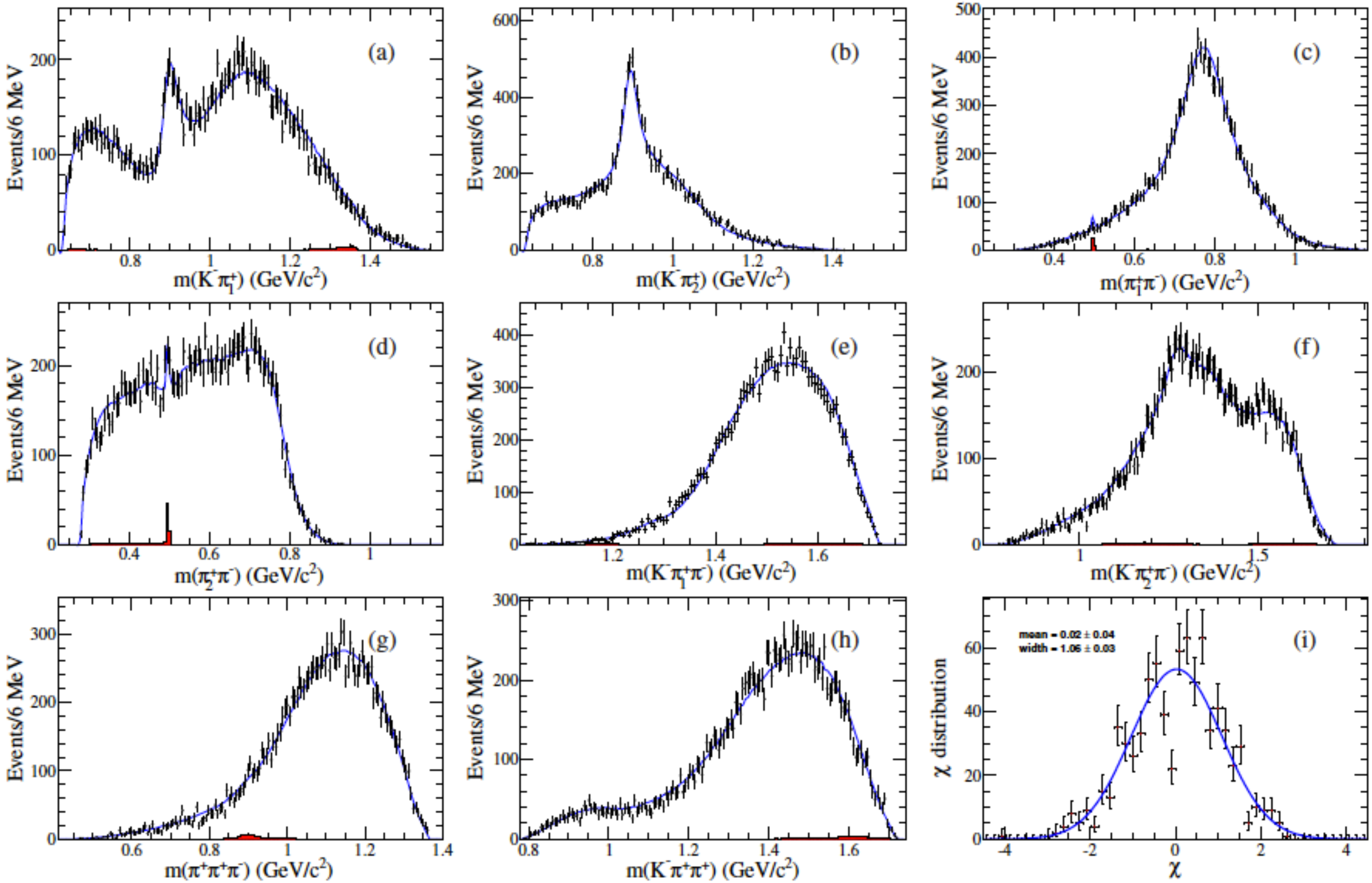
- Initial state propagation into final states by S-wave scattering process
  - Describe using scattering data
  - Assumes two-body system isolated

$$F_u(s) = \sum_l [I - iK(s)\rho(s)]^{-1}_{uv} P_v(s).$$

Eur. Phys. J. A16, 229-258 (2003)

PhysRevD.78.034023 (2008)

Amplitude analysis of $D^0 \to K^-\pi^+\pi^+\pi^-$

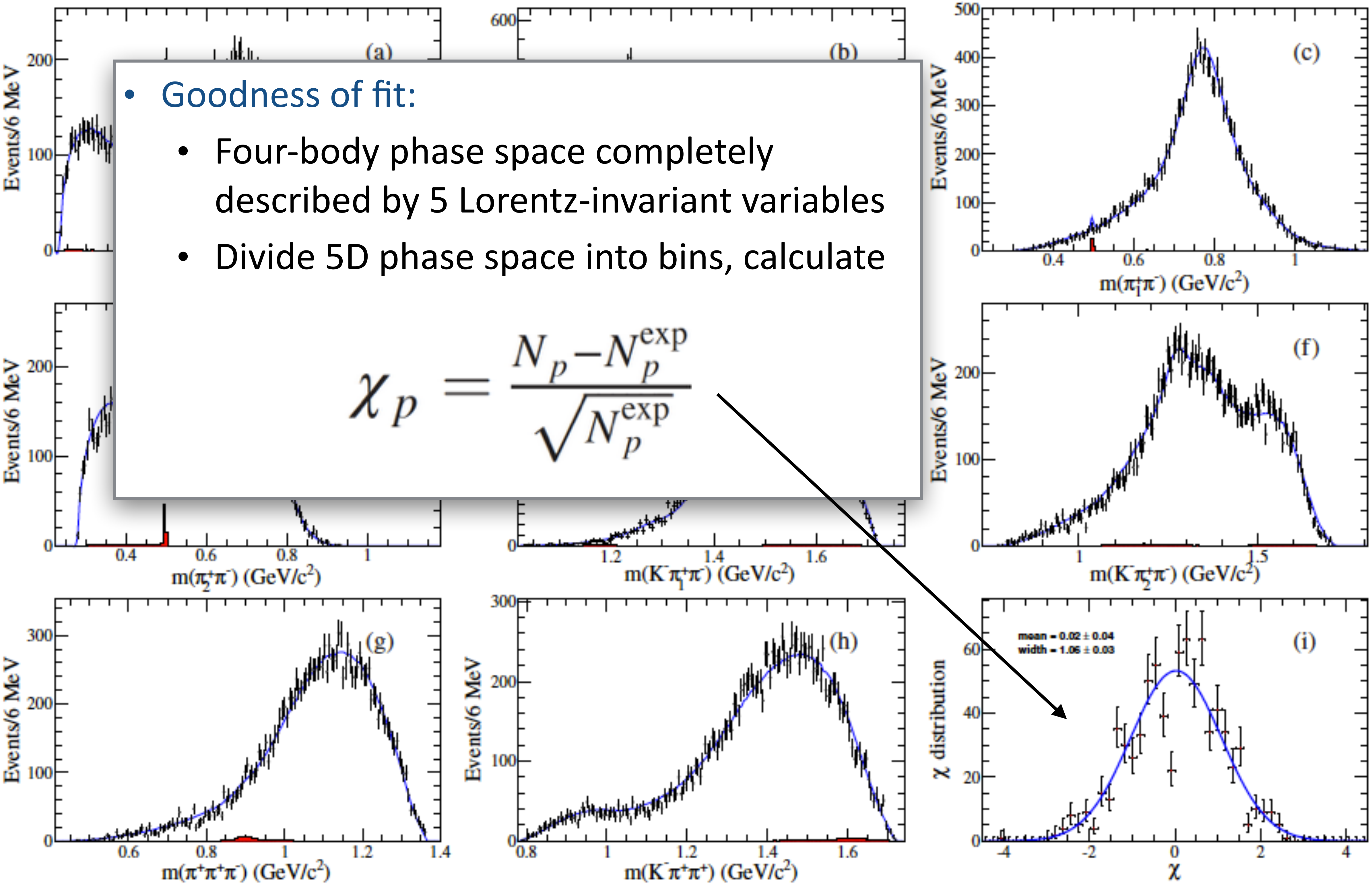


PhysRevD.95.072010 (2017)

Amplitude analysis of $D^0 \to K^-\pi^+\pi^+\pi^-$

- Goodness of fit:
  - Four-body phase space completely described by 5 Lorentz-invariant variables
  - Divide 5D phase space into bins, calculate

$$\chi_p = \frac{N_p - N_p^{\text{exp}}}{\sqrt{N_p^{\text{exp}}}}$$



PhysRevD.95.072010 (2017)

Amplitude analysis of $D^0 \to K^-\pi^+\pi^+\pi^-$

Component	Amplitude	Significance (σ)
$D^0 \to \bar{K}^{*0}\rho^0$	$D^0[S] \to \bar{K}^{*0}\rho^0$	>10.0
	$D^0[P] \to \bar{K}^{*0}\rho^0$	>10.0
	$D^0[D] \to \bar{K}^{*0}\rho^0$	>10.0
$D^0 \to K^- a_1^+(1260),\ a_1^+(1260) \to \rho^0\pi^+$	$D^0 \to K^- a_1^+(1260),\ a_1^+(1260)[S] \to \rho^0\pi^+$	>10.0
	$D^0 \to K^- a_1^+(1260),\ a_1^+(1260)[D] \to \rho^0\pi^+$	7.4
$D^0 \to K_1^-(1270)\pi^+,\ K_1^-(1270) \to \bar{K}^{*0}\pi^-$	$D^0 \to K_1^-(1270)\pi^+,\ K_1^-(1270)[S] \to \bar{K}^{*0}\pi^-$	4.3
	$D^0 \to K_1^-(1270)\pi^+,\ K_1^-(1270)[D] \to \bar{K}^{*0}\pi^-$	9.6
$D^0 \to K_1^-(1270)\pi^+,\ K_1^-(1270) \to K^-\rho^0$	$D^0 \to K_1^-(1270)\pi^+,\ K_1^-(1270)[S] \to K^-\rho^0$	>10.0
$D^0 \to K^-\pi^+\rho^0$	$D^0 \to (\rho^0 K^-)_A\pi^+,\ (\rho^0 K^-)_A[D] \to K^-\rho^0$	9.6
	$D^0 \to (K^-\rho^0)_P\pi^+$	7.0
	$D^0 \to (K^-\pi^+)_{\text{S-wave}}\rho^0$	5.1
	$D^0 \to (K^-\rho^0\pi^+)_V\pi^+$	6.8
$D^0 \to \bar{K}^{*0}\pi^+\pi^-$	$D^0 \to (\bar{K}^{*0}\pi^-)_P\pi^+$	8.5
	$D^0 \to \bar{K}^{*0}(\pi^+\pi^-)_S$	8.9
	$D^0 \to (\bar{K}^{*0}\pi^-)_V\pi^+$	9.7
$D \to K^-\pi^+\pi^+\pi^-$	$D^0 \to ((K^-\pi^+)_{\text{S-wave}}\pi^-)_A\pi^+$	>10.0
	$D^0 \to K^-((\pi^+\pi^-)_S\pi^+)_A$	>10.0
	$D^0 \to (K^-\pi^+)_{\text{S-wave}}(\pi^+\pi^-)_S$	>10.0
	$D^0[S] \to (K^-\pi^+)_V(\pi^+\pi^-)_V$	8.8
	$D^0 \to (K^-\pi^+)_{\text{S-wave}}(\pi^+\pi^-)_V$	5.8
	$D^0 \to (K^-\pi^+)_V(\pi^+\pi^-)_S$	>10.0
	$D^0 \to (K^-\pi^+)_T(\pi^+\pi^-)_S$	6.8
	$D^0 \to (K^-\pi^+)_{\text{S-wave}}(\pi^+\pi^-)_T$	9.7

About 40% comes from nonresonant four-body ($D^0 \to K^-\pi^+\pi^-\pi^+$) and three-body ($D^0 \to K^-\pi^+\rho^0$ and $D^0 \to K^{*-}\pi^+\pi^-$) decays

PhysRevD.95.072010 (2017)

Amplitude analysis of $D^0 \to K^-\pi^+\pi^+\pi^-$

Component	An
$D^0 \to \bar{K}^{*0}\rho^0$	$D^0[S$
	$D^0[P$
	$D^0[D$
$D^0 \to K^- a_1^+(1260)$, $a_1^+(1260) \to \rho^0\pi^+$	$D^0 \to K^- a_1^+(1260$
	$D^0 \to K^- a_1^+(1260$
$D^0 \to K_1^-(1270)\pi^+$, $K_1^-(1270) \to \bar{K}^{*0}\pi^-$	$D^0 \to K_1^-(1270)\pi^+$
	$D^0 \to K_1^-(1270)\pi^+,$
$D^0 \to K_1^-(1270)\pi^+$, $K_1^-(1270) \to K^-\rho^0$	$D^0 \to K_1^-(1270)\pi^+$
$D^0 \to K^-\pi^+\rho^0$	$D^0 \to (\rho^0 K^-)_A\pi^+$
	$D^0 \to$
	$D^0 \to (K$
	$D^0 \to ($
$D^0 \to \bar{K}^{*0}\pi^+\pi^-$	$D^0 \to ($
	$D^0 \to$
	$D^0 \to ($
$D \to K^-\pi^+\pi^+\pi^-$	$D^0 \to ((K^-$
	$D^0 \to K^-$
	$D^0 \to (K^-\pi$
	$D^0[S] \to (K$
	$D^0 \to (K^-\pi$
	$D^0 \to (K$
	$D^0 \to (K$
	$D^0 \to (K^-\pi$

The amplitudes listed below are tested when determining the nominal fit model, but not used in our final fit result.

(1) *Cascade amplitudes*
- (a) $K_1^-(1270)(\rho^0 K^-)\pi^+$, $\rho^0 K^-$ *D*-wave
- (b) $K_1^-(1400)(\bar{K}^{*0}\pi^-)\pi^+$, $\bar{K}^{*0}\pi^-$ *S* and *D*-waves
- (c) $K^{*-}(1410)(\bar{K}^{*0}\pi^-)\pi^+$
- (d) $K_2^{*-}(1430)(\bar{K}^{*0}\pi^-)\pi^+$, $K_2^{*-}(1430)(K^-\rho^0)\pi^+$
- (e) $K^{*-}(1680)(\bar{K}^{*0}\pi^-)\pi^+$, $K^{*-}(1680)(K^-\rho^0)\pi^+$
- (f) $K_2^{*-}(1770)(\bar{K}^{*0}\pi^-)\pi^+$, $K_2^{*-}(1770)(K^-\rho^0)\pi^+$
- (g) $K^- a_2^+(1320)(\rho^0\pi^+)$
- (h) $K^-\pi^+(1300)(\rho^0\pi^+)$
- (i) $K^- a_1^+(1260)(f_0(500)\pi^+)$

(2) *Quasi-two-body amplitudes*
- (a) $\bar{K}^{*0} f_0(500)$
- (b) $\bar{K}^{*0} f_0(980)$

(3) *Three-body amplitudes*
- (a) $\bar{K}^{*0}(\pi^+\pi^-)_V$ *S*, *P*- and *D*-waves
- (b) $(K^-\pi^+)_V\rho^0$ *S*, *P* and *D*-waves
- (c) $\bar{K}_2^{*0}(1430)(\pi^+\pi^-)_S$
- (d) $\bar{K}_2^{*0}(1430)\rho^0$
- (e) $\bar{K}^{*0} f_2(1270)$
- (f) $(K^-\pi^+)_S f_2(1270)$
- (g) $K^-(\rho^0\pi^+)_V$
- (h) $K^-(\rho^0\pi^+)_P$
- (i) $K^-(\rho^0\pi^+)_A$
- (j) $K^-(\rho^0\pi^+)_T$
- (k) $(\bar{K}^{*0}\pi^-)_T\pi^+$
- (l) $(K^-\rho^0)_T\pi^+$
- (m) $(\bar{K}^{*0}\pi^-)_A\pi^+$, $\bar{K}^{*0}\pi^-$ *S* and *D*-waves

(4) *Four-body nonresonance amplitudes*
- (a) $(K^-\pi^+)_T(\pi^+\pi^-)_V$ *P*- and *D*-waves
- (b) $(K^-\pi^+)_V(\pi^+\pi^-)_T$ *P*- and *D*-waves
- (c) $(K^-\pi^+)_V(\pi^+\pi^-)_V$ *P*- and *D*-waves
- (d) $(K^-(\pi^+\pi^-)_S)_A\pi^+$

About 40% comes from nonresonant four-body ($D^0 \to K^-\pi^+\pi^-\pi^+$) and three-body ($D^0 \to K^-\pi^+\rho^0$ and $D^0 \to K^-\pi^+\pi^-$) decays

PhysRevD.95.072010 (2017)

Common challenges for amplitude analyses

- Requires many many fits!
  - Must consider additional resonances as a source of systematic uncertainty
  - These fits also have many free parameters
  - With every increasing statistics, this becomes a computational problem
- A set of amplitudes can be “sufficient”, but how do we know it is “correct”?
- Usually neglected: phase space dependent systematics
  (e.g. momentum dependent tracking efficiency)
- How to treat detector resolution?
  - Difficult for unbinned fits
  - Not a worry for broad resonances, but what about narrow ones?
- How do we know if we found the global minimum?
- How to judge goodness of fit?
- How to deal with multiple solutions?

Summary

- BES III has impressive data sets for light hadron spectroscopy, charm at threshold, XYZ physics, etc.
- Amplitude analysis plays a key role in the BESIII physics program
  - Many new results, with much more to come!

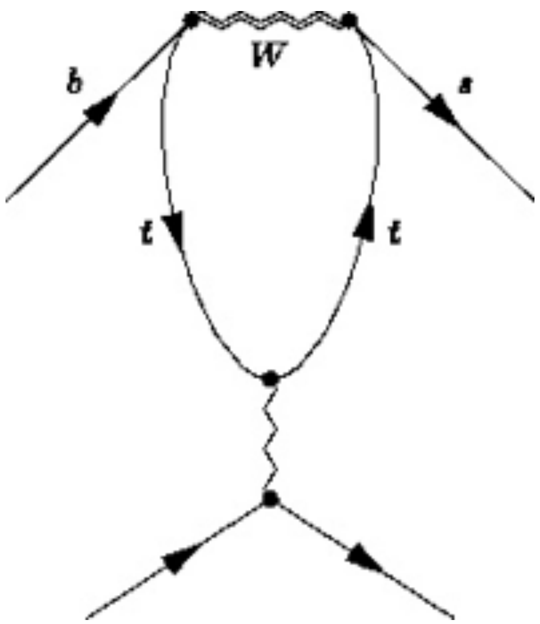
- Especially with increasing statistics, challenging and interesting problems
  - We have beautiful data that are extremely hard to fit very well

Summary

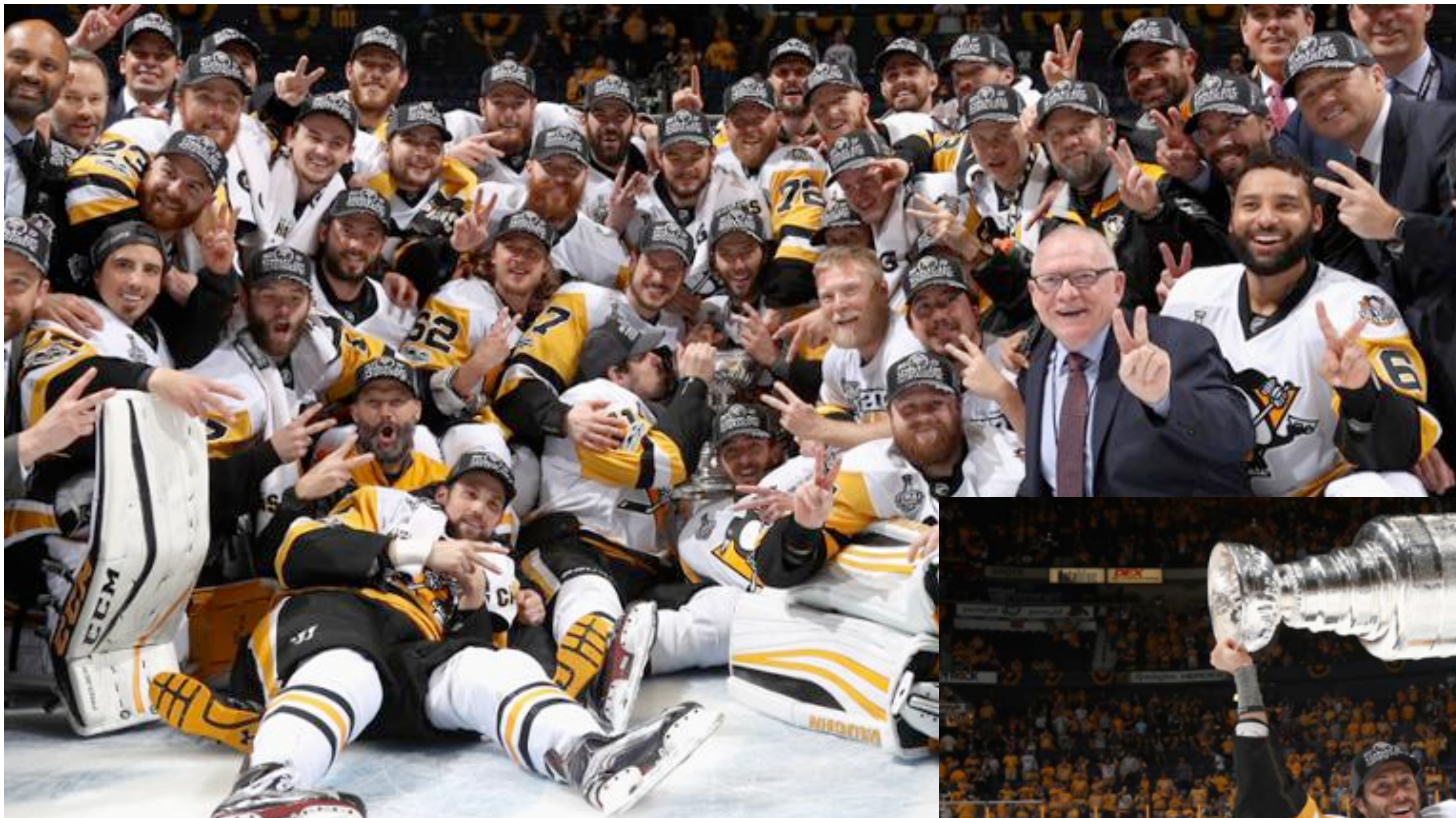
- BES III has impressive data sets for light hadron spectroscopy, charm at threshold, XYZ physics, etc.
- Amplitude analysis plays a key role in the BESIII physics program
  - Many new results, with much more to come!

- Especially with increasing statistics, challenging and interesting problems
  - We have beautiful data that are extremely hard to fit very well
  - We need you!
    …to come up with unique solutions

Go Penguins!



Penguins repeat Stanley Cup with Game 6 win

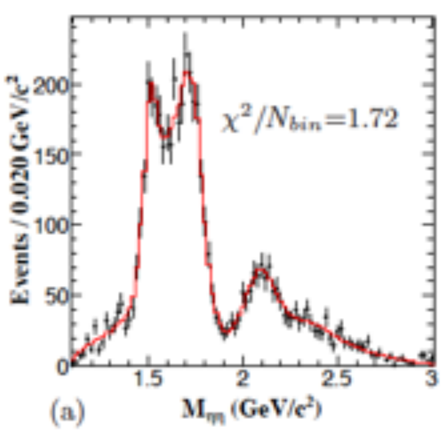


Extra slides

BESIII at BEPCII

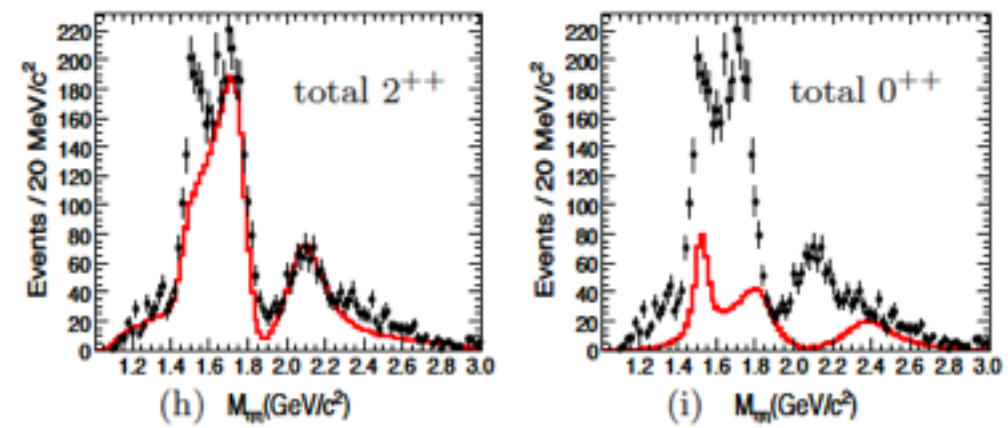
- The physics goals of BESIII cover a diverse range:
  - **Light hadron spectroscopy,** charm physics, τ physics, charmonium physics

PRD 87, 092009 (2013)



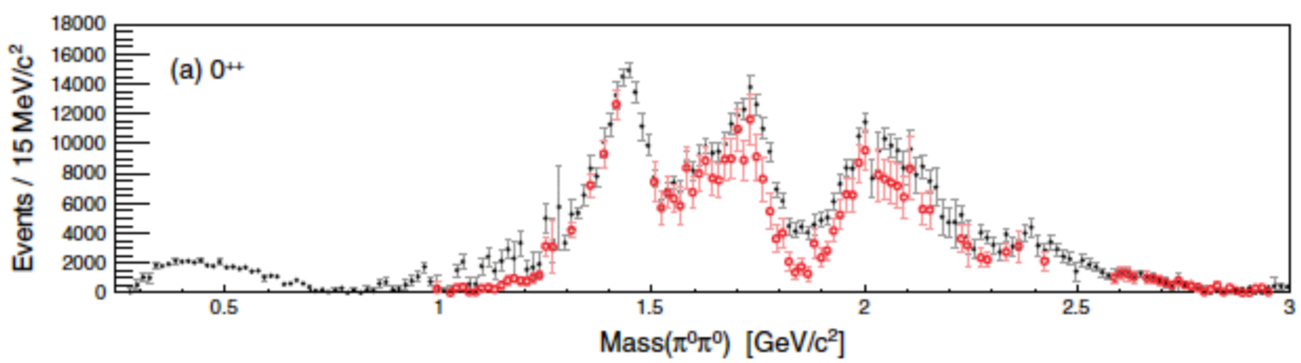
Partial wave analysis of $J/\psi \to \gamma\eta\eta$

- Mass dependent fit with Breit-Wigner lineshapes
- Study existence and dominance of iscoscalar scalar and tensor states



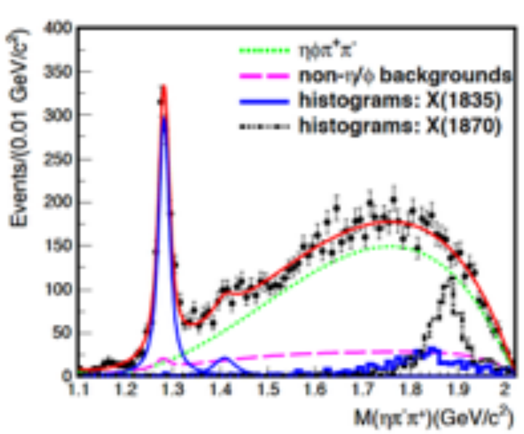
PRD 92 052003 (2015)

Amplitude analysis of the $\pi^0\pi^0$ system produced in radiative J/ψ decays



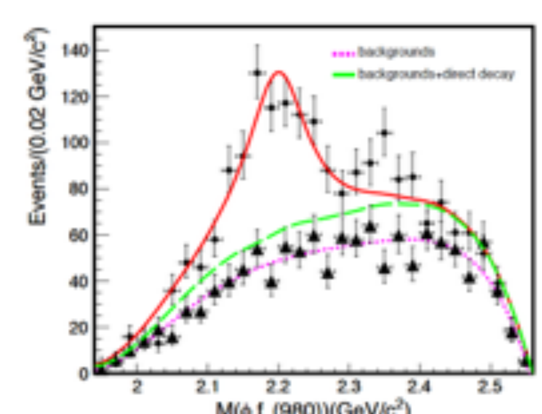
- Mass independent fit to extract a piecewise function that describes the dynamics of the $\pi^0\pi^0$ system is determined as a function of $M_{\pi0\pi0}$

PRD 91, 052017 (2015)



$J/\psi \to \eta\phi\pi^+\pi^-$

- Observed Y(2175): possible strangeonium counterpart of Y(4260)
- Observed η(1295): existence is questionable



BESIII at BEPCII

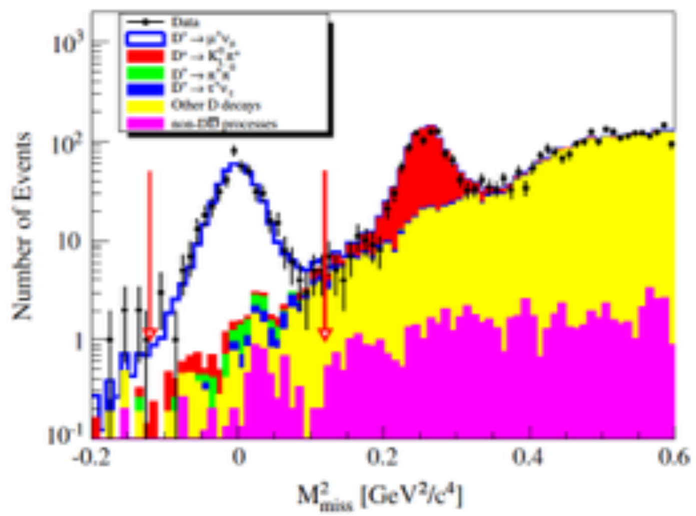
- The physics goals of BESIII cover a diverse range:
  - Light hadron spectroscopy, **charm physics, τ physics,** charmonium physics

Precision measurements of $B(D^+ \to \mu^+\nu_\mu)$

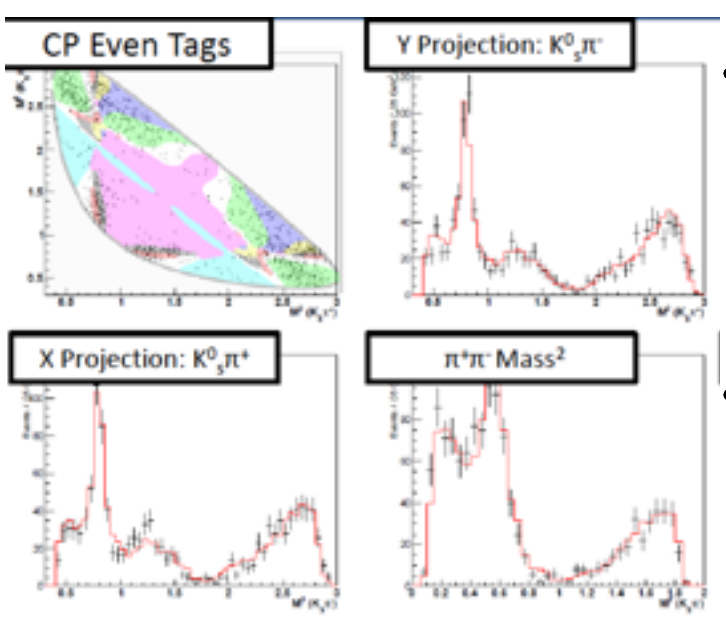
$B(D^+ \to \mu^+\nu_\mu) =$ BESIII: PRD 89, 051104(R) (2014)

$[3.71 \pm 0.19(\text{stat}) \pm 0.06(\text{sys})] \times 10^{-4}$

- Using $|V_{cd}|$ from global SM fit, $f_{D+} = (203.2 \pm 5.3 \pm 1.8)$ MeV
- Using lattice QCD prediction for f_{D+}, $|V_{cd}| = 0.2210 \pm 0.0058 \pm 0.0047$
- In either case, these are the most precise results for these quantities



Measurement of the relative strong-phase



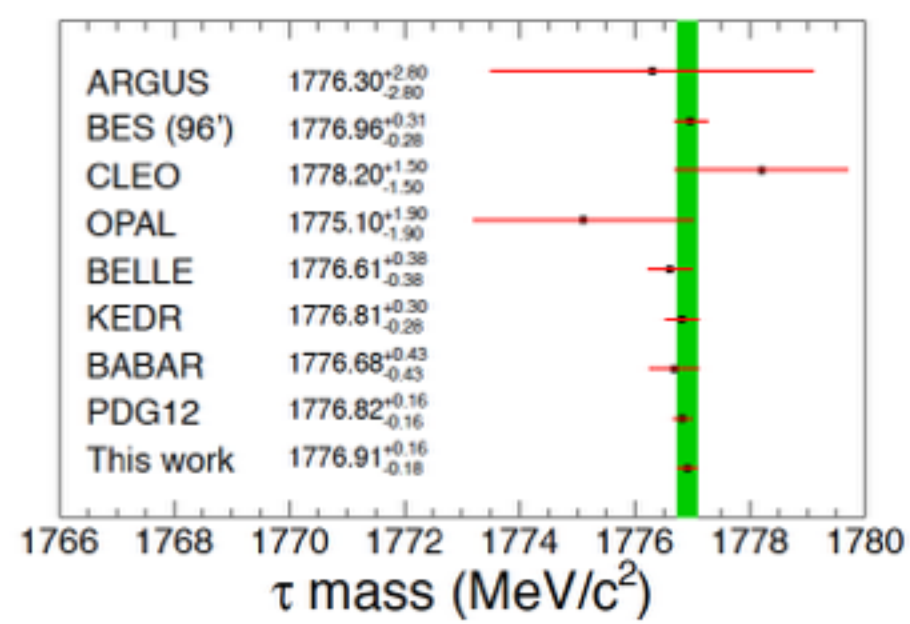
- Model independent measurement of the strong phase difference between $\bar{D}^0$ and D^0 decays to $K^0\pi^+\pi^-$

Preliminary

- Significant improvement in a previously statistically limited measurement

Precision Measurement of the Mass of the τ Lepton

BESIII: PRD 90, 012001 (2014)

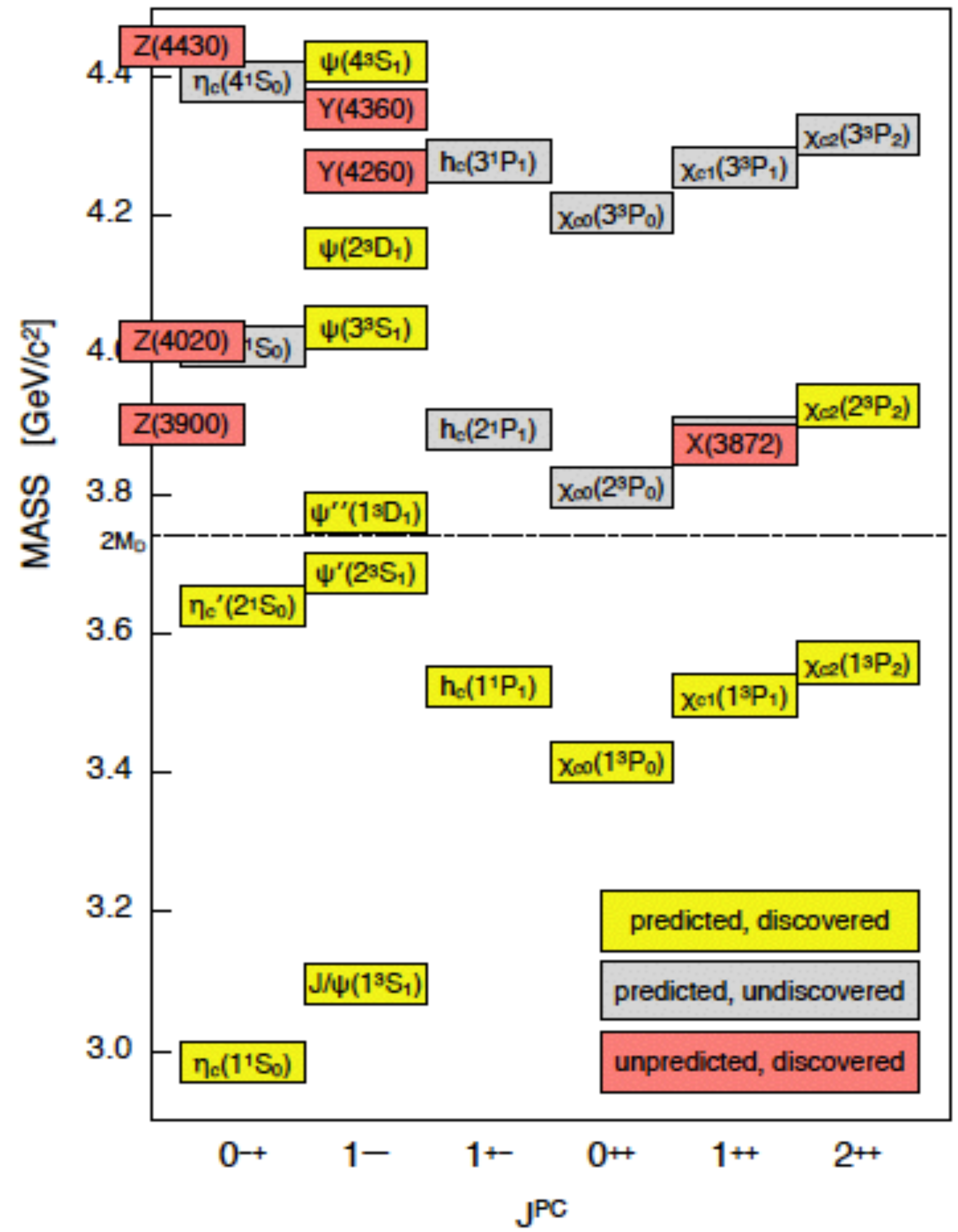


BESIII at BEPCII

- The physics goals of BESIII cover a diverse range:
  - Light hadron spectroscopy, charm physics, τ physics, **charmonium physics**

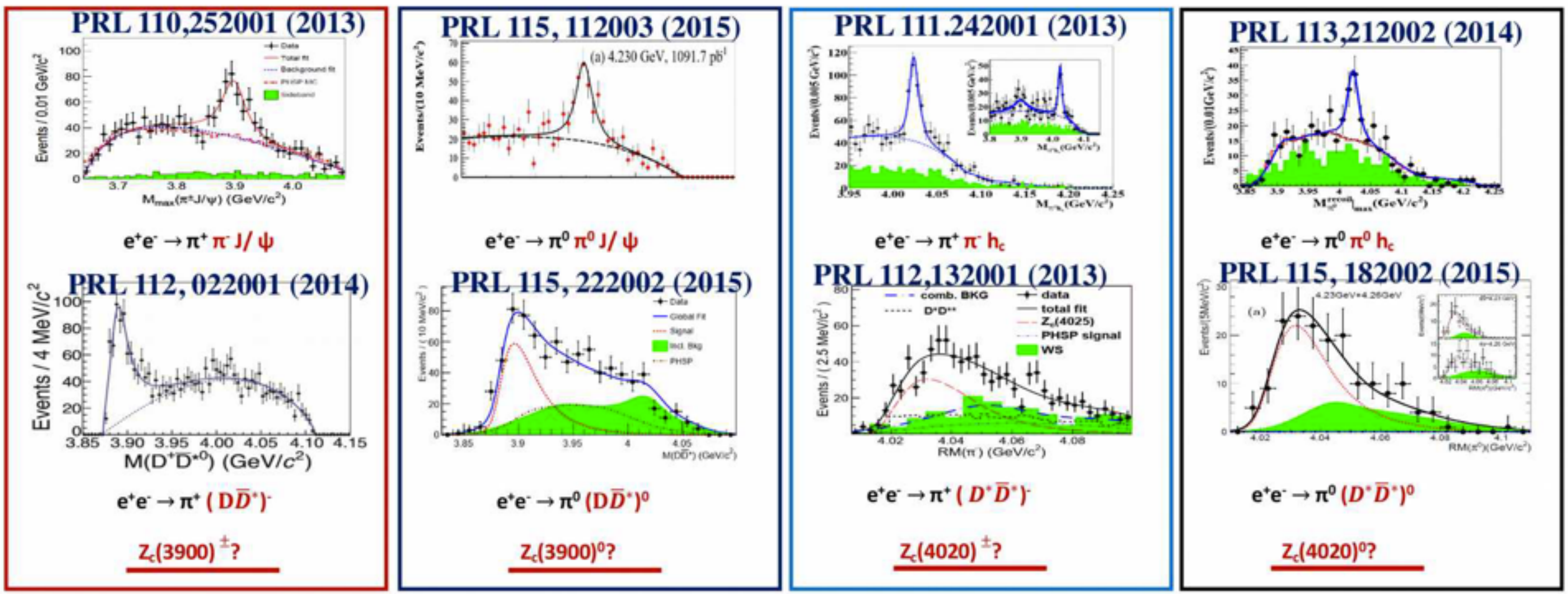
- XYZ physics:
  - $Z_c(3900)^{\pm}$ to $\pi^+\pi^-J/\psi$ (2013)
  - $Z_c(3900)^0$ to $\pi^0\pi^0J/\psi$ (2015)
  - $Z_c(3885)^{\pm}$ to $(DD^*)^{\pm}$ (2014)
  - $Z_c(3885)^0$ to $(DD^*)^0$ (2015)
  - $Z_c(4020)^{\pm}$ to $\pi^+\pi^-$hc (2013)
  - $Z_c(4020)^0$ to $\pi^0\pi^0h_c$ (2014)
  - $Z_c(4025)^{\pm}$ to $(D^*D^*)^{\pm}$ (2013)
  - $Z_c(4025)^0$ to $(D^*D^*)^0$ (2015)
  - Observation of X(3823) (2015)
  - Y states in $\pi^+\pi^-$ J/ψ (2017) and $\pi^+\pi^-h_c$ (2017)
  - …

Figure by R. Mitchell



Summary of Z states observed at BESIII

- Several Z states have been measured in $c\bar{c}$ and open charm final states
- Isospin triplet appears to be established for all of them
- Masses and widths are comparable in measurements to πJ/ψ and D(*)D*



Amplitude analysis

- Use the Intensity function to calculate a (properly normalized) probability to find an event at some position in phase space $\vec{x}$, with model parameters θ:

$$f(\vec{x}|\theta) = \frac{\eta(\vec{x})I(\vec{x}|\theta)}{\int \eta(\vec{x})I(\vec{x}|\theta)dx},$$

and fold in Poisson statistics to obtain a likelihood:

$$L(\vec{x},\theta) = \frac{(e^{-\mu}\mu^N)}{N!}\prod_{i=1}^{N}\frac{\eta(\vec{x}_i)I(\vec{x}_i|\theta)}{\int \eta(\vec{x})I(\vec{x}|\theta)d\vec{x}}.$$

where $\mu = \int \eta(\vec{x})I(\vec{x}|\theta)d\vec{x}$

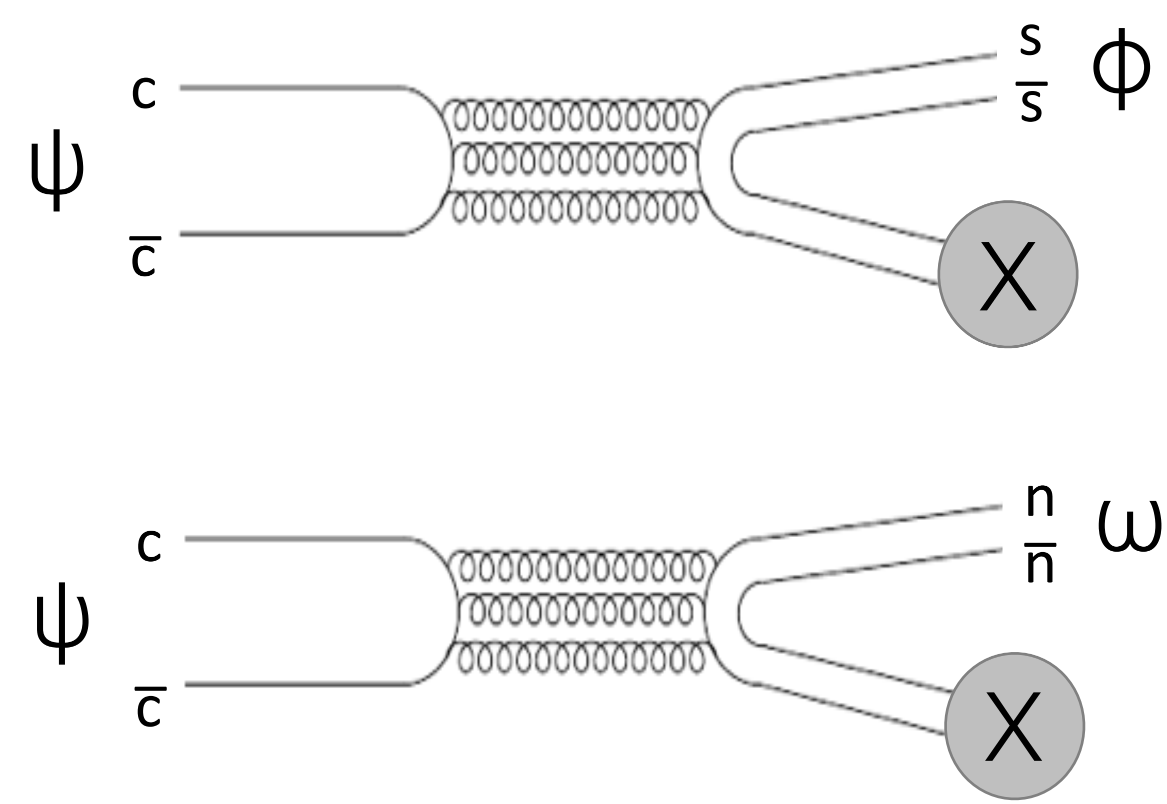
- Take the natural log of the likelihood and cancel like terms (drop terms that are constant in θ):

$$\ln L = \sum_{i=1}^{N}\ln I(\vec{x}_i|\theta) - \int \eta(\vec{x})I(\vec{x}|\theta)d\vec{x},$$

This term is approximated using a phase space MC sample

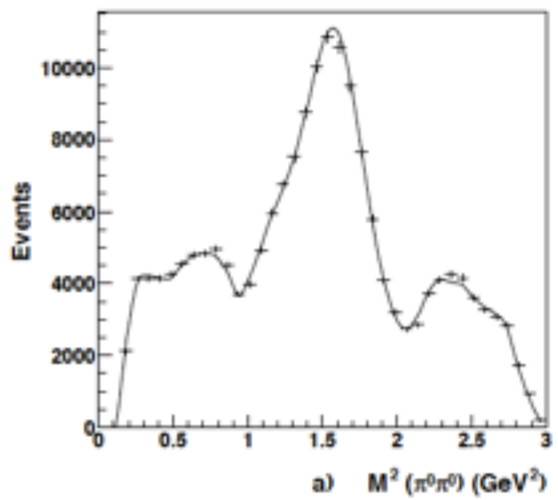
Hadron spectroscopy with charmonium decays

- BESIII has world leading samples of J/ψ and ψ' decays
- "Glue-rich" environment
  - The J/ψ and ψ' masses are below open charm threshold, so OZI suppressed processes dominate
  - Suppression factor on radiative decays due to fine structure constant only about a factor of 10
  - Radiative decays about 8% of the total cross section
- (Naive) Flavor-tagging with decays to light mesons

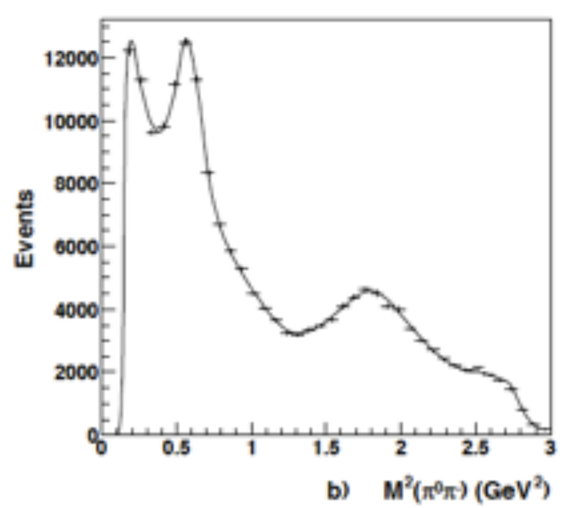


K-matrix (for S-wave)

Reconstruct 00^{++} wave based on several data sources

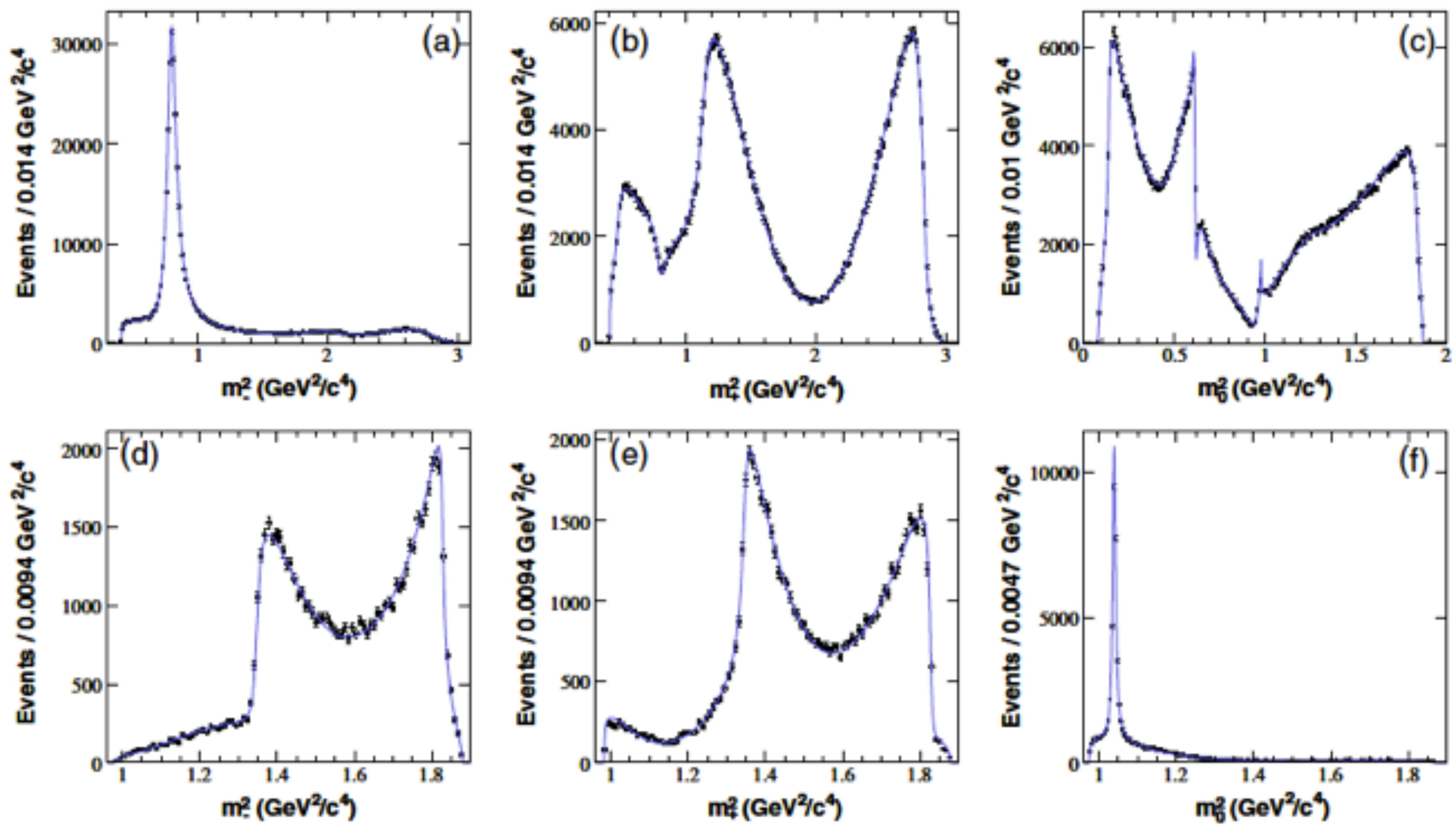


Crystal Barrel data: $p\bar{p}$ annihilation into $\pi^0\pi^0\pi^-$ in liquid D_2



Use parameterization in amplitude analysis of $D \to K_S\pi^+\pi^-$

BaBar: $B \to D^{(*)}K^{(*)}$, $D \to K_S\pi^+\pi^-$ and $K_SK^+K^-$



Sum of fit fractions with K-matrix (isobar) model of S-wave: 103.6% (122.5%)

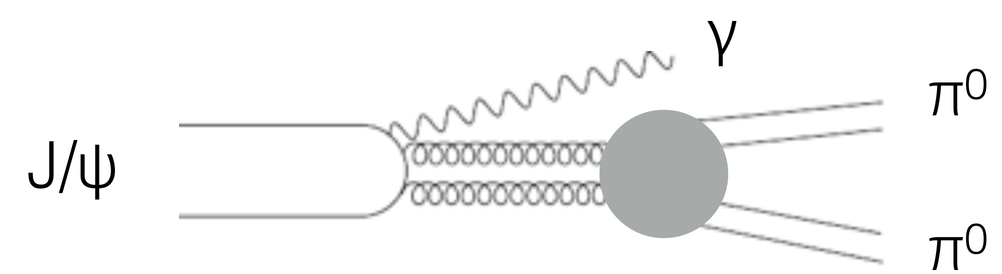
χ^2/DOF = 1.11 (1.20)

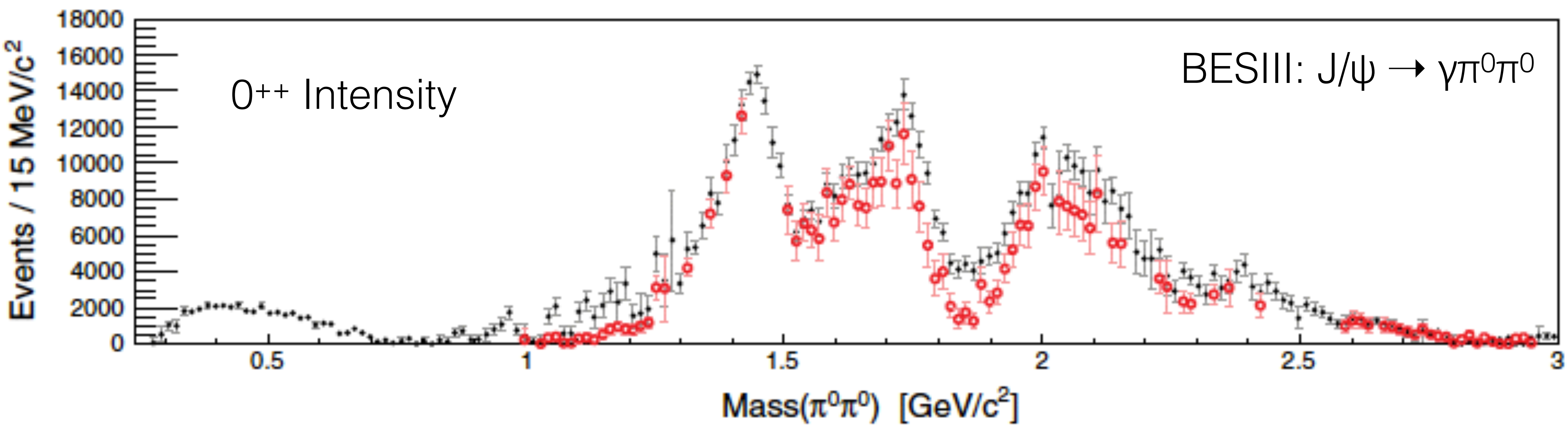
Eur. Phys. J. A16, 229-258 (2003)

PhysRevD.78.034023 (2008)

Could also extract S-wave information from J/ψ decays?

- Extract scalar spectrum from $J/\psi \rightarrow \gamma PP$ (eg. $\gamma\pi^0\pi^0$) in a model independent way
  - Easily produced in e^+e^- collisions
  - No interaction with final state photon
  - Very clean neutral channel (backgrounds ~2%) and relatively simple amplitude analysis (J^{PC} = even^{++} only)
  - Results may be combined with those of similar reactions for a more comprehensive study of the light scalar meson spectrum

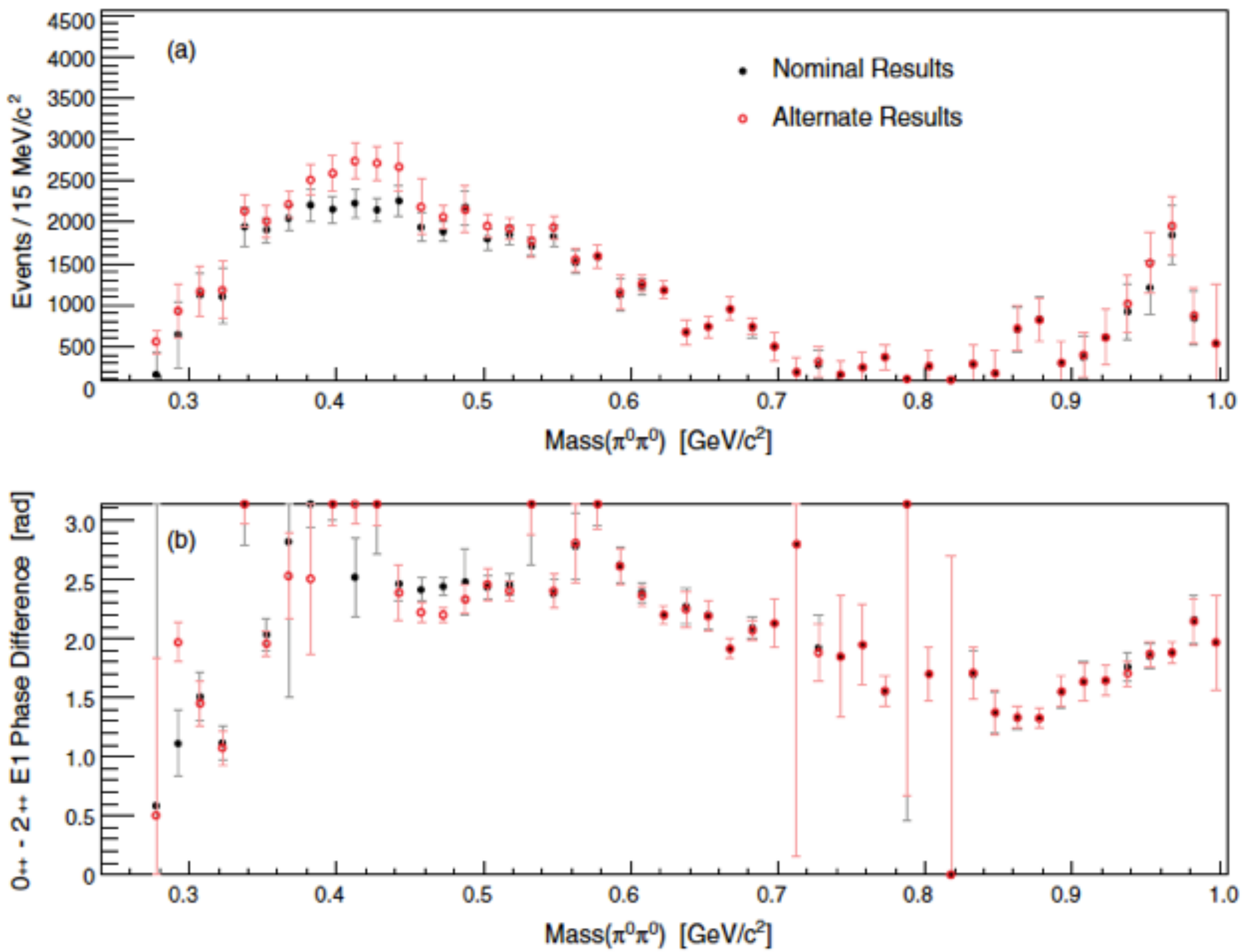




PhysRevD.92.052003 (2015)

$J/\psi \to \gamma\pi^0\pi^0$: Alternate Results

- Nominal results include subtraction of γη(‘) backgrounds
- Repeat analysis without background subtraction (assume only signal events)
- Difference between the nominal and alternate results gives a very conservative estimate of systematic effect from γη(‘) backgrounds



PhysRevD.92.052003 (2015)

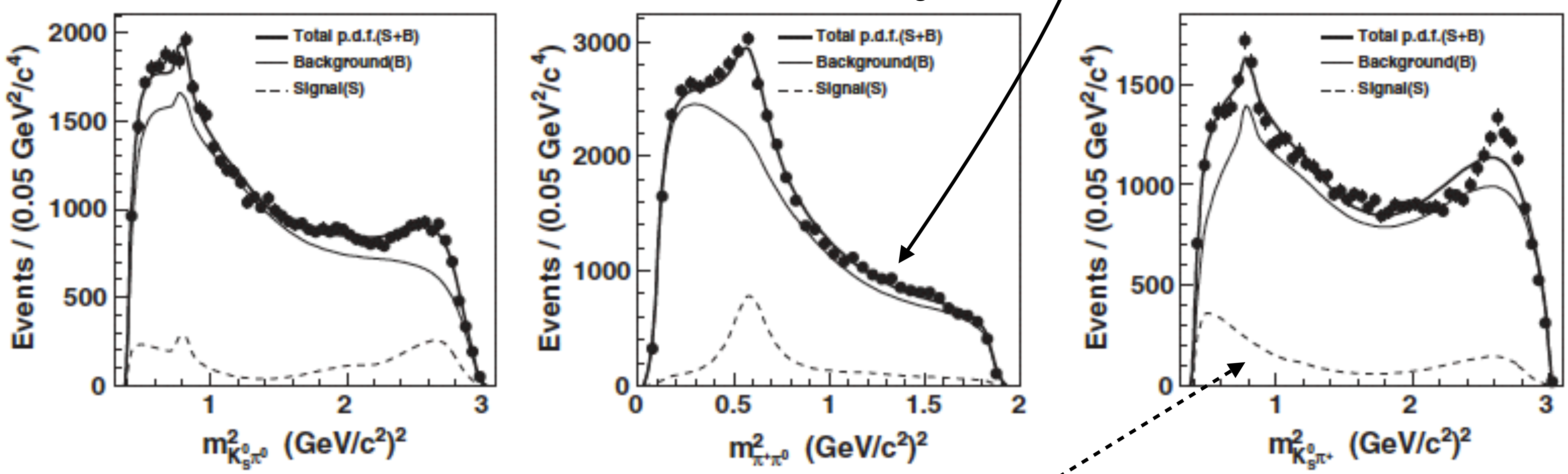
Note on background subtraction

- Not all events in the data sample are signal events!
- Approximate the effect of backgrounds using a MC sample or sidebands (better description, but comes with challenges) and remove with a term in the likelihood
  - Parametrize backgrounds and include in PDF

$$\mathcal{P}(x,y) = f_S \frac{|\mathcal{M}(x,y)|^2 \varepsilon(x,y)}{\int_{DP} |\mathcal{M}(x,y)|^2 \varepsilon(x,y)dxdy} + f_{B1} \frac{B_1(x,y)}{\int_{DP} B_1(x,y)dxdy} + f_{B2} \frac{B_2(x,y)}{\int_{DP} B_2(x,y)dxdy}$$

$$f_S + f_{B1} + f_{B2} \equiv 1$$

BESIII: $D^+ \rightarrow K_S \pi^+ \pi^0$

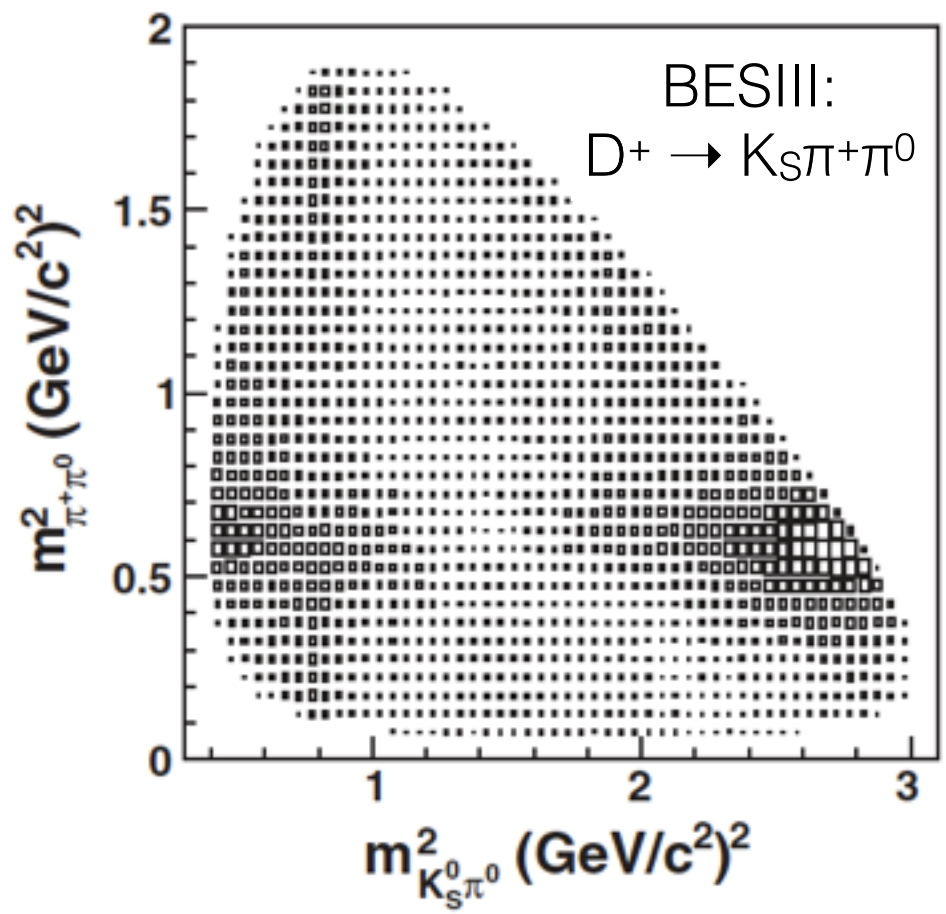


Tail of signal distribution

PhysRevD.89.052001 (2014)

Importance of final state interactions

- Long distance strong interaction effects can cause significant changes in decay rates and phases of decay amplitudes
  - Rich substructure in Dalitz plot spectra indicate complexity of FSI



- Use weak three-body decays of open heavy flavor mesons to study interference between intermediate resonances
  - More kinematic freedom than two-body final states
  - Intermediate resonances dominate and cause non-uniform distribution of events in Dalitz plot

- Better understanding of final state interactions in D decays is important to reduce uncertainties related to D^0-$\overline{D}^0$ mixing parameters and of the angle γ

PhysRevD.89.052001 (2014)