ELECTROMAGNETIC INT.

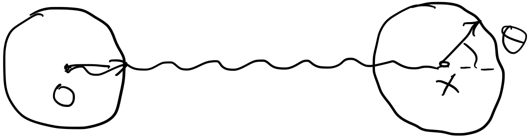
OF HADRONS

EM INT. OF HADRONS

- EM IN SM

$$\mathcal{L}_{QUARK} = \bar{q}\left(i\gamma^\mu \partial_\mu - m\right) q$$

$$q(x) \xrightarrow{U(1)} q(x)\, e^{i\Theta(x)}$$



$$\frac{\Delta\Theta}{\Delta z} \longrightarrow ?$$

$$\partial^\mu \Theta$$

$$\partial_\mu q \longrightarrow \left[\partial_\mu q + i(\partial_\mu \Theta) q\right] \cdot e^{i\Theta}$$

$$\partial_\mu q \Longrightarrow D_\mu q \equiv \left[\partial_\mu + i e_q A_\mu\right] q$$

$$A_\mu(x) \xrightarrow{U(1)} A_\mu - \frac{1}{e_q}\partial_\mu \Theta$$

$$D_\mu q \xrightarrow{U(1)} e^{i\Theta(x)} D_\mu q$$

$$\bar{q}\, i\gamma^\mu D_\mu q \longrightarrow \text{inv.}$$

$$\bar{q} q \longrightarrow \text{inv}$$

$$\mathcal{L}_{QUARK} \Longrightarrow \bar{q}\left(i\gamma^\mu D_\mu - m\right) q$$

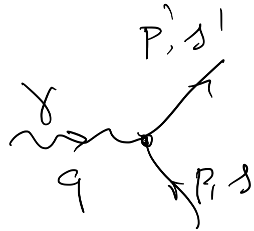
$$= \underbrace{\bar{q}\left(i\gamma^\mu \partial_\mu - m\right) q}_{\mathcal{L}_{QUARK}} - \underbrace{e_q \left(\bar{q}\gamma^\mu q\right)}_{\mathcal{L}_{INT}} A_\mu$$

$$e_u = +\frac{2}{3} e \qquad e_d = -\frac{1}{3} e$$

$$e^2/4\pi \simeq 1/137$$

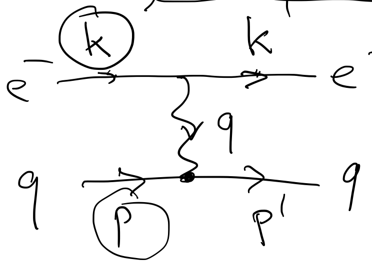
$$\mathcal{L}_{INT} = -e\, \bar{q}\gamma^\mu q\, A_\mu$$

$$= -e\, J^\mu_{em}\, A_\mu$$



- $\partial_\mu J^\mu_{em}(x) = 0 \longrightarrow q_\mu J^\mu = 0$

- DIRAC $\boxed{\langle p', s' | J^\mu_{em}(0) | p, s \rangle = \bar{u}\gamma^\mu u}$

- 

$$s, t = q^2 \qquad \boxed{-q^2 = Q^2}$$

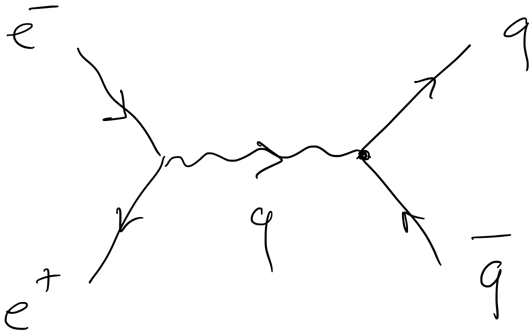
$$Q^2 > 0$$

$$p'^2 = p^2 = m^2$$

$$(p+q)^2 \Longrightarrow q^2 = -2p\cdot q$$

$$\underset{LAB}{=} -2Mq^0 < 0$$

$$p^\mu(M, 0)$$



$$q^2 = (k + k')^2$$

$$\xi = s > 0$$

$\underline{\text{CM}}$ $\quad k + k' = (\sqrt{s}, 0)$

$\rightarrow q_\mu J^\mu = 0$

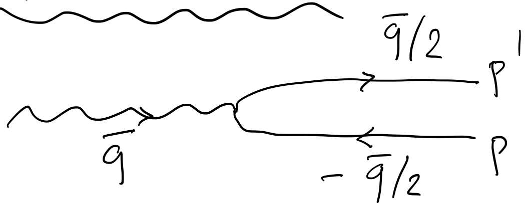
$\rightarrow$ **GORDON ID**

$$\bar{u}(p', s')\, \gamma^\mu\, u(p, s)$$

$$= \bar{u}(p', s') \left\{ \underbrace{\frac{(p + p')^\mu}{2m}} + \underbrace{i \sigma^{\mu\nu} \frac{q_\nu}{2M}} \right\} u(p, s)$$

$$\sigma^{\mu\nu} = \frac{i}{2} \left[\gamma^\mu, \gamma^\nu \right]$$

→ BREIT - FRAME



$q^2 < 0 \qquad Q^2 \equiv -q^2$

$q\,(0,\ 0,\ 0,\ Q)$

$p\,(E_p,\ 0,\ 0,\ -Q/2)$

$p'\,(E_p,\ 0,\ 0,\ +Q/2)$

$$E_p = \sqrt{\frac{Q^2}{4} + M^2} = M\sqrt{1+\tau}$$

$$\tau \equiv Q^2/4M^2$$

- $\langle p' s' \,|\, J^0_{em}(0) \,|\, p\ s \rangle_{\text{BREIT}}$

$$= u^{+}(\underbrace{p'}, \underbrace{s'})\ u(\underbrace{p}, \underbrace{s})$$

$$= \delta_{ss'}\,\underbrace{(2M)}$$

$s = \pm\frac{1}{2}$

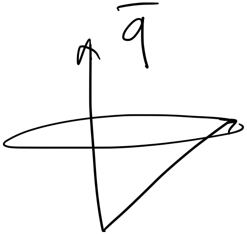
$s' = \pm\frac{1}{2}$

- $\langle p' s' | J^{i}_{em}(0) | p\, s \rangle_{\text{BREIT}}$

$$= \chi^{\dagger}_{s'}\, 2i \left(\frac{\vec{\sigma}}{2} \times \vec{q} \right)^{i} \chi_{s}$$

SPIN FLIP

$$\vec{\mu} = \left(\frac{e}{2M} \right) g\, \vec{S}$$



$$\vec{S} = \hbar \frac{\vec{\sigma}}{2}$$

$$\boxed{g = 2}$$

$\Rightarrow$ COMPOSITE / HADRONS

$$\langle p' s' | J^\mu_{em}(0) | p\, s \rangle = \bar{u}(p' s')\, \Gamma^\mu\, u(p\, s)$$

$$\hookrightarrow q_\mu \langle | J^\mu_{em} | \rangle = 0$$

$$\Gamma^\mu = \gamma^\mu \quad (\text{POINT})$$

$$\Gamma^\mu \quad (p+p')^\mu, \quad \sigma^{\mu\nu} q_\nu, \quad \cancel{q^\mu}$$

P, T

$$\boxed{\Gamma^\mu = F_1(q^2)\gamma^\mu + F_2(q^2)\, i\sigma^{\mu\nu} \frac{q_\nu}{2M}}$$

$$p^2, \; q^2, \; p \cdot q \longrightarrow q^2, p'^2$$

$$F_1^p(q^2=0) = 1$$

$$\underbrace{F_2^p(q^2=0)} = \kappa^p = \underline{\underline{1.79}} \qquad \underline{(1933)}$$

$$\mu^p = 1 + \kappa^p = \underline{2.79}$$

$\rightarrow$ <u>ISOSPIN</u> (u, d)

$$\underbrace{J^\mu_{em}(0)} = \frac{2}{3}\bar{u}\gamma^\mu u - \frac{1}{3}\bar{d}\gamma^\mu d$$

$$= \frac{1}{6}\left(\underline{\bar{u}\gamma^\mu u + \bar{d}\gamma^\mu d}\right) \leftarrow \text{ISOSCALAR}$$

$$+ \frac{1}{2}\left(\underline{\bar{u}\gamma^\mu u - \bar{d}\gamma^\mu d}\right) \leftarrow \text{ISOVECTOR}$$

$\underline{\underline{SU(2)_V}}$ $\qquad m_u = m_d$

$$\begin{pmatrix} \mathbb{1} & \\ & -\mathbb{1} \end{pmatrix}$$

$$q = \begin{pmatrix} u \\ d \end{pmatrix}$$

$$J^{\mu}_{em}(0) = \frac{1}{6}\,\bar{q}\,\mathbb{1}\,q + \frac{1}{2}\,\bar{q}\,\tau_3\,q$$

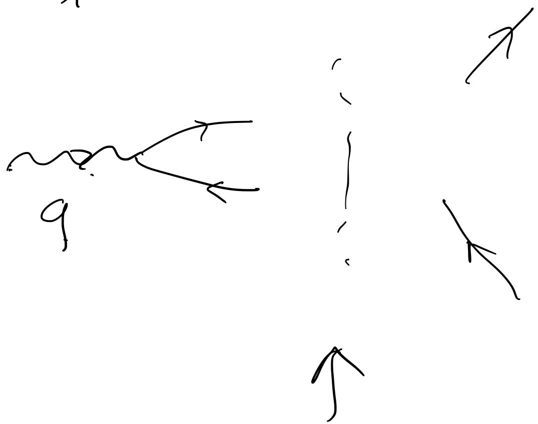
$\hookrightarrow$

$$\eta = \begin{pmatrix} p \\ n \end{pmatrix}$$

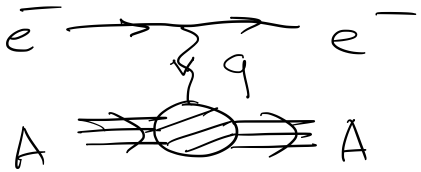
$$\langle \eta(p' s') \,|\, J^{\mu}_{em}(0) \,|\, \eta(p s) \rangle$$

$$= \bar{\eta}(p' s') \left\{ \frac{1}{2}\,\Gamma^{\mu}_{S} + \frac{1}{2}\,\tau_3\,\Gamma^{\mu}_{V} \right\} \eta(p s)$$

$$\begin{cases} F_1^p = \dfrac{1}{2}\left(F_1^S + F_1^V\right) \\ F_1^n \qquad\qquad\qquad\qquad - \end{cases}$$

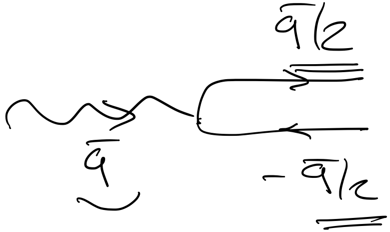


⟹ 2) PHYSICS INTERPRETATION OF FFs



$\underbrace{|\bar{q}|}$

$\underline{\underline{A \gg 1}}$



$$\hookrightarrow \langle N(p',s') | J^0_{em}(0) | N(p,s) \rangle_{\text{BREIT}} = \underline{(2M)}\, \underbrace{\delta_{ss'}}\, \underbrace{G_E(q^2)}$$

$$\| \; G_E = F_1 - \tau F_2$$

$$\tau \equiv Q^2/4M^2$$

$$\langle N(p',s') | J^i_{em}(0) | N(p,s) \rangle$$

$$= \chi^\dagger_{s'}\, i(\bar{\sigma} \times \bar{q})\, \chi_s\, \underbrace{G_M(q^2)}$$

$$\| \; G_M = F_1 + F_2$$

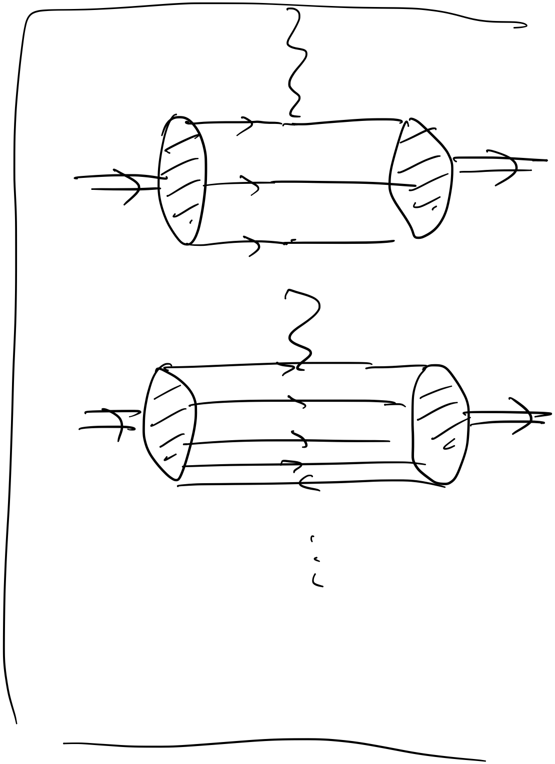
$$\int d^3\bar{r} \; \rho_{ch}(r) = \underline{1}$$

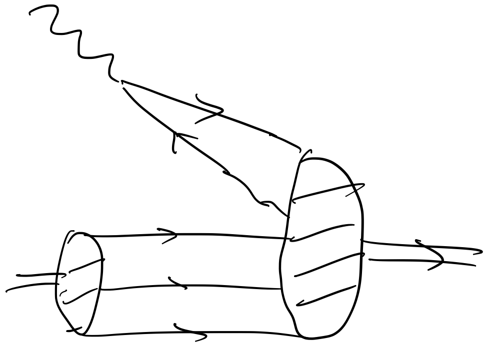
$$\tilde{\rho}_{ch}(|\bar{q}|) = \int d^3\bar{r} \; e^{i\bar{q}\cdot\bar{r}} \rho_{ch}(r)$$

$\parallel\!|$ ← $\underline{\underline{\text{NON REL.}}}$

$$G_E(q^2) \longrightarrow Q^2 = |\bar{q}|_B^2$$

↳ HADRONS $m_u \simeq m_d \simeq 0$ (M)



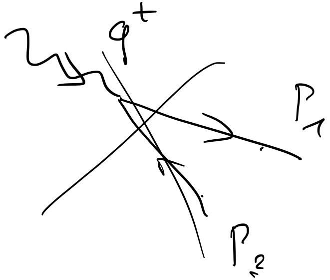


NON-DIAGONAL

NO DENSITY INT.

$F_1(q^2)$, $F_2(q^2)$

LIGHT-FRONT FRAME

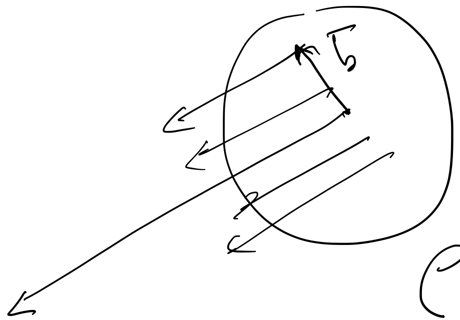


$$a^\pm = a^0 \pm a^3$$

$$\boxed{q^+ = 0}$$

$$(0, 0, 0, Q)$$

$$\left\{ \begin{array}{l} P_1^+ > 0 \\ P_2^+ > 0 \end{array} \right.$$

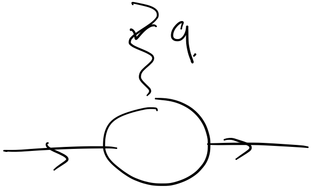


$P \quad z$

$$\vec q = \vec q_\perp$$

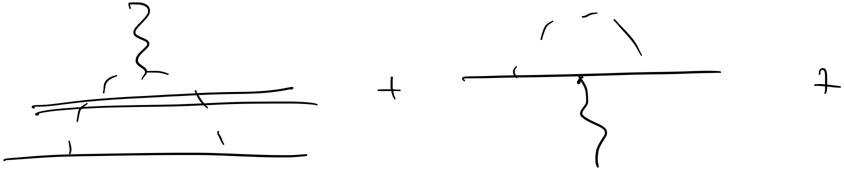
$$\rho_0(b) = \int \frac{d^2 \vec q_\perp}{(2\pi)^2} \, e^{i \vec q_\perp \cdot \vec b} \, F_1(-\vec q_\perp^{\,2})$$

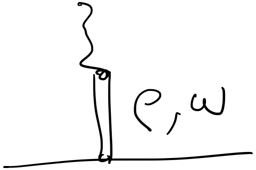
$\Longrightarrow$ ANALYTIC STRUCTURE



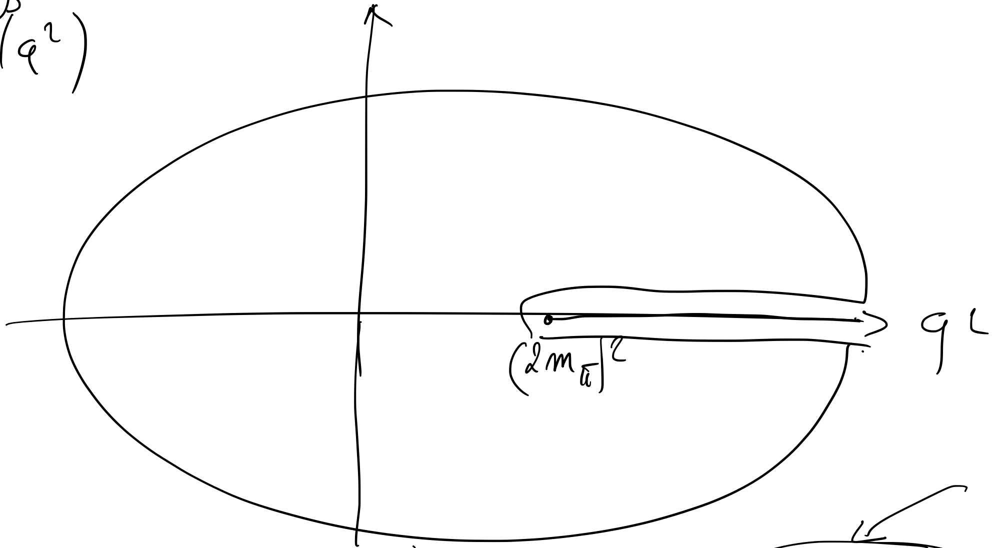
$F_1(q^2)$

$\boxed{\text{EFT}}$





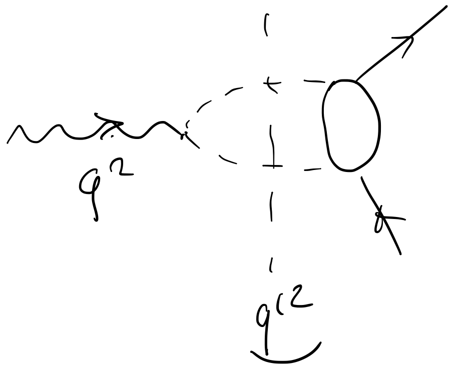
$F_1^{V,S}(q^2)$

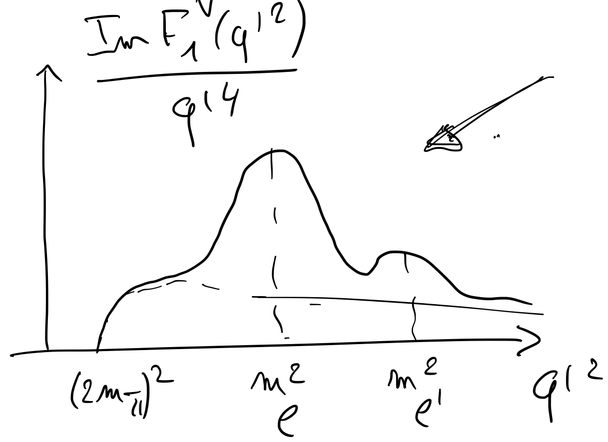


$$F_1(q^2) = \frac{1}{\pi}\int_{\underbrace{(2m_\pi)^2}}^{\infty} dq'^2 \, \frac{\mathrm{Im}\, F_1(q'^2)}{q'^2 - q^2}$$

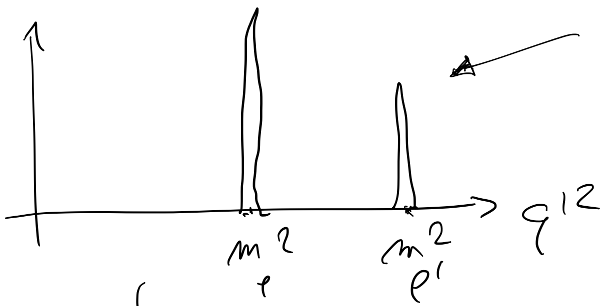
$$\underline{F_1(0) = 1}$$

$$F_1^V(q^2) = 1 + \frac{q^2}{\pi}\int_{(2m_\pi)^2}^{\infty} dq'^2 \, \frac{\mathrm{Im}\, F_1^V(q'^2)}{q'^2(q'^2 - q^2)}$$





$$\frac{1}{\pi}\,\mathrm{Im}\,F_1^V(q'^2) = a_\rho\,\delta(q'^2 - m_\rho^2) + a_{\rho'}\,\delta(q'^2 - m_{\rho'}^2)$$



$$F_1^V(q^2) = \frac{a_\rho}{m_\rho^2 - q^2} + \frac{a_{\rho'}}{m_{\rho'}^2 - q^2} = \frac{q^2\cancel{(-a_\rho - a_{\rho'})} + (a_{\rho'} m_\rho^2 + a_\rho m_{\rho'}^2)}{(m_\rho^2 - q^2)(m_{\rho'}^2 - q^2)}$$

$$F_1(0) = 1$$

$$\frac{a_\rho}{m_\rho^2} + \frac{a_{\rho'}}{m_{\rho'}^2} = \underline{1}$$

$$\rightsquigarrow \boxed{a_{\rho'} \simeq -a_\rho}$$

$$a_\rho \left(\frac{1}{m_\rho^2} - \frac{1}{m_{\rho'}^2} \right) = 1$$

$$a_\rho \frac{(m_{\rho'}^2 - m_\rho^2)}{m_\rho^2 m_{\rho'}^2} = 1$$

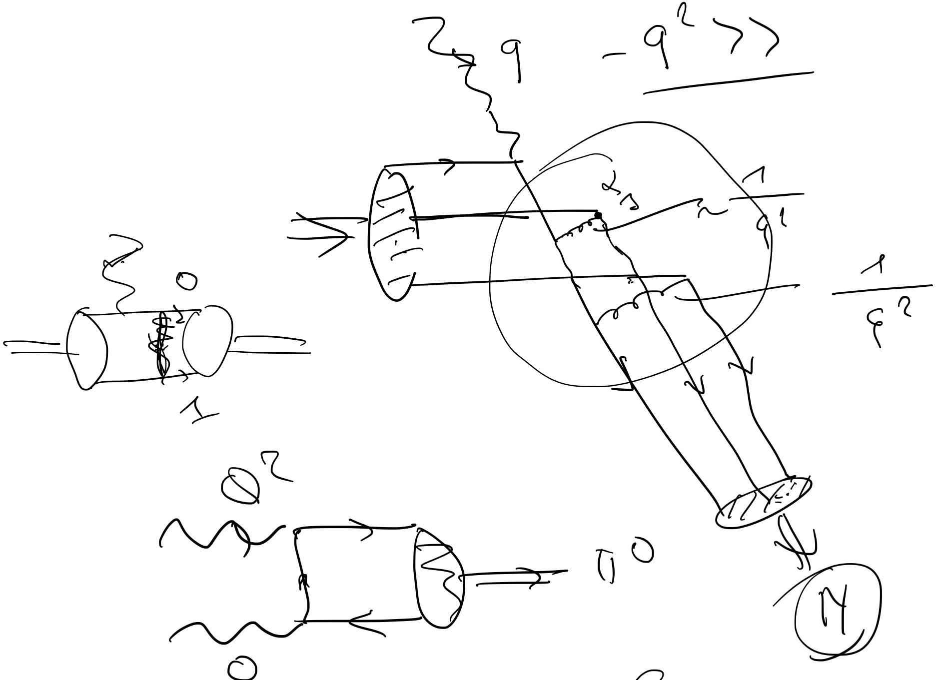
DIPOLE

$$\hookrightarrow \boxed{F_1^V(q^2) = \frac{1}{\left(1 - \frac{q^2}{m_\rho^2}\right)\left(1 - \frac{q^2}{m_{\rho'}^2}\right)}}$$

$$= \frac{1}{\left(1 - \frac{q^2}{\Lambda^2}\right)^2}$$

$\Lambda \simeq 0.843$ GeV

→ PERTURBATIVE QCD



$$F_1^p(q^2) \sim \frac{1}{q^4}$$

$$F^\pi(q^2) \sim \frac{1}{q^2}$$

$$Q^2 \gtrsim 10 \text{ GeV}^2$$

$$F \sim \frac{1}{Q^2} \int dx \, \frac{DA(x)}{x}$$

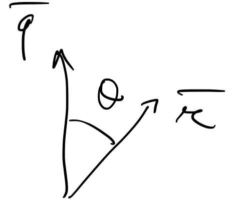
$$\rho_0 \, e^{-\Lambda r}$$

$\hookrightarrow$ $$\int d^3\bar{r} \; \rho_0 \, e^{-\Lambda r} = \rho_0 \frac{4\pi}{\Lambda^3} \int_0^\infty d r\Lambda \; r^2 \Lambda^2 e^{-\overbrace{\Lambda r}^{\alpha}}$$

$$= \rho_0 \frac{4\pi}{\Lambda^3} \underbrace{\int_0^\infty d\alpha \; \alpha^2 e^{-\alpha}}_{2} = 1$$

$\hookrightarrow$

$$\rho_0 \int d^3\bar{r}\; e^{\underbrace{i\bar{q}\cdot\bar{r}}_{|\bar{q}|\, r \cos\theta}}\; e^{-\Lambda r}$$



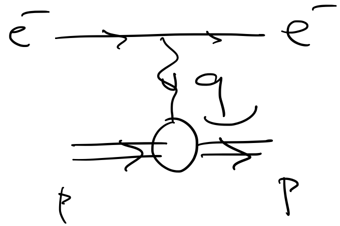
$$\rho_0 (2\pi) \int_0^\infty dr\; r^{\cancel{2}}\; e^{-\Lambda r} \int_{-1}^{1} dx\; e^{irqx}$$

$$\frac{1}{iq\cancel{r}}\left(e^{irq} - e^{-irq} \right)$$

$$e^{(-\Lambda + iq) r} - e^{-(\Lambda + iq) r}$$

$$\frac{1}{\left(1 + |\bar{q}|^2/\Lambda^2\right)^2} \sim \frac{1}{(iq-\Lambda)^2} - \frac{1}{(iq+\Lambda)^2} \qquad \frac{1}{\left(\Lambda^2 - (iq)^2\right)^2}$$

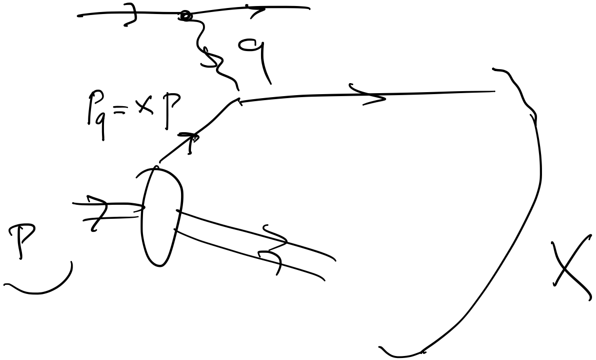
$\Rightarrow$



ELASTIC

$\Rightarrow -q^2 \gg$

$d\sigma \sim L_{\mu\nu} \underline{W^{\mu\nu}}$



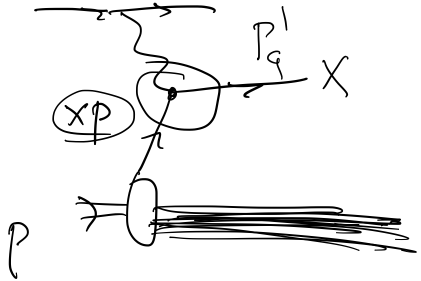
DIS

$$W^{\mu\nu} \equiv \frac{1}{2\pi} \sum_X \frac{(2\pi)^4 \, \delta^4(p+q-p_X)}{} \cdot \langle p | J^{\mu +}_{em}(0) | X \rangle \langle X | J^{\nu}_{em}(0) | p \rangle$$

$X = p \longrightarrow$ ELASTIC

$X \neq p \longrightarrow$ IN

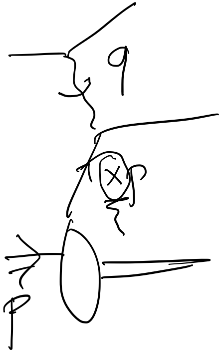
$\hookrightarrow \boxed{Q^2 \gg 0}$



LO

$$\sum_X = \int \frac{d^3 \bar{p}'_q}{(2\pi)^3 2E'_q} \sum_{s'}$$

$$\int \frac{d^{3} p_q'}{(2\pi)^3\, 2E_q'} \; (2\pi)^4 \, \delta^{(4)}\Big(q + \underbrace{p_q}_{xp} - p_q'\Big)$$



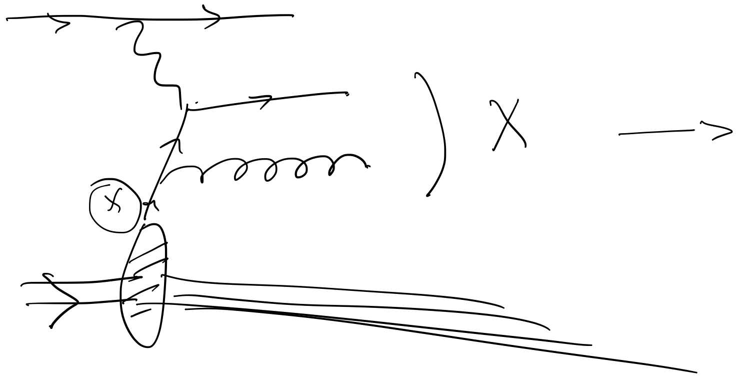
$$(2\pi) \; \boxed{\frac{1}{2E_q'} \, \delta\left(q^0 + x p^0 - p_q'^{\,0}\right)}$$

$$\delta\Big(p_q'^{\,2} - \underbrace{\cancel{m_q^2}}_{0}\Big)$$

$$\frac{1}{2a}\Big(\delta(x-a) + \delta(x+a)\Big)$$

$$\delta(x^2 - a^2)$$

$$\delta\left((xp+q)^2\right) = \delta\Big(\underbrace{\cancel{x^2 p^2}}_{M^2 \ll Q^2} + \underbrace{q^2}_{-Q^2} + 2p\cdot q\, x\Big) \rightarrow \frac{1}{2p\cdot q}\,\delta\Big(x - \underbrace{\frac{Q^2}{2p\cdot q}}_{x_B}\Big)$$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