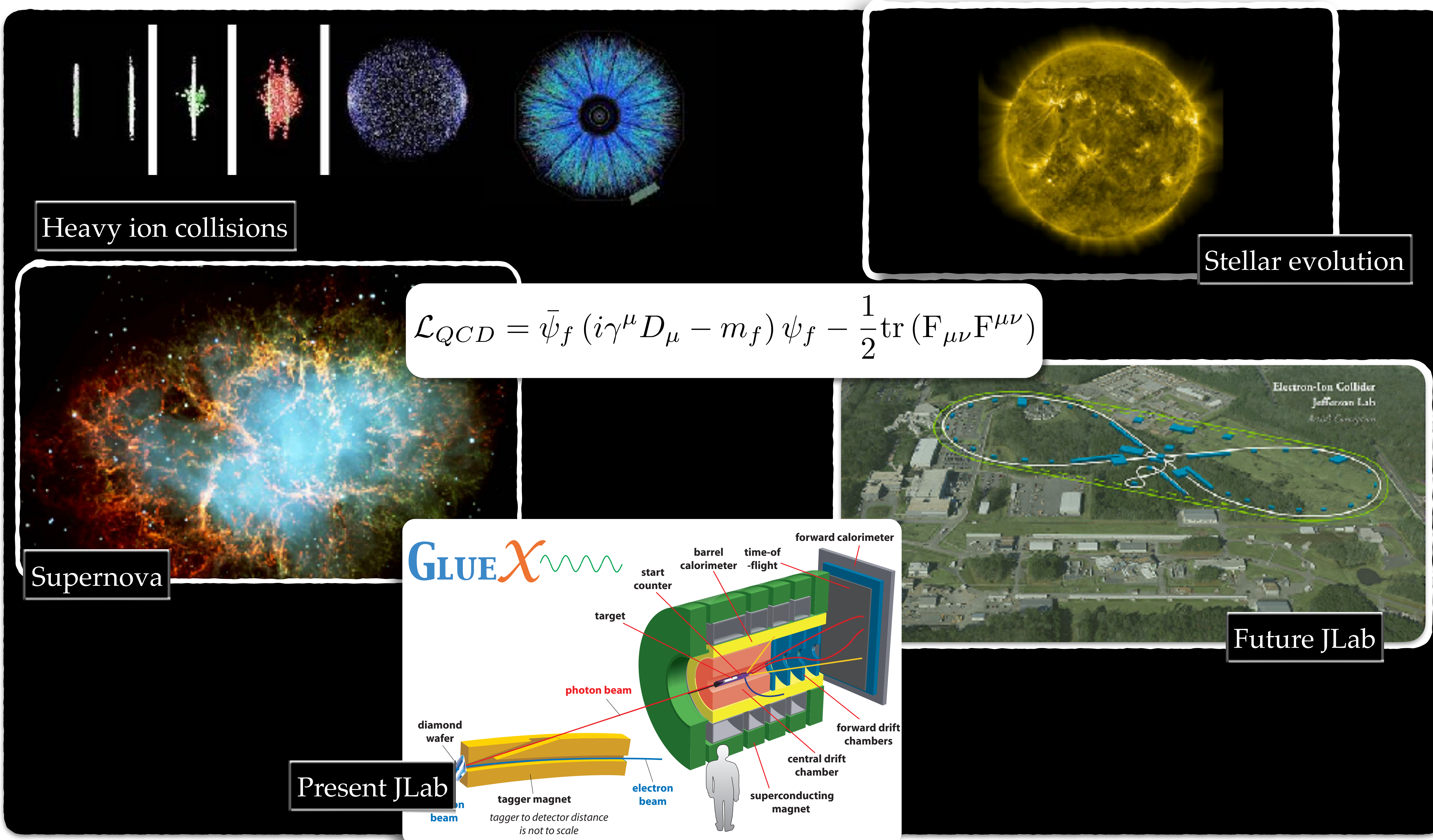
Scattering from lattice QCD

Raúl Briceño

hadspec Jefferson Lab OLD DOMINION UNIVERSITY

Quantum Chromodynamics

"The fundamental theory of the strong nuclear force"



$$\mathcal{L}_{QCD} = \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f - \frac{1}{2} \mathrm{tr} \left(\mathrm{F}_{\mu\nu} \mathrm{F}^{\mu\nu} \right)$$

Resonances in experiments

- the vast majority of QCD states are resonances

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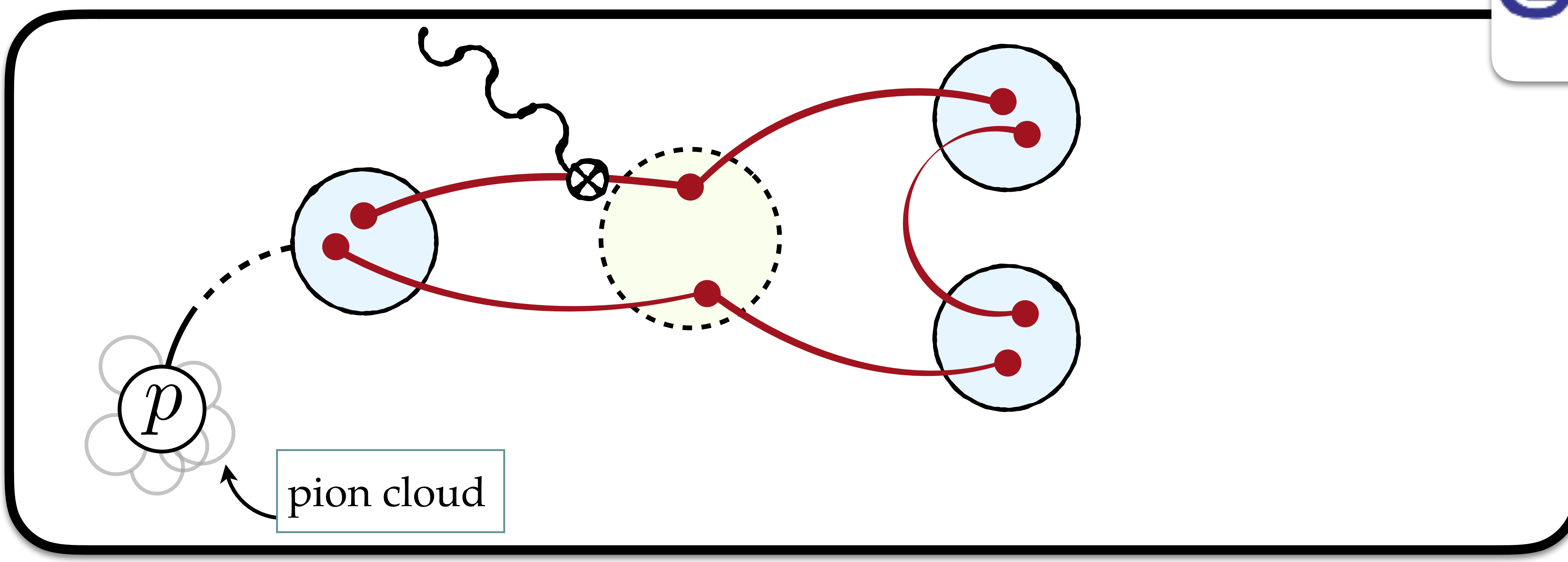

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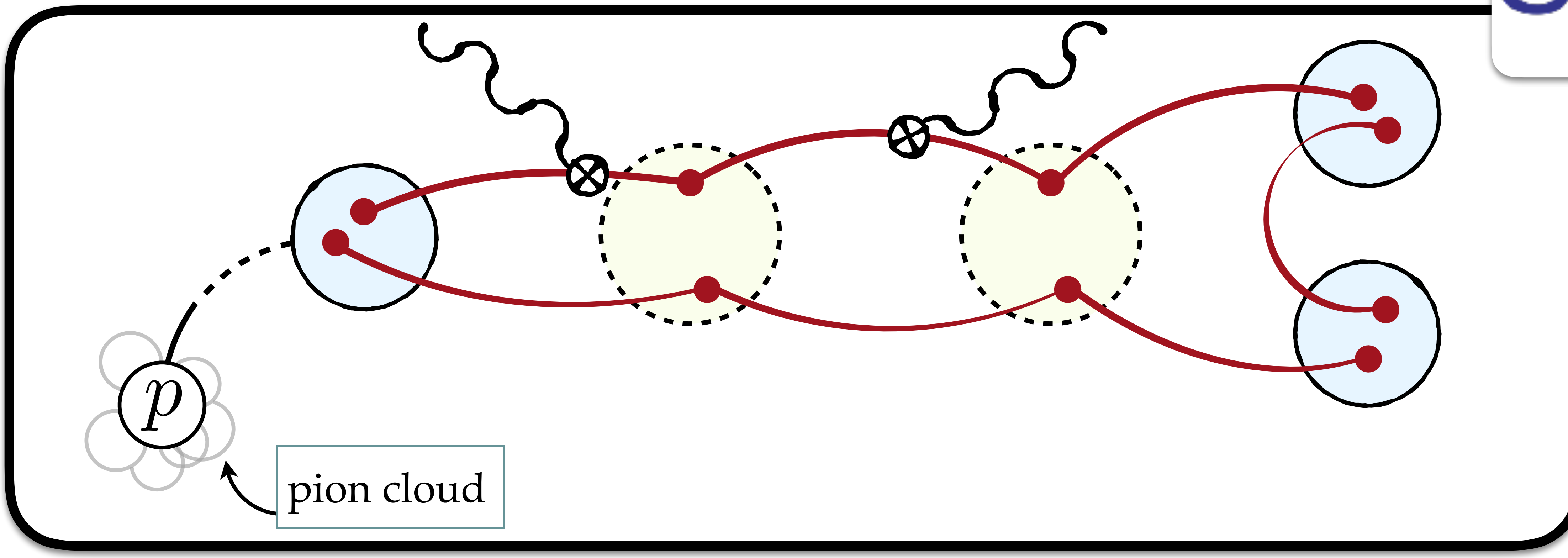
Resonances in experiments

- Probing resonances experimentally is *"hard"*



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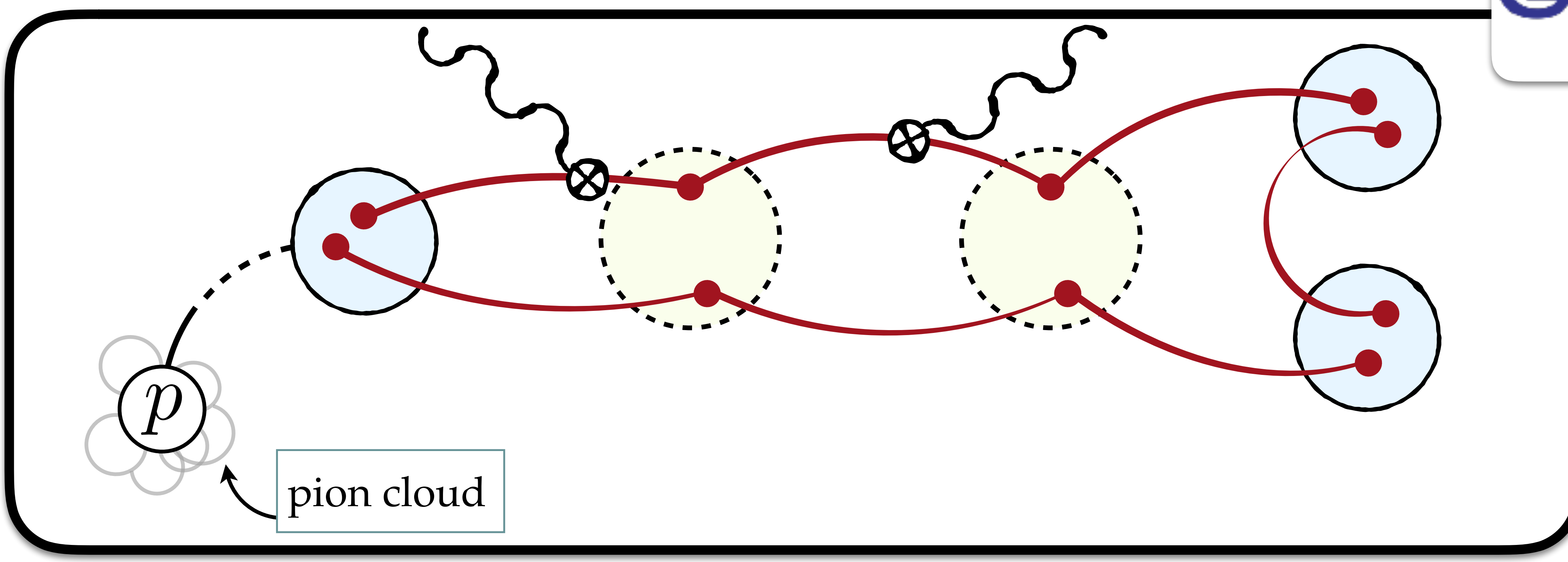
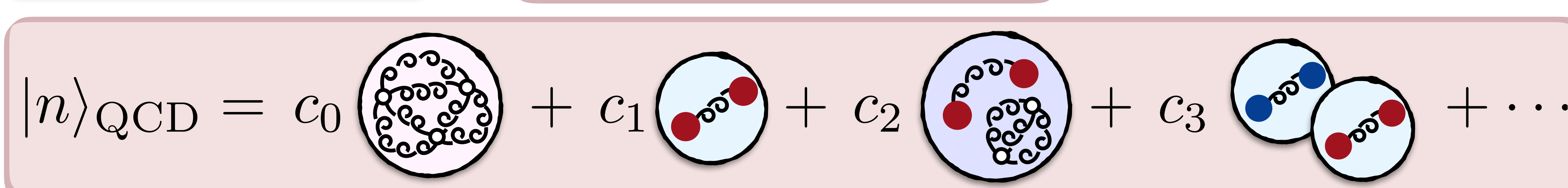


experimental demands

- confirmation
- production mechanism [couplings]
- identification of prominent decay channels
- couplings to decay channels

theoretical demands

- structural understanding



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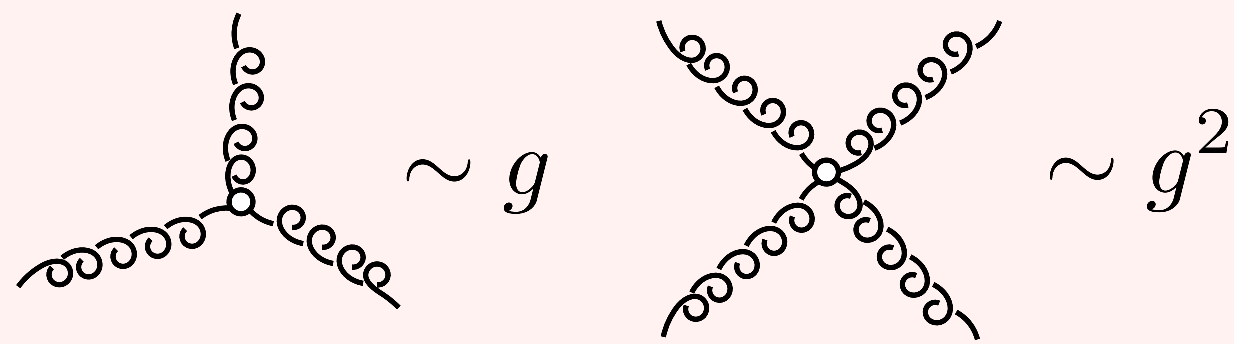

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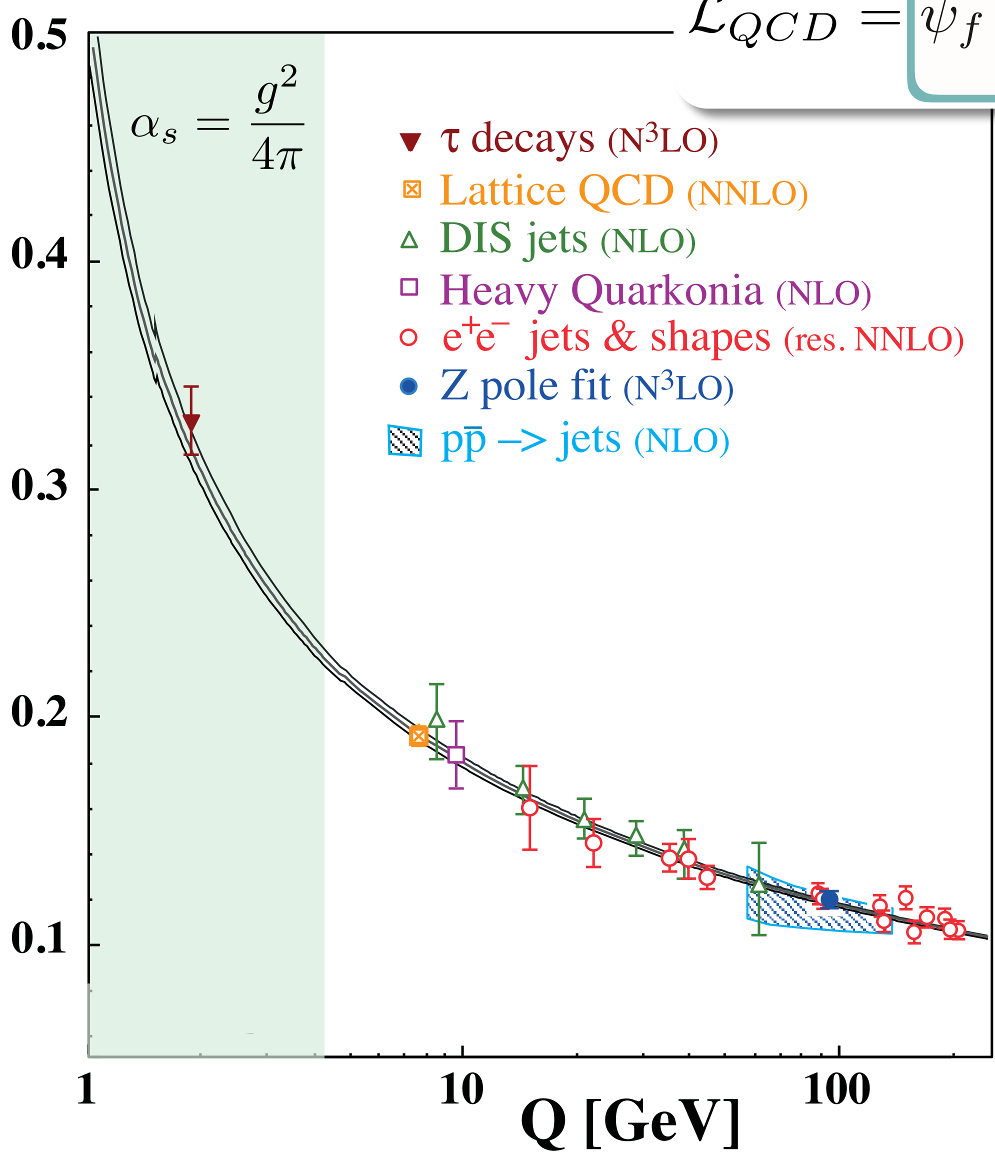
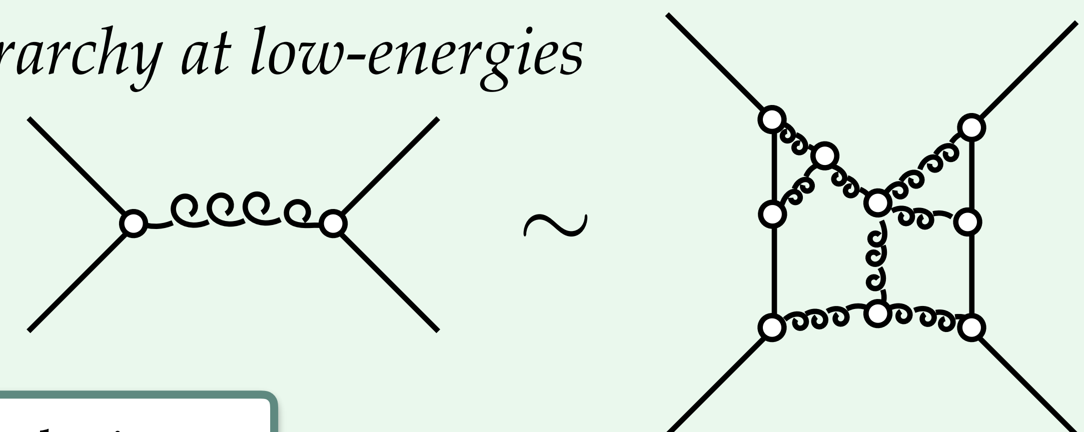


No hierarchy at low-energies



non-perturbative....

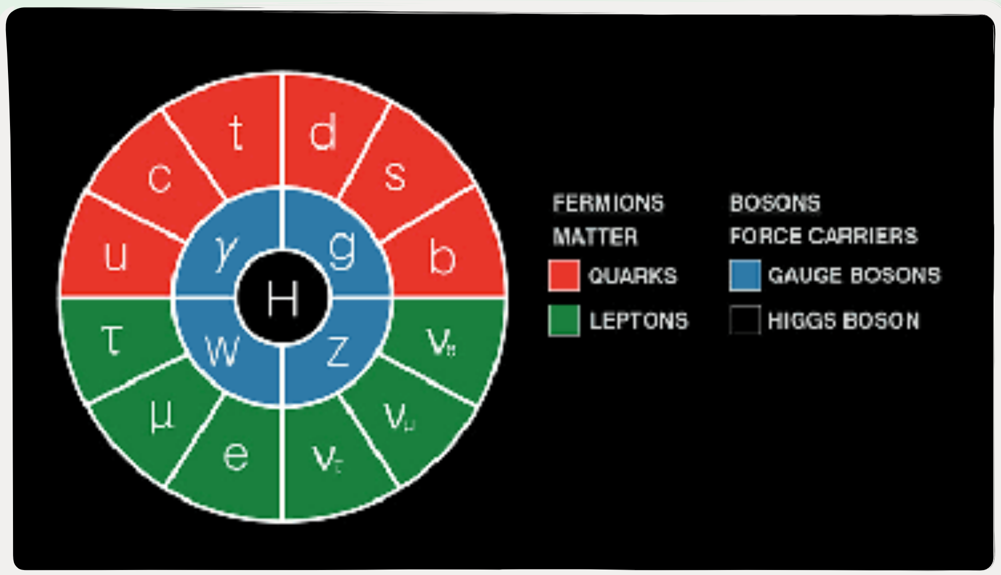
- *confinement?*
- *origin of mass?*
- *formation of matter*
- ...

Desperate for a Nobel Prize?

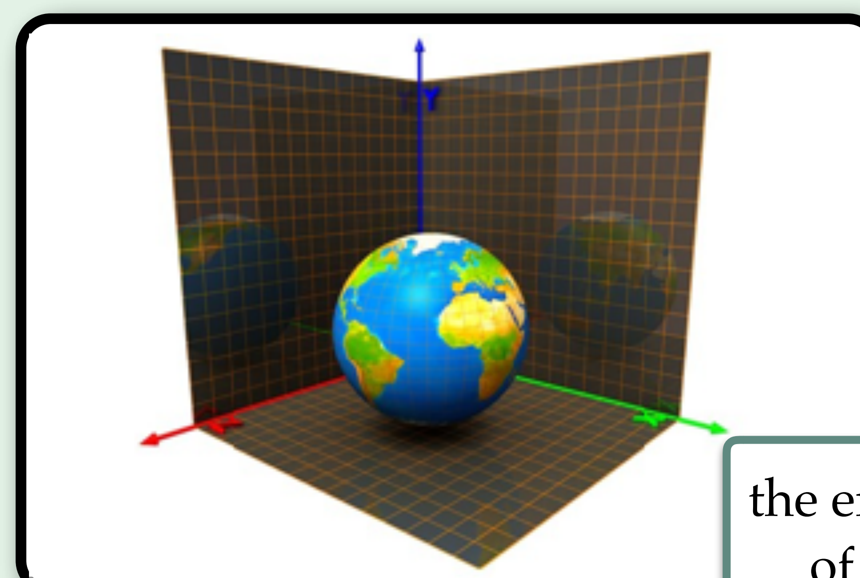
Lattice QCD

In summary:

- QCD is non-perturbative
- Solution: *be smart* and *let computers do the hard work!*

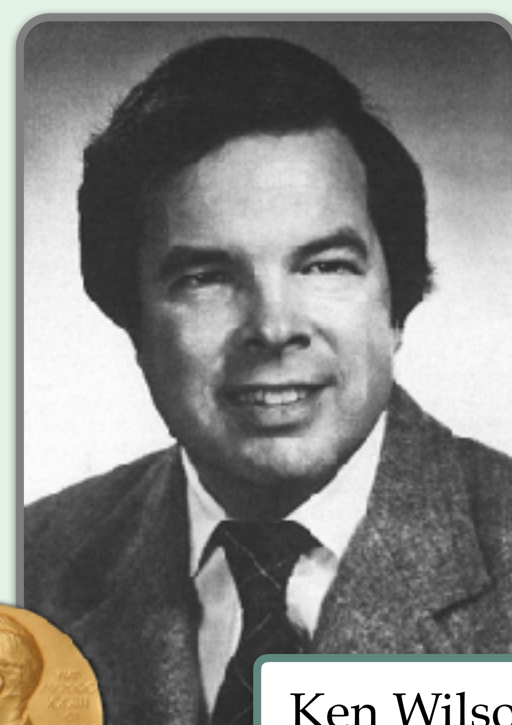


JLab's cluster



the emergence of nature

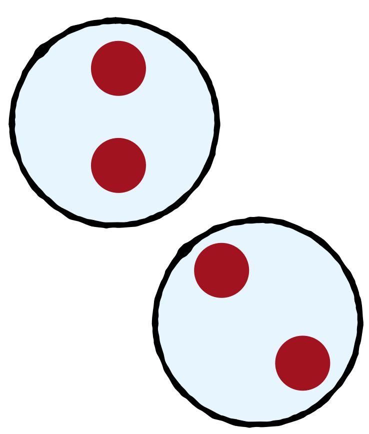
Richard Feynman



Ken Wilson

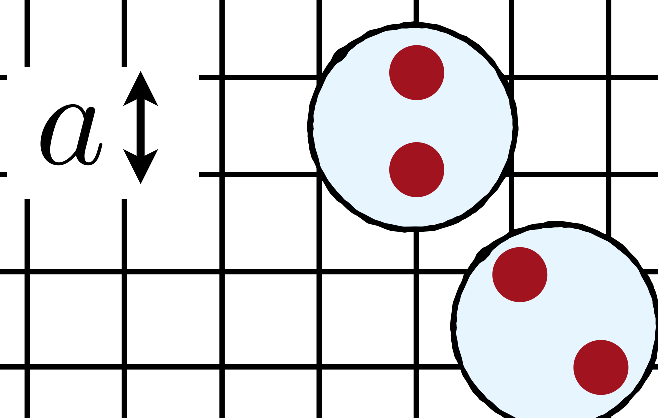
Lattice QCD

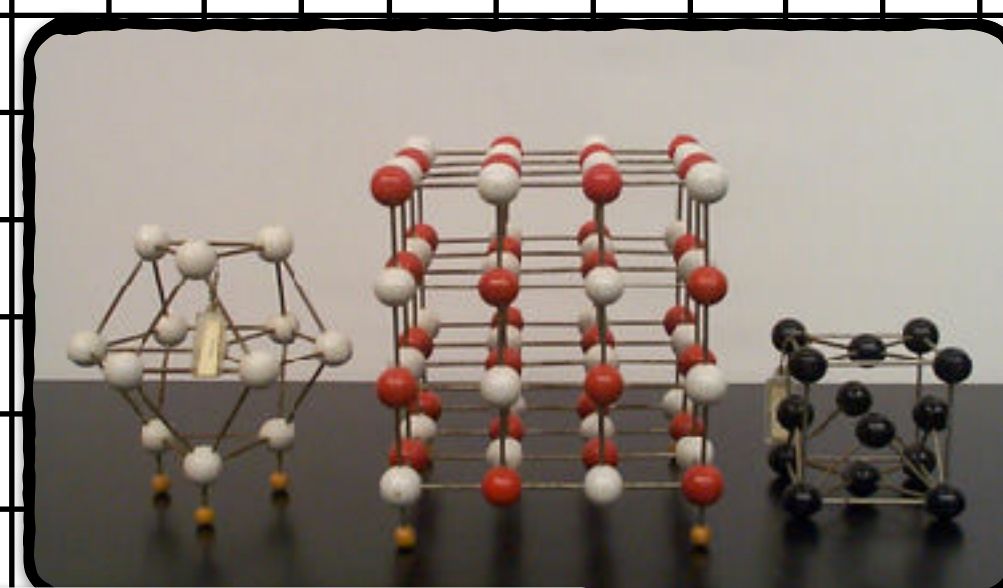
- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monte Carlo sampling



Lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monte Carlo sampling
- lattice spacing: $a \sim 0.03 - 0.15$ fm

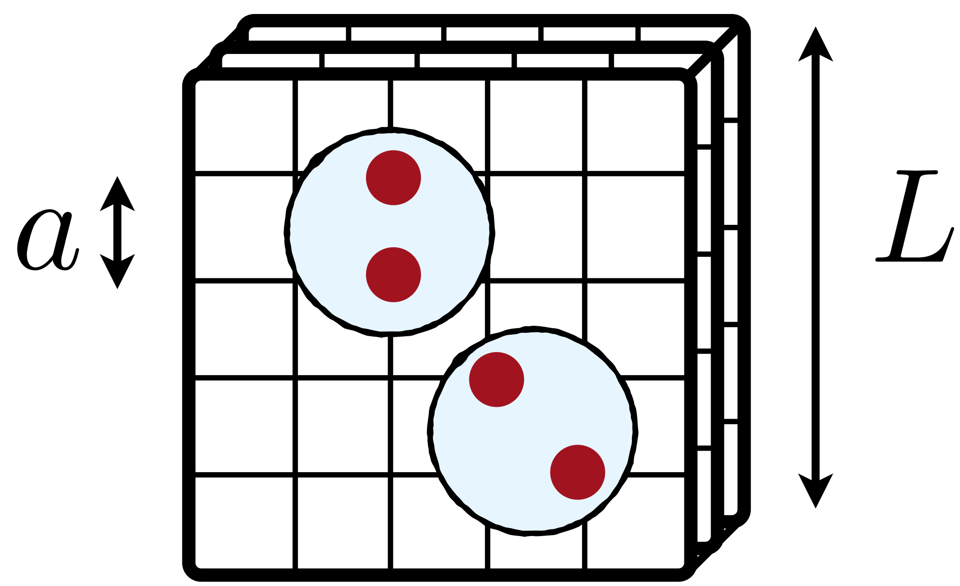




more familiar lattices

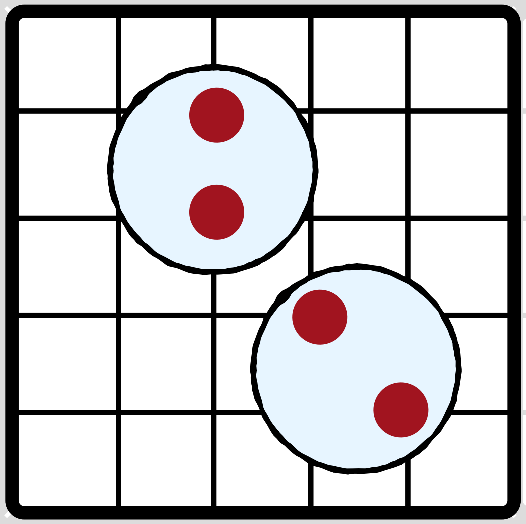
Lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monte Carlo sampling
- lattice spacing: $a \sim 0.03 - 0.15$ fm
- finite volume



Lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
- Monte Carlo sampling
- lattice spacing: $a \sim 0.03 - 0.15$ fm
- finite volume [periodic…]



Never free!

No asymptotic states!

No scattering!

Lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \to -it_E$
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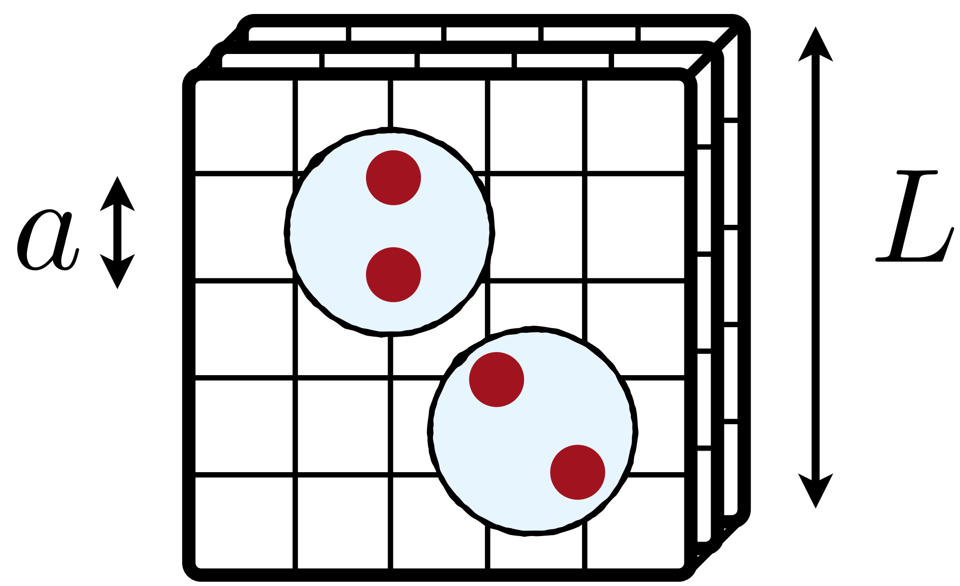
- lattice spacing: $a \sim 0.03 - 0.15$ fm
- finite volume
- quark masses: $m_q \to m_q^{\text{phys.}}$



Advantage over experiment!

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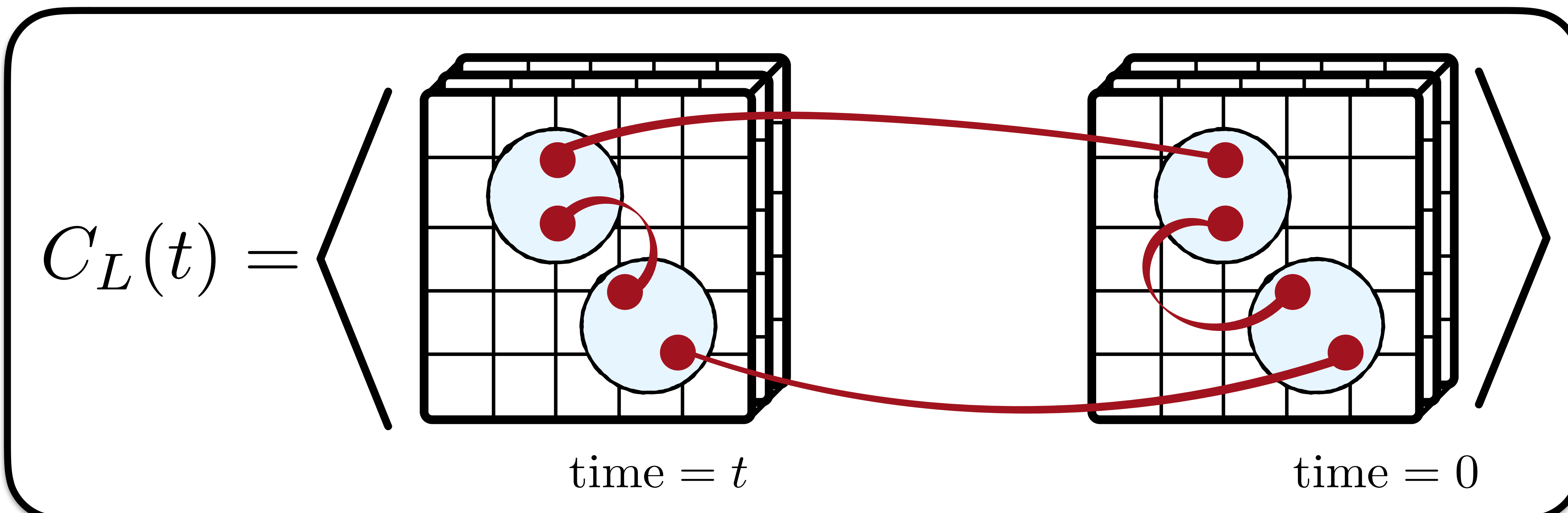
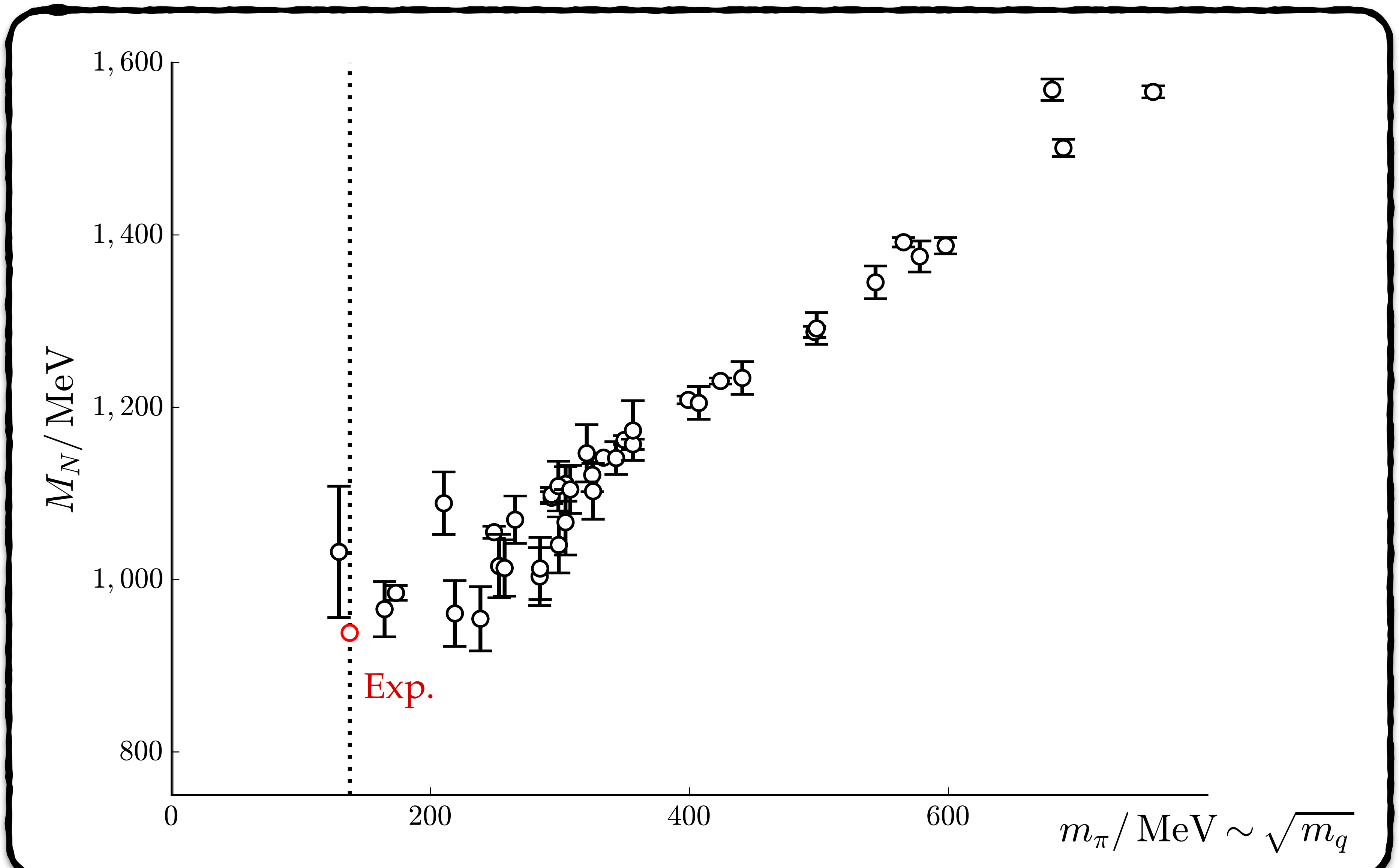


Status of spectroscopy

- *Simple* properties of QCD stable states [non-composite states]
  - physical or lighter quark masses [down to m_π~120 MeV] ✓
  - non-degenerate light-quark masses: N_f=1+1+1+1 ✓
  - dynamical QED ✓



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  - non-degenerate light-quark masses: $N_f=1+1+1+1$ ✓
  - dynamical QED ✓
- Frontier of lattice: multi-particle physics
  - scattering / reactions
  - composite states
    - bound states
    - hadronic resonances

Formal development:
- under way
- more needed

Benchmark calculations:
- exploratory
- proof of principle
- unphysical quark masses
- …

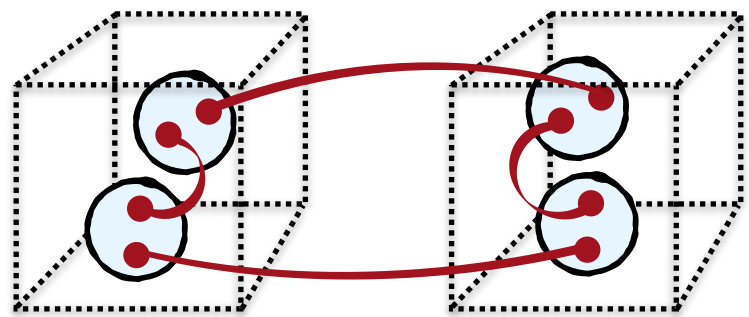
Questions?

meet Jazzi. Jazzi likes long walks in the park and chasing squirrels.

Outline

QCD: m_q / Λ_{QCD} →

Correlation functions

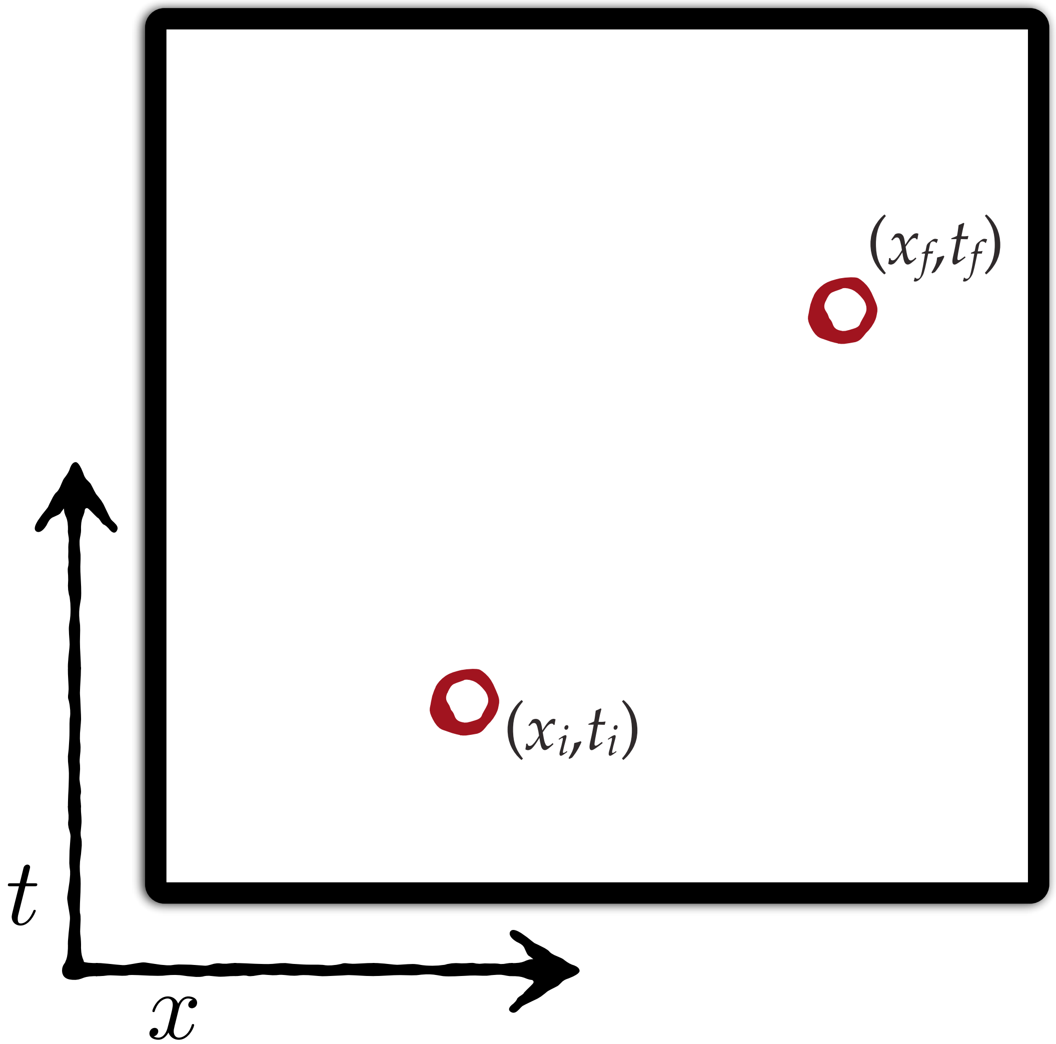


Path-integral in quantum mechanics

- Imagine a world where quarks are free to propagate

I measured a quark at (x_i,t_i), the probability of finding it at (x_f,t_f) is:

$$\langle x_f, t_f | x_i, t_i \rangle = \langle x_f | e^{i\hat{H}t_f} e^{-i\hat{H}t_i} | x_i \rangle = \langle x_f | e^{i\hat{H}(t_f - t_i)} | x_i \rangle$$



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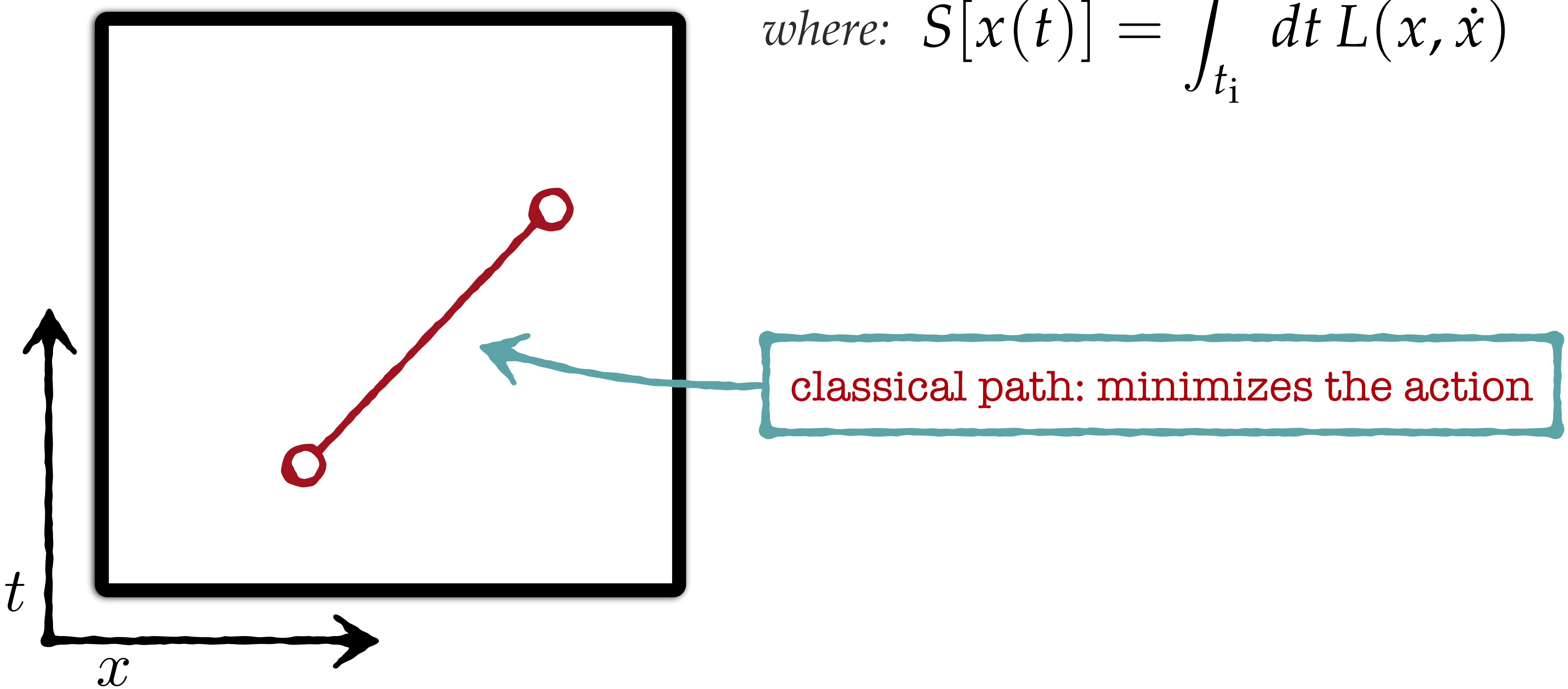
where: $S[x(t)] = \int_{t_i}^{t_f} dt\, L(x, \dot{x})$



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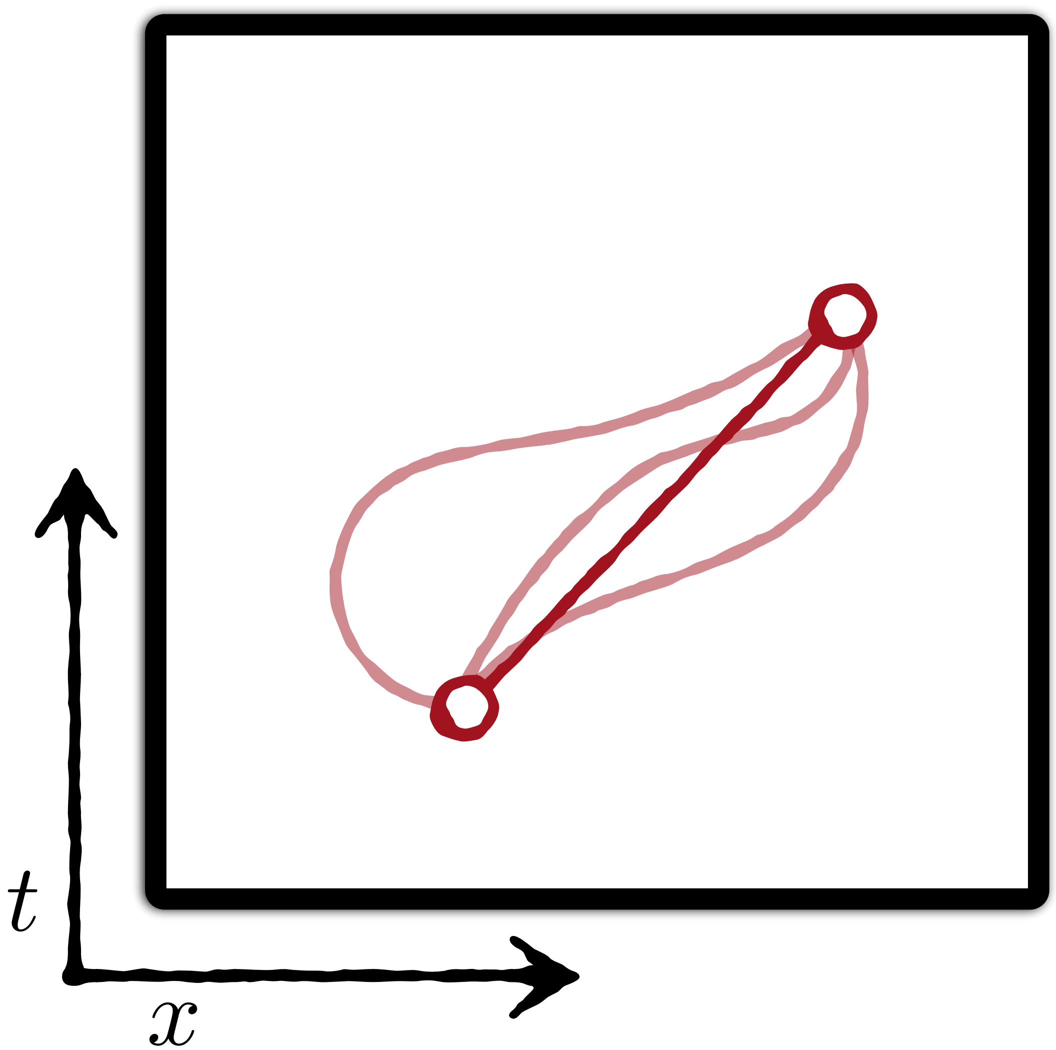

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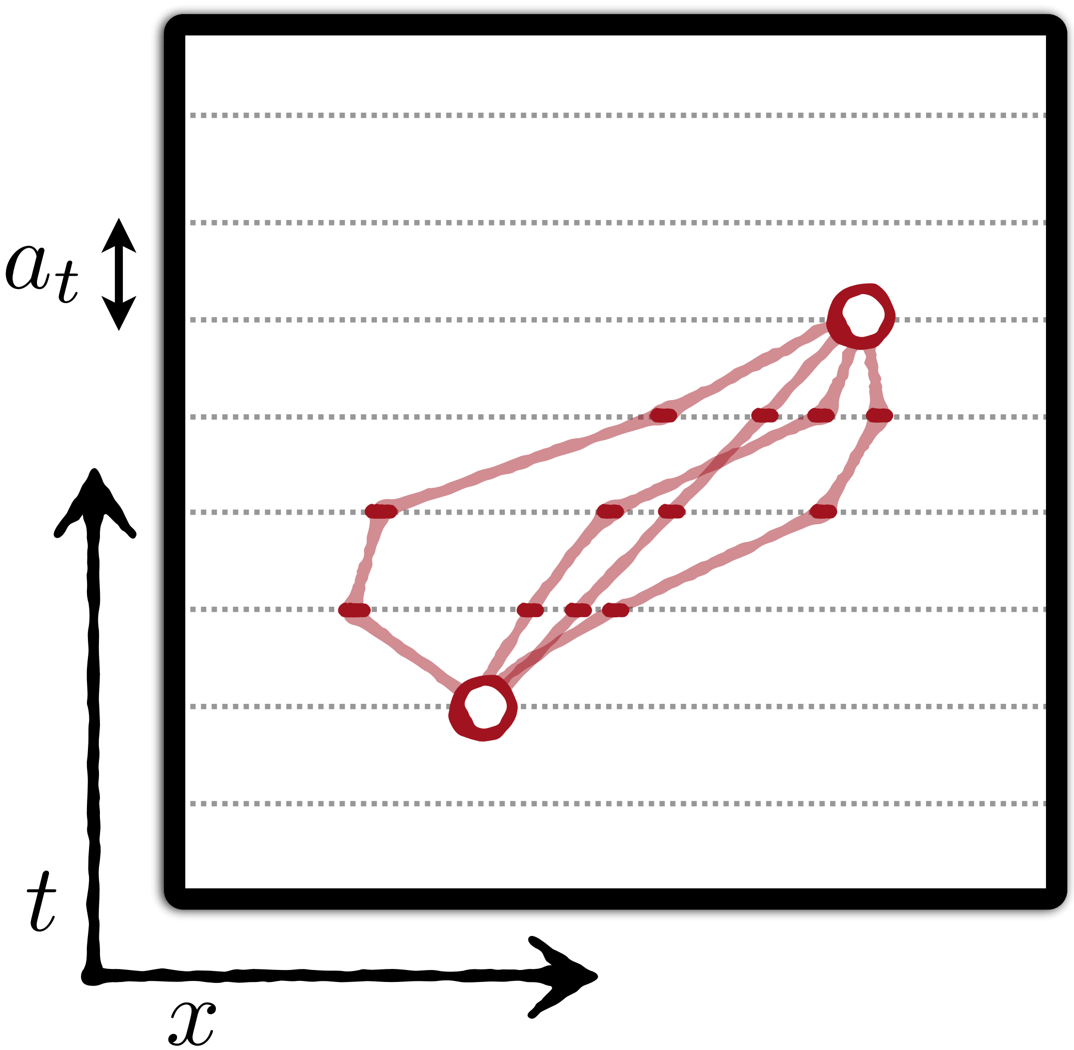

remember, this is how you derive the path integral representation

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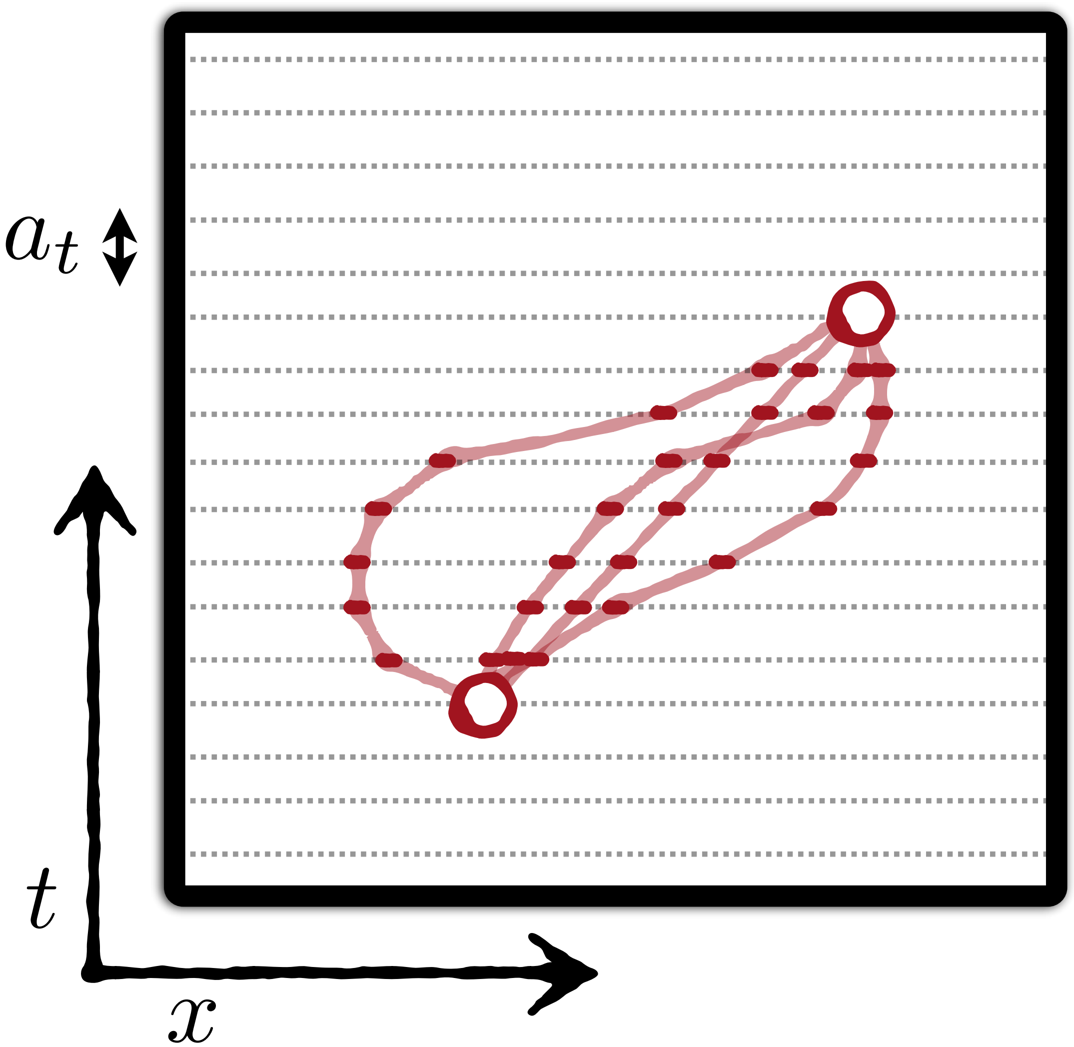

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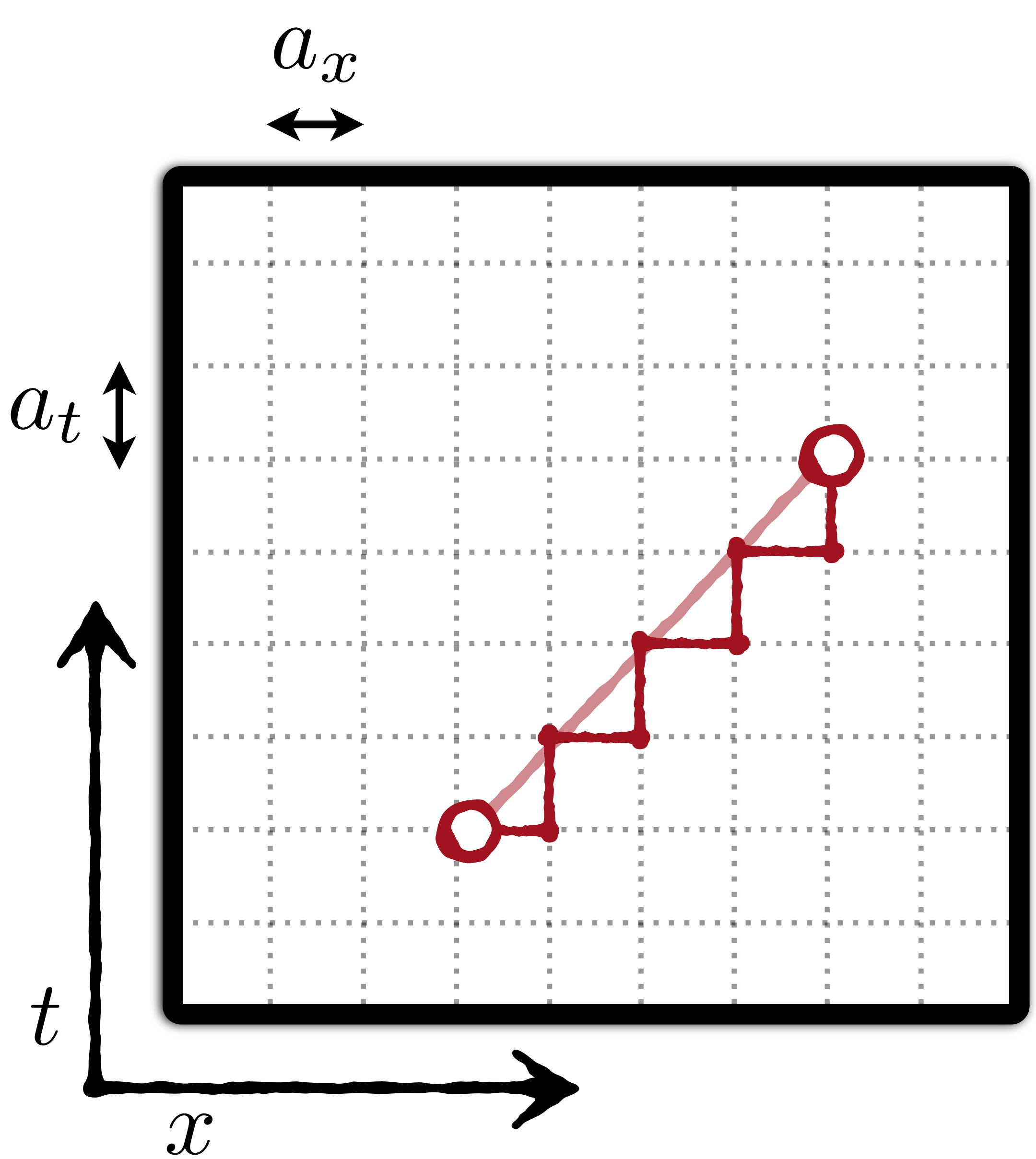
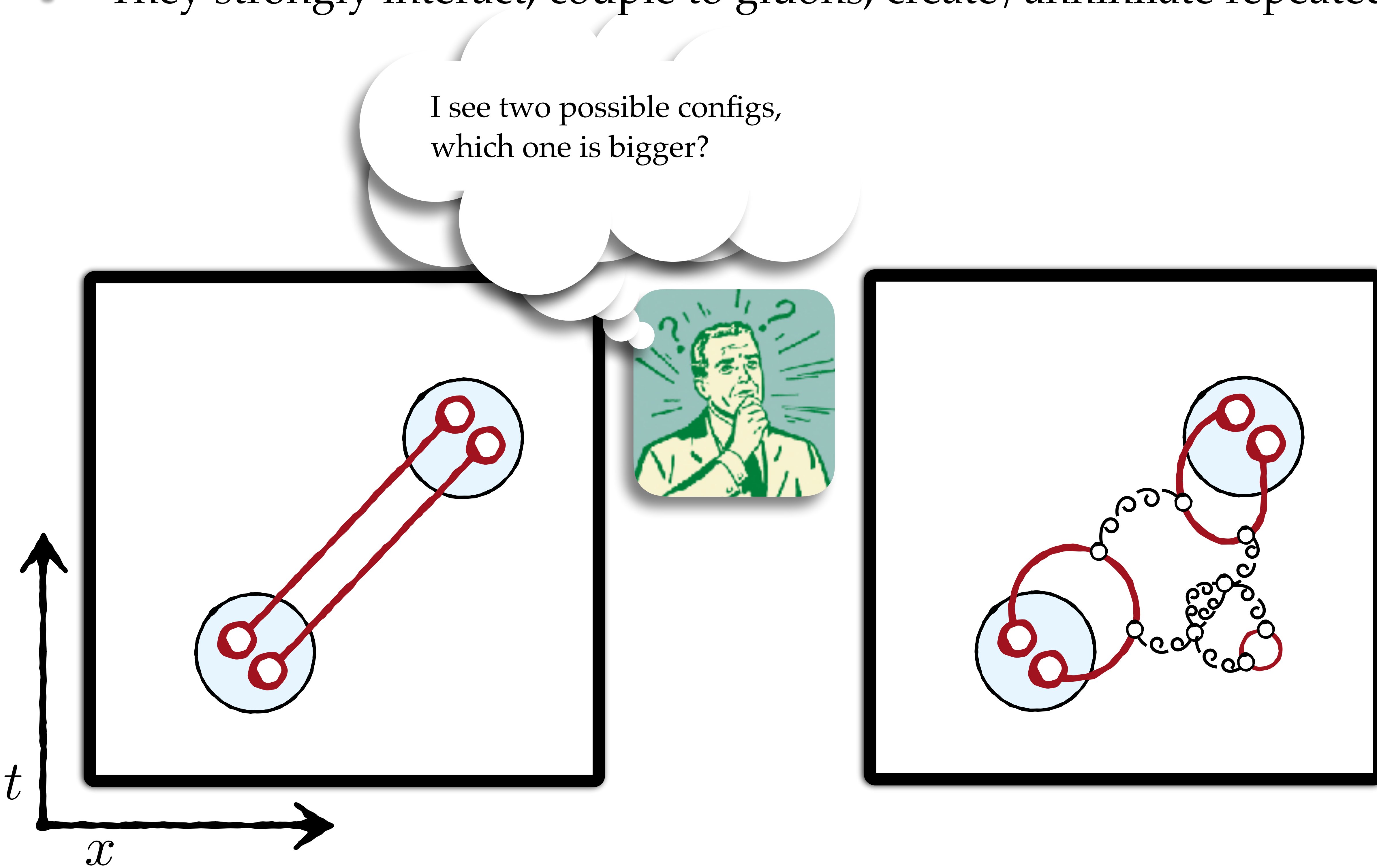


can evaluate this numerically by introducing a mesh in spacetime

Path-integral in quantum field theory

- Quarks aren't free, they live inside bound states
- They strongly interact, couple to gluons, create/annihilate repeatedly



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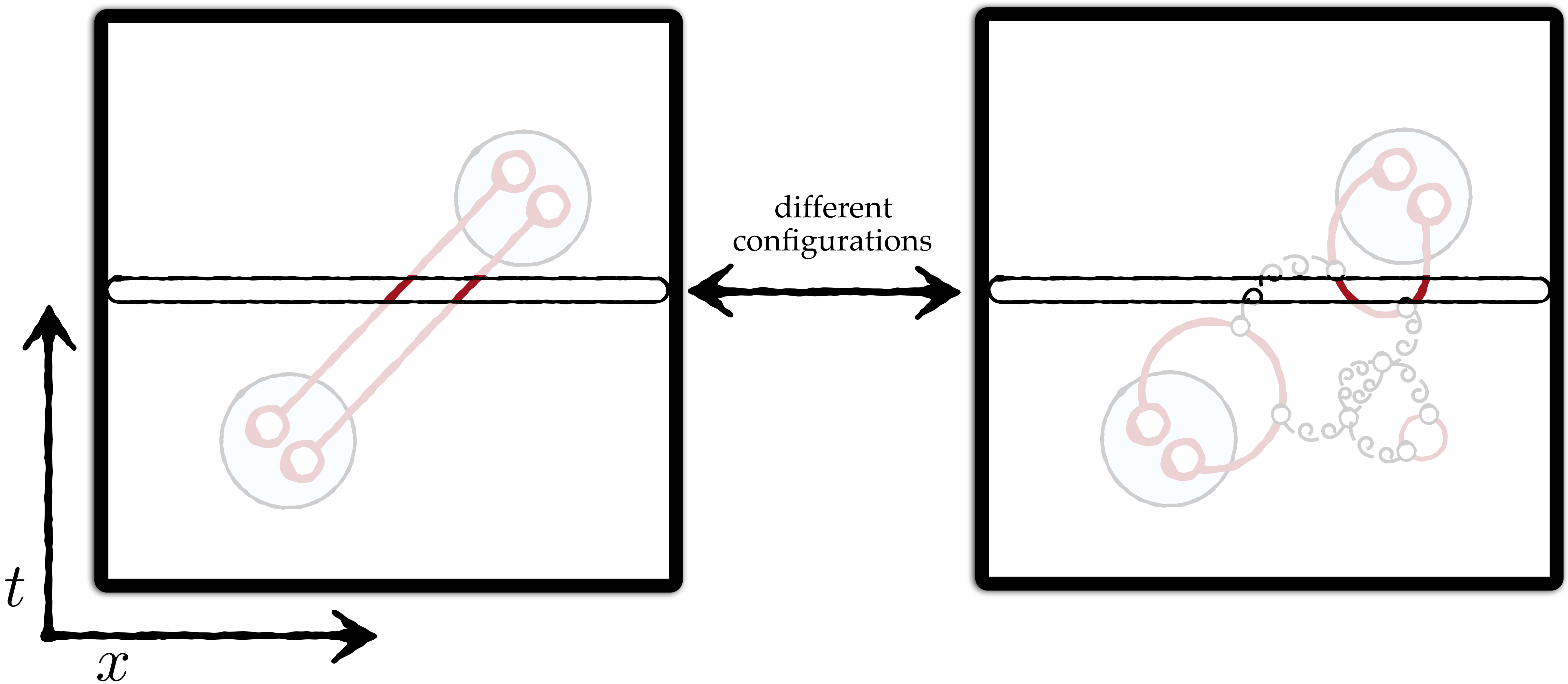
- Need to "sum" over all configurations
- Path integral: $Z = \int \mathcal{D}\varphi(x)\ e^{iS[\varphi]}$

$$S = \int d^3x\, dt\, \mathcal{L}$$



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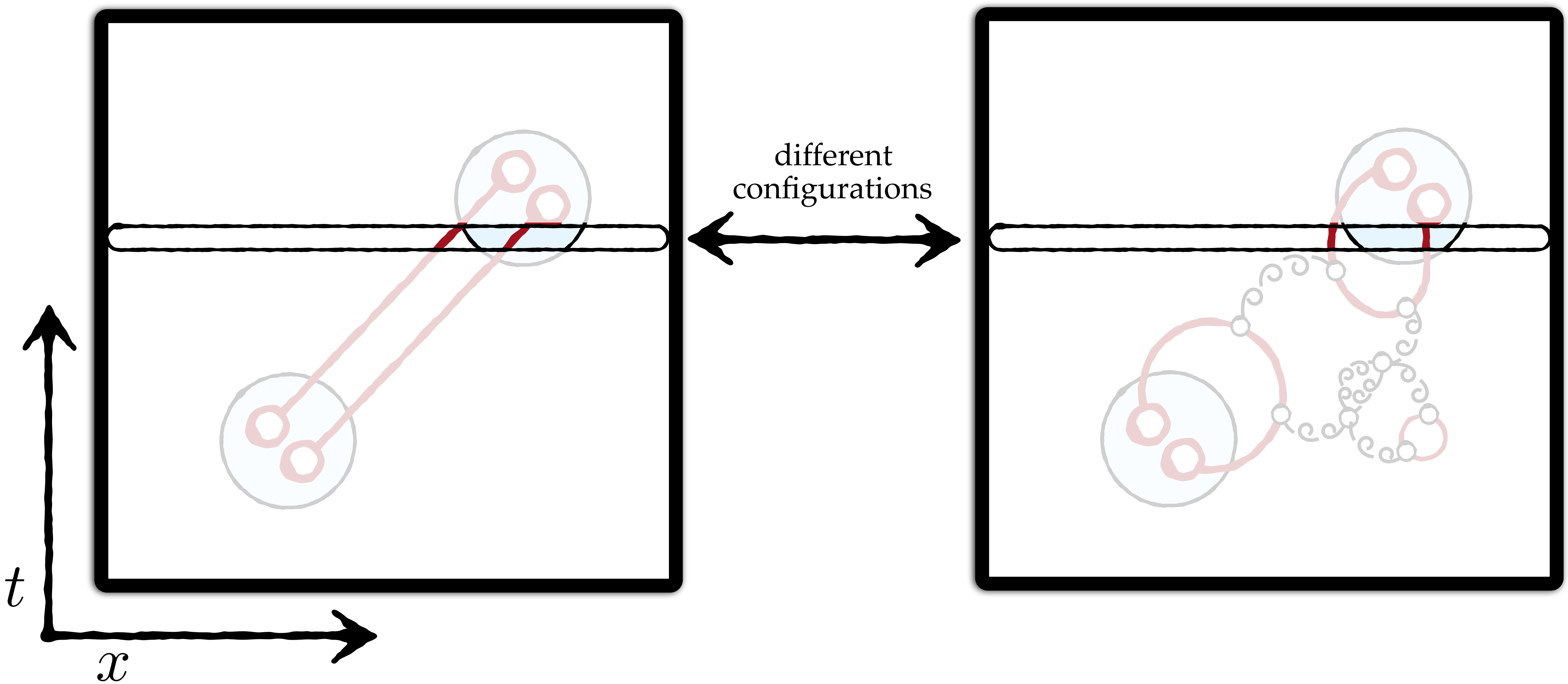
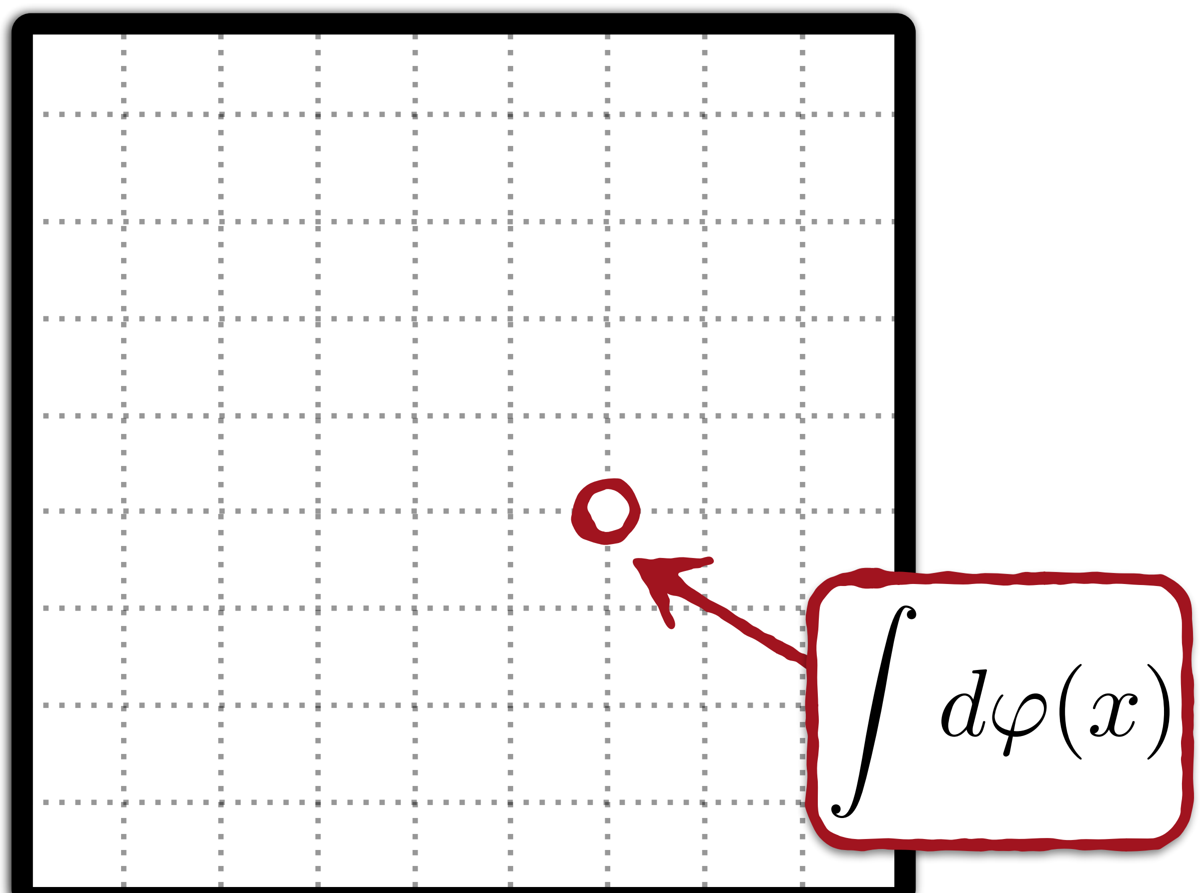
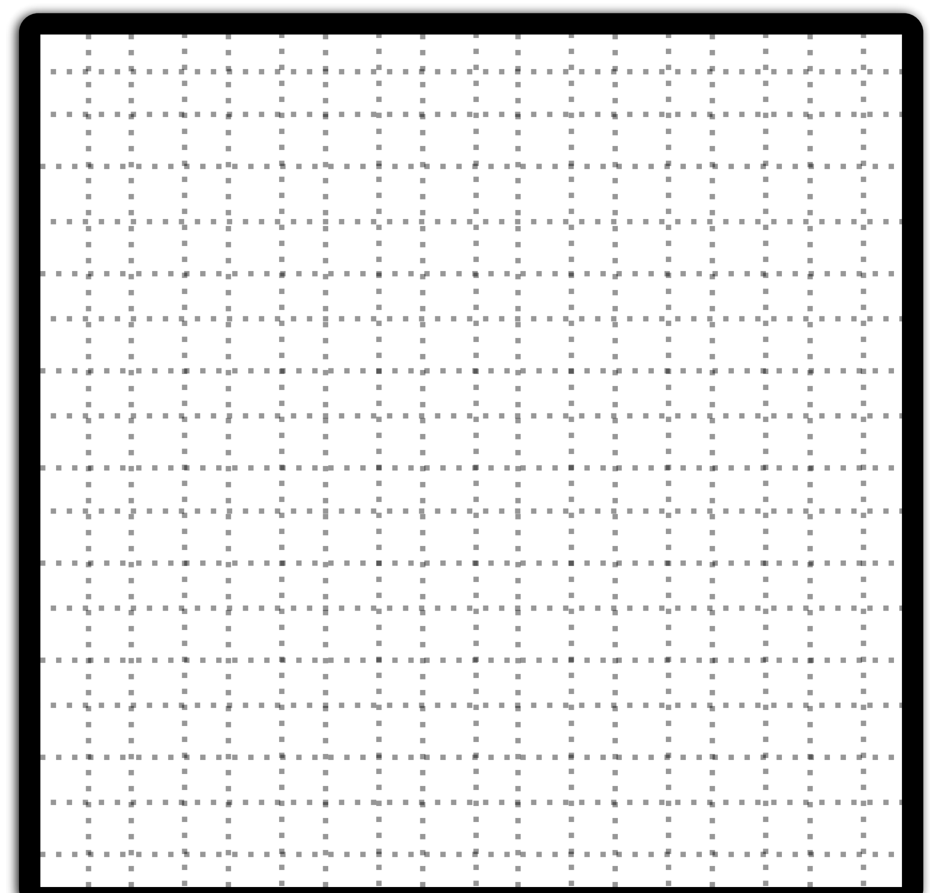


Path-integral in QFT - some details

$$Z = \int \mathcal{D}\varphi(x)\; e^{iS[\varphi]}$$

- integral and measure: $\displaystyle \int \mathcal{D}\varphi(x) \equiv \prod_{x\in V} \int d\varphi(x)$
- "only" rigorously defined by first discretizing





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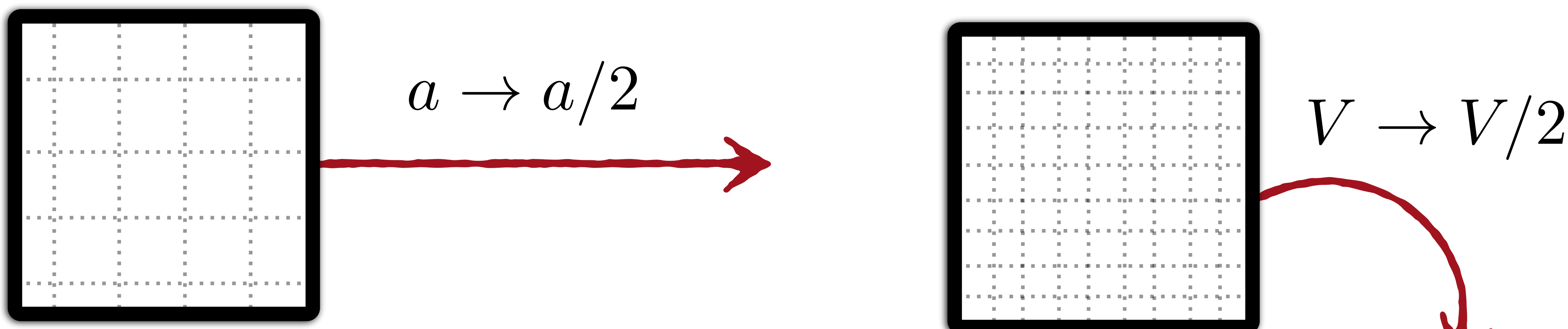
- "only" rigorously defined by first discretizing
- **correlation functions give us access to:**
  - **masses, decay constants, form factors, scattering amplitudes,…!!!**
- two-point correlation functions:

$$\langle \hat{\mathcal{O}}(t)\hat{\mathcal{O}}^\dagger(0)\rangle = \frac{1}{Z}\int \mathcal{D}\varphi(x)\ e^{iS[\varphi]}\mathcal{O}(t)\mathcal{O}^\dagger(0)$$

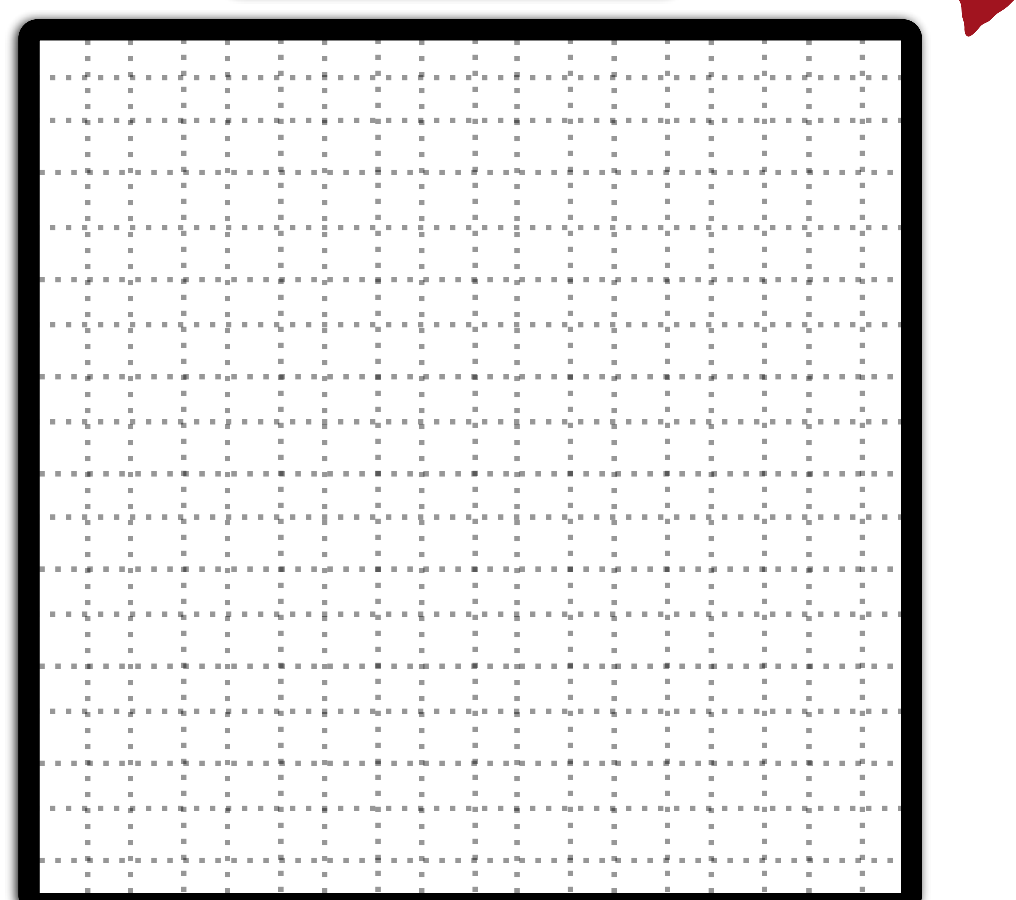
- where $\hat{\mathcal{O}}$ is some generic operator.
  - e.g., $\hat{\mathcal{O}} = \hat{\varphi},\ \hat{\varphi}^2,\ \hat{\varphi}^3,\ \dots, \partial^\mu\varphi, \dots$

Truncation and discretization of volume

- integral and measure: $\int \mathcal{D}\varphi(x) \equiv \prod_{x \in V} \int d\varphi(x)$



- number of integrals $\sim \frac{V}{a^d} = \frac{T}{a_t}\left(\frac{L}{a_s}\right)^{d-1}$



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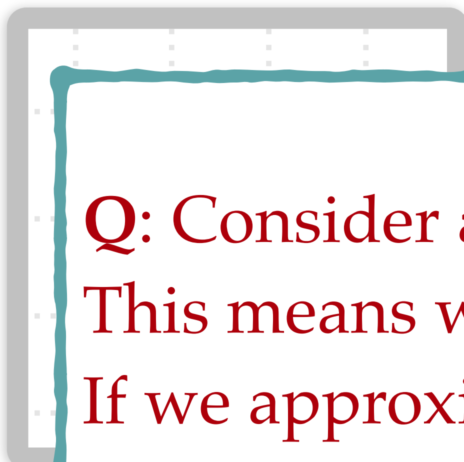
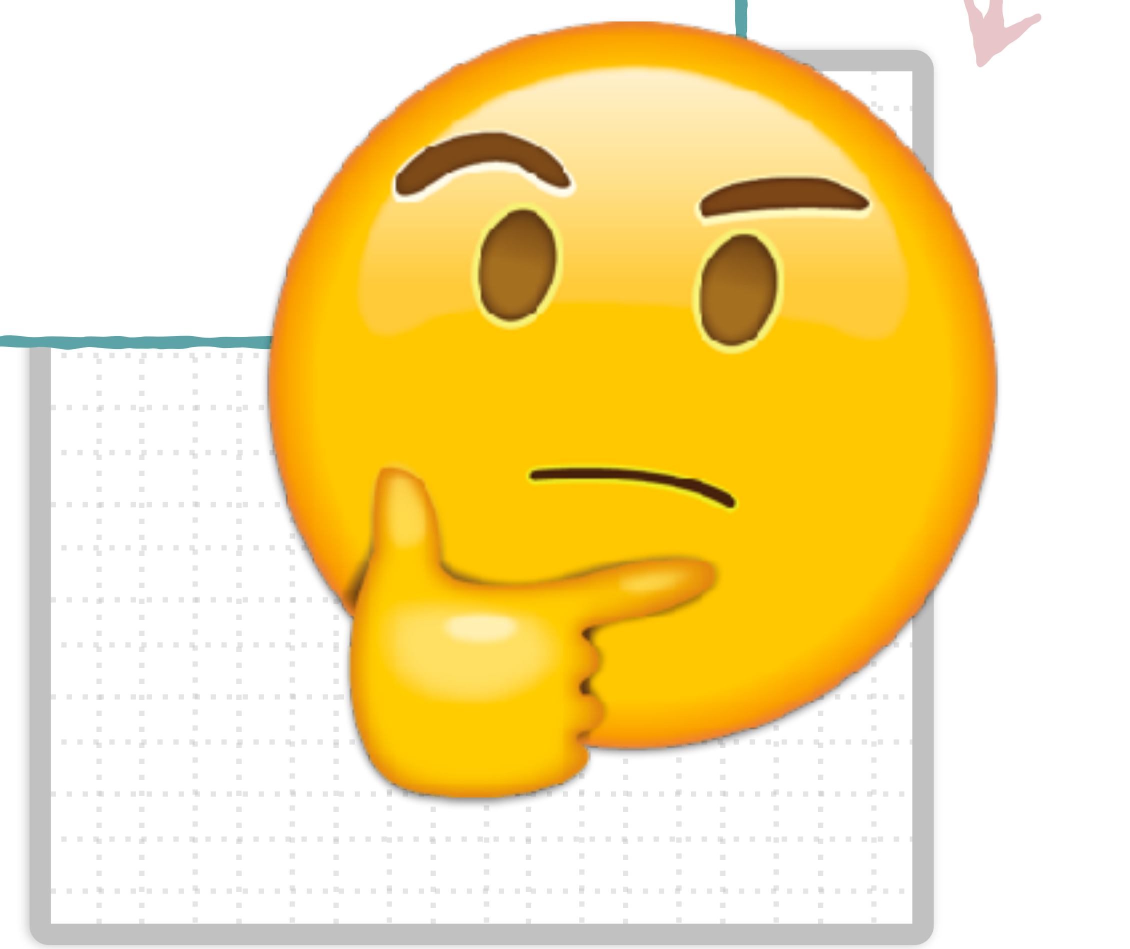


$V \to V/2$



Q: Consider a scalar field in a volume with 10^4 points.
This means we need to evaluate that many integrals.
If we approximate each integral, by a 10 point mesh, how many terms would be have to add?

Truncation and discretization of volume

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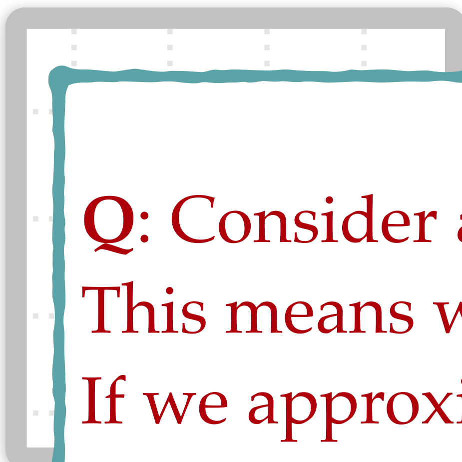

$V \to V/2$

- numb

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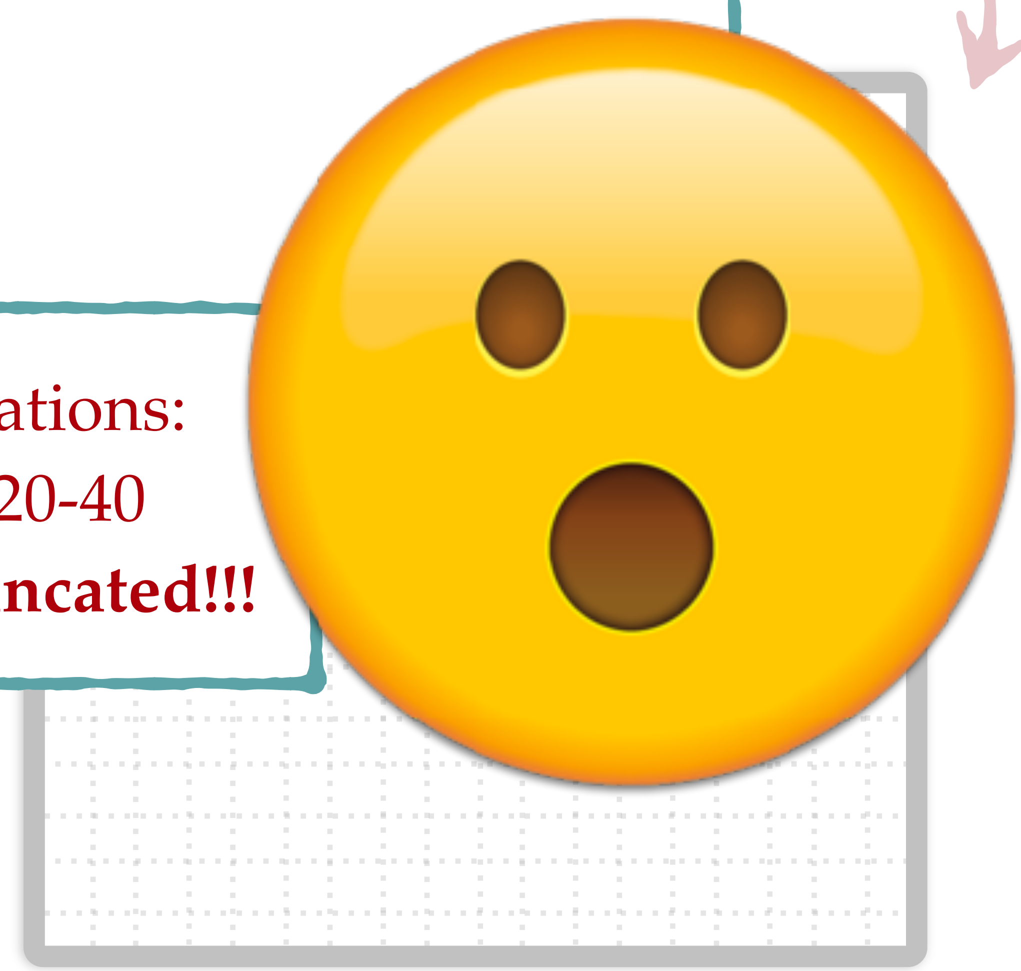
A: 10^{1000} terms!

modern day calculations:
$T/a_t \sim 200$, $L/a_s \sim 20\text{-}40$
!!!spacetime must truncated!!!



Truncation and discretization of volume

- Very explicit example: assume we have two points in space and we want to do a 3-point mesh, where the fields just take values of 1,0, or -1.

$$\int d\varphi(x_1) \int d\varphi(x_2) f(\varphi(x_1), \varphi(x_2))$$

$$\sim \int d\varphi(x_1) \left[f(\varphi(x_1), 1) + f(\varphi(x_1), 0) + f(\varphi(x_1), -1) \right]$$

$$\left.\begin{aligned} &\sim f(1,1) + f(1,0) + f(1-1) \\ &+ f(0,1) + f(0,0) + f(0-1) \\ &+ f(-1,1) + f(-1,0) + f(-1-1) \end{aligned}\right\} \; 3^2 = 9 \text{ terms}$$

Evaluating integrals probabilistically

- could we instead evaluate correlation functions statistically?

$$\langle \hat{\mathcal{O}}(t)\hat{\mathcal{O}}^\dagger(0)\rangle = Z^{-1}\int \mathcal{D}\varphi(x)\; e^{iS[\varphi]}\mathcal{O}(t)\mathcal{O}^\dagger(0)$$

$$\equiv Z^{-1}\int \mathcal{D}\varphi(x)\; \underline{e^{iS[\varphi]}} f[\varphi, t]$$

if only this were positive definite

- it is, if we Wick rotate onto imaginary time: $t \to -it$
  - making Minkowski spacetime replaced by a Euclidean one
  - example...

Wick rotation

- consider a 2D scalar field theory…

$$\mathcal{L}_M = \frac{1}{2}\ \left((\partial_t\varphi)^2 - (\partial_x\varphi)^2 - m_0^2\varphi^2\right) - \mathcal{V}[\varphi]$$

- Wick rotate: $t \to -it$

$$\begin{aligned} e^{iS_M[\varphi]} = exp\left[i\int dx\,dt\mathcal{L}_M[\varphi]\right] &= exp\left[i\int dx\,dt\,\frac{1}{2}\ \left((\partial_t\varphi)^2 - (\partial_x\varphi)^2 - m_0^2\varphi^2 - 2\mathcal{V}[\varphi]\right)\right] \\ &\to exp\left[i\int dx\,(-idt)\,\frac{1}{2}\ \left(-(\partial_t\varphi)^2 - (\partial_x\varphi)^2 - m_0^2\varphi^2 - 2\mathcal{V}[\varphi]\right)\right] \\ &= exp\left[-\int dx\,dt\,\frac{1}{2}\ \left((\partial_t\varphi)^2 + (\partial_x\varphi)^2 + m_0^2\varphi^2 + 2\mathcal{V}[\varphi]\right)\right] \equiv e^{-S_E[\varphi]} \end{aligned}$$

- Euclidean correlation function:

$$\langle\hat{\mathcal{O}}(t)\hat{\mathcal{O}}^\dagger(0)\rangle_E = \frac{\int\mathcal{D}\varphi(x)\ e^{-S_E[\varphi]}\mathcal{O}(t)\mathcal{O}^\dagger(0)}{\int\mathcal{D}\varphi(x)\ e^{-S_E[\varphi]}} \equiv Z_E^{-1}\int\mathcal{D}\varphi(x)\ e^{-S_E[\varphi]}\mathcal{O}(t)\mathcal{O}^\dagger(0)$$

Monte-Carlo technique

- let's evaluate the path integral, statistically…

$$Z_E = \int \mathcal{D}\varphi(x)\, e^{-S_E[\varphi]}$$

probability for a field configuration

- generate field configurations according to the probability
- obtain an ensemble of configurations: $\{\varphi_1, \varphi_2, \varphi_3, \ldots, \varphi_N\}$
- means:

$$Z_E^{-1} \int \mathcal{D}\varphi(x)\; e^{-S_E[\varphi]} f[\varphi, t] \approx \overline{f} \equiv \frac{1}{N} \sum_{n=1}^{N} f[\varphi_n, t]$$

- uncertainties:

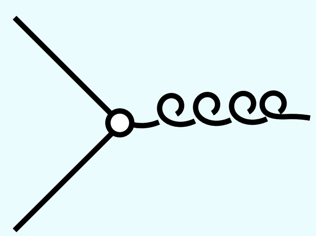
$$\sigma_f \equiv \sqrt{\frac{1}{N(N-1)} \sum_{n=1}^{N} \left(f[\varphi_n, t] - \overline{f}\right)^2}$$

systematically improvable!

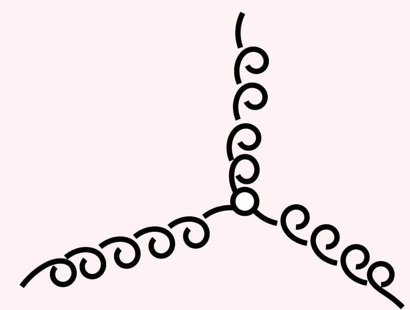
Some words about QCD

- QCD is not a simple scalar field theory
- there are quarks and gluons…

$$\mathcal{L}_{QCD} = \sum_{f=u,d,s,c,b} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f\right) \psi_f - \frac{1}{2}\mathrm{tr}\left(\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu}\right)$$



$\sim g$



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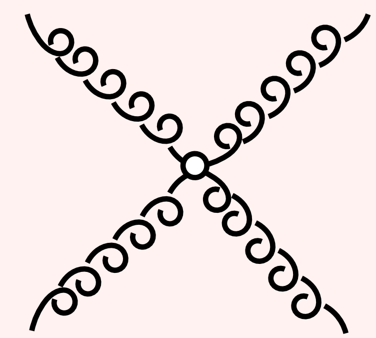

$\sim g^2$

top decays too quickly to hadronize…so forget it!

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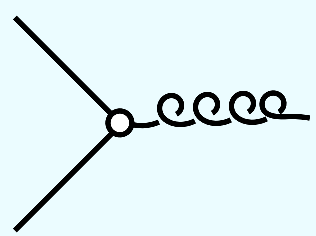
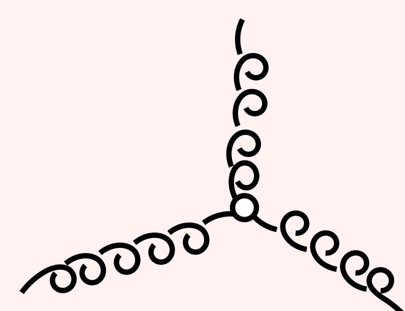
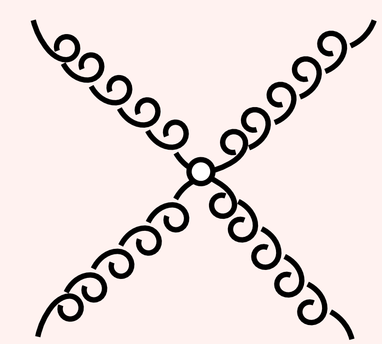


$\sim g$



$\sim g$



$\sim g^2$

quark field: $\left[\psi_f\right]_{\alpha,c}$

Dirac spin index
$\alpha = 1\ldots4$

color index
$c = 1\ldots3$

gluon field: $\left[A_\mu\right]_{cc'}$

Lorentz index
$\mu = 1\ldots4$

matrix in color

Some words about QCD

- QCD is not a simple scalar field theory
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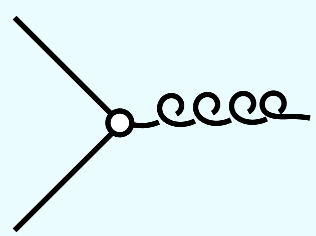
$$\mathcal{L}_{QCD} = \sum_{f=u,d,s,c,b} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f\right)\psi_f - \frac{1}{2}\mathrm{tr}\left(\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu}\right)$$



$\sim g$

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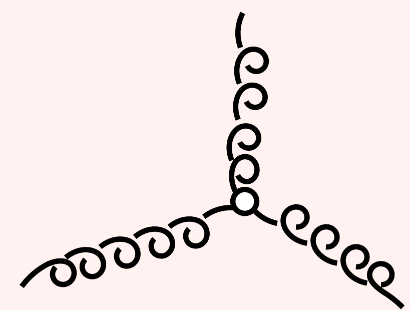
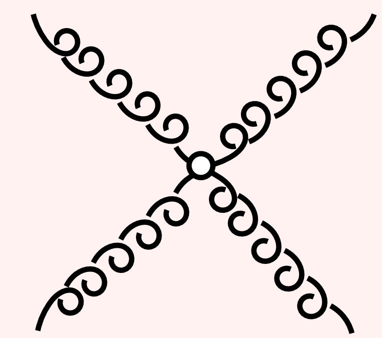


$\sim g$



$\sim g^2$

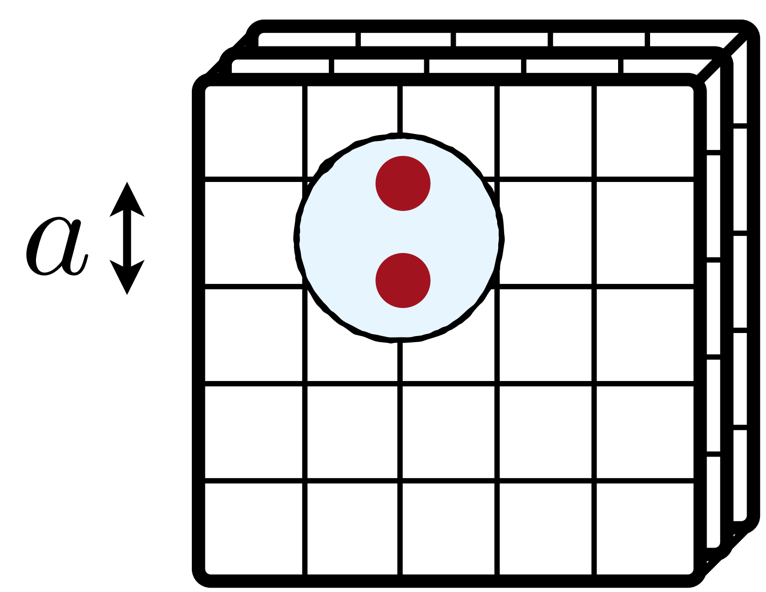
gluon field: $\left[A_\mu\right]_{cc'}$

$$Z_{QCD} = \int \mathcal{D}A\,\mathcal{D}q\,\mathcal{D}\bar{q}\;\, e^{-S_E}$$

… our fields live in a [time *T*] x [volume *V*] x [flavor *f*] x [color 3] x [Dirac 4] - dim ‘space’

Lattice spacing & quark masses

- Parameter of QCD: m_u, m_d, m_s, m_c, m_b, m_t, g
- Dimensional transmutation: m_u/Λ_{QCD}, m_d/Λ_{QCD}, m_s/Λ_{QCD}, … , m_t/Λ_{QCD}
- QCD does not have an inherent mass scale
- QCD can predict masses of hadron in units of Λ_{QCD}
- Phenomenologically, we fix Λ_{QCD}
- Lattice QCD: am_u, am_d, am_s, am_c, am_b , am_t



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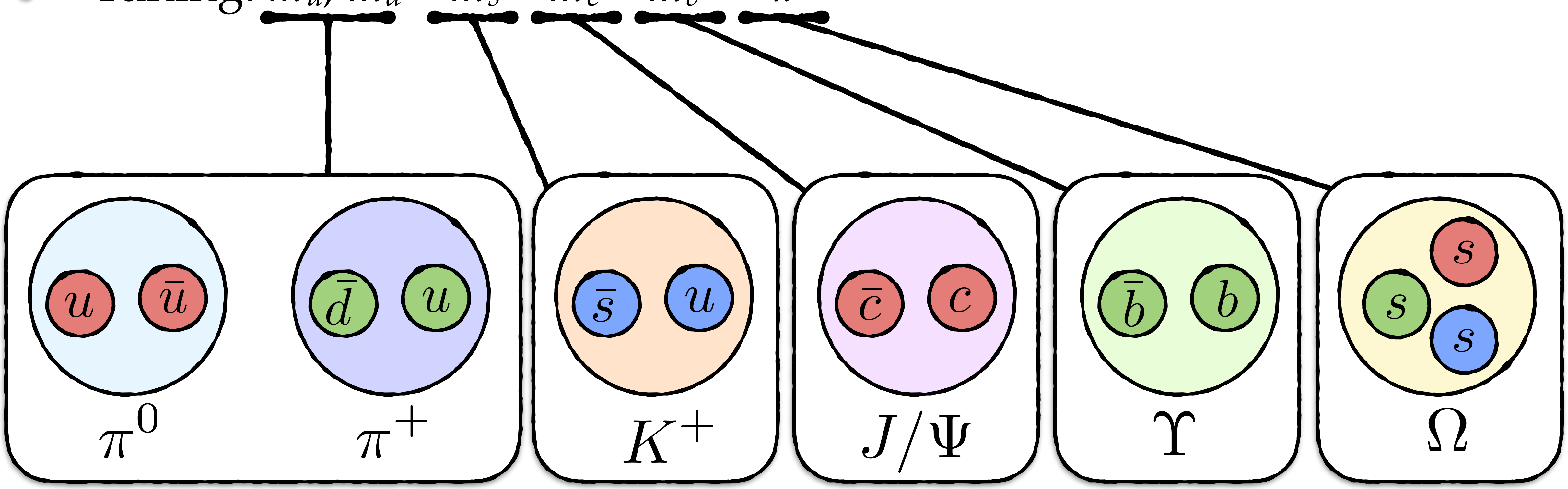
- Tuning: m_u, m_d m_s m_c m_b a



Lattice spacing & quark masses

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- Phenomenologically, we fix Λ_{QCD}
- Lattice QCD: am_u, am_d, am_s, am_c, am_b , am_t
- Tuning
- In general: $m_h(a) = m_h(0)+O(a)$ […but we won't discuss this…]

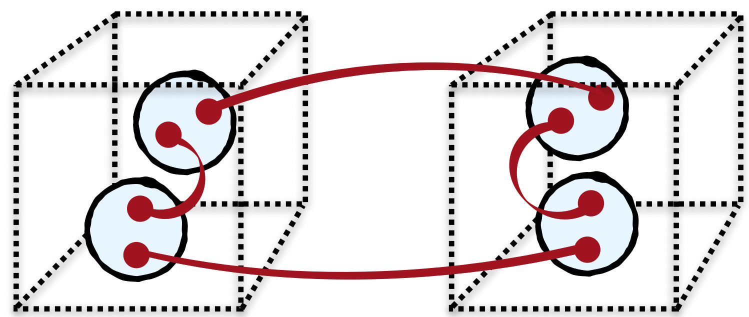
Questions?

Jazzi prefers python, but will code in C++ if need be...

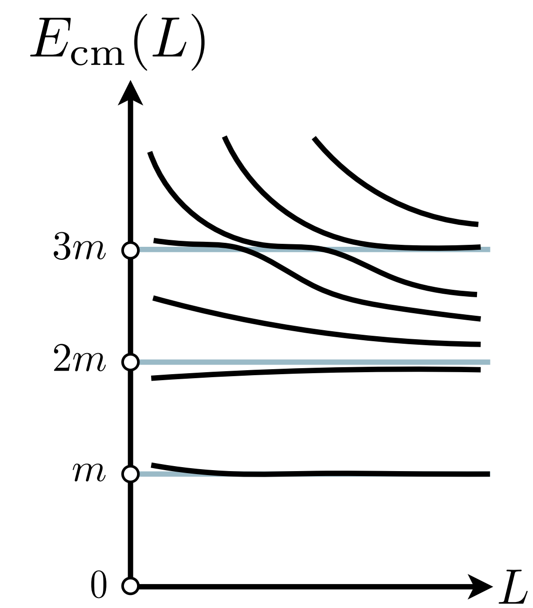
Outline

QCD: m_q/Λ_{QCD}

Correlation functions



Finite-volume spectrum:



Time evolution in Euclidean spacetime

- The time-dependence of: $\langle \mathcal{O}(t)\mathcal{O}^\dagger(0)\rangle \equiv \langle\Omega|\mathcal{O}(t)\mathcal{O}^\dagger(0)|\Omega\rangle$

  $|\Omega\rangle$: QCD vacuum [assumed to have zero energy]

- Heisenberg picture in Minkowski spacetime

$$\mathcal{O}_M(t) = e^{it\hat{H}}\mathcal{O}(0)e^{-it\hat{H}}$$

- Wick rotation onto Euclidean spacetime: $t \rightarrow -it$

$$\mathcal{O}_E(t) = e^{t\hat{H}}\mathcal{O}(0)e^{-t\hat{H}}$$

- Euclidean correlation functions:

$$\langle\Omega|\mathcal{O}(t)\mathcal{O}^\dagger(0)|\Omega\rangle = \langle\Omega|e^{t\hat{H}}\mathcal{O}(0)e^{-t\hat{H}}\mathcal{O}^\dagger(0)|\Omega\rangle$$

Time evolution in Euclidean spacetime

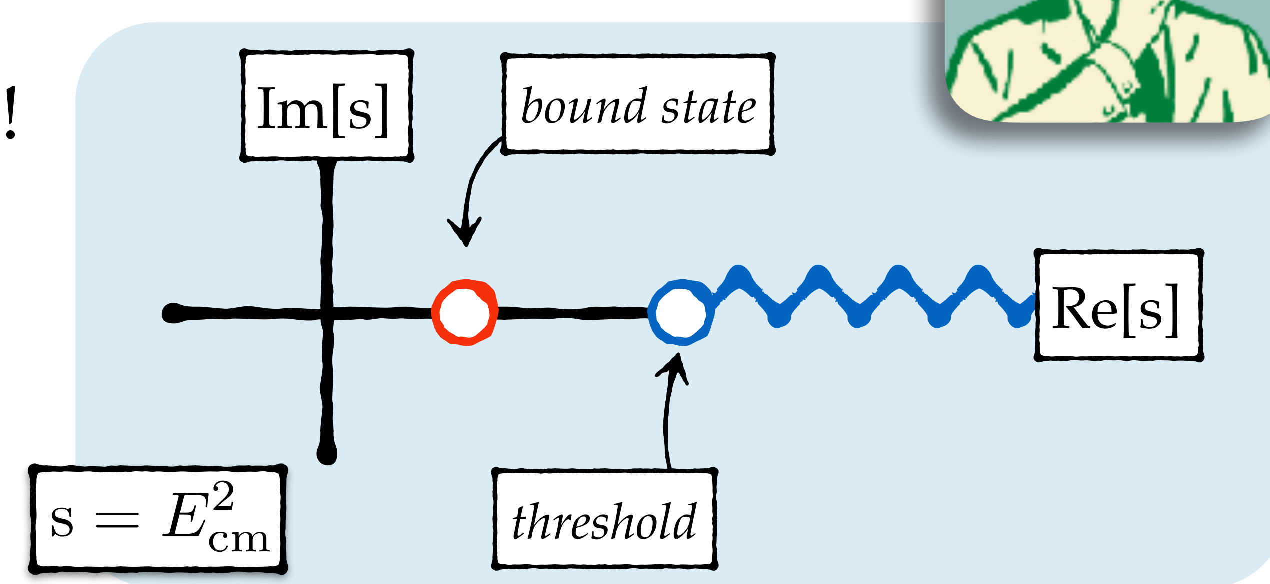
- We would like to introduce eigenstates of the Hamiltonian

$$\hat{H}|n\rangle = |n\rangle E_n$$

such that…
$$C(t) = \langle\Omega|e^{t\hat{H}}\mathcal{O}(0)e^{-t\hat{H}}\mathcal{O}^\dagger(0)|\Omega\rangle$$
$$= \sum_n e^{tE_\Omega} e^{-tE_n}\langle\Omega|\mathcal{O}(0)|n\rangle\langle n|\mathcal{O}^\dagger(0)|\Omega\rangle$$
$$= \sum_n e^{-tE_n}|\langle\Omega|\mathcal{O}(0)|n\rangle|^2$$

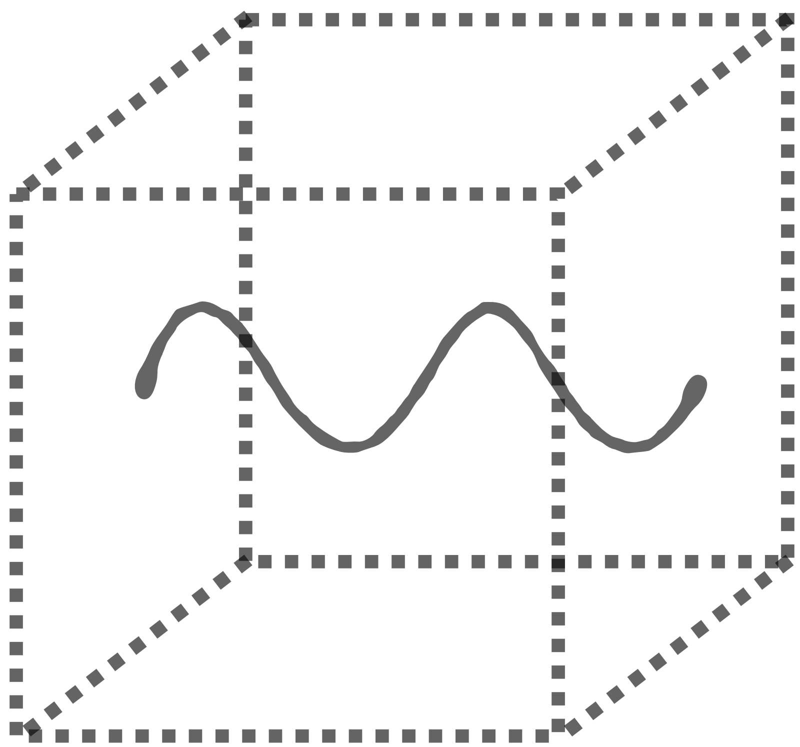
except, the spectrum is continuous!

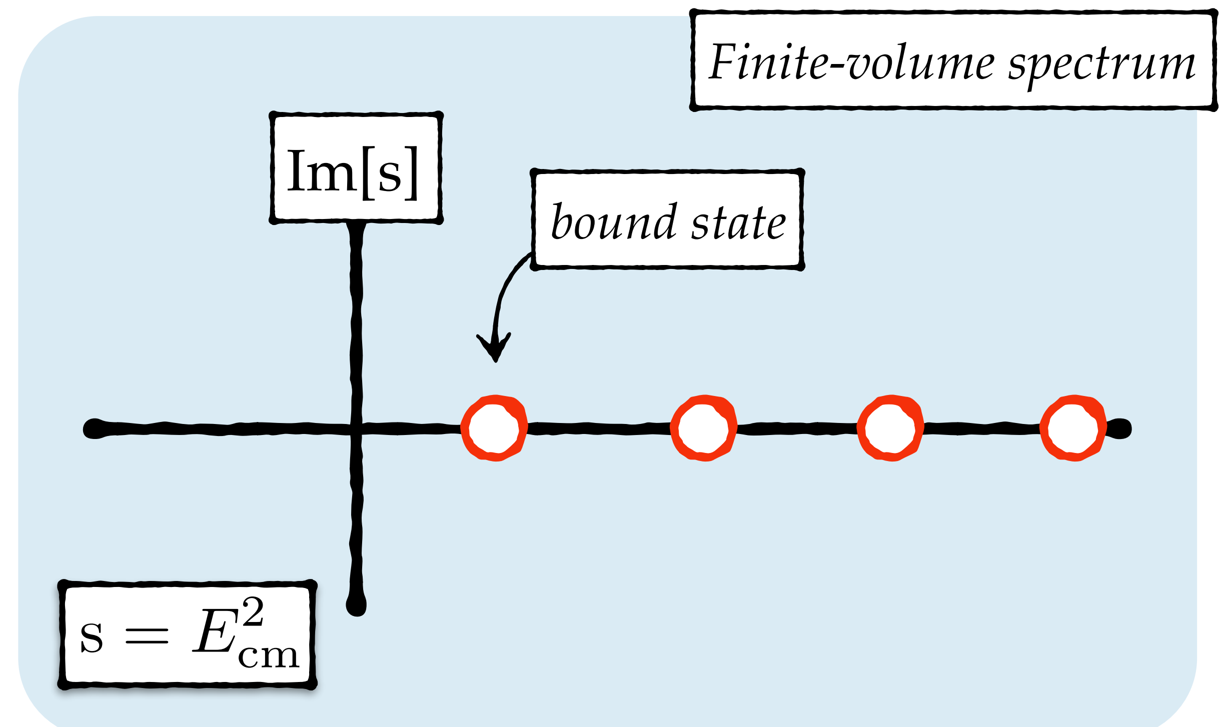
Should we be integrating…?



Time evolution in finite Euclidean spacetime

- Remember, we have placed the theory in a finite-volume





"only a discrete number of modes can exist in a finite volume"

consequently, we can rigorously write

$$C(t) = \sum_n e^{-tE_n} |\langle \Omega | \mathcal{O}(0) | n \rangle|^2$$

Ground state masses

- In principle, each correlation function has access to infinite number of states

- A simple limit

$$\lim_{t\to\infty} C(t) = \lim_{t\to\infty} \sum_n e^{-tE_n} |\langle\Omega|\mathcal{O}(0)|n\rangle|^2$$

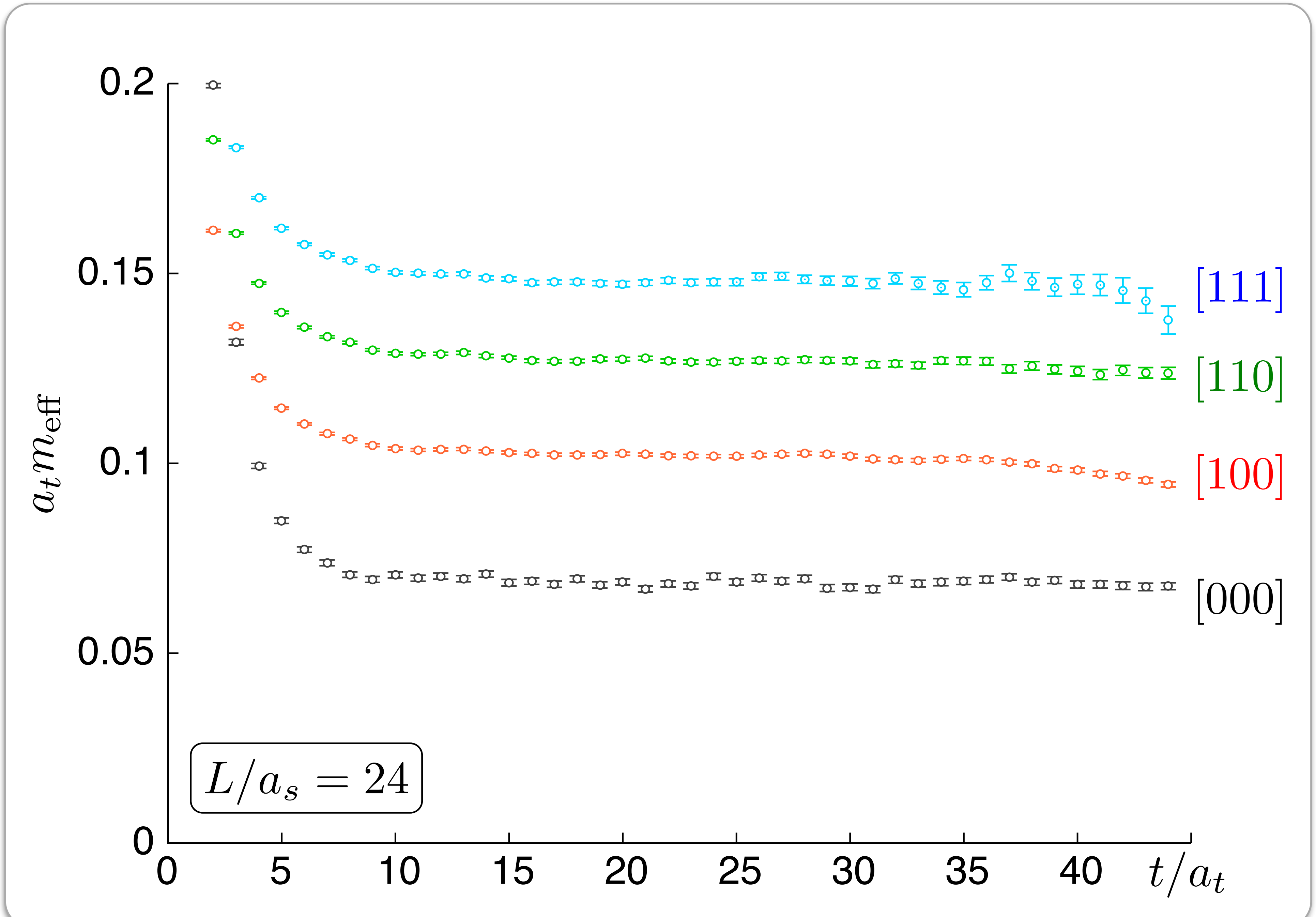
$$= e^{-tE_0} |\langle\Omega|\mathcal{O}(0)|0\rangle|^2 + \mathcal{O}(e^{-t(E_1-E_0)})$$

- This motivates:

$$m_{\text{eff}}(t) = \log \frac{C(t)}{C(t+1)}$$

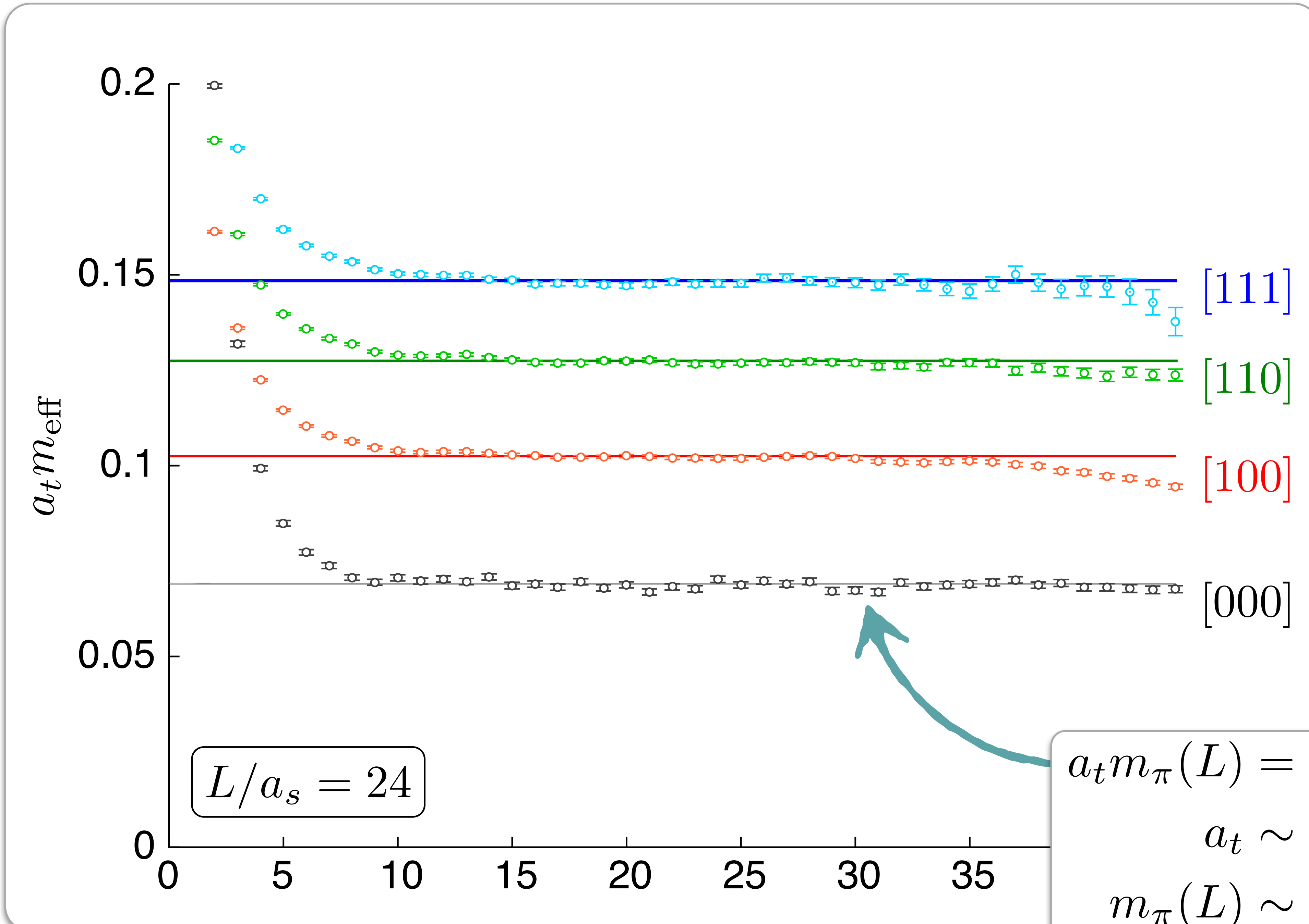
$$\to \log \frac{e^{-tE_0}}{e^{-(t+1)E_0}} = \log e^{E_0} = E_0$$

The π spectrum



$$a_t m_{\text{eff}}(t) = \log \frac{C(t)}{C(t+1)}$$

The π spectrum



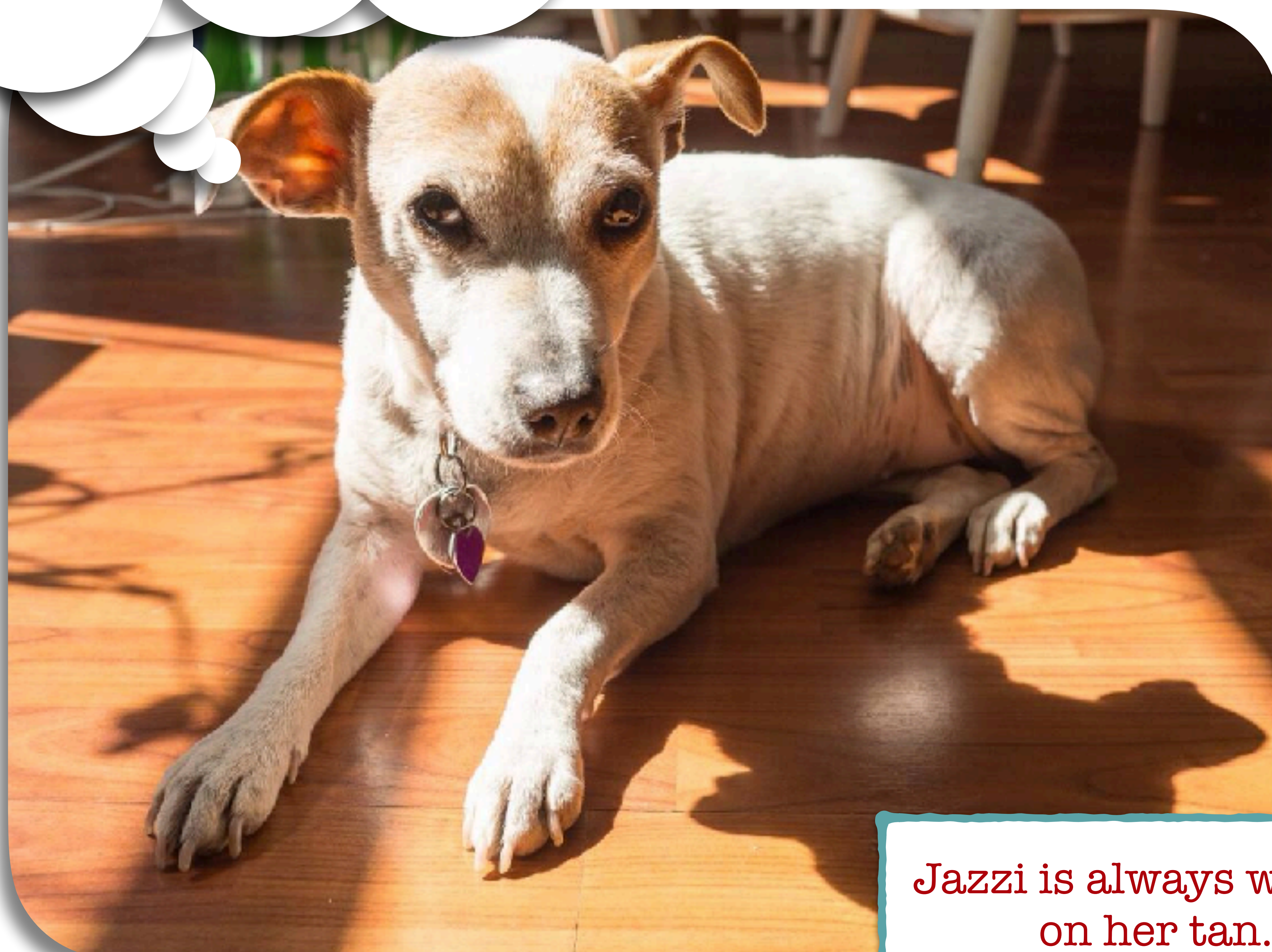
$$a_t m_\pi(L) = 0.0690(2)$$

$$a_t \sim 0.035 \text{ fm}$$

$$m_\pi(L) \sim 394\,\text{MeV}$$

$$a_t m_{\text{eff}}(t) = \log \frac{C(t)}{C(t+1)}$$

Questions?

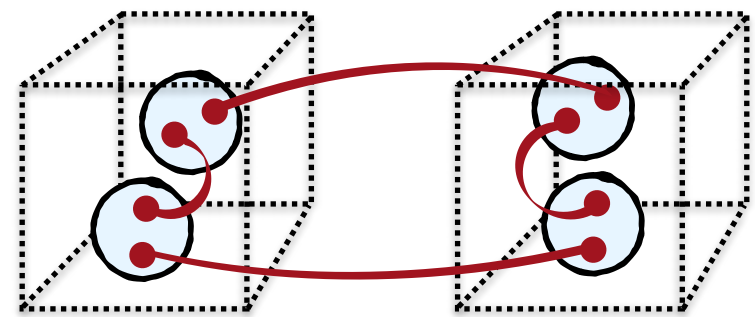


Jazzi is always working on her tan…

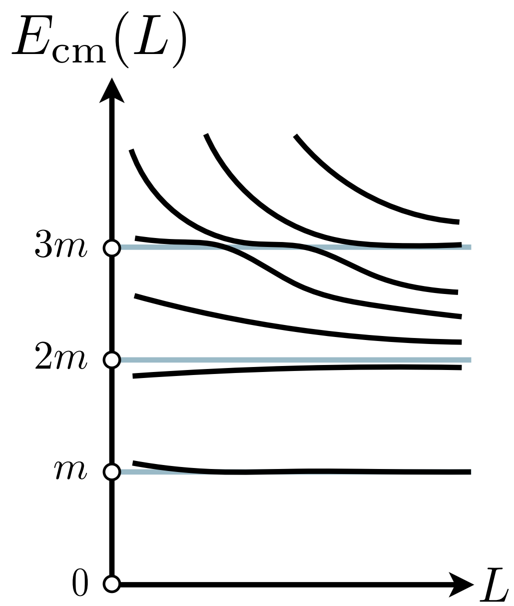
Outline

QCD: m_q/Λ_{QCD}

Correlation functions

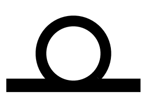


Finite-volume spectrum:

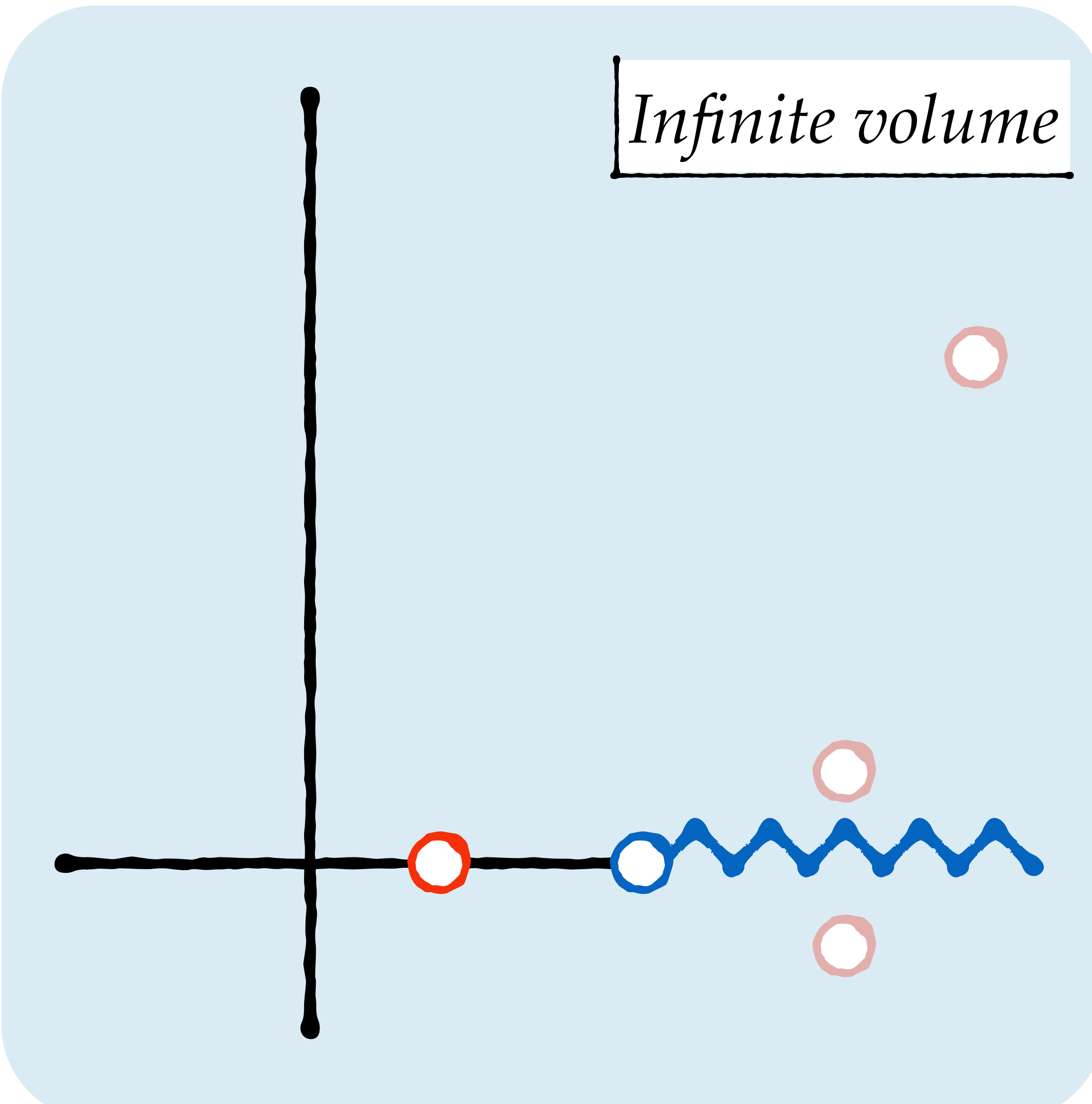


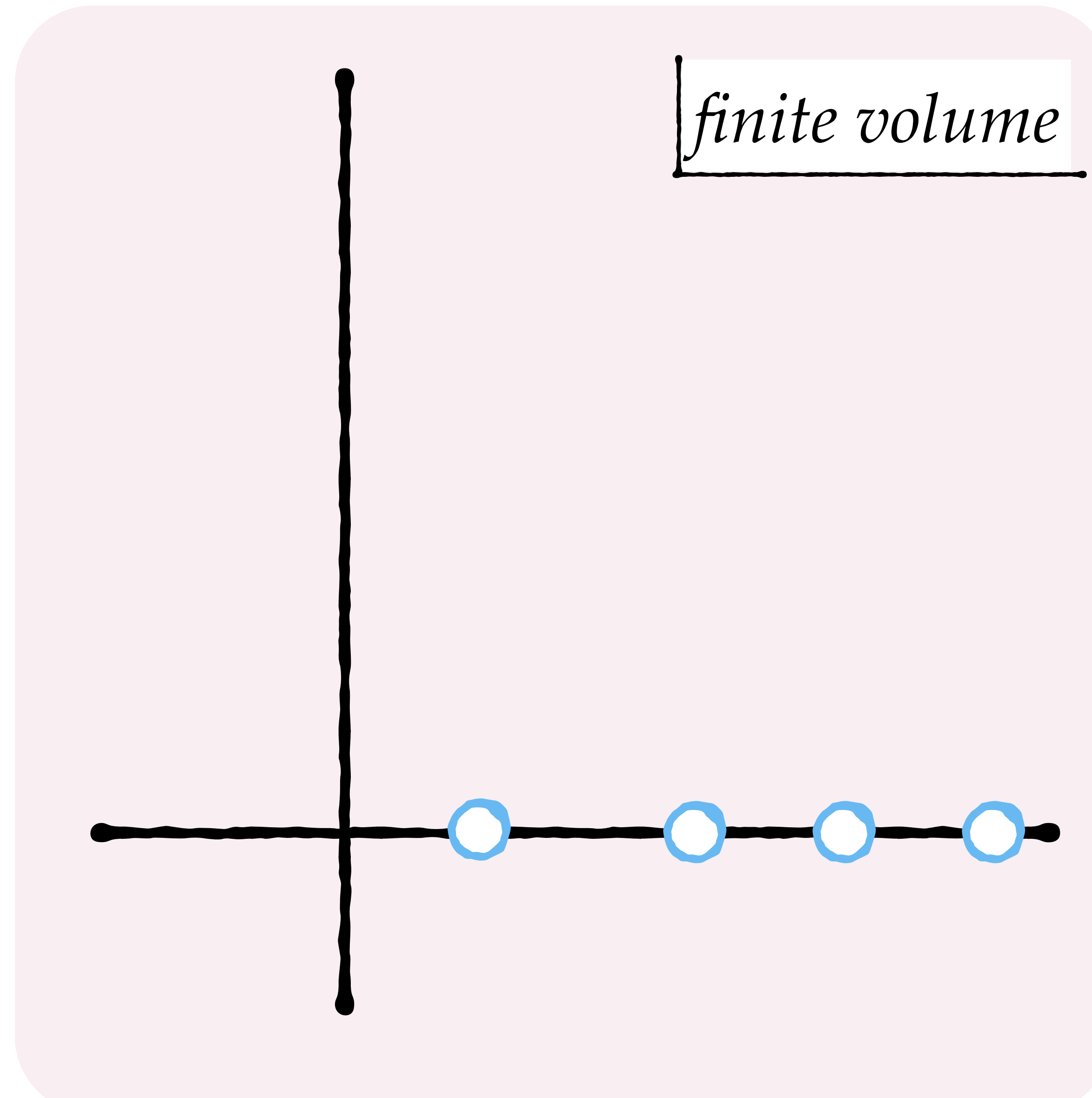
Interpretation of spectrum:

if $E_{\rm cm}(L) \ll 2m$: Masses of hadrons

 $\longleftrightarrow$ $E_{\rm cm}(L) = \sqrt{p^2 + m^2} + \mathcal{O}(e^{-m_\pi L})$

Finite vs. infinite volume spectrum



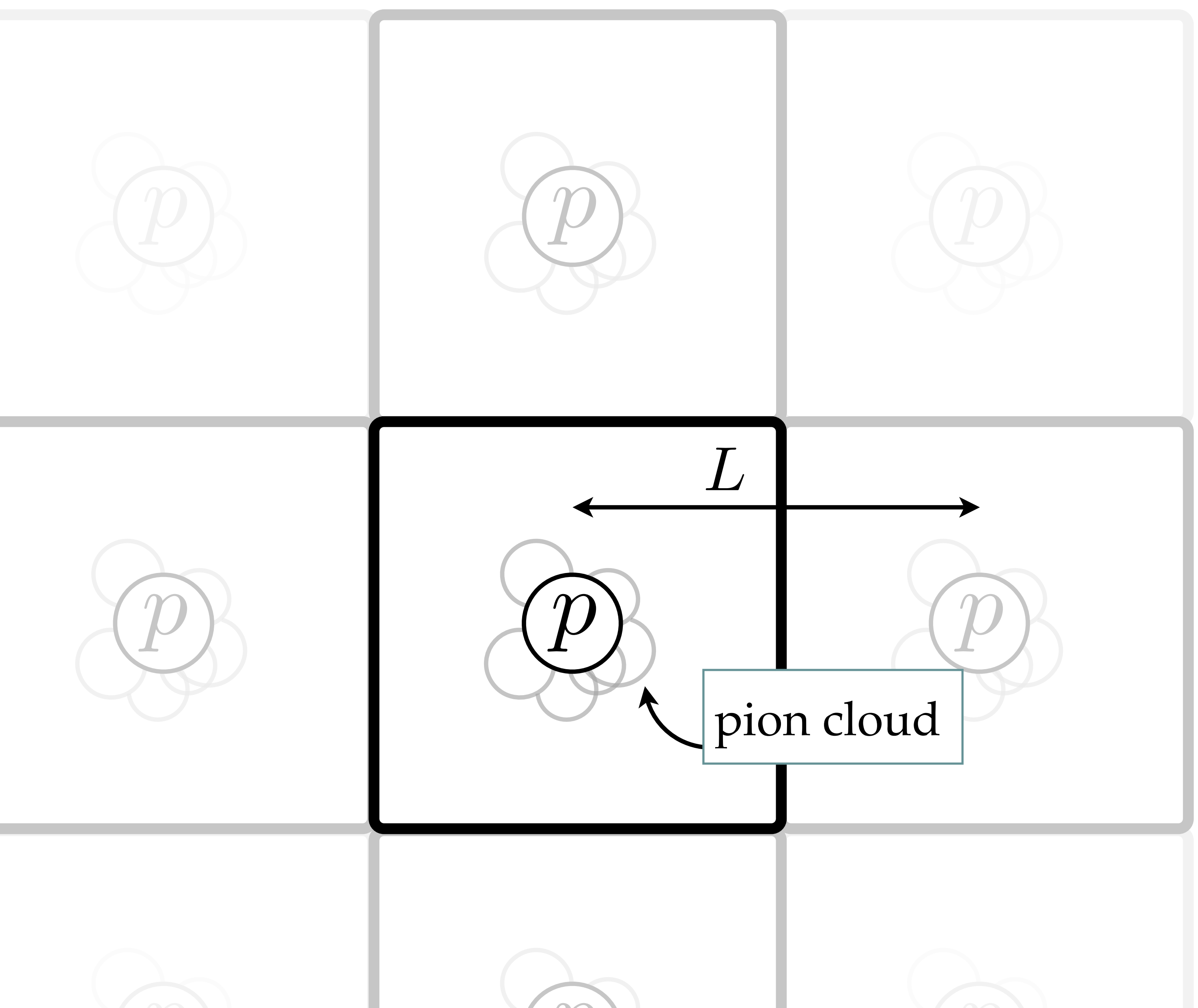


both pictures *are* QCD:

"Two analytic manifestations of QCD"

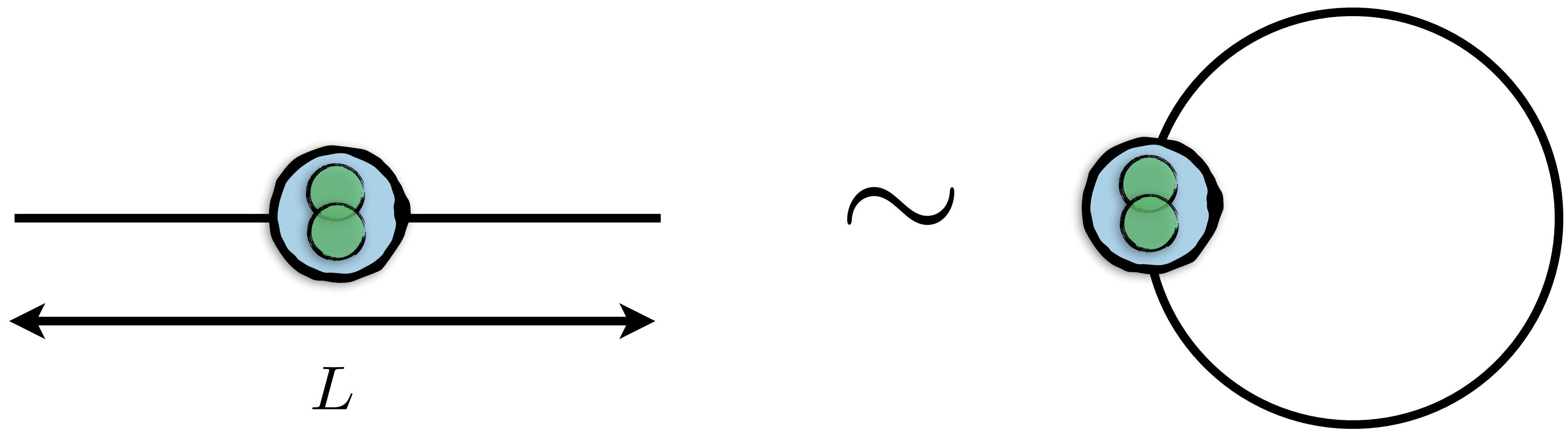
Infrared limit of the theory

- Finite-volume arise from the interactions with mirror images
- Assuming $L >> $ size of the hadrons ~ $1/m_\pi$
  - This is a purely infrared artifact
  - We can determine these artifact using hadrons are the degrees of freedom
- Note $m_\pi L$ is a natural parameter



Physics in a 1+1 Dimensions

- Free particle wave function: $\varphi_p(x) = e^{ipx}$
- Periodicity



$$\varphi_p(L + x) = e^{ip(x+L)} = \varphi_p(x) = e^{ipx}$$

- Discretized momentum and spectrum: $p = \dfrac{2\pi n}{L}$
- Question: What happens to the masses determined in a finite-volume?

Exponentially suppressed corrections

$$m_\pi(L) = m_\pi(\infty) + \mathcal{O}\left(e^{-m_\pi L}\right)$$

CHOOSE YOUR OWN ADVENTURE®

sketchy quantum mechanical derivation

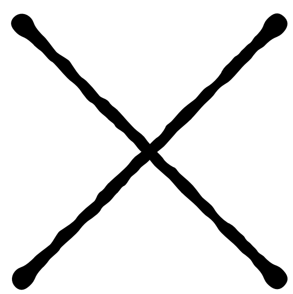
sketchy quantum field theoretical derivation

Scalar field theory in 1+1D box

- Toy model for mesons: $$\mathcal{L}_M = \frac{1}{2}\left((\partial_t\varphi)^2 - (\partial_x\varphi)^2 - m_0^2\varphi^2\right) - \frac{\lambda}{4!}\varphi^4$$

- Feynman rules:

$$= \Delta_0(p^2) = \frac{i}{p^2 - m_0^2 + i\epsilon}$$



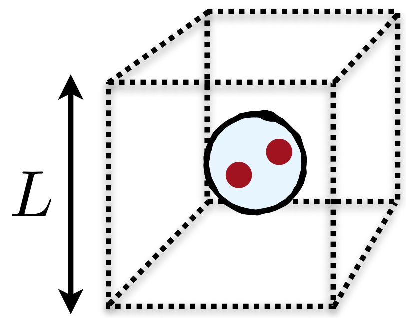
$$= i\lambda$$

- Integrals over momenta become sums:

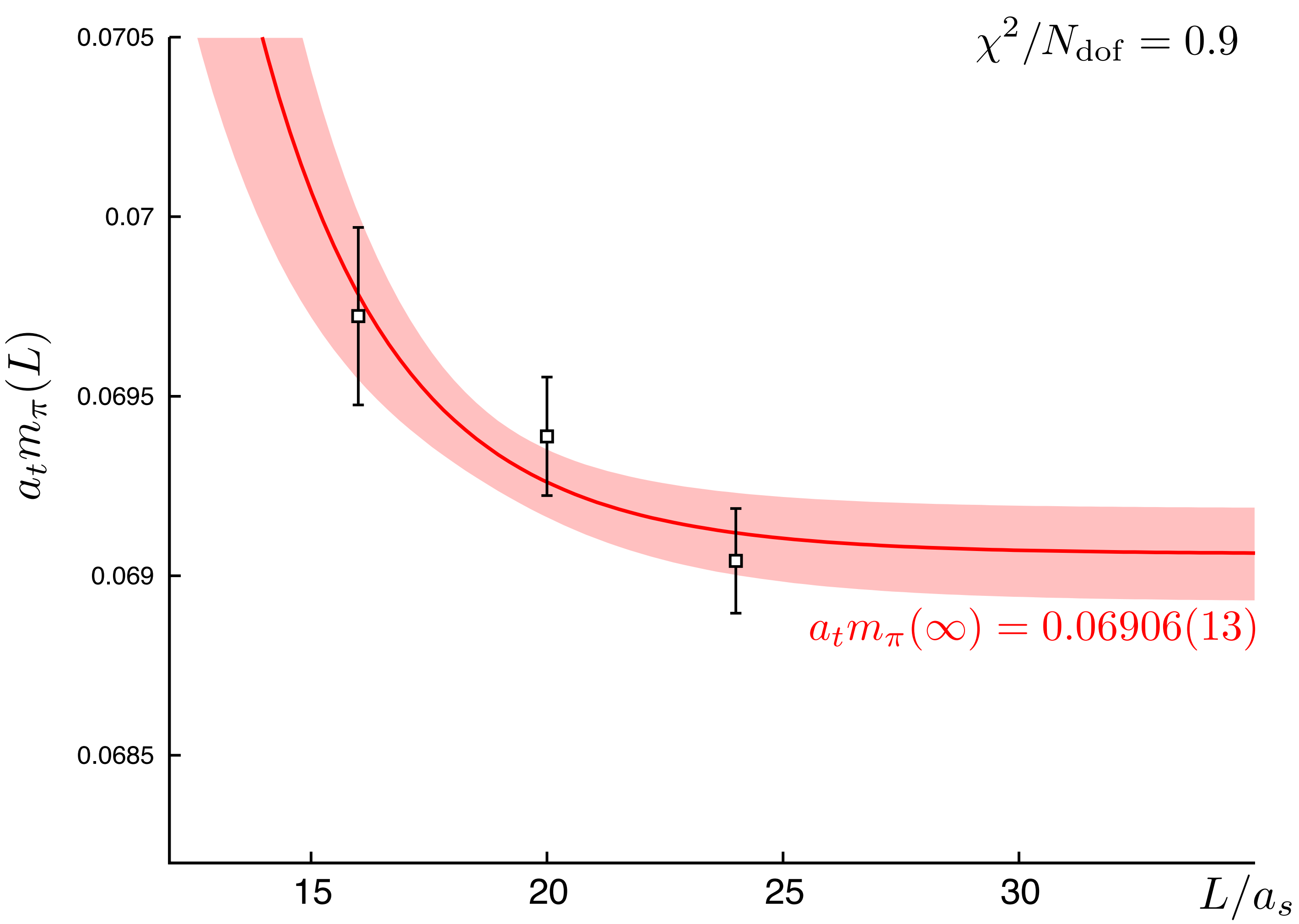
$$\int \frac{dk}{2\pi} \to \sum_k \frac{\Delta k}{2\pi} = \sum_k \frac{\Delta k}{2\pi} = \sum_k \frac{2\pi\Delta n}{2\pi L} = \frac{1}{L}\sum_k$$

- Integrals over energy are still integrals

The π in a box

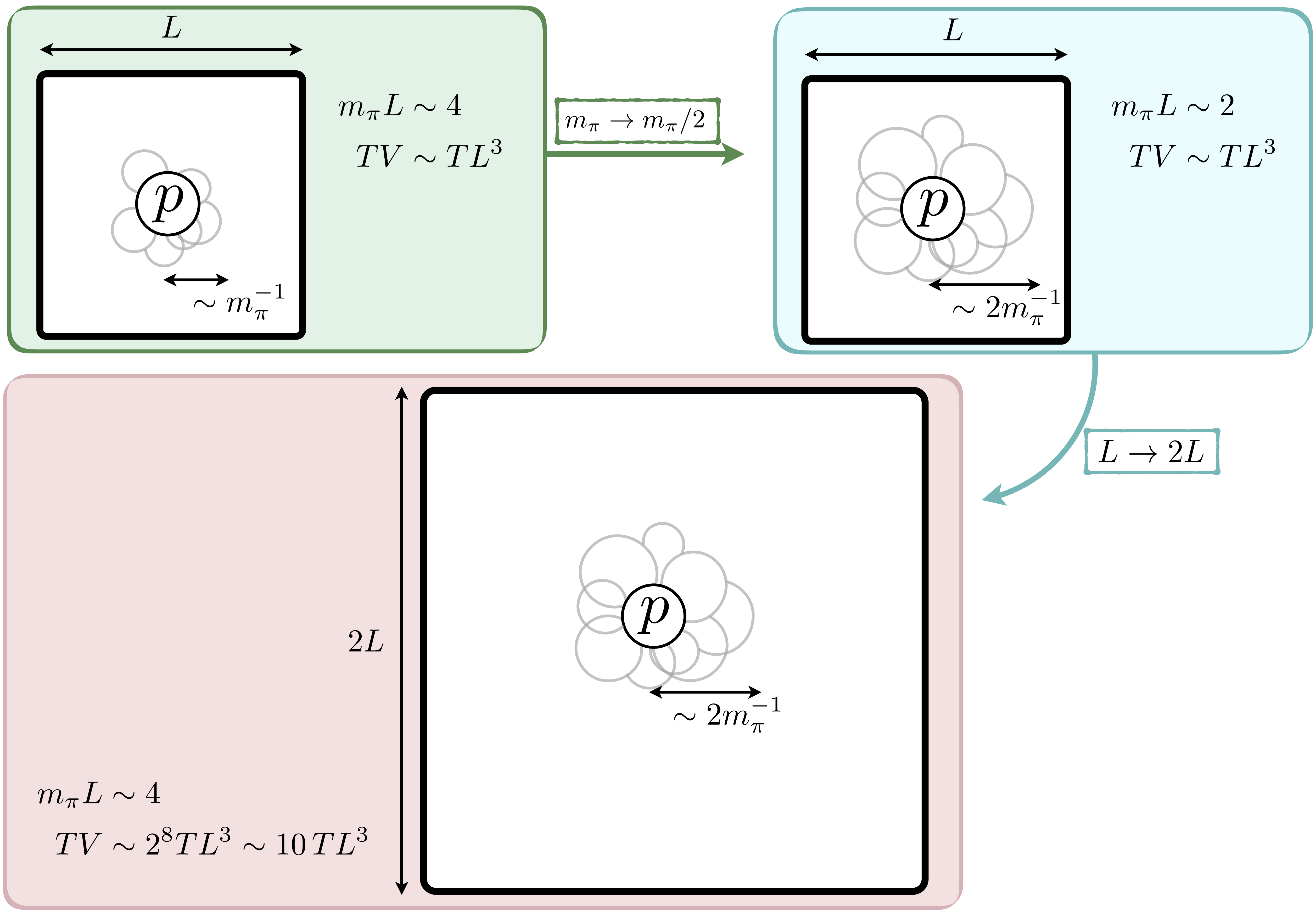


$$m_\pi(L) = m_\pi + c\,\frac{e^{-m_\pi L}}{(m_\pi L)^{3/2}}$$



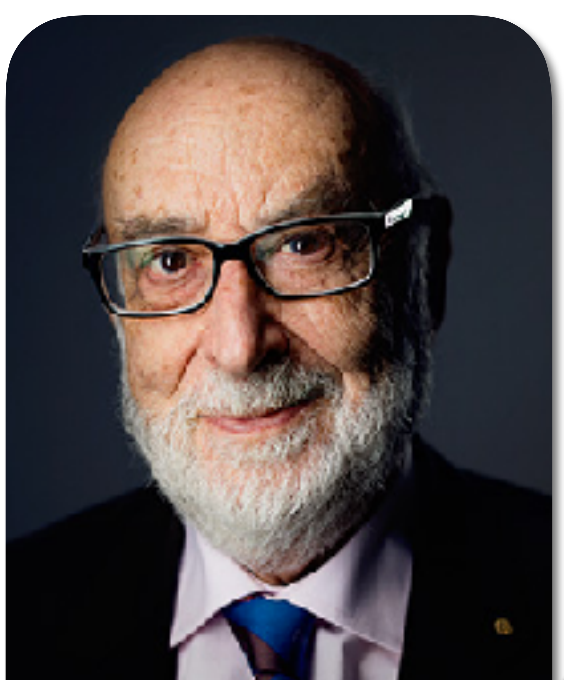
$$a_s \sim 0.12\ \mathrm{fm} \longrightarrow m_\pi L \sim 3.8,\ \ 4.7,\ \ 5.6$$

Challenge with light quark masses



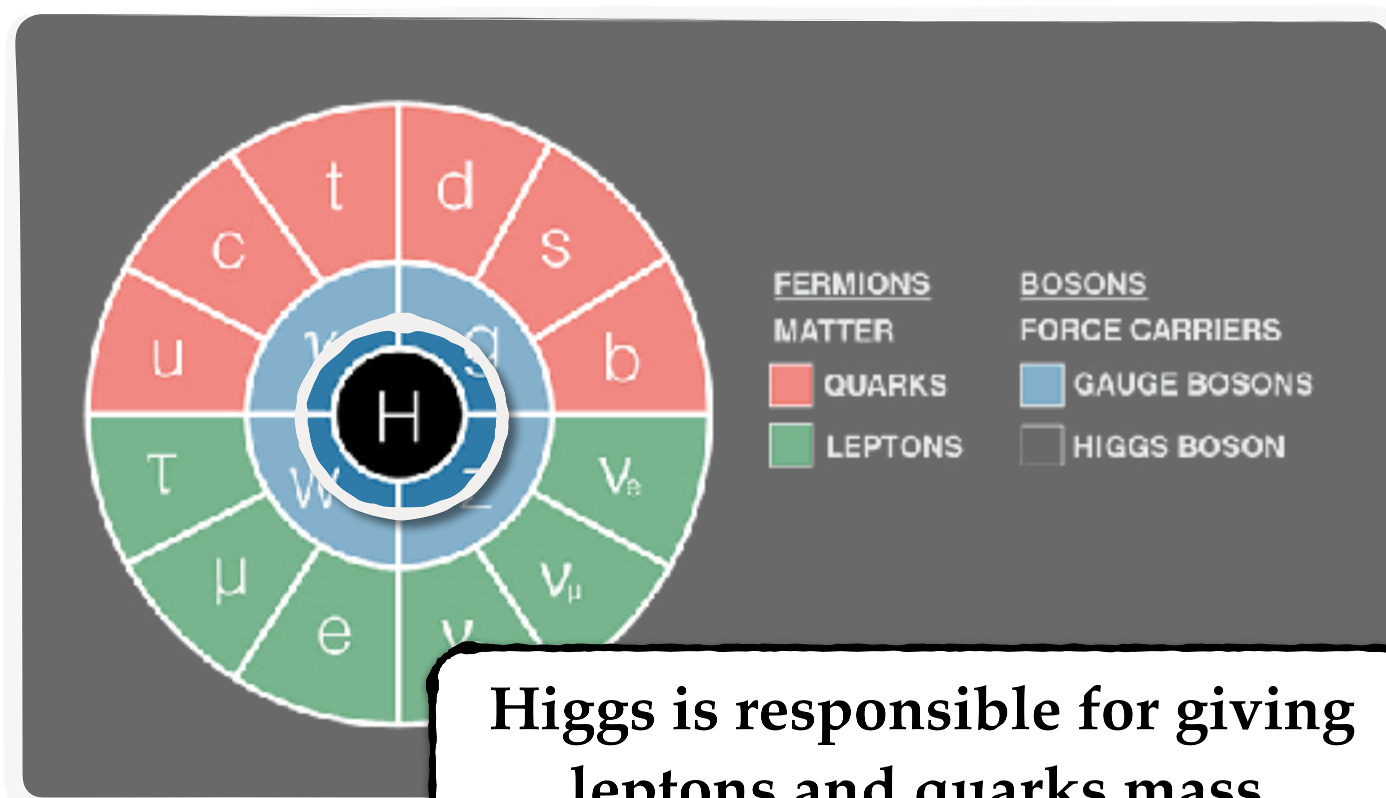
the origin of mass

2012, LHC @ CERN



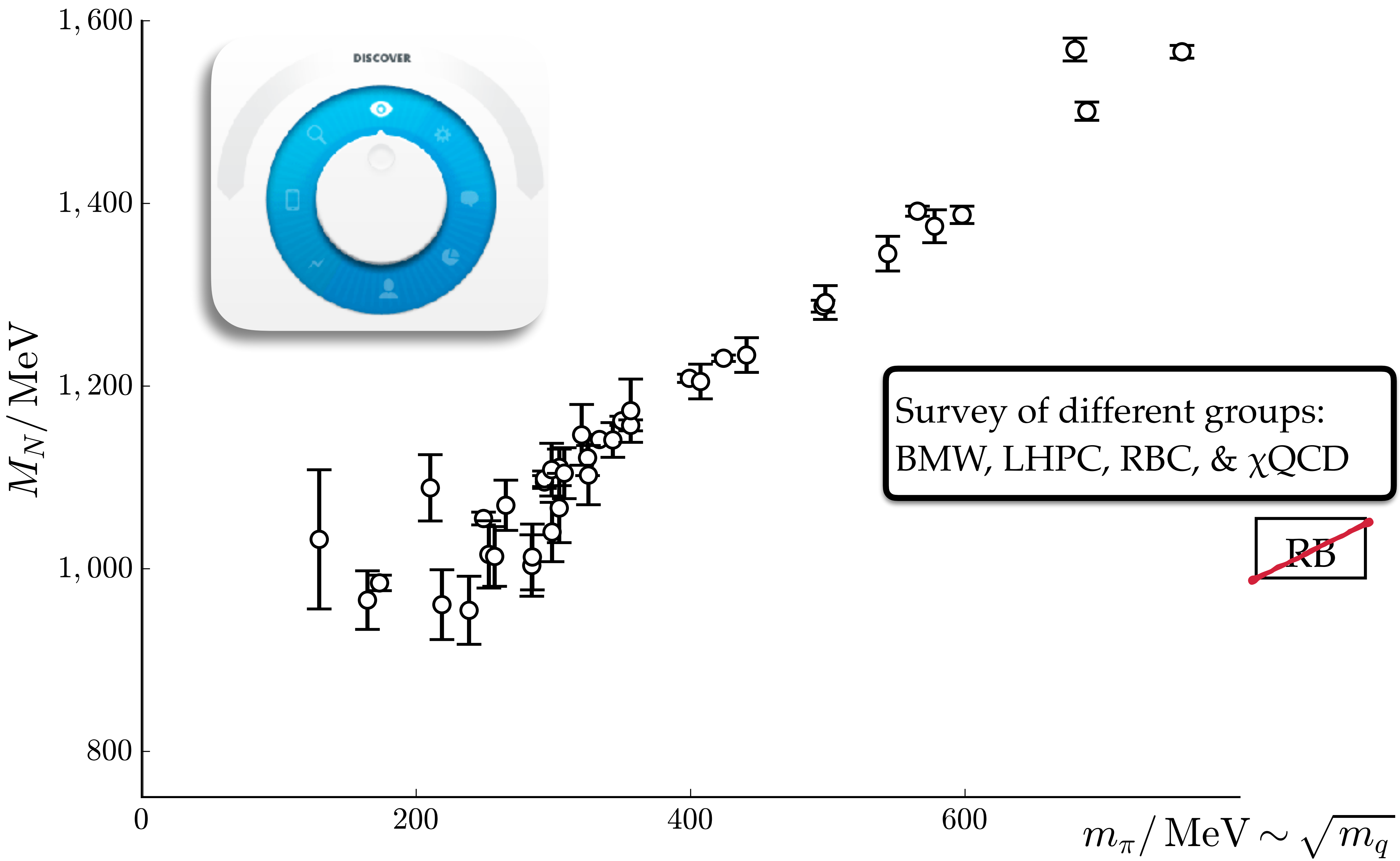
Francois Englert

Peter Higgs

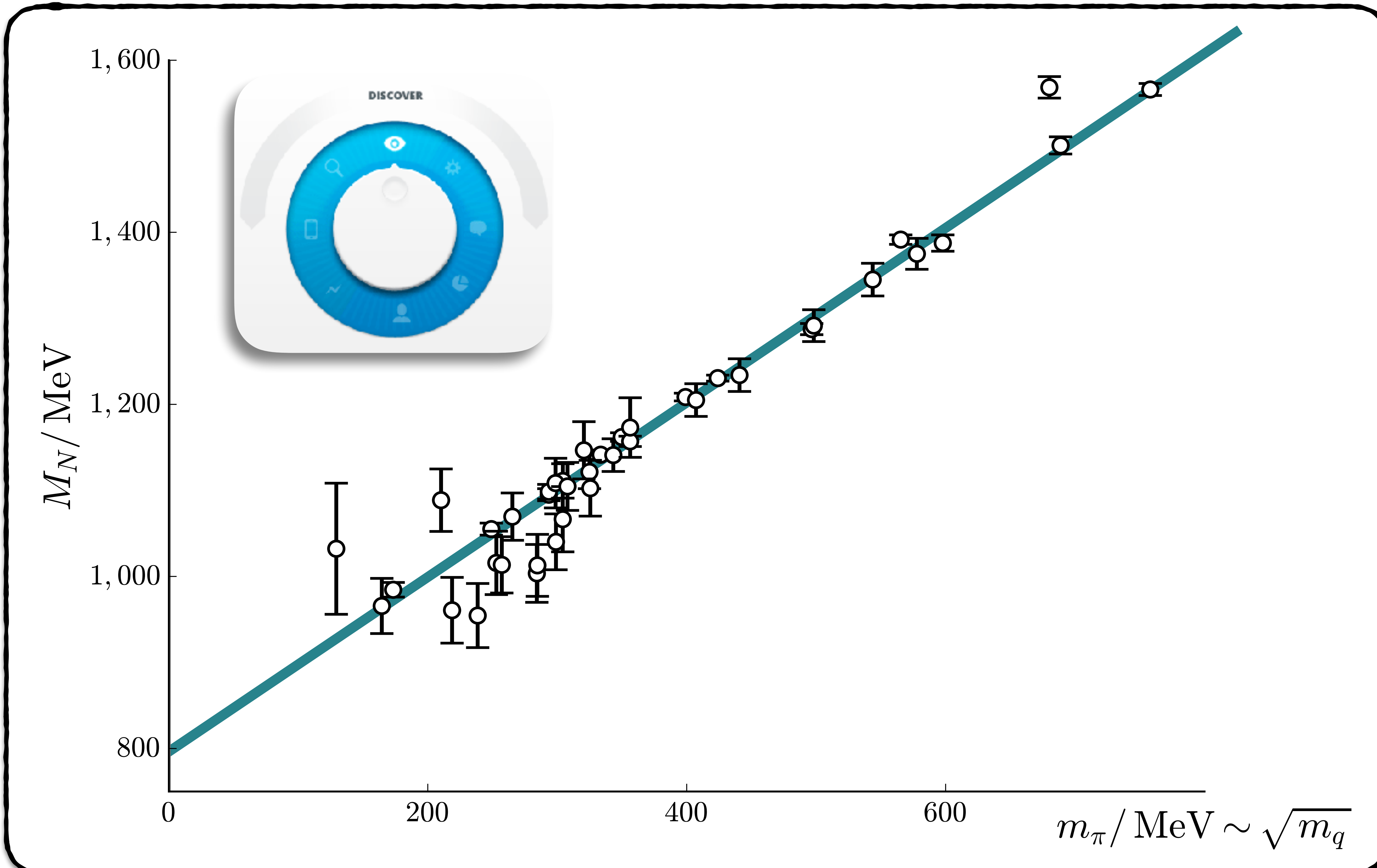


Higgs is responsible for giving leptons and quarks mass.

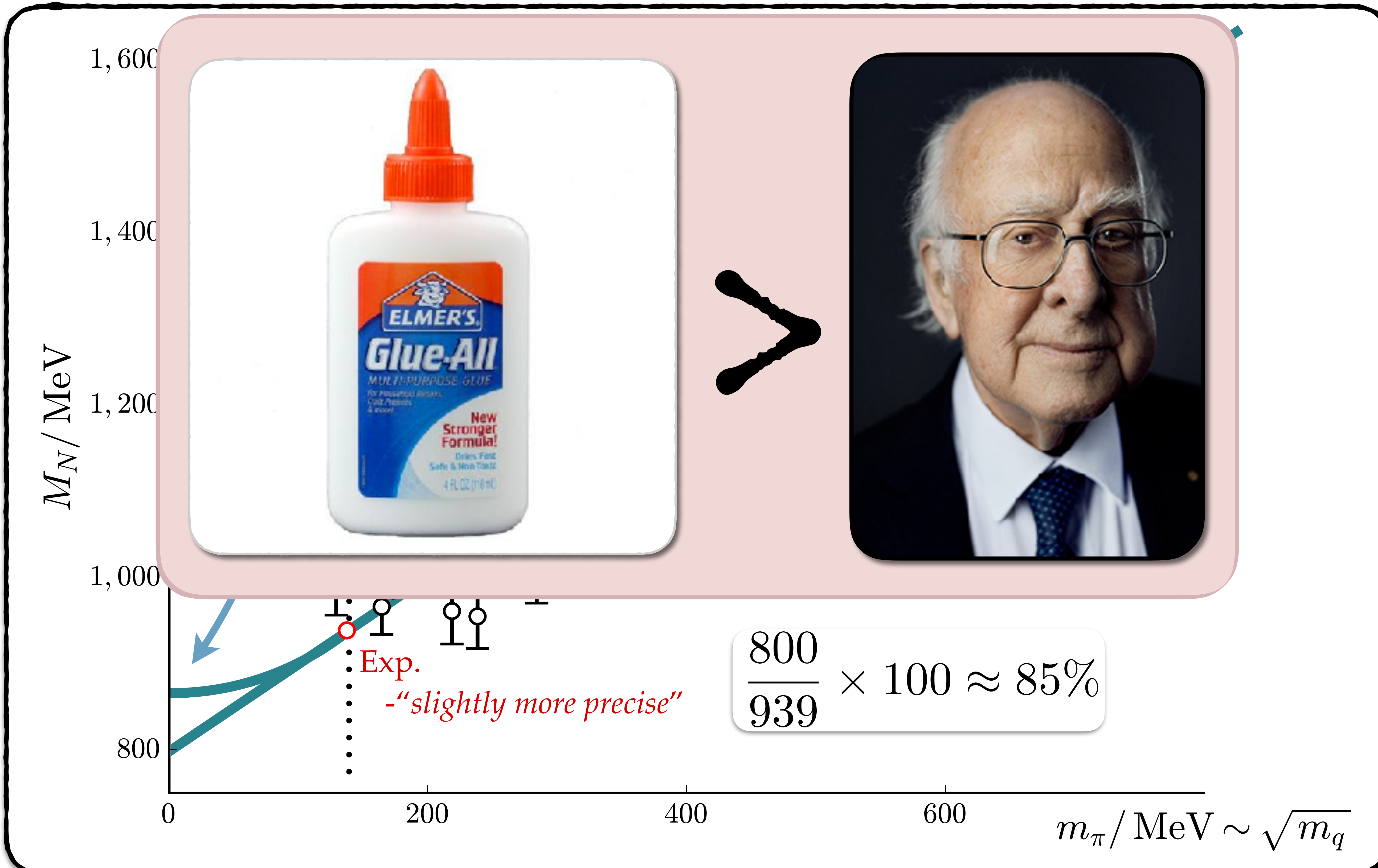
the origin of mass



the origin of mass



the origin of mass



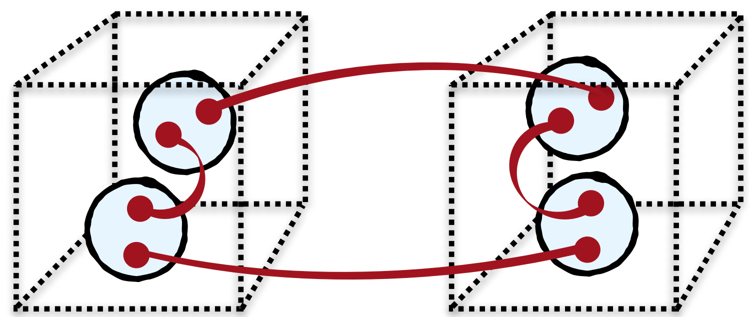
Jazzi loves car rides!

Questions?

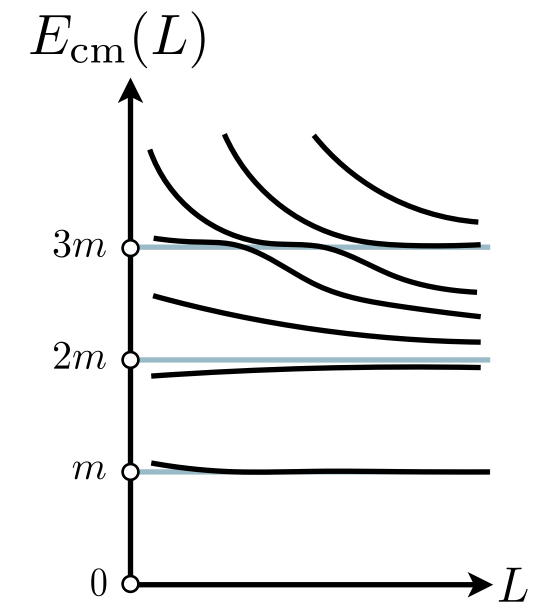
Outline

QCD: m_q/Λ_{QCD}

Correlation functions

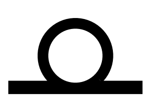


Finite-volume spectrum:



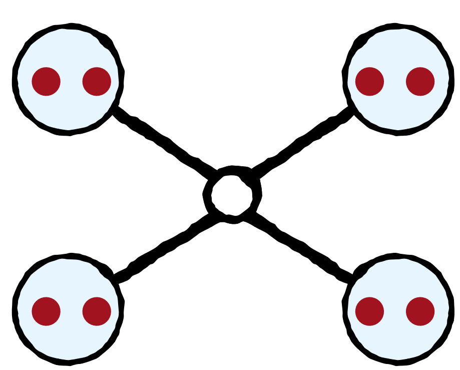
Interpretation of spectrum:

if $E_{\mathrm{cm}}(L) \ll 2m$: Masses of hadrons

 $\longleftrightarrow$ $E_{\mathrm{cm}}(L) = \sqrt{p^2 + m^2} + \mathcal{O}(e^{-m_\pi L})$

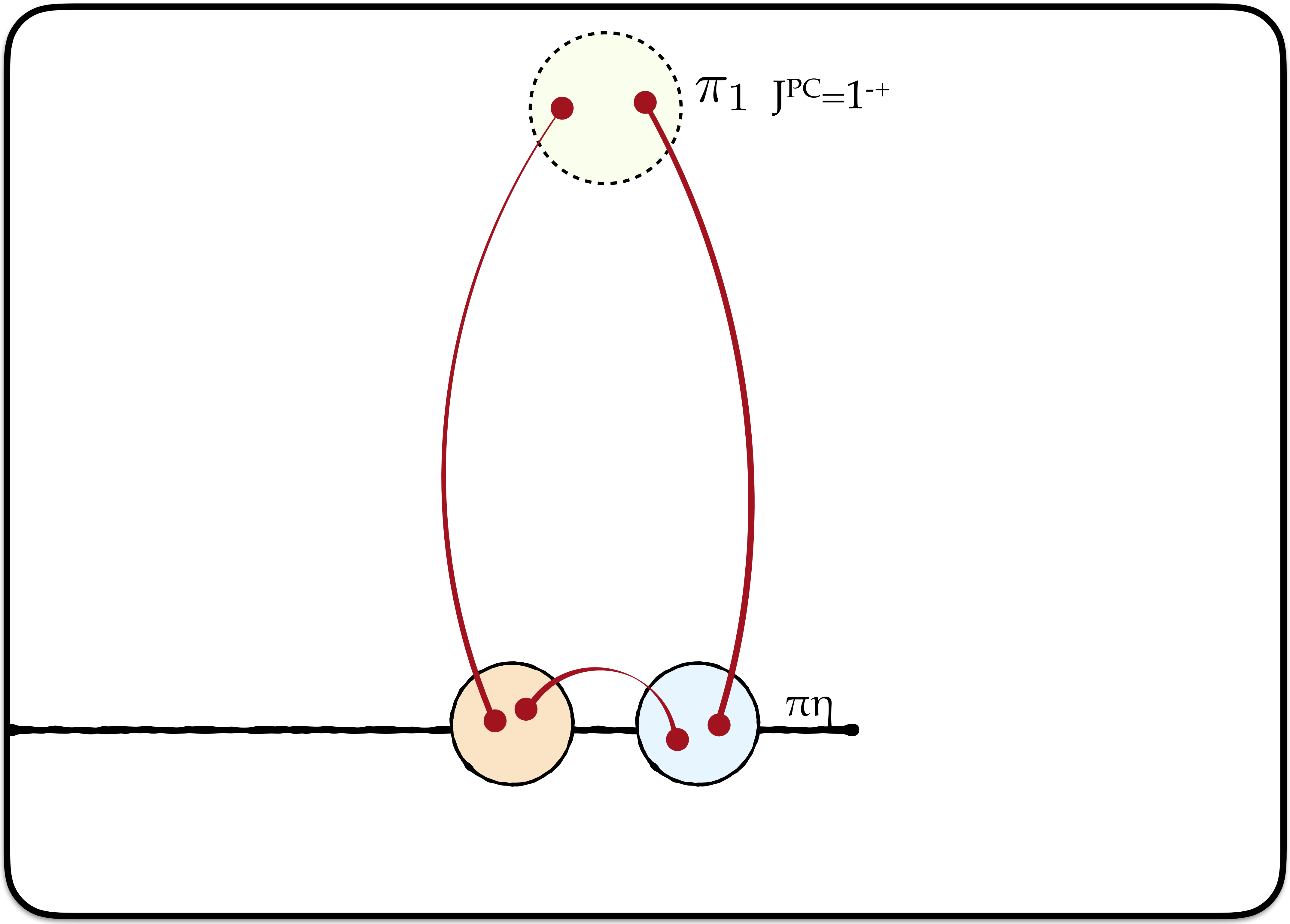
else:

$E_{\mathrm{cm}}(L) \longleftrightarrow$

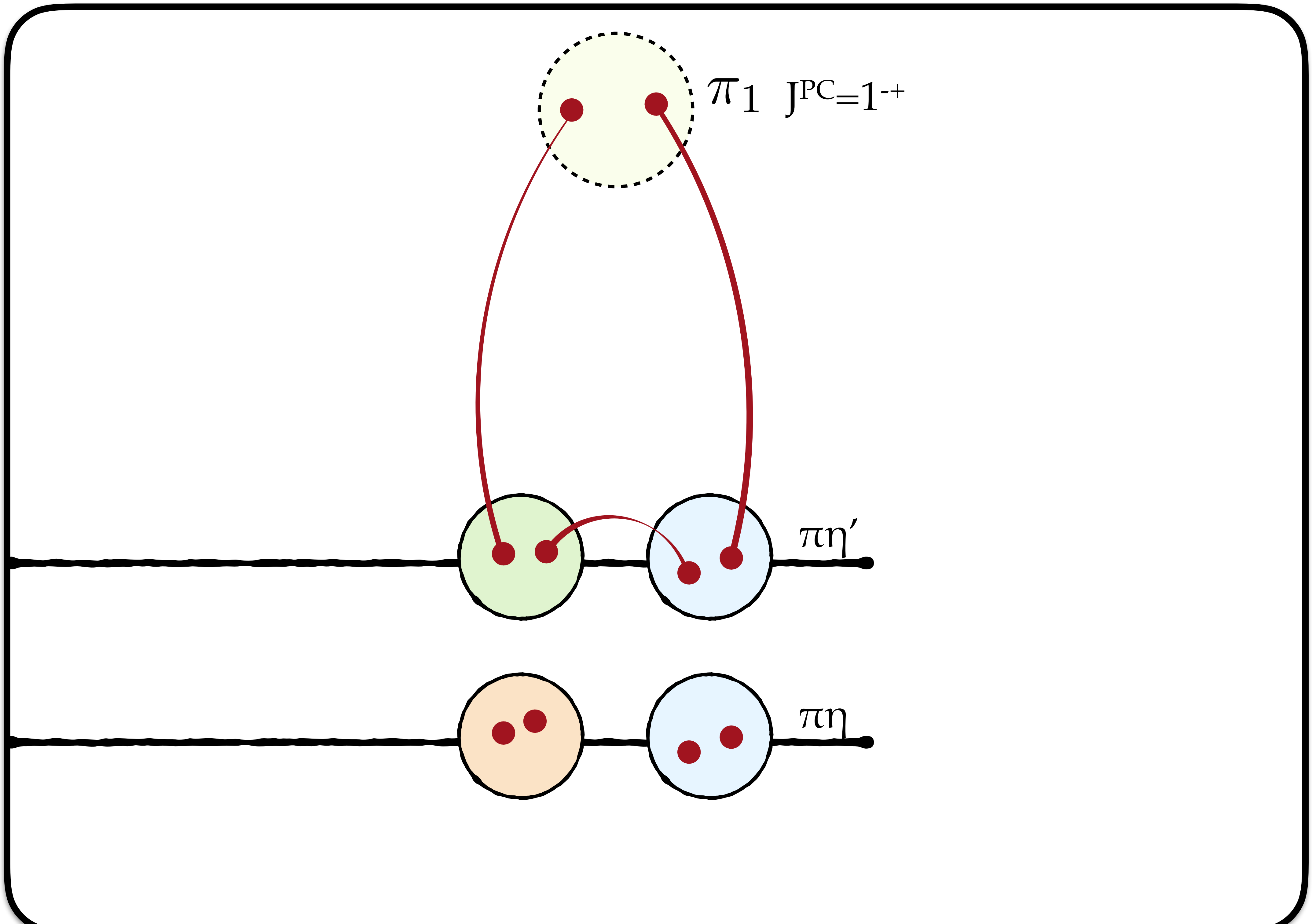


Scattering amplitudes

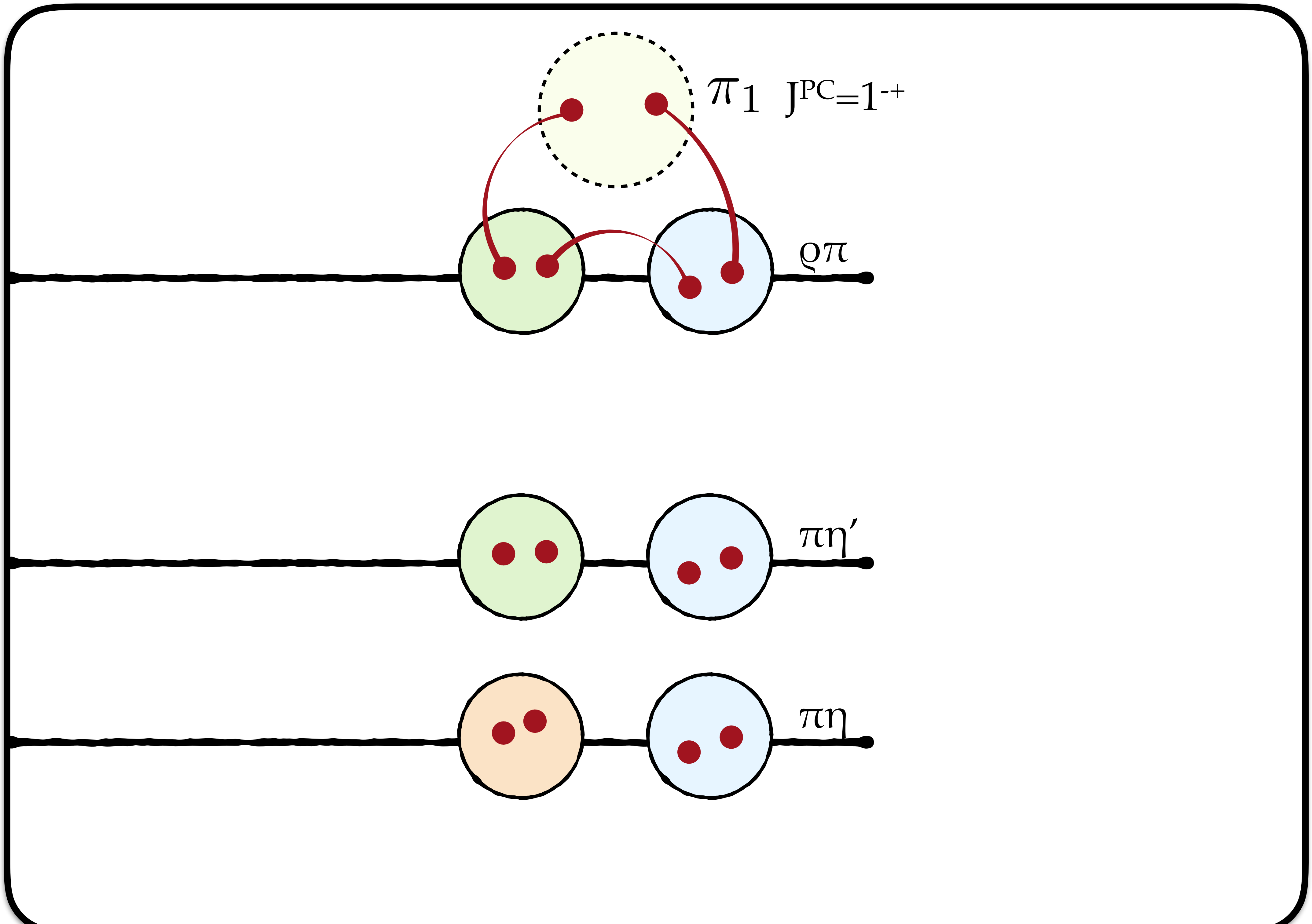
Exotics



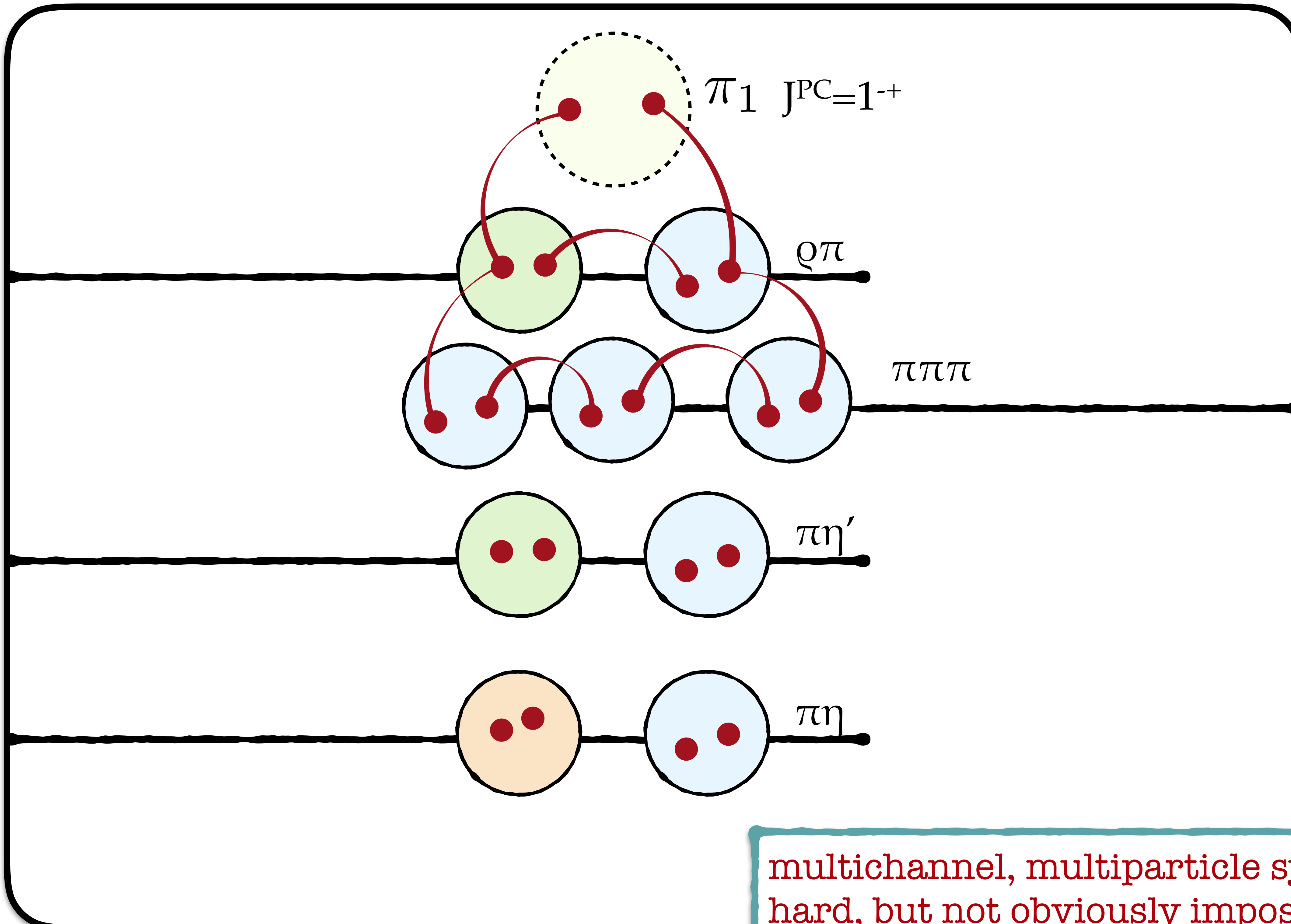
Exotics



Exotics



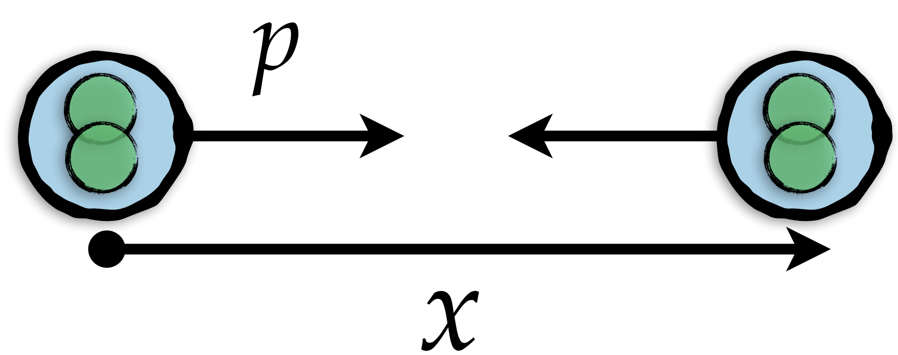
Exotics



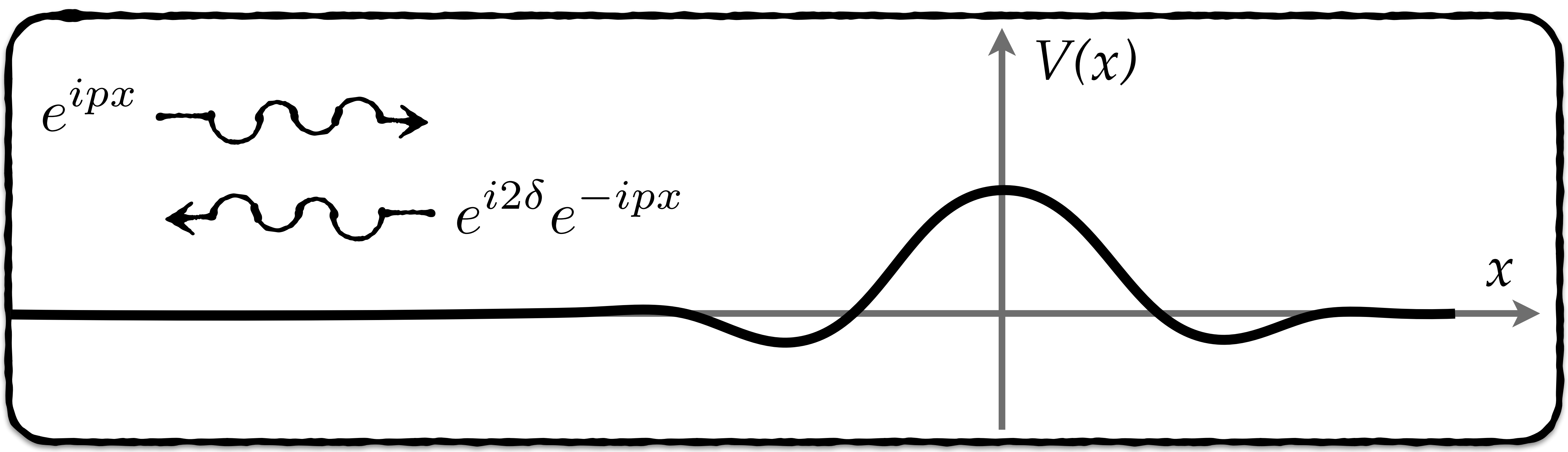
multichannel, multiparticle system!
hard, but not obviously impossible...
let's take it one step at a time...

Two particles in 1+1 Dimensions

- Two-particle plane waves: $\varphi_{k_1 k_2}(x_1, x_2) = e^{ip_1 x_1} e^{ip_2 x_2}$
- In the c.m. frame: $\varphi_p(x_1, x_2) = e^{ip(x_1 - x_2)} \equiv e^{ipx}$



- Imagine a finite-range potential that depends on x:



Two particles in 1+1 Dimensions

- Outside the potential, we get

$$\varphi_p(x) \sim e^{ipx} + Ae^{i2\delta}e^{-ipx} = e^{i\delta}\left(e^{-i(p|x|+\delta)} + A\,e^{i(p|x|+\delta)}\right), \qquad (x<0)$$

$$\varphi_p(x) \sim e^{-ipx} + Be^{i2\delta}e^{ipx} = e^{i\delta}\left(e^{-i(p|x|+\delta)} + B\,e^{i(p|x|+\delta)}\right), \qquad (x<0)$$

- Imposing exchange symmetry: $\varphi_p(x) = \varphi_p(-x)$

  we obtain: $\varphi_p(x) \sim \cos(p|x| + \delta)$

- More explicitly:

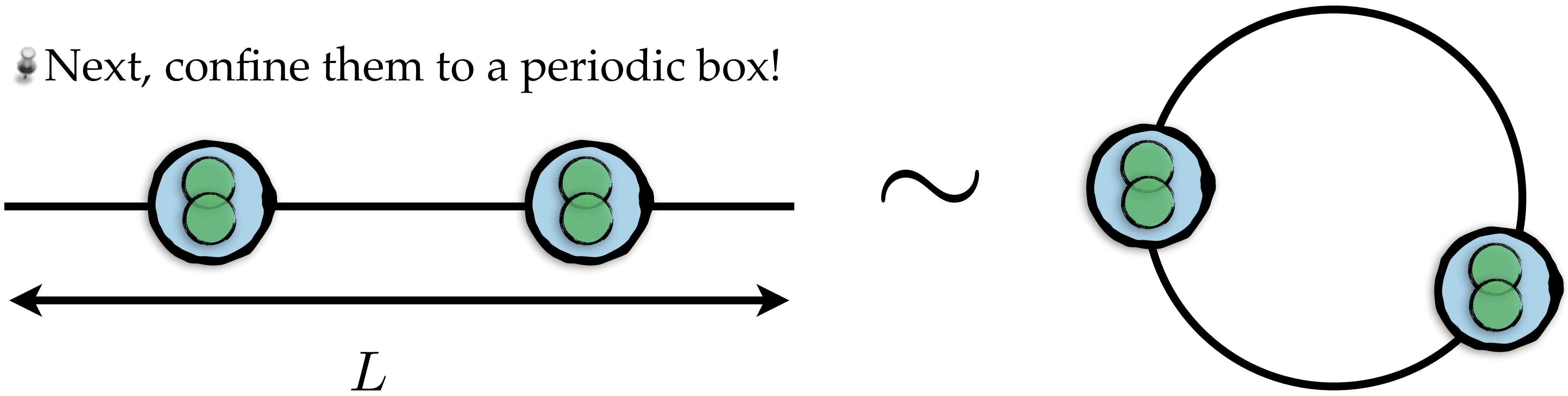
$$\varphi_p(x) \sim \cos(-px+\delta), \qquad (x<0)$$
$$\sim \cos(px+\delta), \qquad (x<0)$$

- Its derivative:

$$\varphi_p'(x) \sim p\sin(-px+\delta), \qquad (x<0)$$
$$\sim -p\sin(px+\delta), \qquad (x<0)$$

Two particles in 1+1 Dimensions

- Next, confine them to a periodic box!



- Imposing periodicity:

$$\varphi_p(L/2) = \varphi_p(-L/2) \Leftrightarrow \cos(p\frac{L}{2} + \delta) = cos(p\frac{L}{2} + \delta)$$

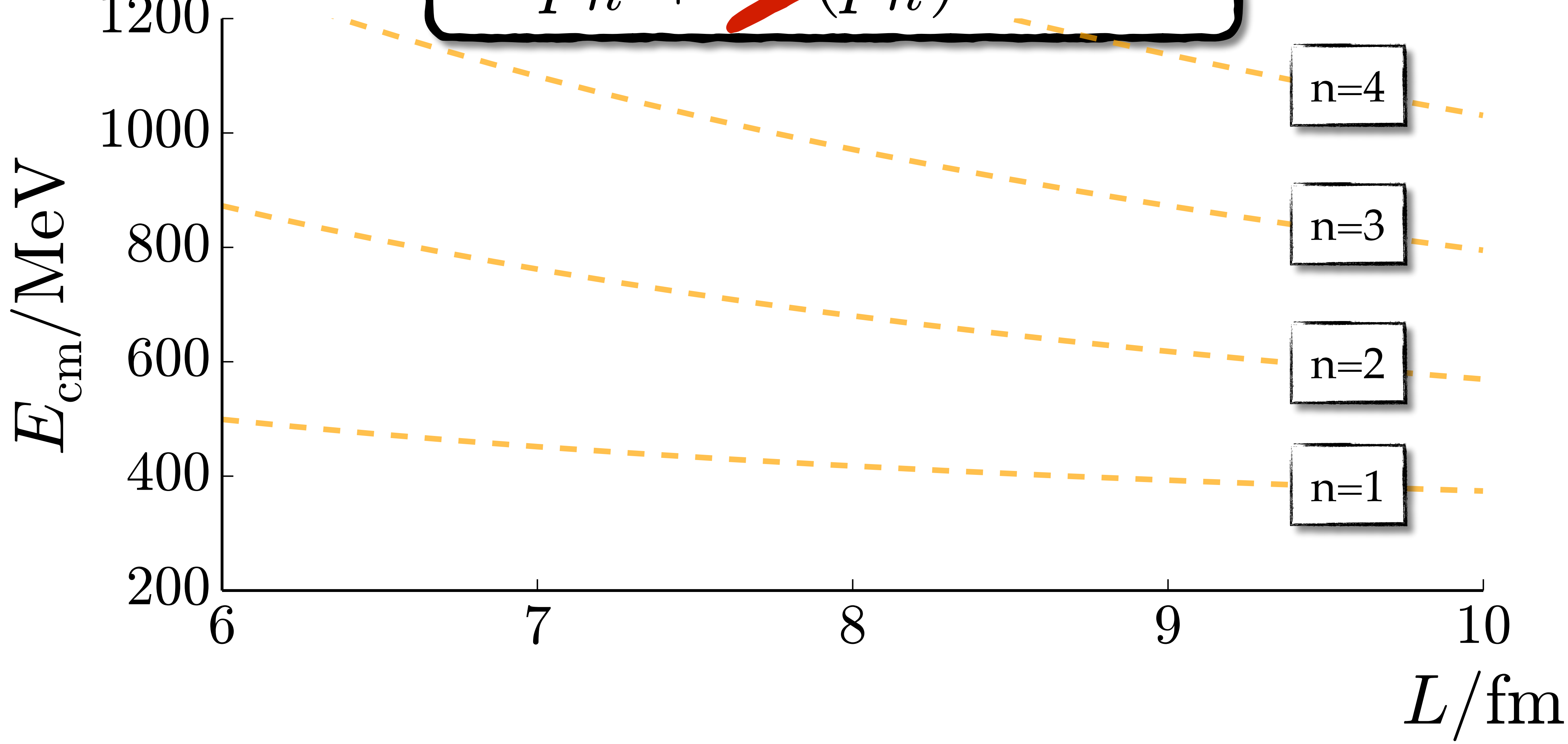
$$\varphi_p'(L/2) = \varphi_p'(-L/2) \Leftrightarrow p\sin(p\frac{L}{2} + \delta) = -p\sin(p\frac{L}{2} + \delta) = 0$$

- Quantization condition:

$$L\ p_n + 2\delta(p_n) = 2\pi n$$

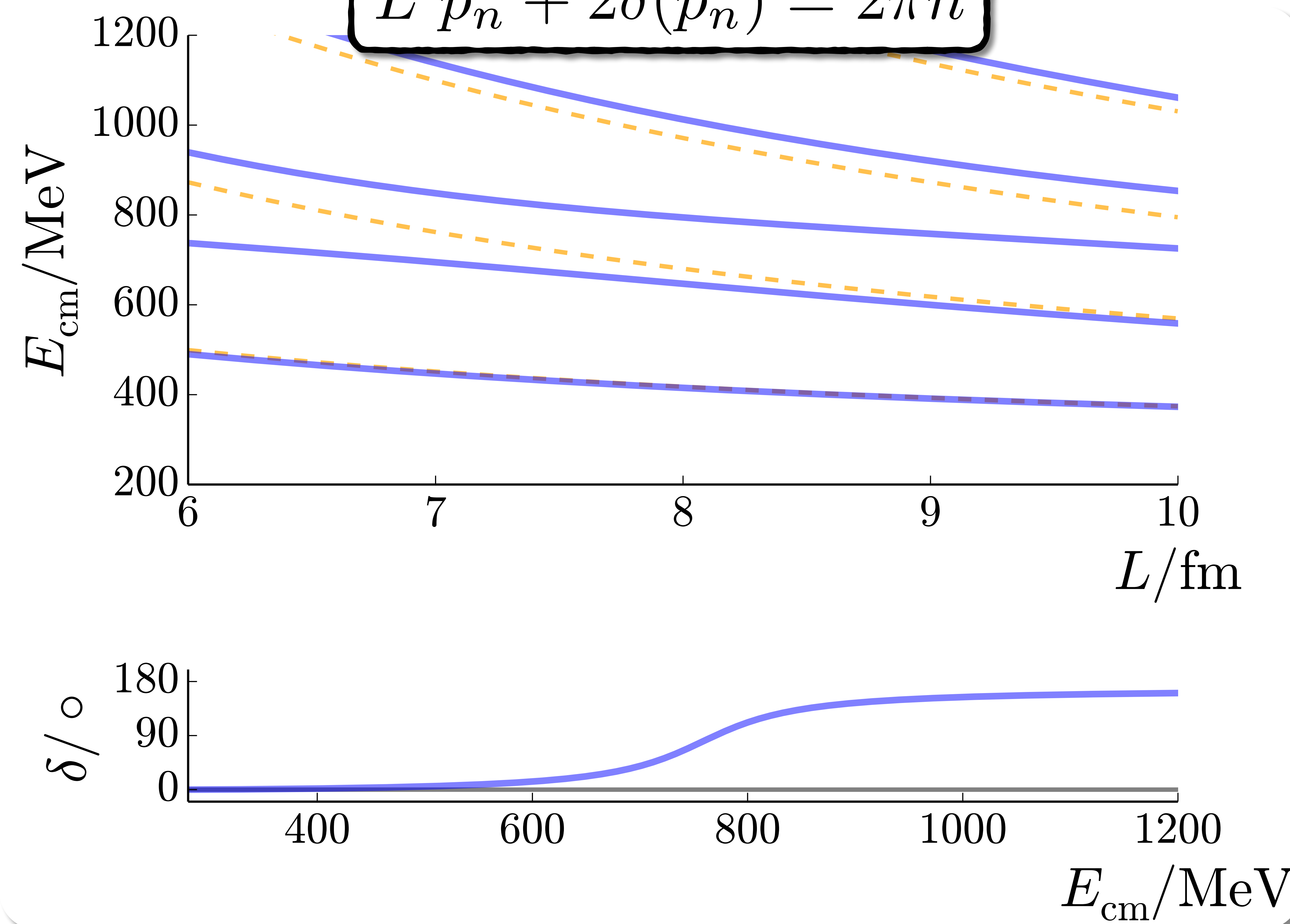
Spectrum in a 1+1D box

$$L\ p_n + 2\delta(p_n) = 2\pi n$$



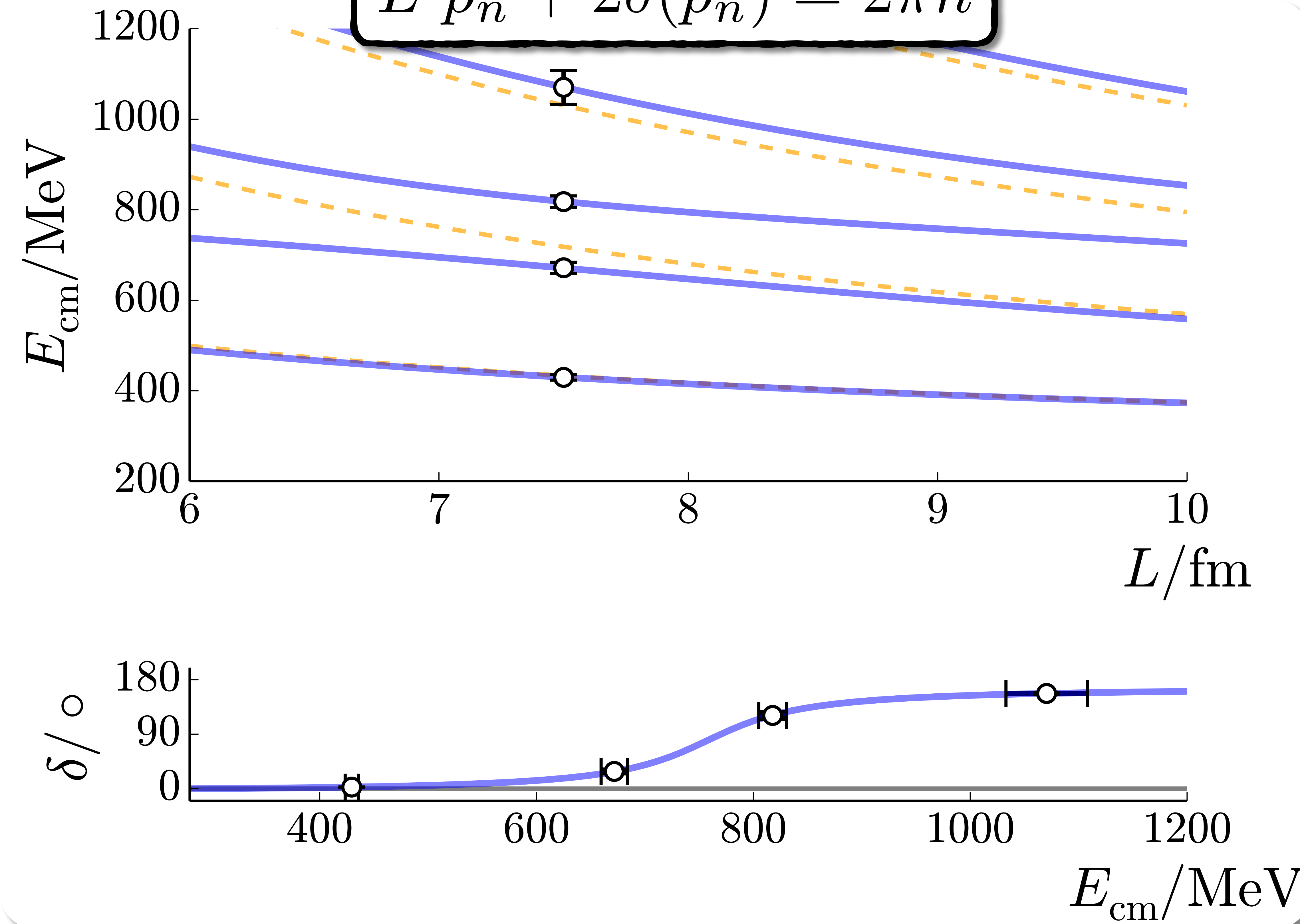
Spectrum in a 1+1D box

$$L\ p_n + 2\delta(p_n) = 2\pi n$$

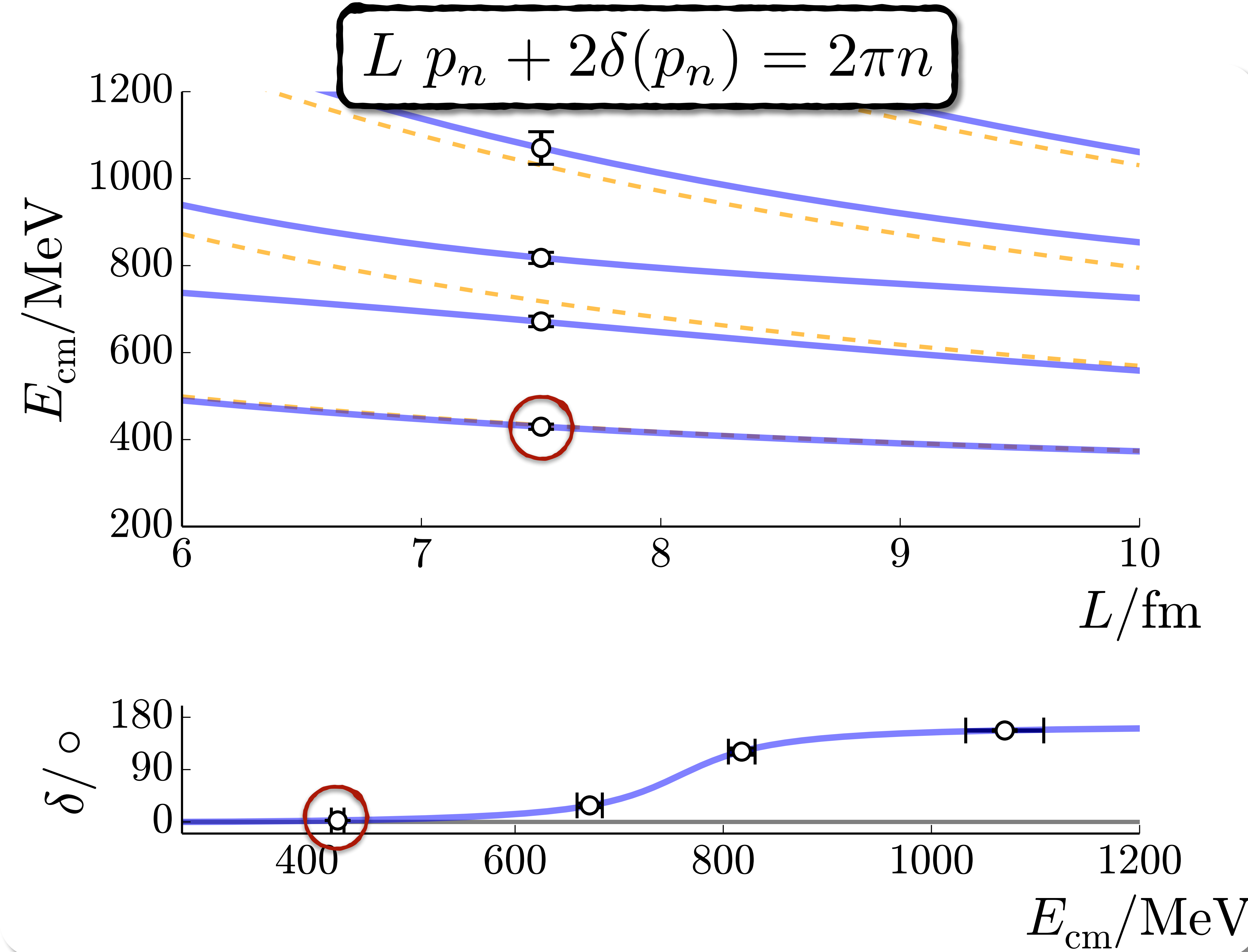


Spectrum in a 1+1D box

$$L\ p_n + 2\delta(p_n) = 2\pi n$$

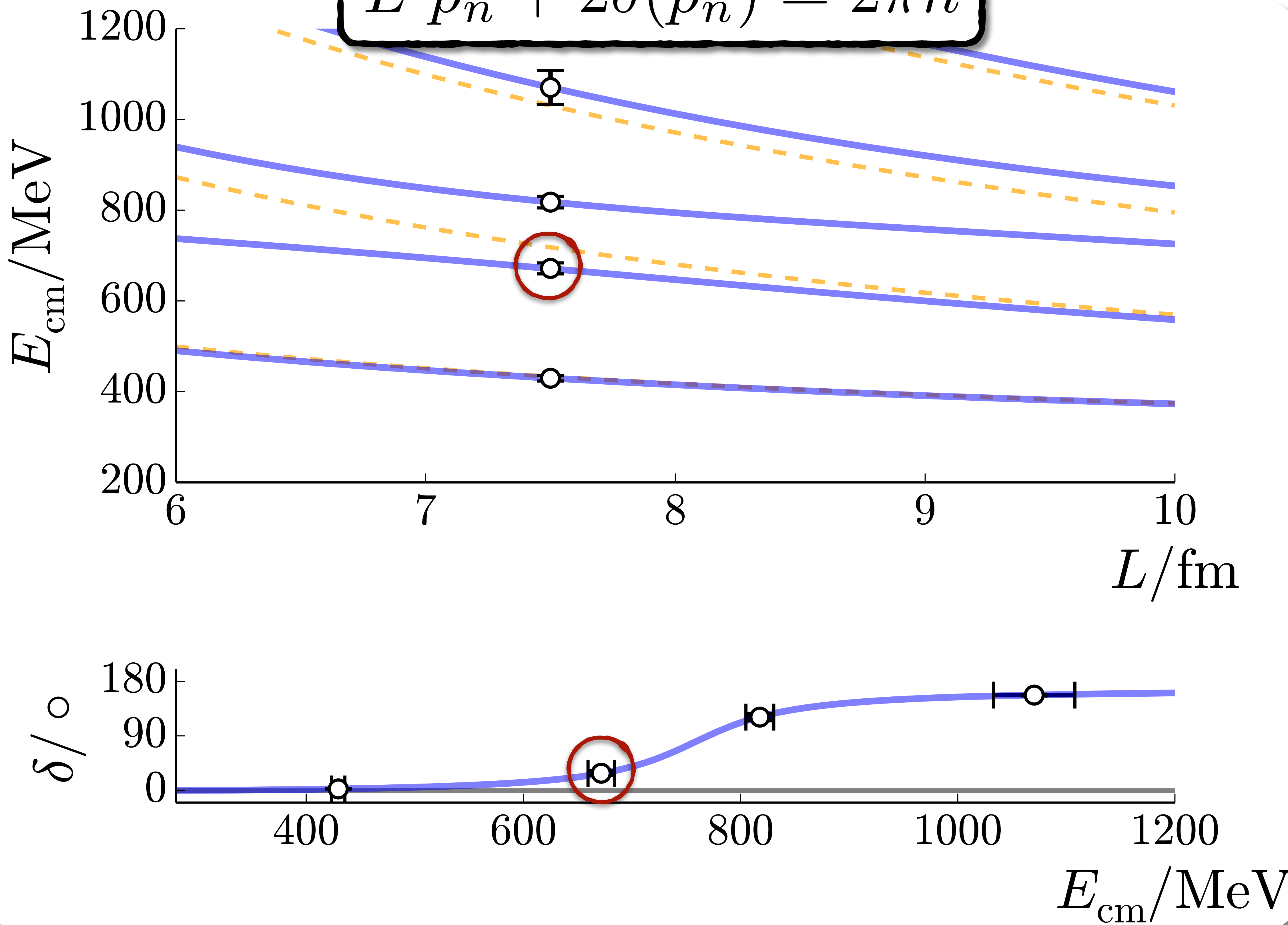


Spectrum in a 1+1D box



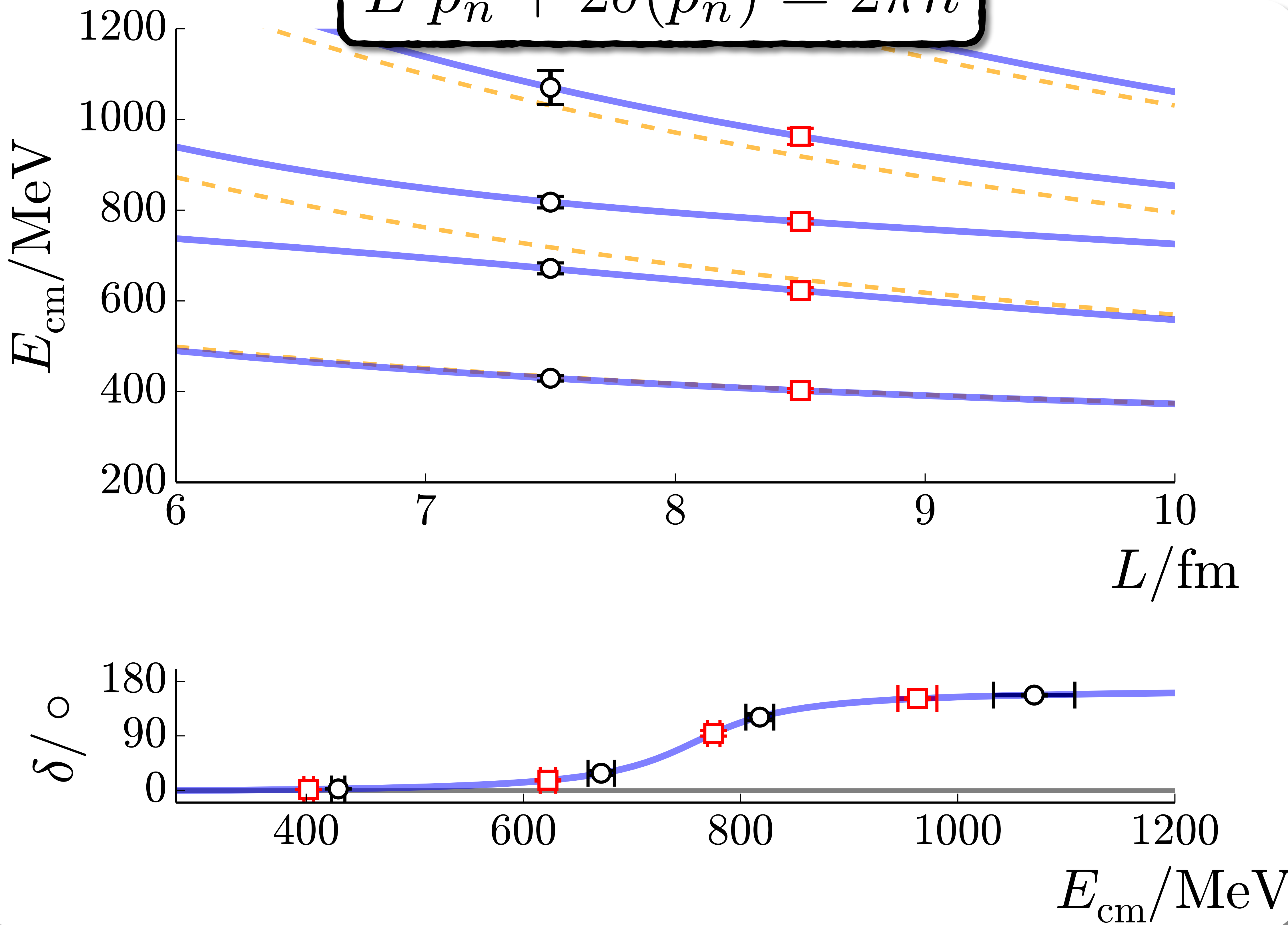
Spectrum in a 1+1D box

$$L\ p_n + 2\delta(p_n) = 2\pi n$$



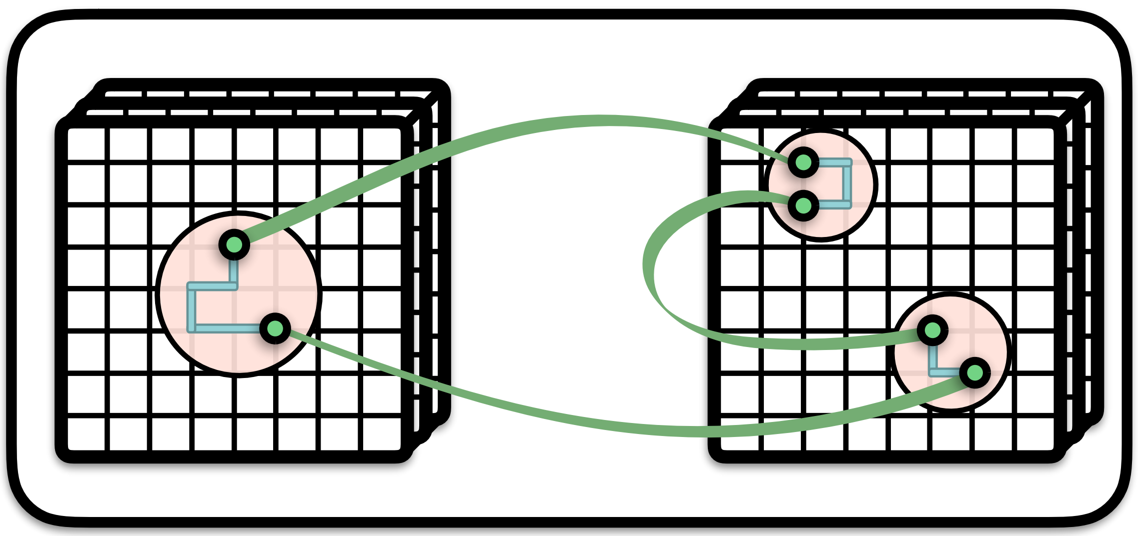
Spectrum in a 1+1D box

$$L\ p_n + 2\delta(p_n) = 2\pi n$$

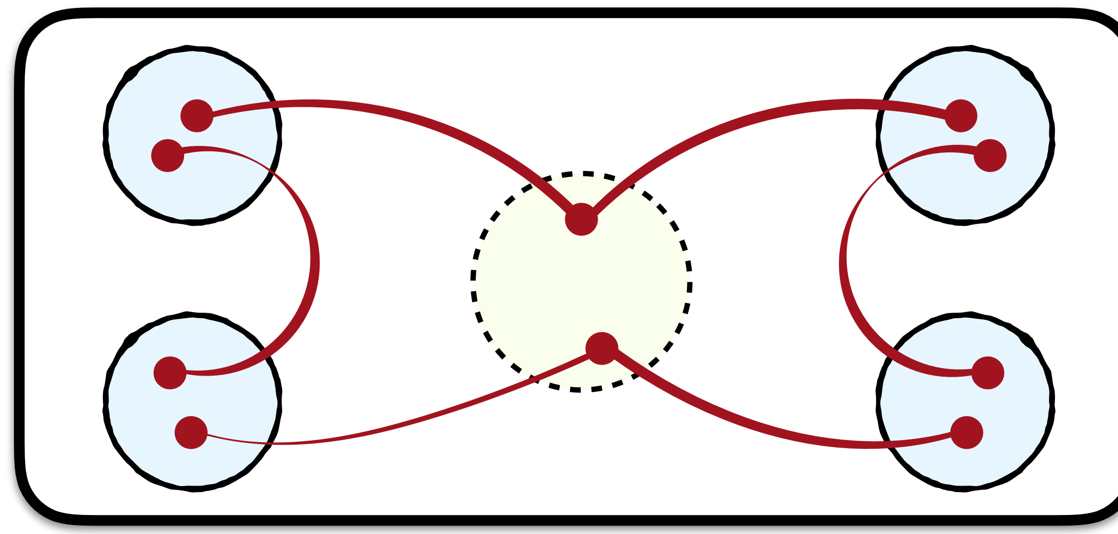


Spectrum in a 3+1D box

$$\det[F_2^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$$



finite volume spectrum



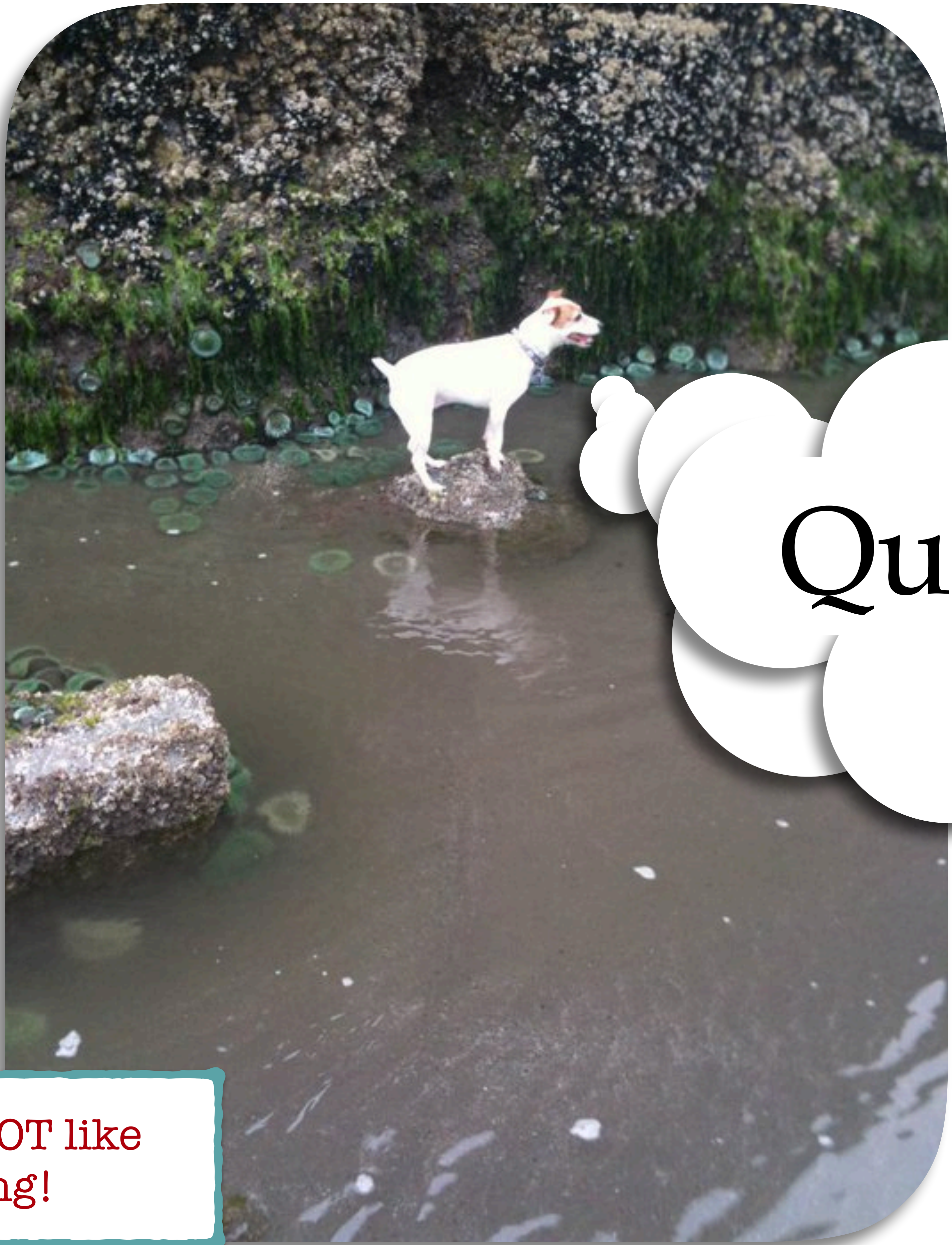
scattering amplitude

E_L = finite volume spectrum

L = finite volume

F_2 = known function

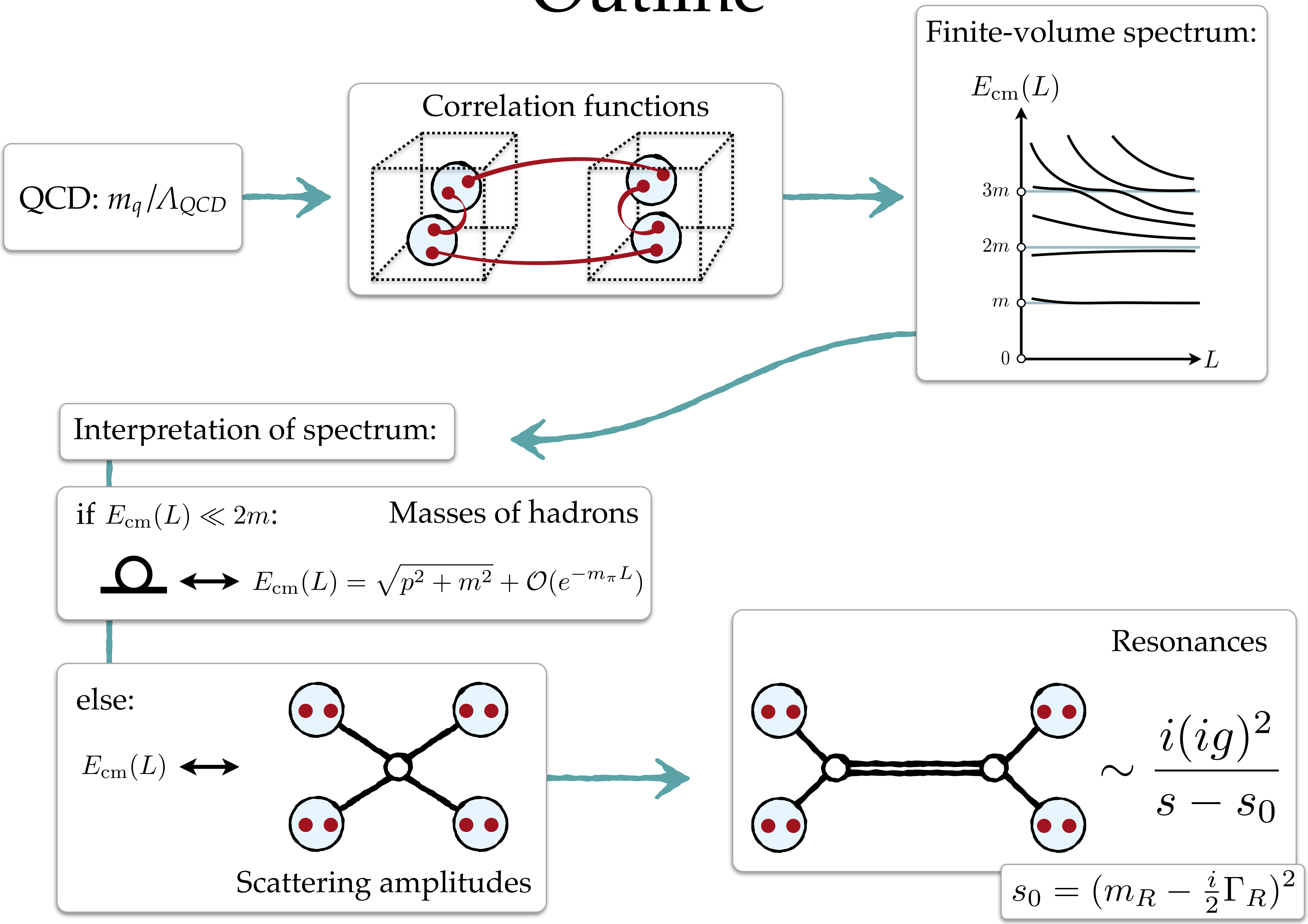
$\mathcal{M}$ = scattering amplitude



Questions?

Jazzi does NOT like swimming!

Outline



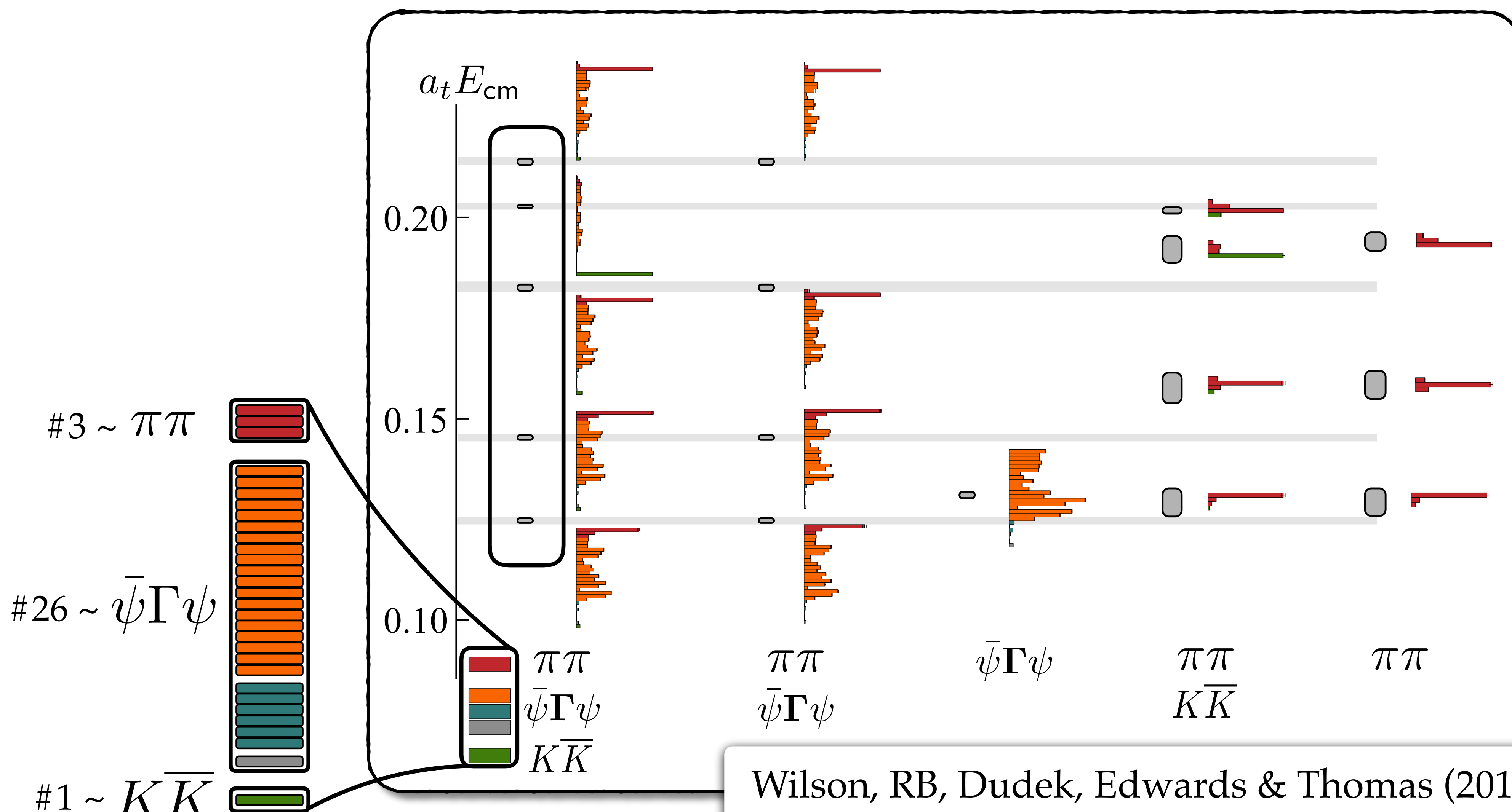
QCD: m_q/Λ_{QCD}
Correlation functions
Finite-volume spectrum:
$E_{\mathrm{cm}}(L)$
$3m$
$2m$
m
0
L
Interpretation of spectrum:
if $E_{\mathrm{cm}}(L) \ll 2m$:
Masses of hadrons
$\longleftrightarrow$ $E_{\mathrm{cm}}(L) = \sqrt{p^2 + m^2} + \mathcal{O}(e^{-m_\pi L})$
else:
$E_{\mathrm{cm}}(L) \longleftrightarrow$
Scattering amplitudes
Resonances
$\sim \frac{i(ig)^2}{s - s_0}$
$s_0 = (m_R - \frac{i}{2}\Gamma_R)^2$

Extracting the spectrum

Two-point correlation functions:

$$C_{ab}^{2pt.}(t,\mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t,\mathbf{P})\mathcal{O}_a^\dagger(0,\mathbf{P})|0\rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

- Use lots of operators
- e.g., $\pi\pi$ isotriplet at rest, m_π=236 MeV

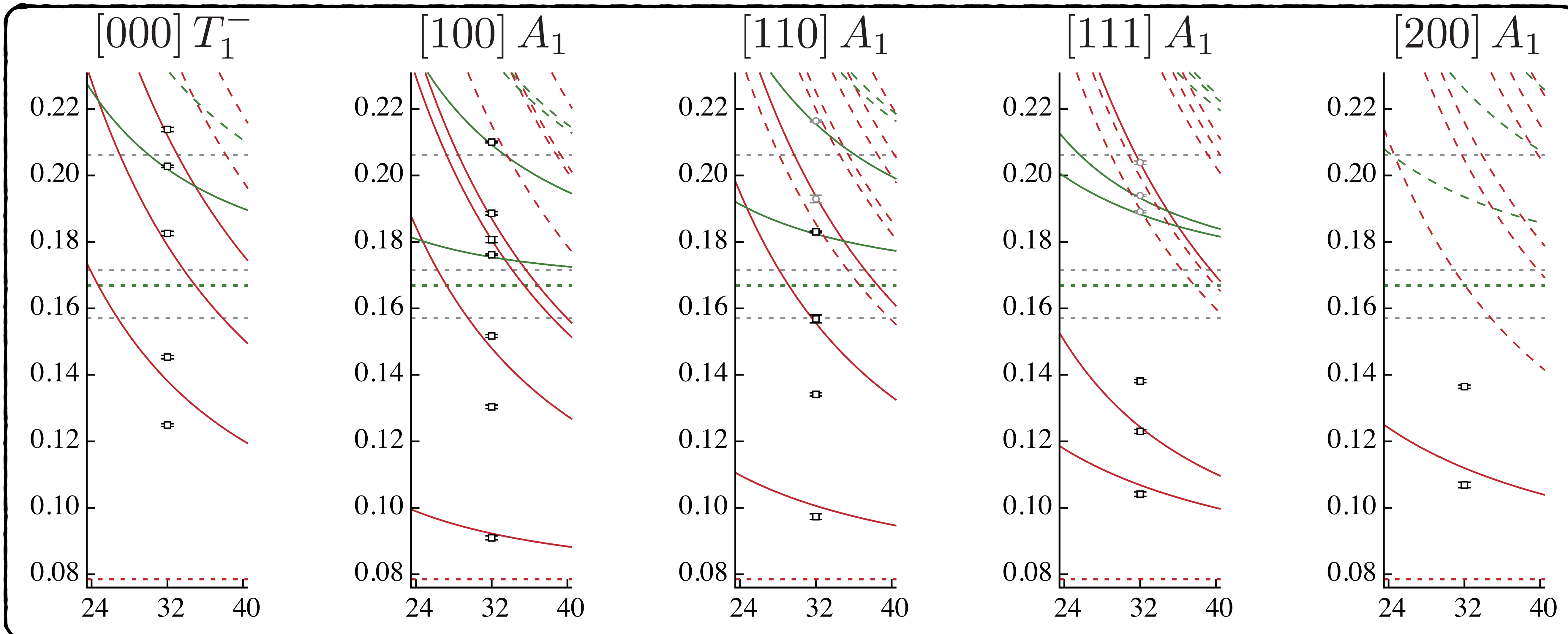


Wilson, RB, Dudek, Edwards & Thomas (2015)

$\pi\pi$ scattering

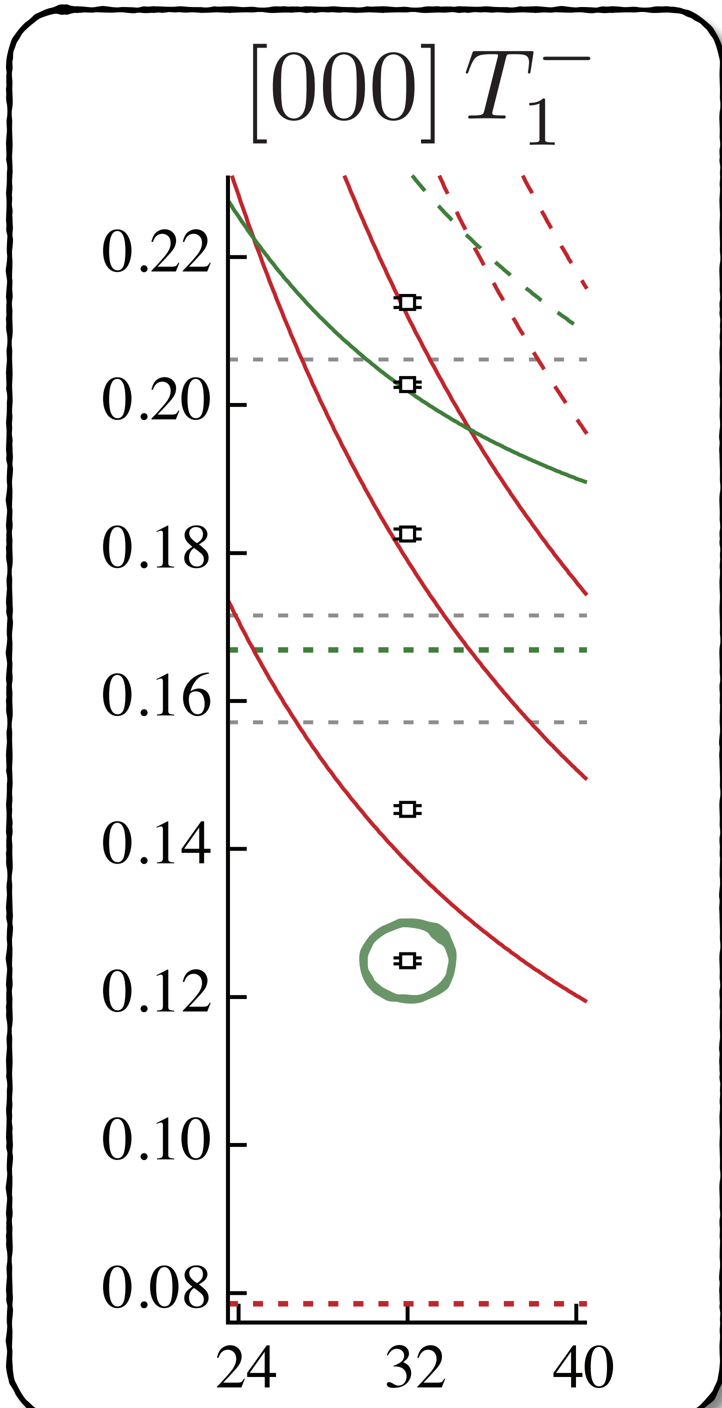
(I=1 channel)

Half of the spectrum:

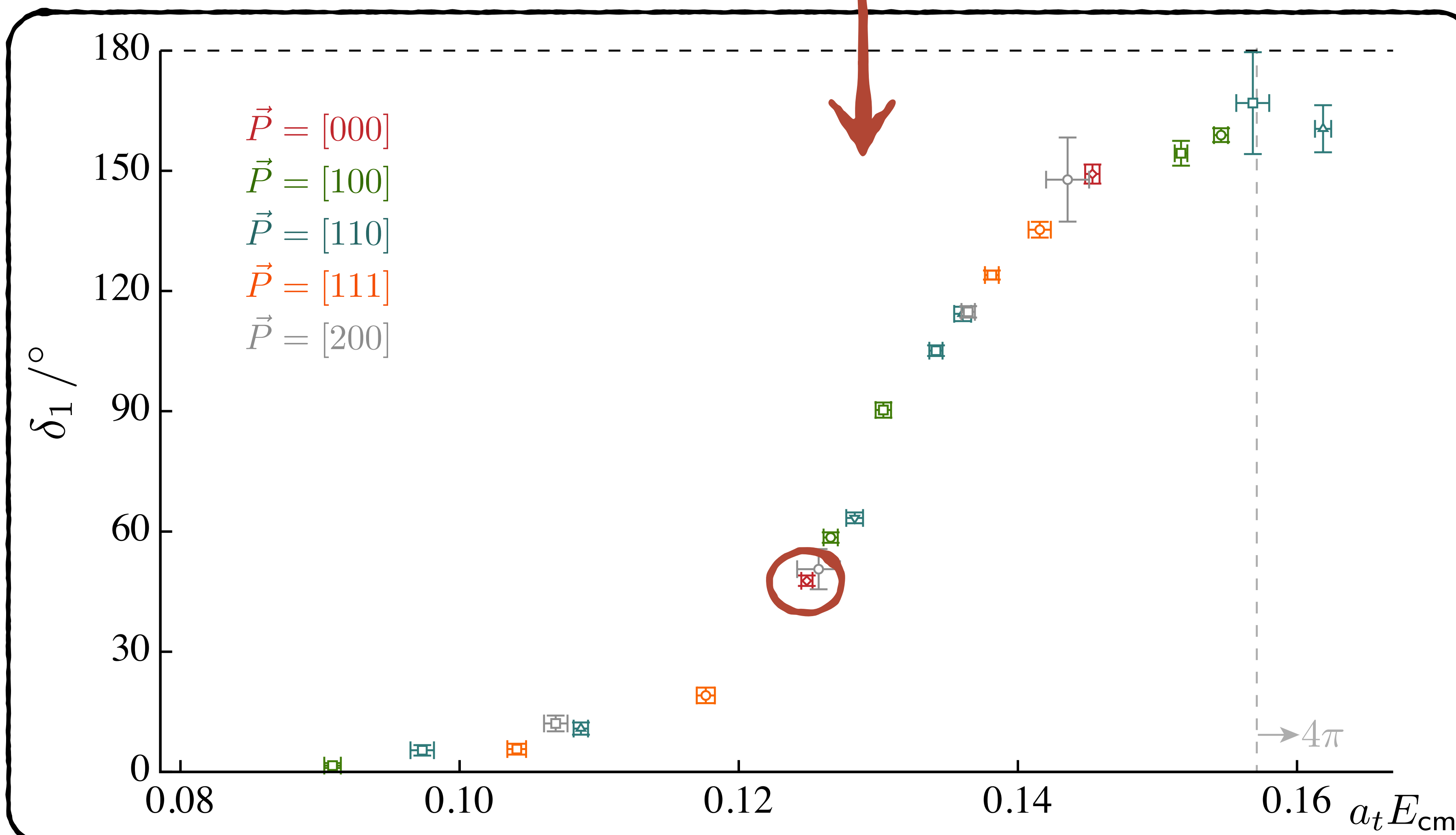


Wilson, RB, Dudek, Edwards & Thomas (2015)

Isovector $\pi\pi$ scattering



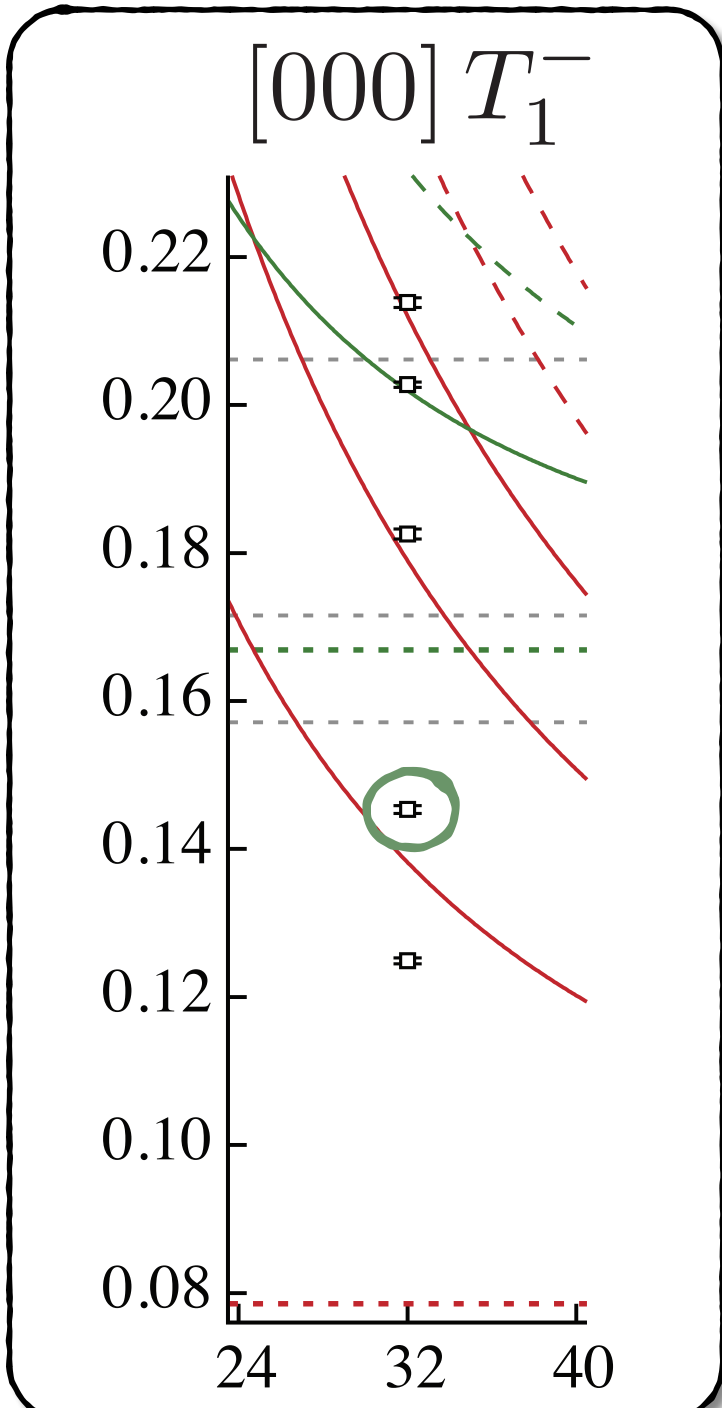
$$\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$$



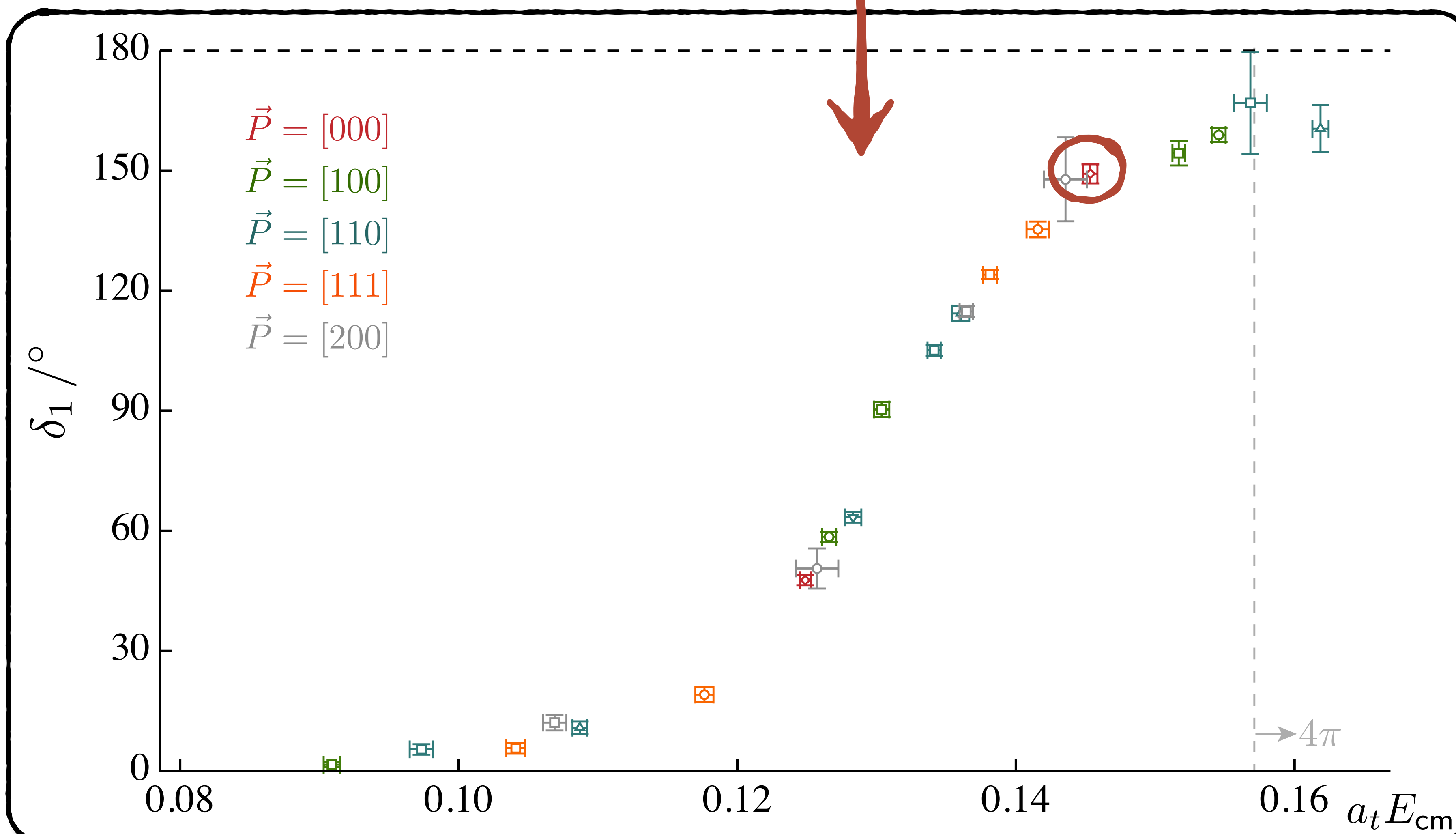
$$\mathcal{M} \propto \frac{1}{\cot \delta_1 - i}$$

Wilson, RB, Dudek, Edwards & Thomas (2015)

Isovector $\pi\pi$ scattering



$$\det[F^{-1}(E_L, L) + \mathcal{M}(E_L)] = 0$$



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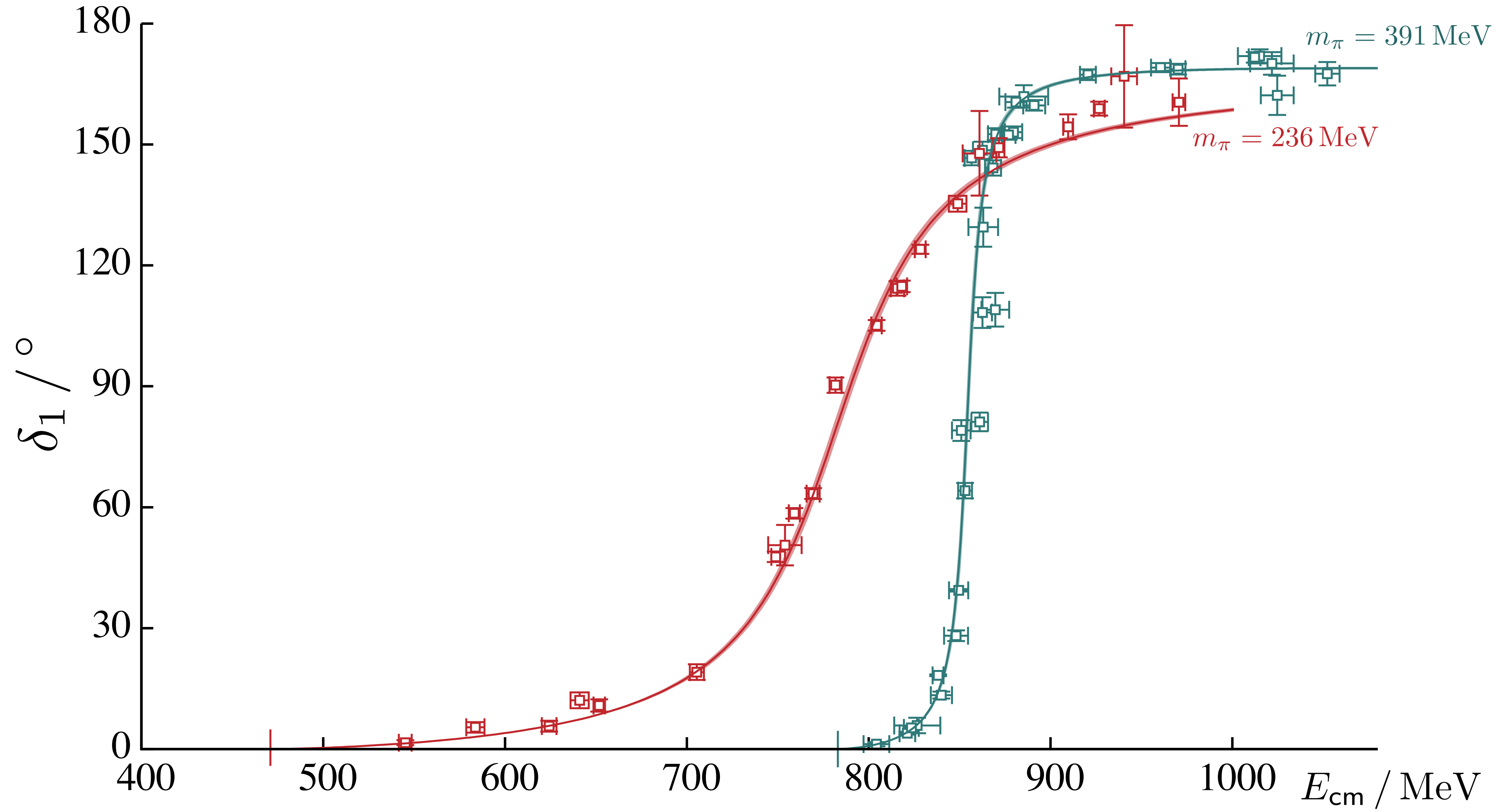
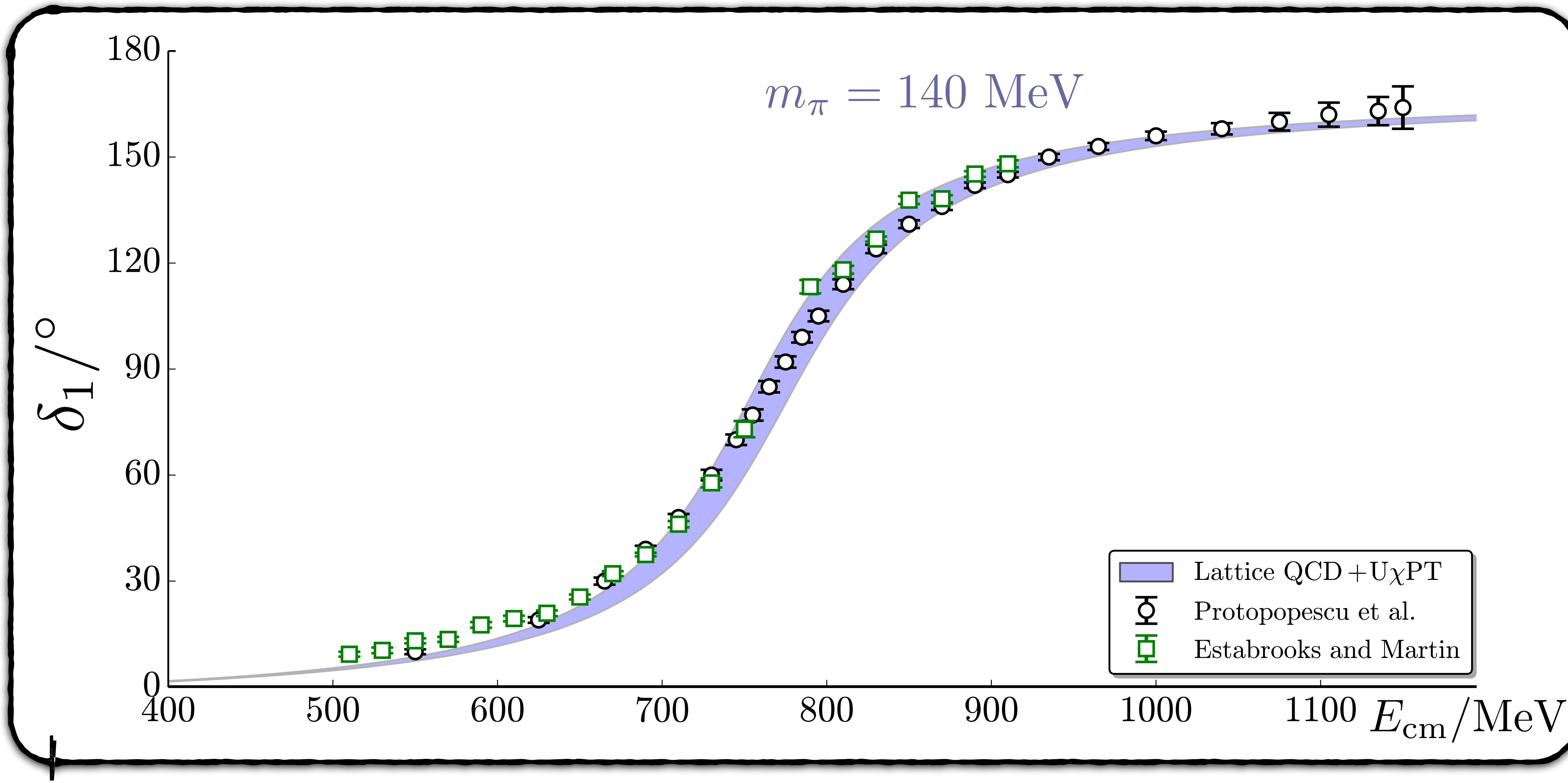
Wilson, RB, Dudek, Edwards & Thomas (2015)

Isovector $\pi\pi$ scattering



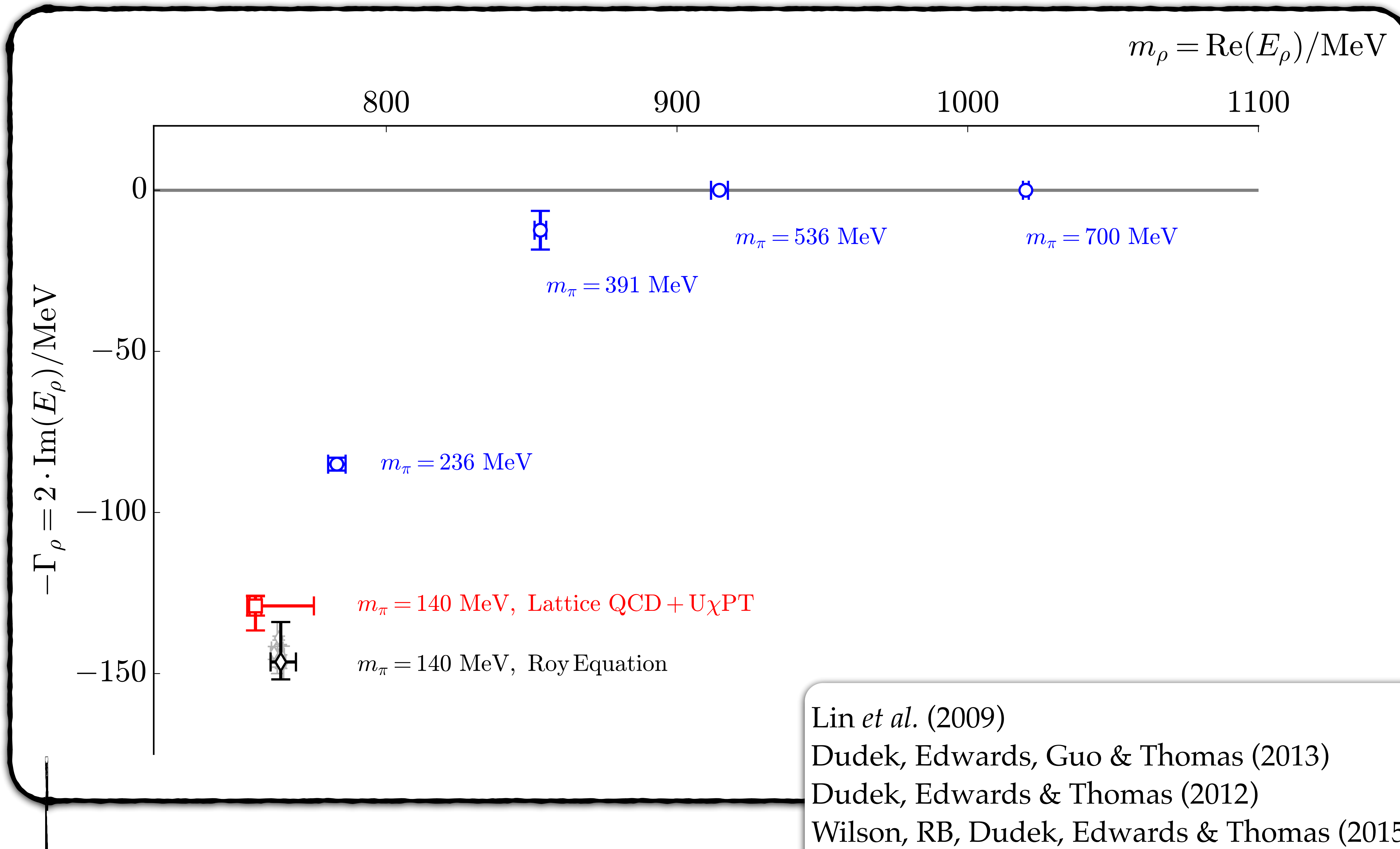
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)

Comparison with experiment



Bolton, RB & Wilson (2016)

The ρ vs m_π



Lin *et al.* (2009)
Dudek, Edwards, Guo & Thomas (2013)
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
Bolton, RB & Wilson (2015)

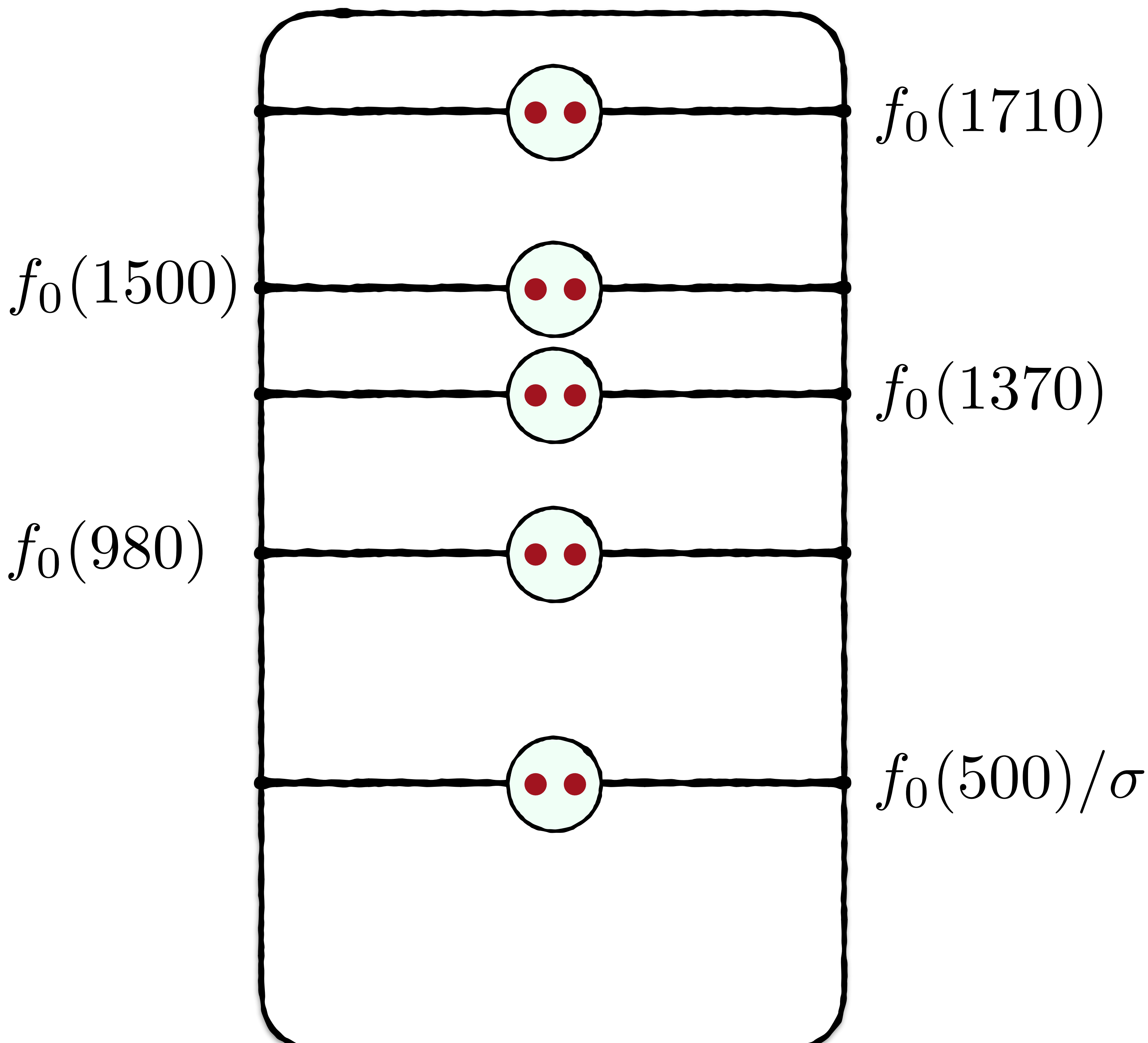
Questions?

Jazzi loves the great outdoors,...

The real frontier

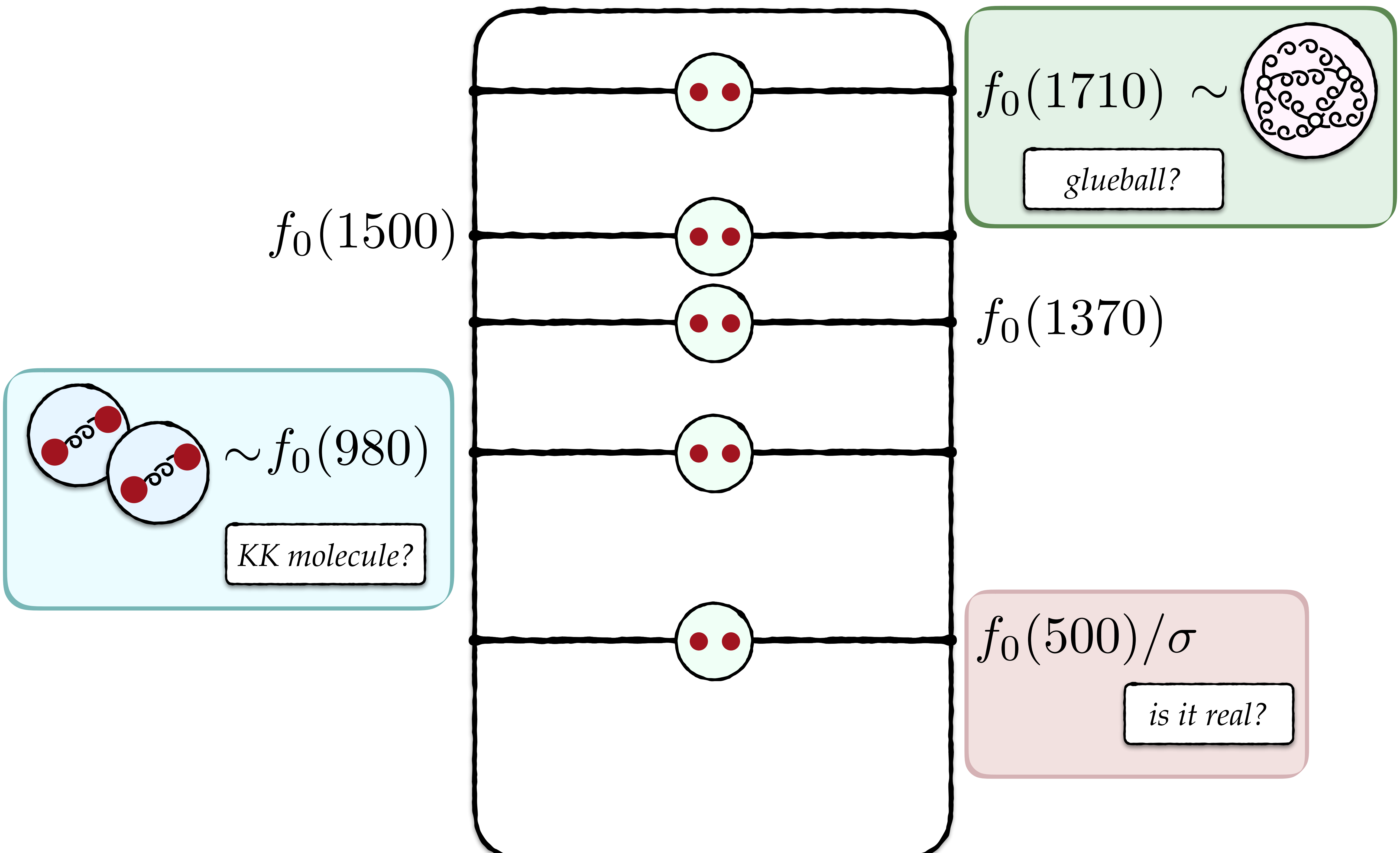
The isoscalar, scalar sector

[i.e., the quantum numbers of the vacuum]



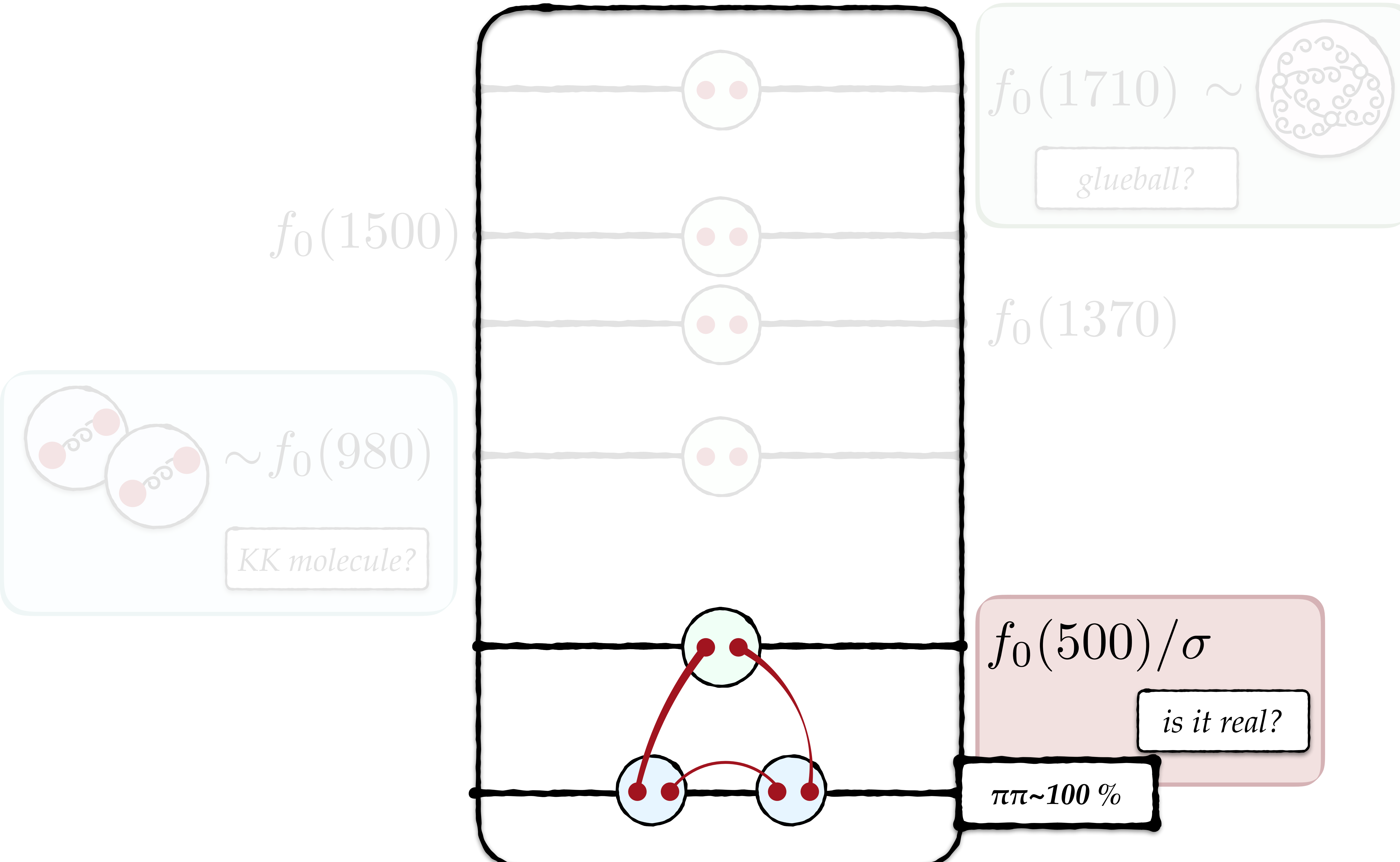
The isoscalar, scalar sector

[i.e., the quantum numbers of the vacuum]



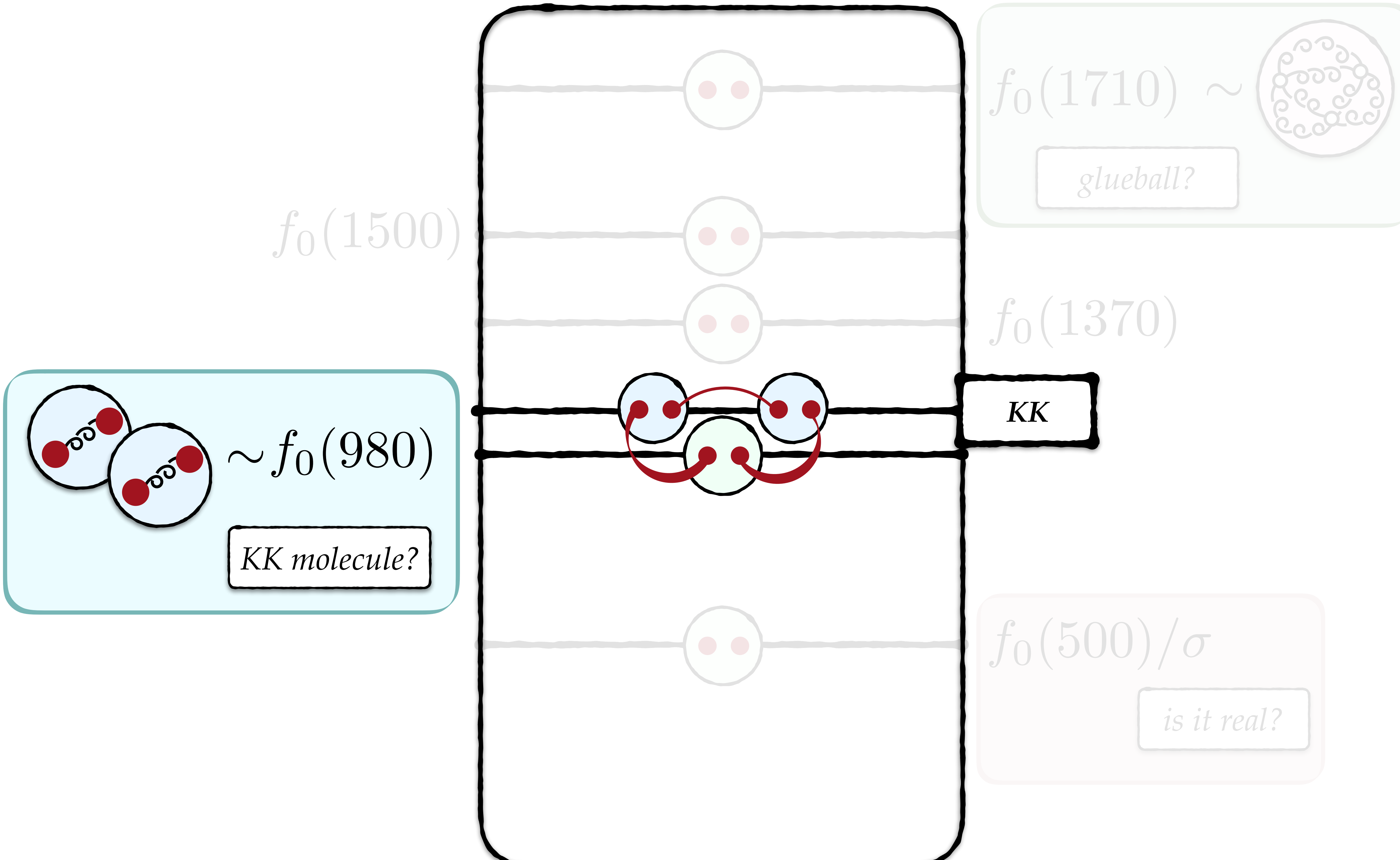
The isoscalar, scalar sector

[i.e., the quantum numbers of the vacuum]



The isoscalar, scalar sector

[i.e., the quantum numbers of the vacuum]



The isoscalar, scalar sector

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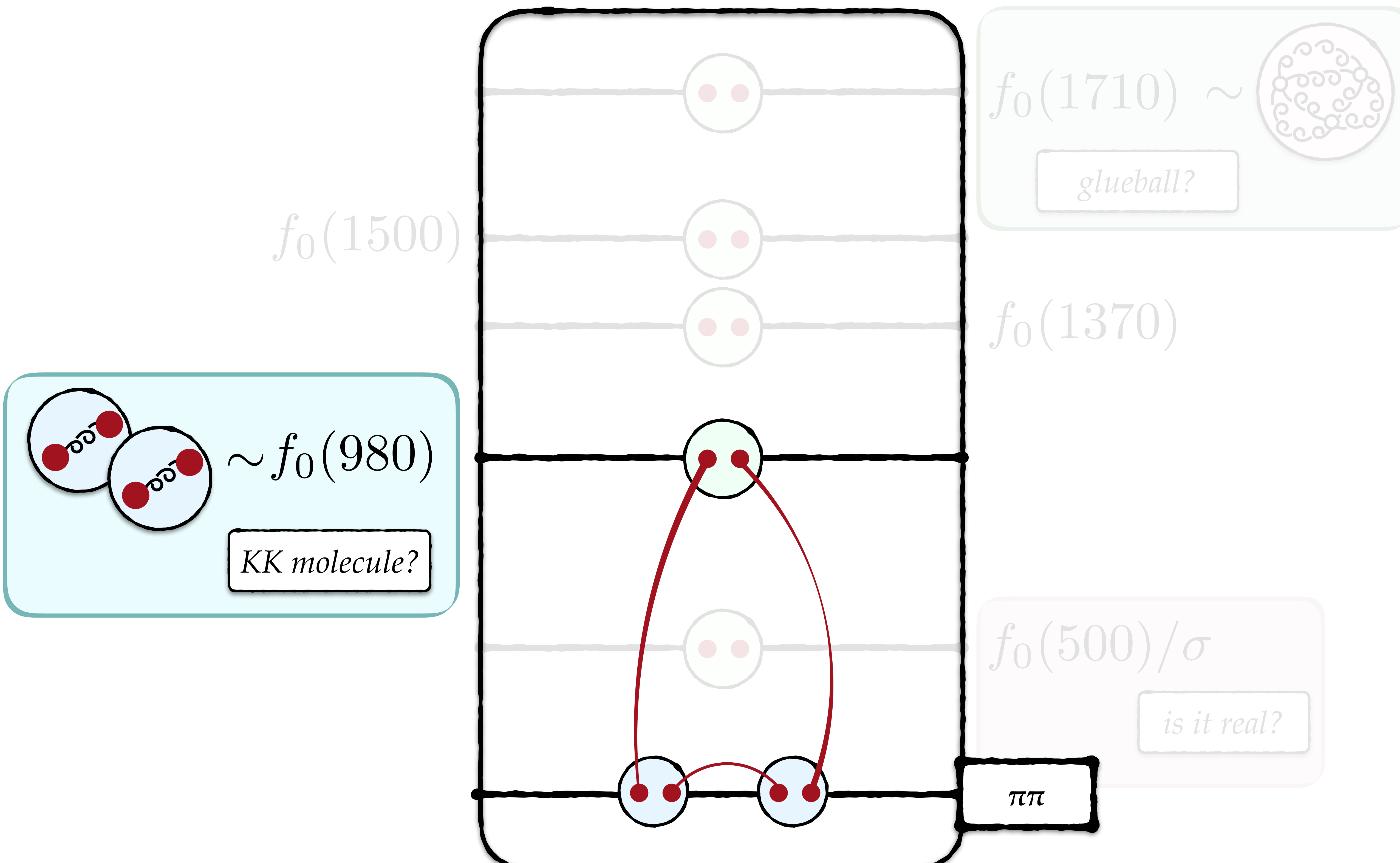
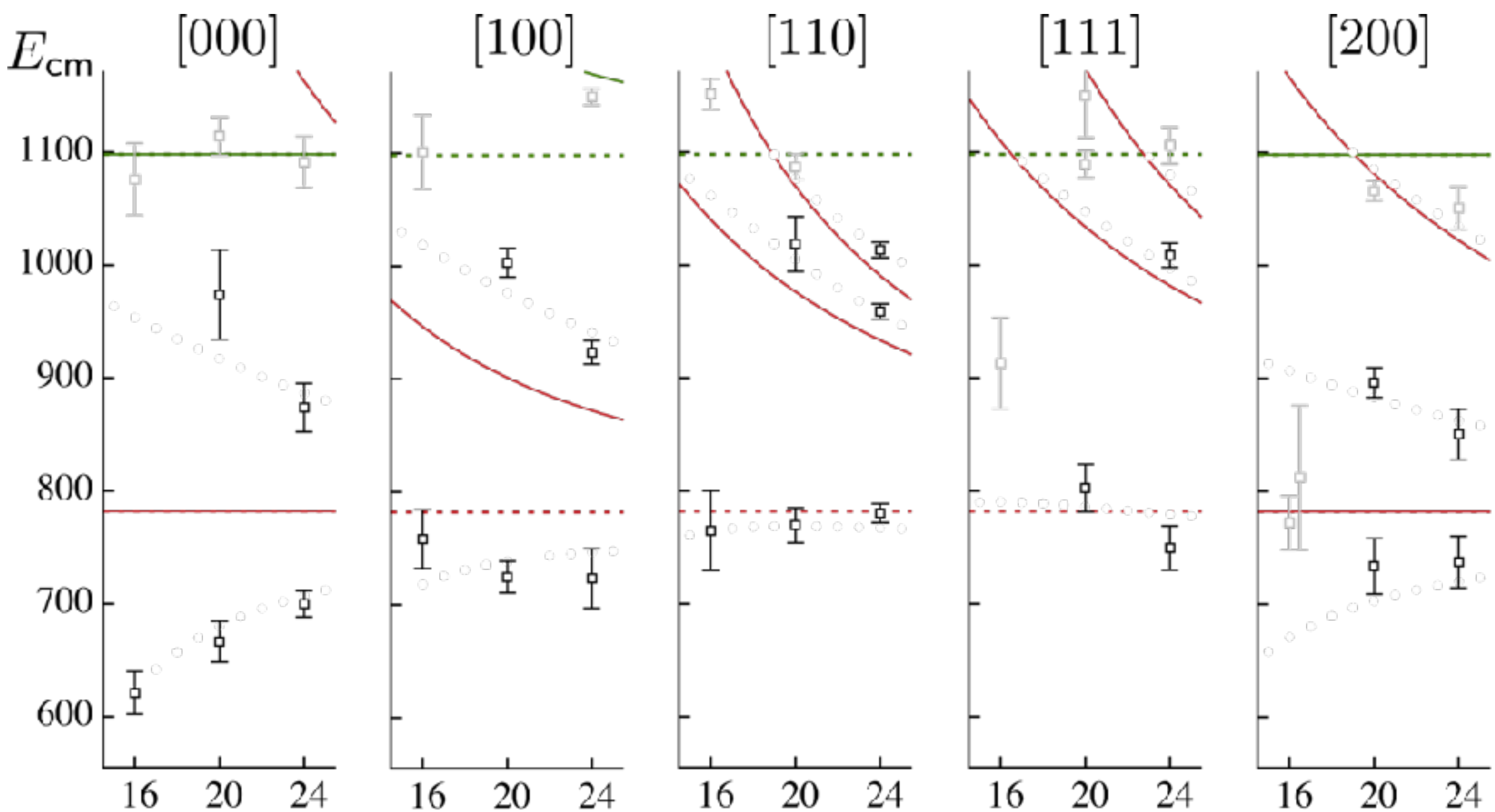


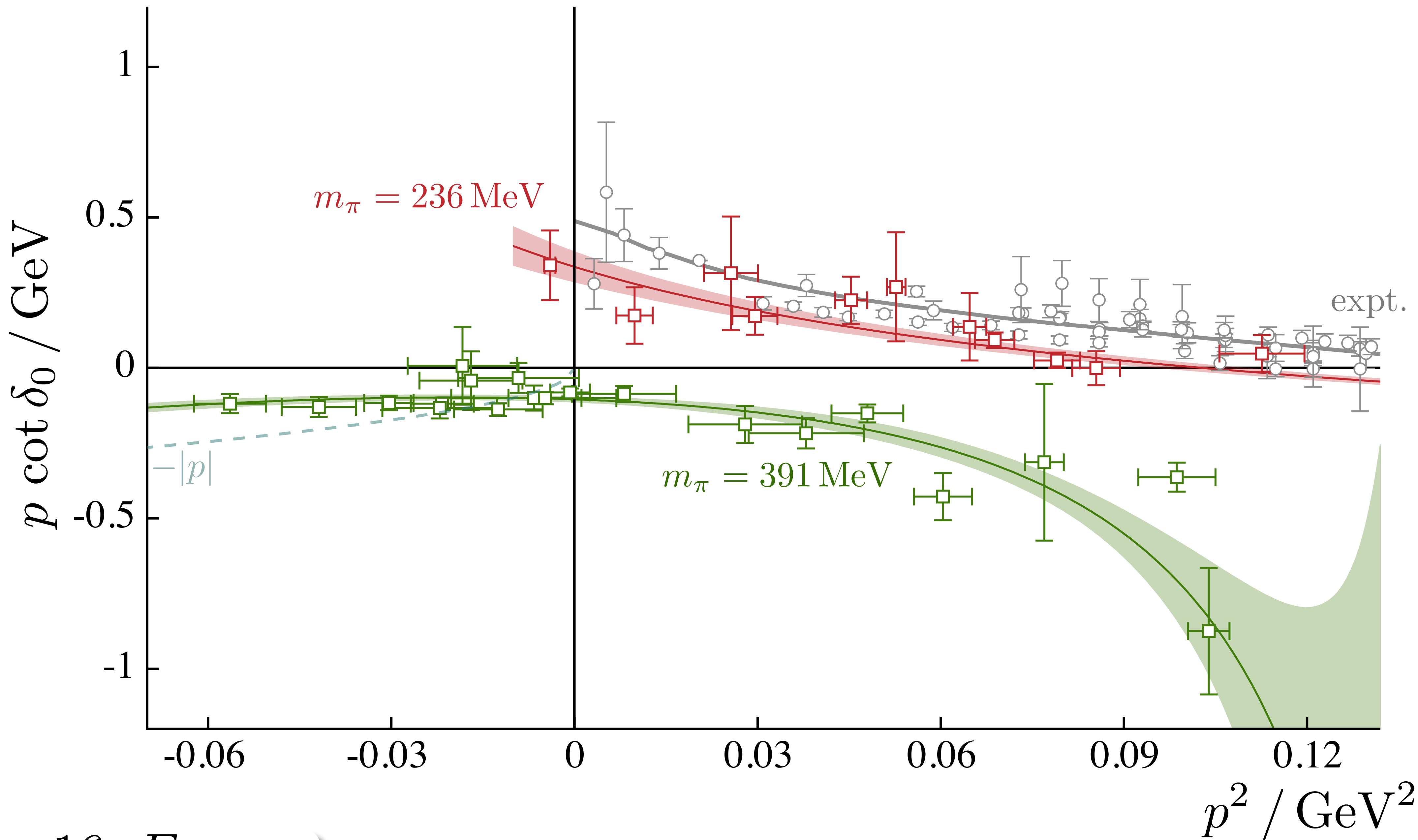
Extracting the spectrum



m_π=391 MeV

RB, Dudek, Edwards, Wilson - PRL (2017)

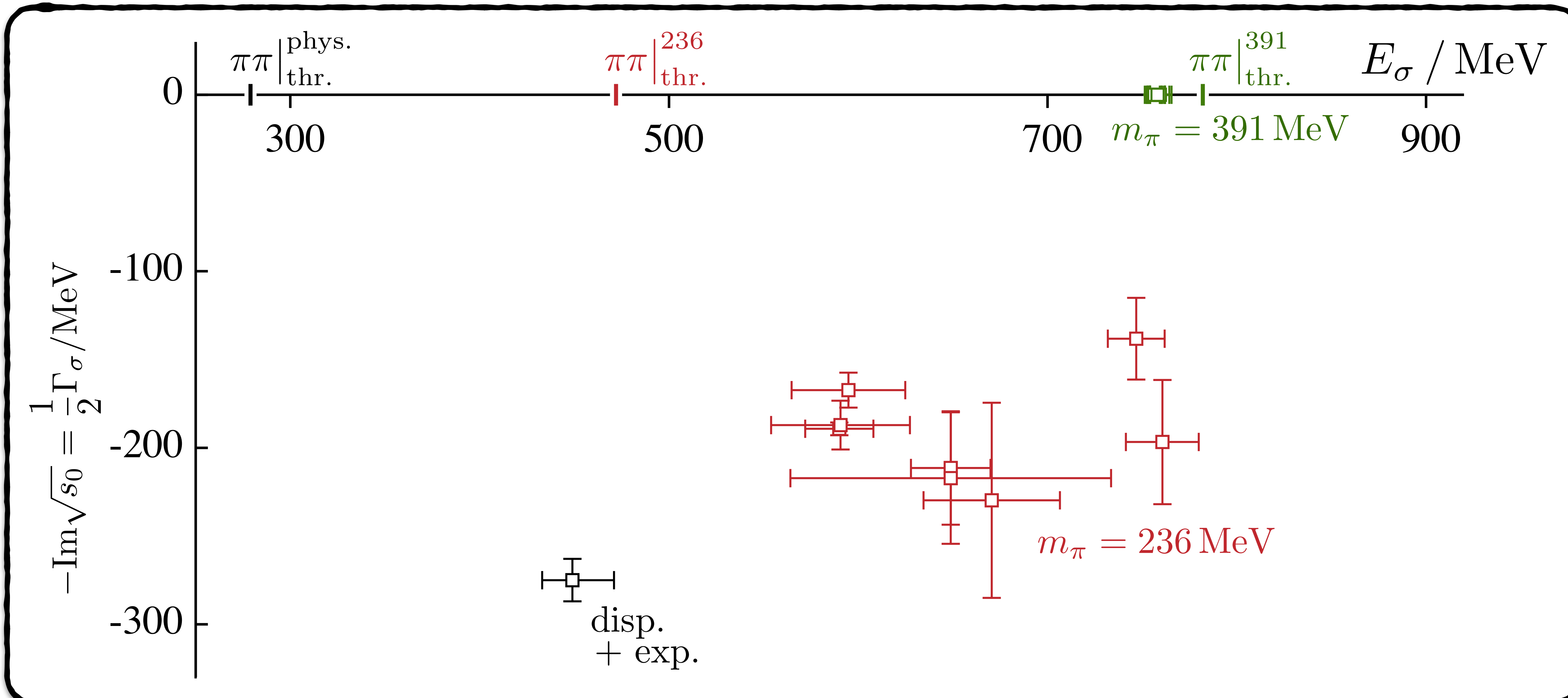
Isoscalar $\pi\pi$ scattering



$$\mathcal{M}_0 = \frac{16\pi E_{\rm cm}}{p \cot \delta_0 - ip}$$

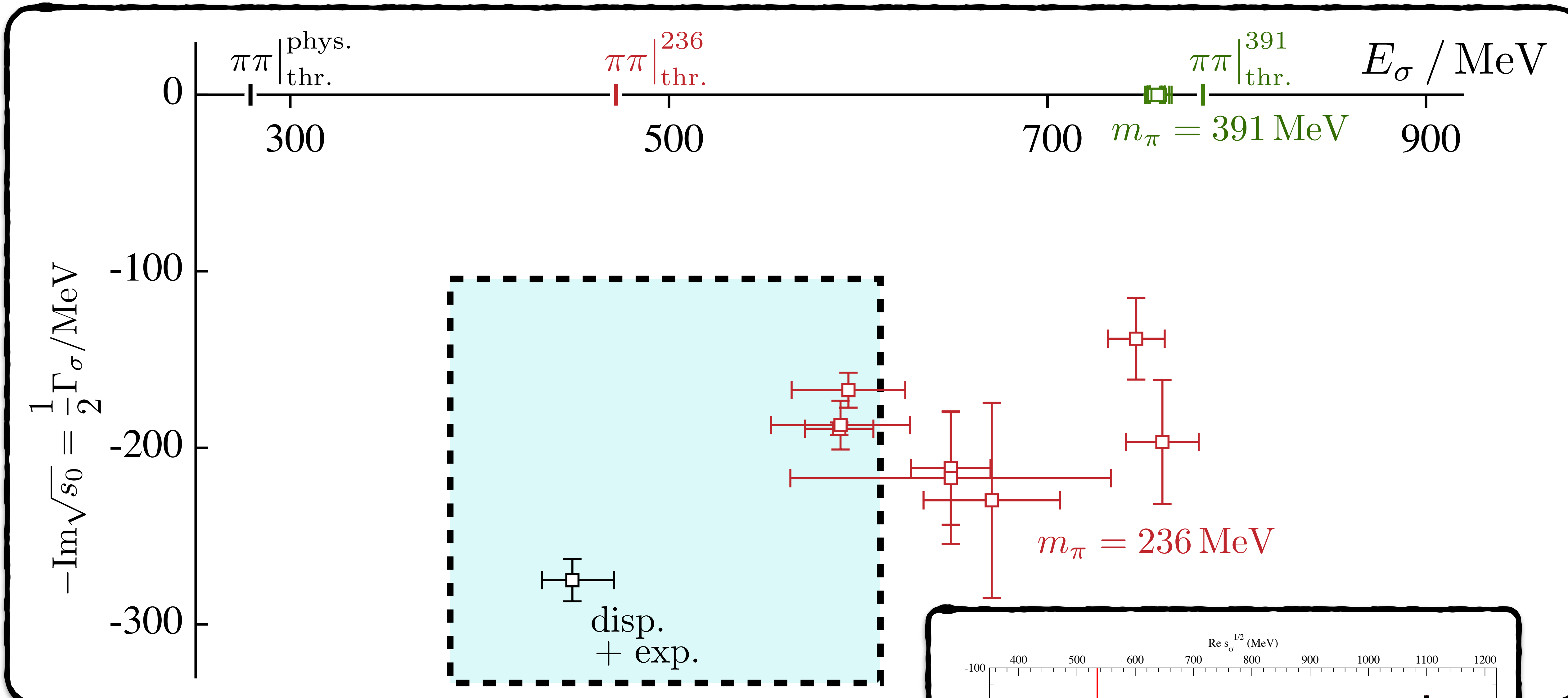
RB, Dudek, Edwards, Wilson - PRL (2017)

The σ / $f_0(500)$ vs m_π

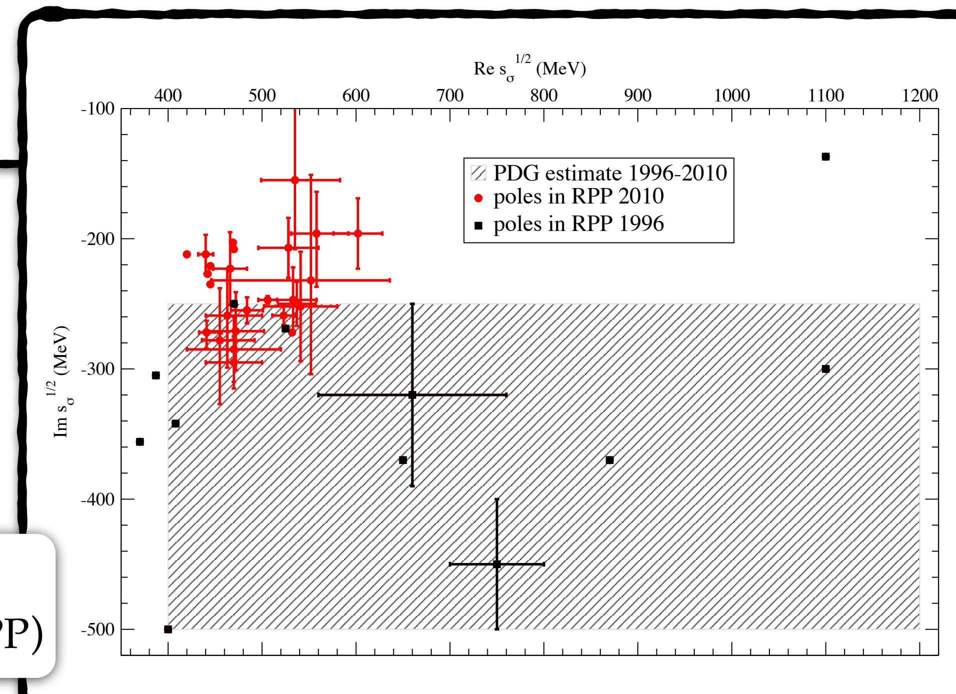


RB, Dudek, Edwards, Wilson - PRL (2017)

The σ / f_0(500) vs m_π

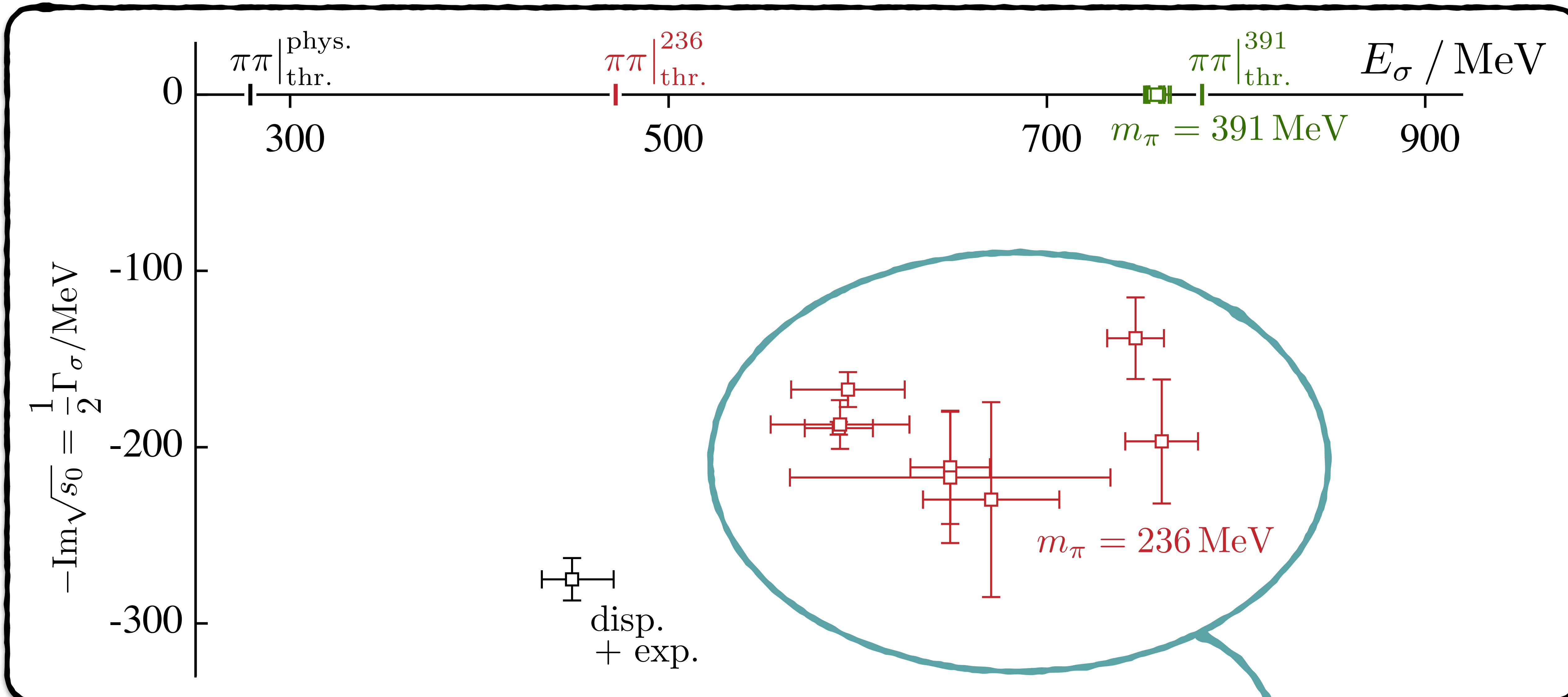


RB, Dudek, Edwards, Wilson - PRL (2017)



J. R. Peláez (2015)
Review of Particle Physics (RPP)

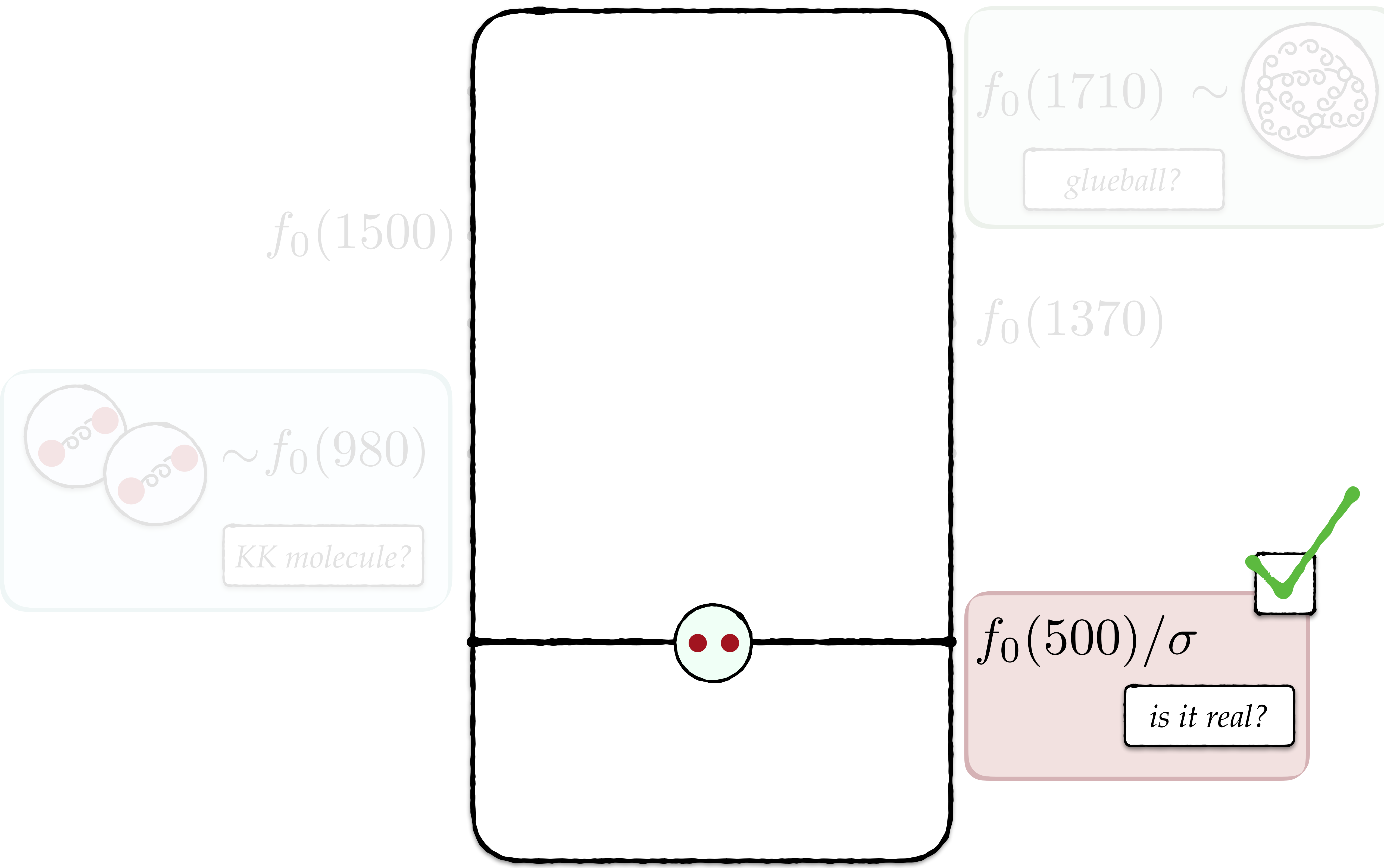
The σ / $f_0(500)$ vs m_π



RB, Dudek, Edwards, Wilson - PRL (2017)

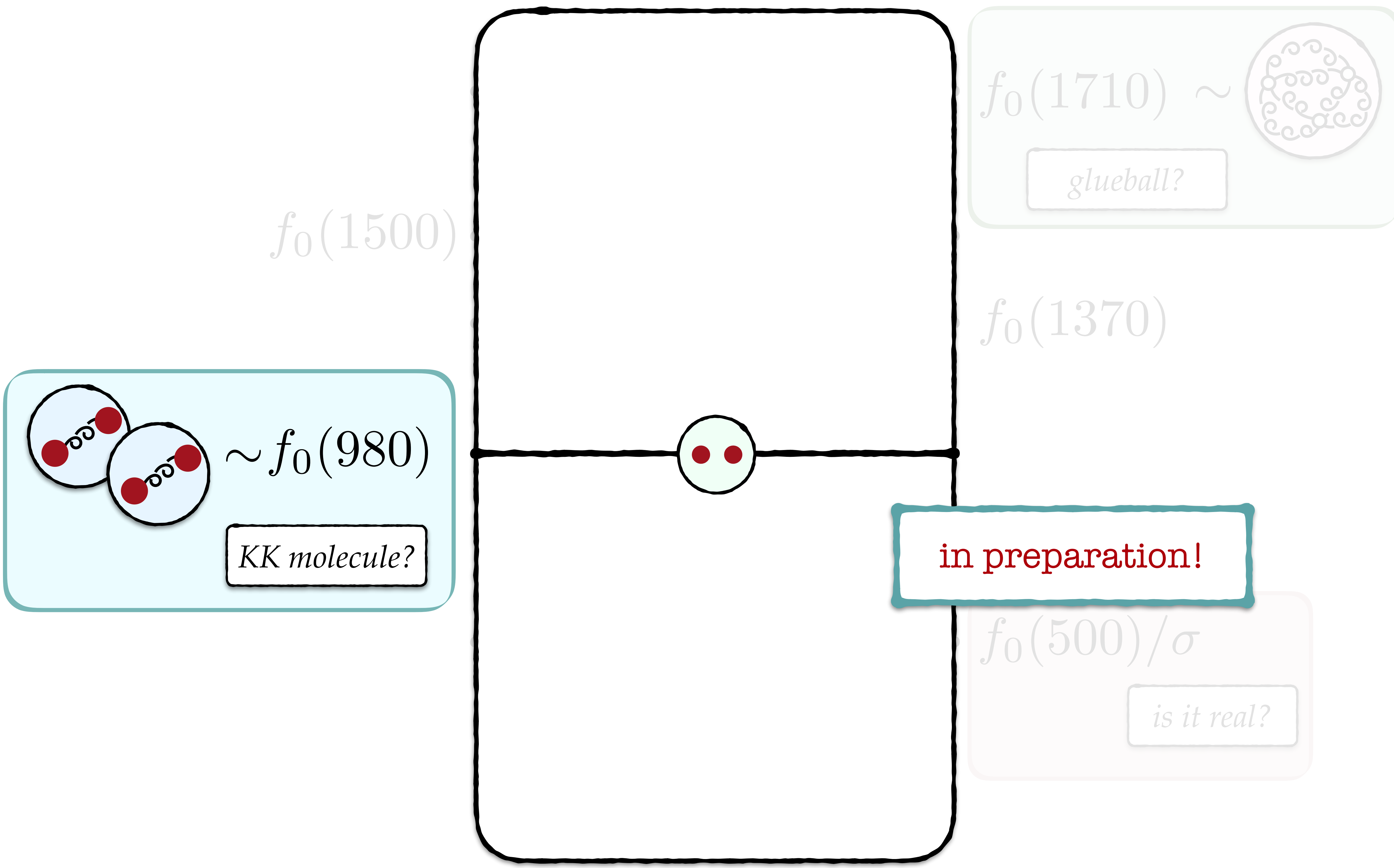
the time is ripe for sophisticated amplitude analysis of lattice QCD results!

The isoscalar, scalar sector



The isoscalar, scalar sector

[i.e., the quantum numbers of the vacuum]

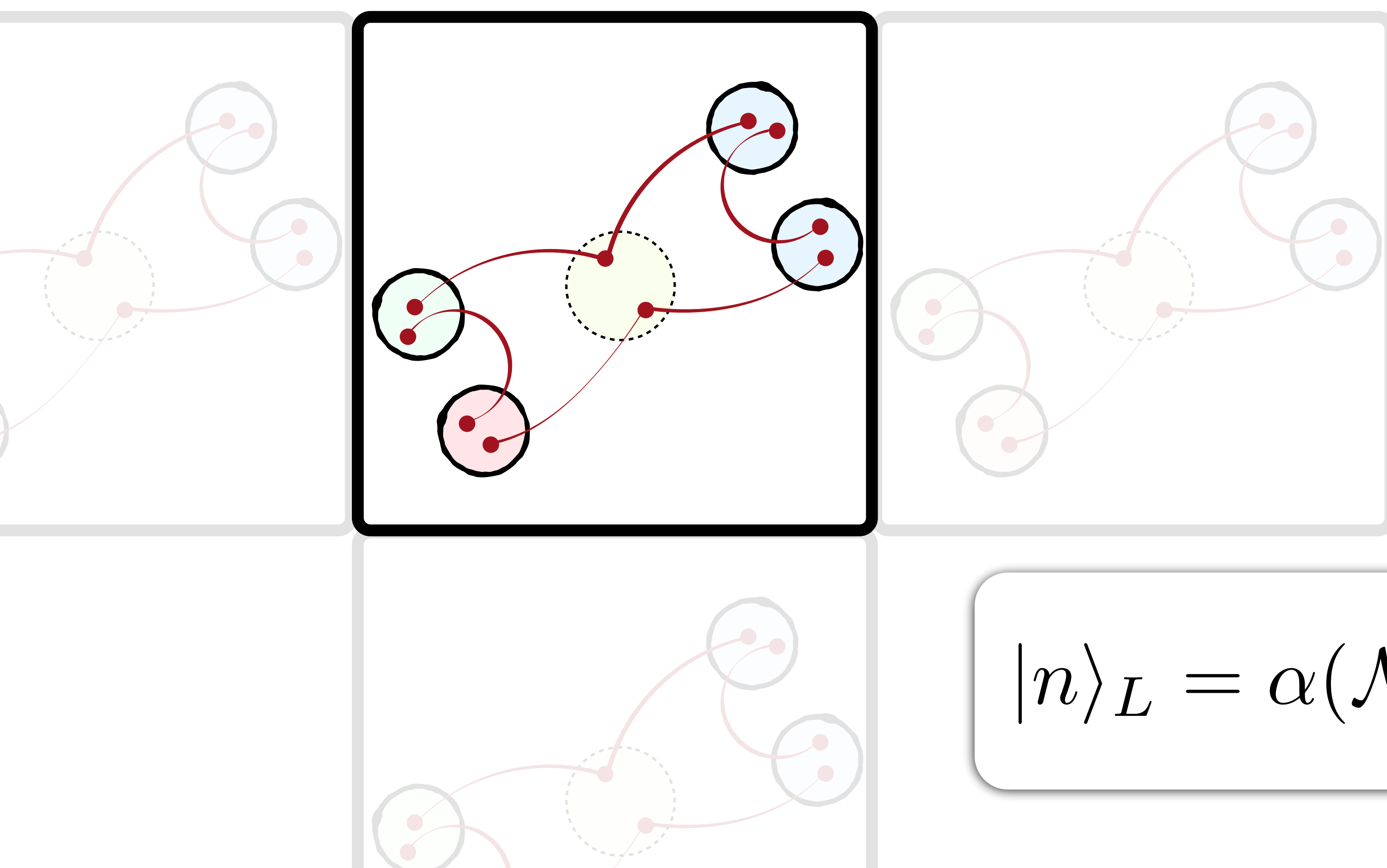


More than one channel open

- *Coupled channels:* e.g., $\pi\eta$, $K\overline{K}$

$$\det\begin{bmatrix} F_{\pi\eta}^{-1}+\mathcal{M}_{\pi\eta,\pi\eta} & \mathcal{M}_{\pi\eta,K\overline{K}} \\ \mathcal{M}_{\pi\eta,K\overline{K}} & F_{K\overline{K}}^{-1}+\mathcal{M}_{K\overline{K},K\overline{K}} \end{bmatrix}=0$$

Hansen & Sharpe / RB & Davoudi (2012)
RB (2014) / RB & Hansen (2015)

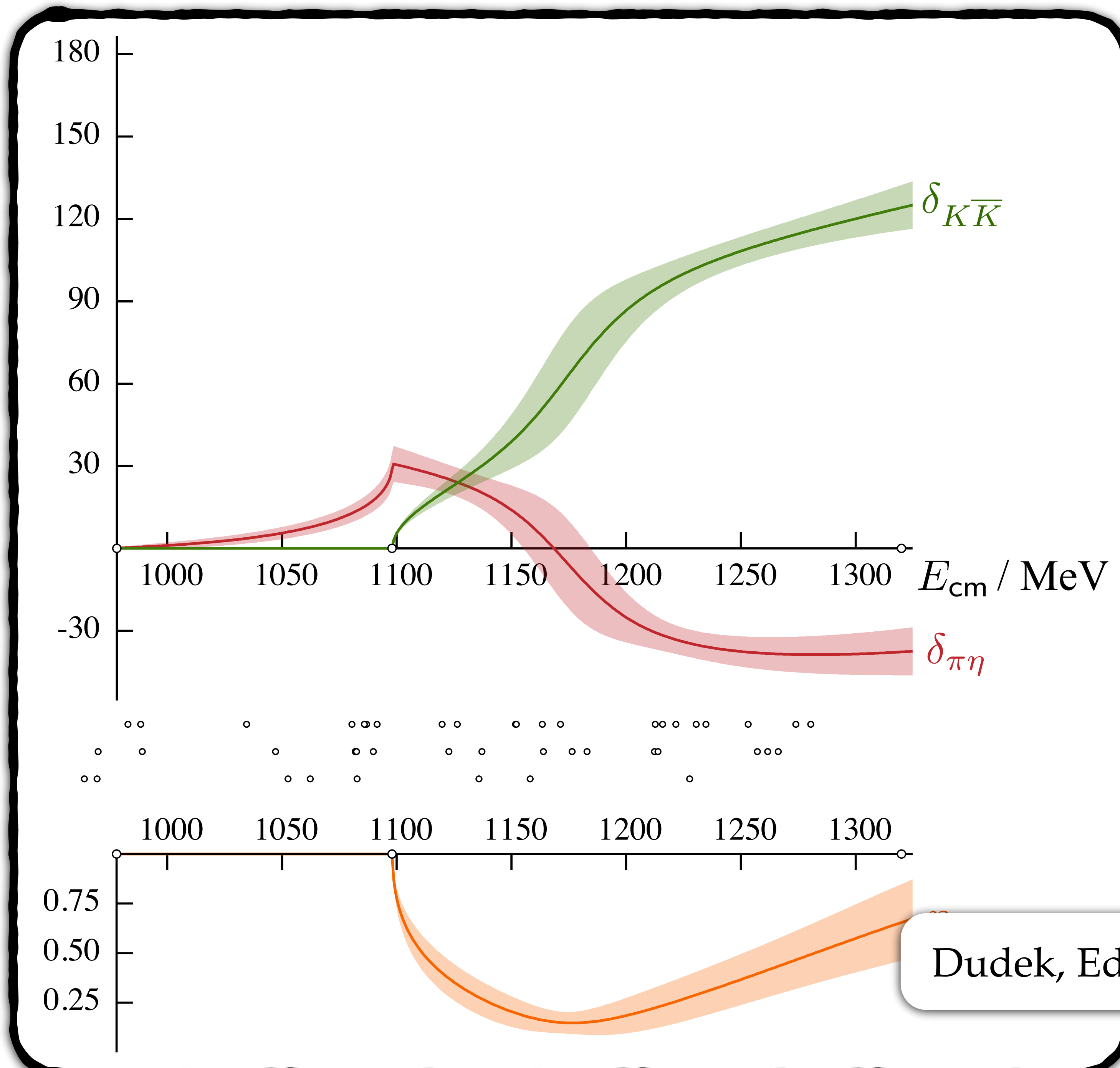


can't pull states apart!

$$|n\rangle_L=\alpha(\mathcal{M},L)|\pi\eta\rangle+\beta(\mathcal{M},L)|K\overline{K}\rangle$$

More than one channel open

- *Coupled channels:* e.g., $\pi\eta$, $K\overline{K}$
- *Practical solution: parametrize scattering amplitude and fit*



m_π=391 MeV

Dudek, Edwards & Wilson (2016)

More than one channel open

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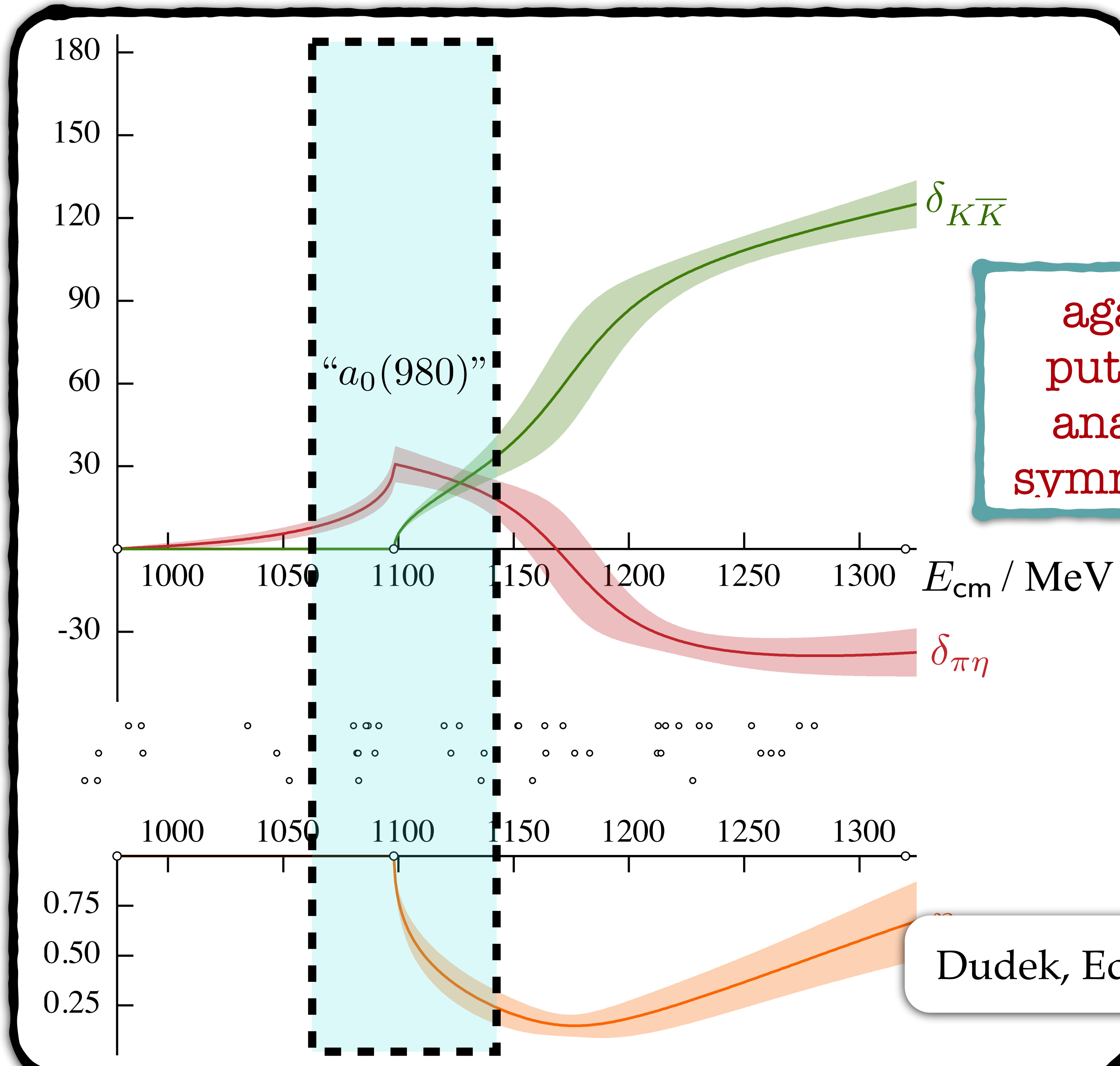


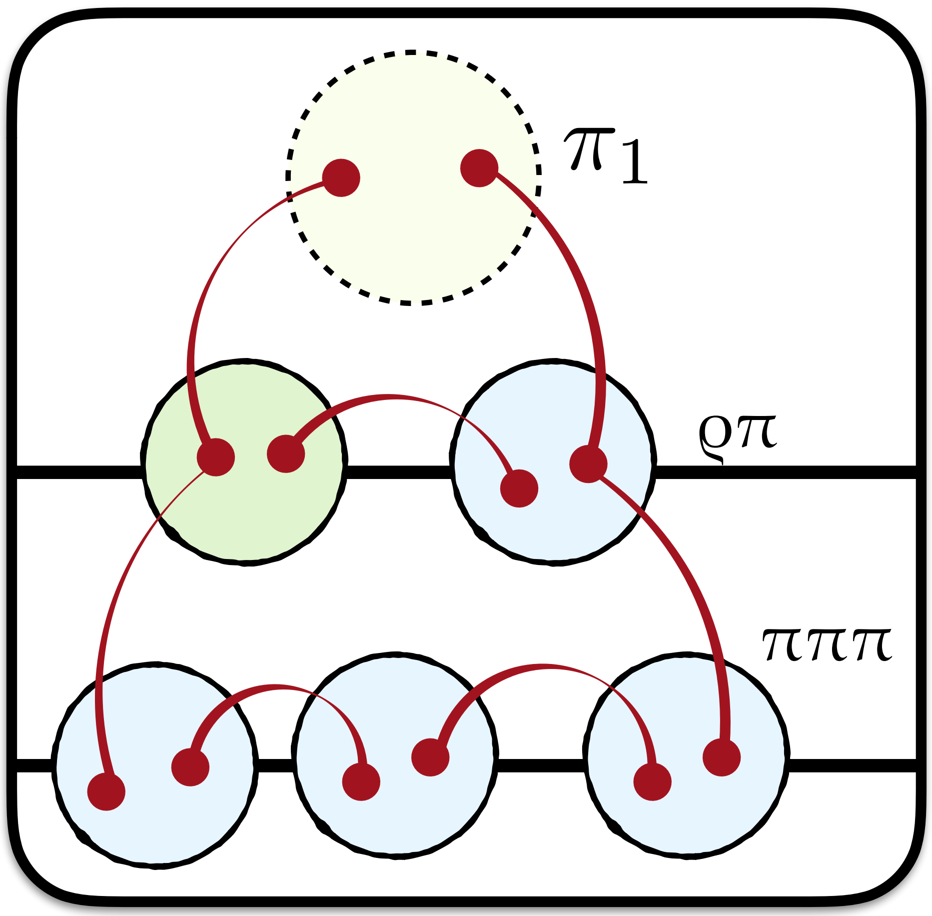
again, we should be putting constrains of analyticity, crossing symmetry…into our fits

m_π=391 MeV

Dudek, Edwards & Wilson (2016)

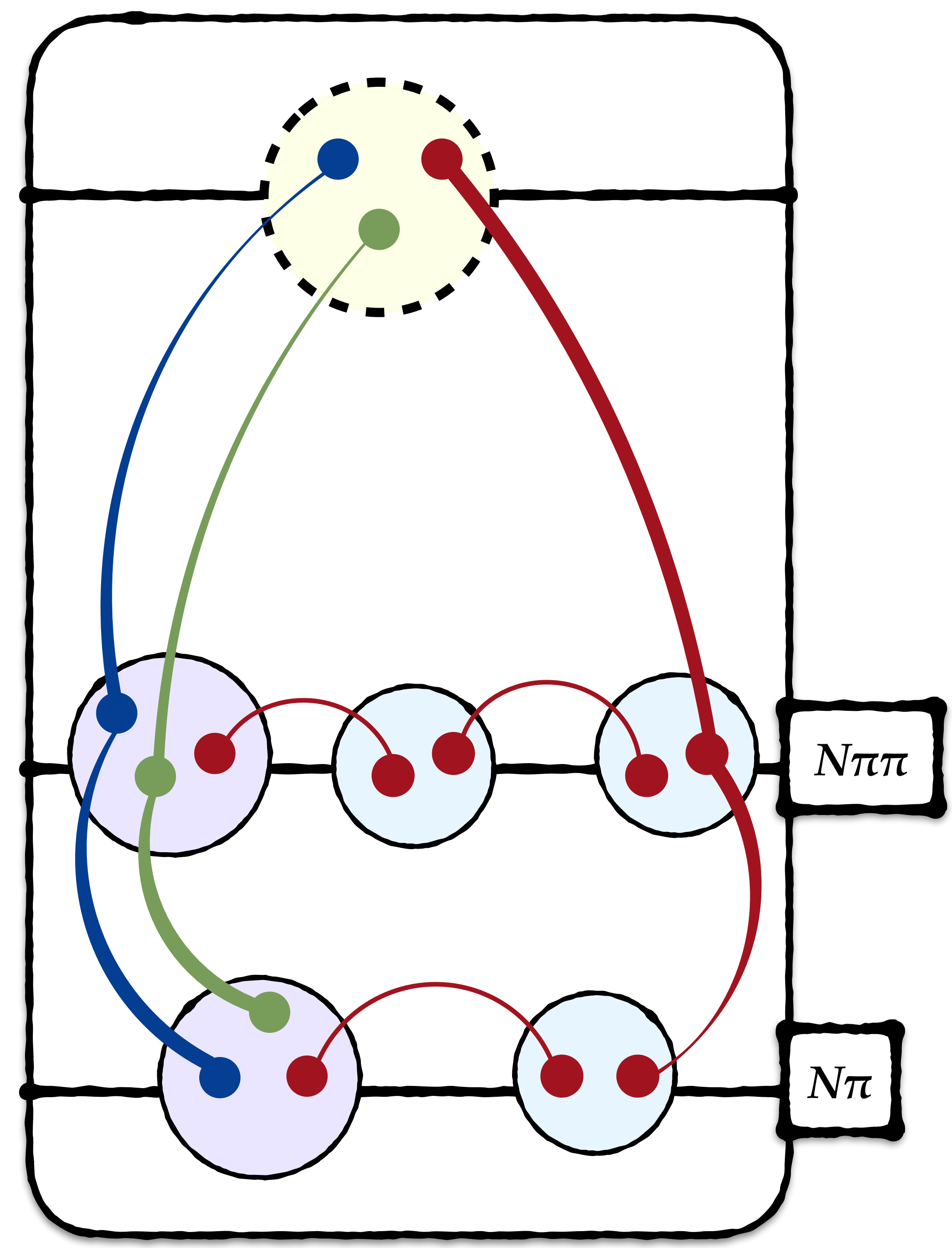
RB

Three-body



Going higher in energy

- *Coupled channels*
- *Beyond two particles:*

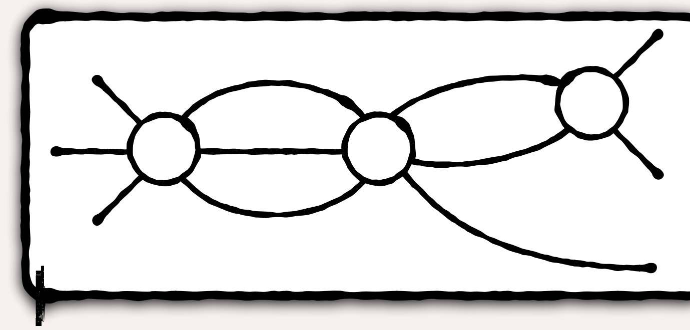


Going higher in energy

- *Coupled channels*
- *Beyond two particles:*

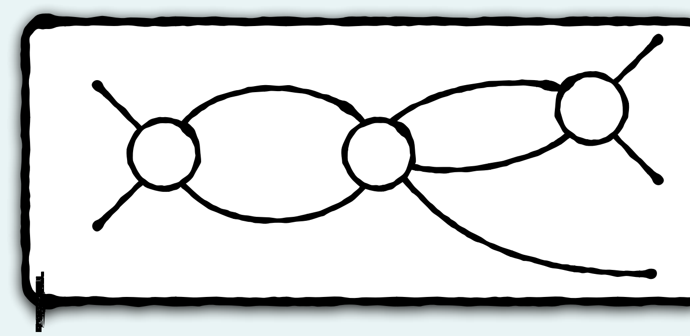
$$\det\left[1+F_3\mathcal{K}_{\mathrm{df},3}\right]=0$$

Hansen & Sharpe (2014)



$$\det\left[1+\begin{pmatrix}F_2 & 0\\ 0 & F_3\end{pmatrix}\begin{pmatrix}\mathcal{K}_2 & \mathcal{K}_{23}\\ \mathcal{K}_{32} & \mathcal{K}_{\mathrm{df},3}\end{pmatrix}\right]=0$$

RB, Hansen & Sharpe (2016)



Quantum Chromodynamics

Jose Rodriguez (Skype/Microsoft)

The end!

above all, Jazzi loves a good nap!