Day 6

Emilie

$m_\eta = 547$ MeV

$\eta \to \pi^+ \pi^-$?

π^+

$J^P = 0^-$

$m_\pi \sim 140$ MeV

$J^P = 0^-$

↳ $l = 0$

$P(\pi^+ \pi^-)$?

$= \eta_{\pi^+} \eta_{\pi^-} (-1)^l$

$= (-1)(-1)(-1)^0 = 1$

$\eta \longrightarrow K^+ K^-$? $\Rightarrow$ forbidden.

↳ $m_K \sim 497$ MeV

$\eta \longrightarrow \gamma\gamma \qquad S_\gamma = 1.$

$P(\gamma) = -1 \qquad \vec{J} = \vec{l} \otimes \vec{S}$

$P(2\gamma) = \eta_\gamma \eta_\gamma (-1)^l$

$\eta \to 2\gamma$ allowed.

$\eta \to \pi^+\pi^-\pi^0$) suppressed by PS

$\eta \to \pi^0\pi^0\pi^0$)

↳ + isospin breaking.

$\eta \to \pi^+\pi^-\pi^+\pi^-$?

⇒ kinematically forbidden.

$\eta \to \mu^+ \mu^-$?

↳ allowed.

$$|\pi^+\rangle = |\underset{I}{1}, \underset{I_3}{1}\rangle$$

$$|\pi^0\rangle = |1, 0\rangle$$

$$|\pi^-\rangle = |1, -1\rangle$$

$$|\pi^+\rangle \otimes |\pi^+\rangle =$$

$$|1,1\rangle \otimes |1,1\rangle$$

$$= C_{"1"} |2,2\rangle$$

$$|\pi^-\rangle \otimes |\pi^-\rangle =$$

$$|1,-1\rangle \otimes |1,-1\rangle$$

$$= |2,-2\rangle$$

$$|\pi^0\rangle \otimes |\pi^0\rangle$$

$$= |1,0\rangle \otimes |1,0\rangle$$

$$= \sqrt{\frac{2}{3}}\,|2,0\rangle + 0\,|1,0\rangle - \frac{1}{\sqrt{3}}\,|0,0\rangle$$

$|\pi^+\rangle \otimes |\pi^0\rangle$

$|\pi^-\rangle \otimes |\pi^0\rangle$

$|2,0\rangle =$

$$I_n = 0$$

$$|0,0\rangle_3 = \frac{1}{\sqrt{3}} |1,1\rangle_1 |1,-1\rangle_2$$

$$- \frac{1}{\sqrt{3}} |1,0\rangle_1 |1,0\rangle_2$$

$$+ \frac{1}{\sqrt{3}} |1,-1\rangle_1 |1,1\rangle_2$$

$$|0,0\rangle = |1,-1\rangle_2$$

$$\frac{1}{\sqrt{3}}\left(|\pi^+\rangle \otimes \left(\frac{1}{\sqrt{2}}|\pi^0\pi^-\rangle - \frac{1}{\sqrt{2}}|\pi^-\pi^0\rangle\right)\right.$$

$$-\frac{1}{\sqrt{3}}\left(|\pi^0\rangle \otimes \left(\frac{1}{\sqrt{2}}\left(|\pi^+\pi^-\rangle - |\pi^-\pi^+\rangle\right)\right.\right.$$

$$+\frac{1}{\sqrt{3}}\left(|\pi^-\rangle \otimes \left(\frac{1}{\sqrt{2}}\left(|\pi^+\pi^0\rangle - |\pi^0\pi^+\rangle\right)\right.\right.$$

$$|0,0\rangle = \frac{1}{\sqrt{6}} \Big(C|\pi^+\pi^0\pi^-\rangle$$

$$= -|\pi^+\pi^-\pi^0\rangle - |\pi^0\pi^+\pi^-\rangle$$

$$+ |\pi^0\pi^-\pi^+\rangle + |\pi^-\pi^+\pi^0\rangle$$

$$- |\pi^-\pi^0\pi^+\rangle \Big)$$

$$\hookrightarrow C|\pi\pi\pi\rangle_{I=0} = |\pi^-\pi^0\pi^+\rangle$$

$$C\,|\pi\pi\pi\rangle_{I=0} = -|\pi\pi\pi\rangle_{I=0}$$

C parity is conserved.

$$C\,\eta = +\eta$$

$\Rightarrow$ isospin is broken.

Isospin symmetry:

$$m_u = m_d$$

$\Rightarrow$ $m_u \neq m_d$ in reality

$$A(\eta \to \pi^+ \pi^- \pi^0) \propto A_1 (m_u - m_d)$$

$3\pi^0$

$$+ \alpha_{em} A_2$$

$$A_C(s,t,u) =$$

$$\langle \pi^+ \pi^- \pi^0 | \eta \rangle =$$

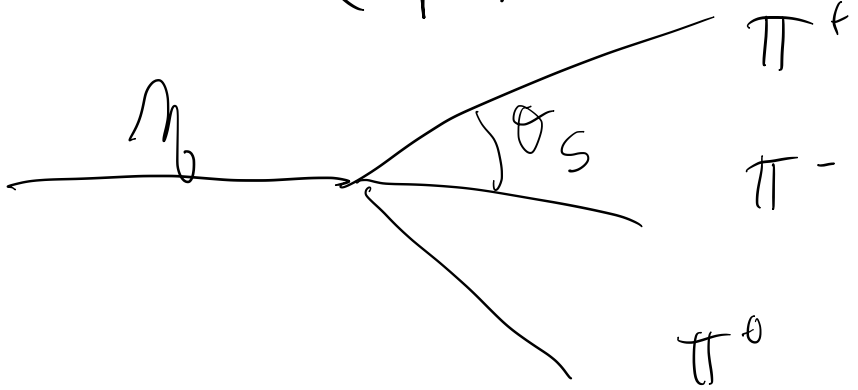
$$i (2\pi)^4 \delta^4 (p_\eta - p_{\pi^+} - p_{\pi^-} - p_{\pi^0})$$

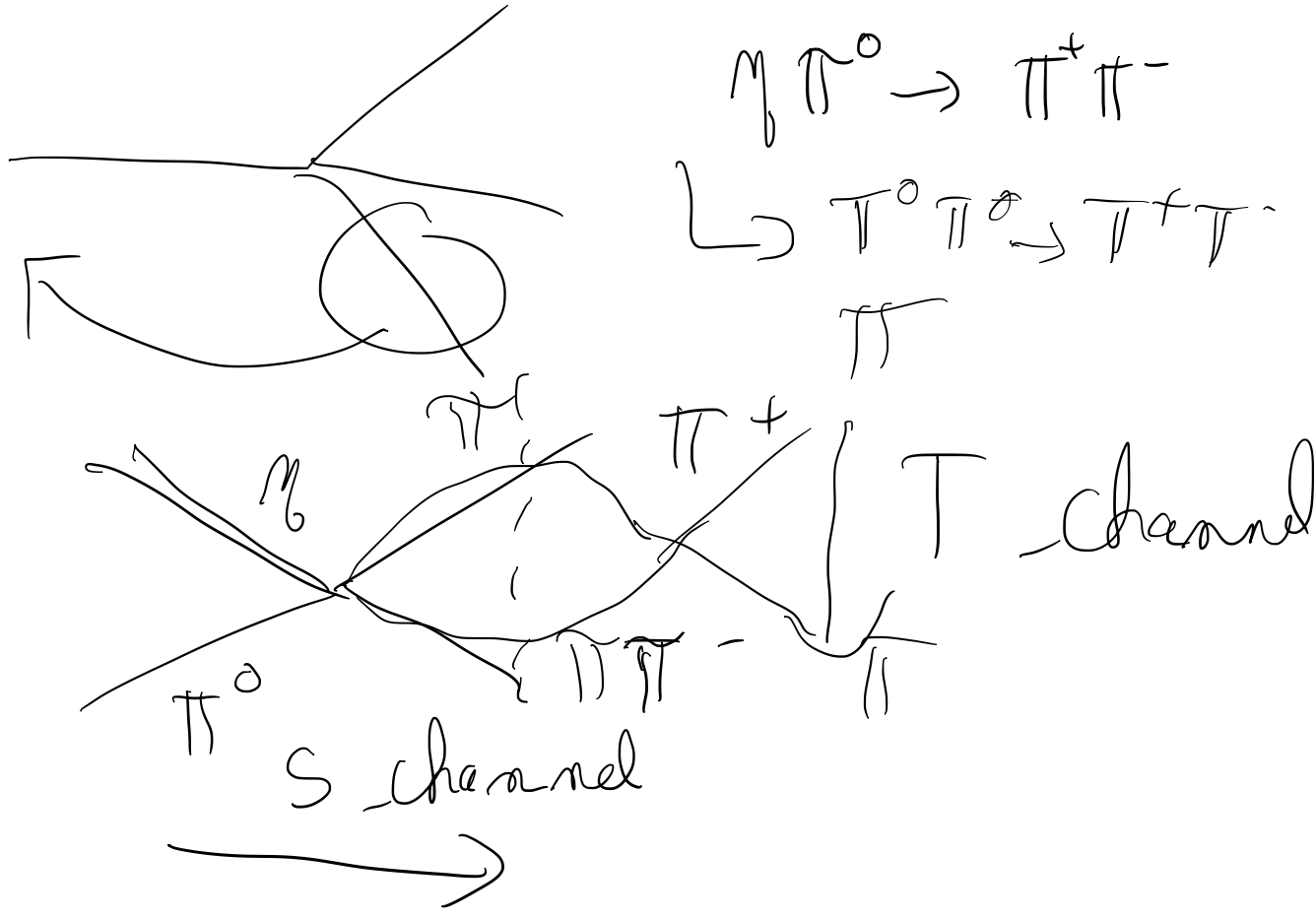
$$A_C(s,t,u)$$

$$s = (p_{\pi^+} + p_{\pi^-})^2 = (p_\eta - p_{\pi^0})^2$$

$$t = (p_{\pi^0} + p_{\pi^-})^2 = \dots$$

$$u = (p_{\pi^0} + p_{\pi^+})^2 = \dots$$



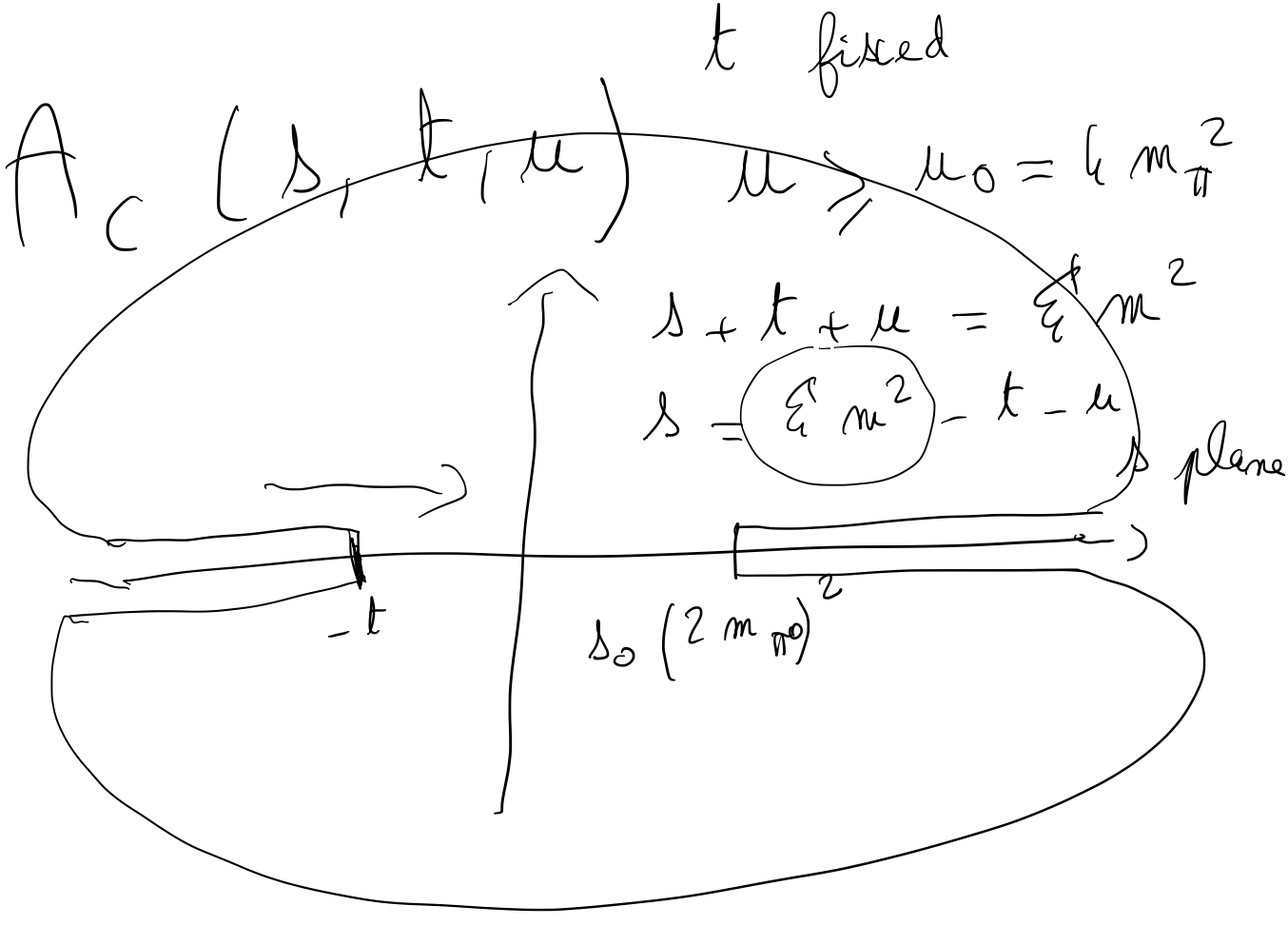

$\eta\pi^0 \to \pi^+\pi^-$
$\pi^0\pi^0 \to \pi^+\pi^-$
π^+
π^+
π^-
η
π^0
T_channel
S_channel

T channel $t \hookrightarrow s$

$\eta \pi^- \rightarrow \pi^0 \pi^-$

U channel $u \hookrightarrow s$

$\eta \pi^+ \rightarrow \pi^0 \pi^+$


t fixed
$A_C(s,t,u)$ $u \geq u_0 = 4m_\pi^2$
$s+t+u = \sum_i m^2$
$s = \sum_i m^2 - t - u$
s plane
$-t$
$s_0 (2m_{\pi^0})^2$

$$A(s,t,u) = P_{n-1}(s,t,u)$$

$$+ \frac{s^n}{2i\pi} \int_{4m_\pi^2}^{\infty} ds' \frac{\text{disc } A(s',t,u(s'))}{s'^n (s'-s)}$$

$$+ \frac{u^n}{2i\pi} \int_{4m_\pi^2}^{\infty} du' \frac{\text{disc } A(s(u'),t,u')}{u'^n (u'-u)}$$

$\ell = 0, I = 0, \quad \ell = 1, I = 1$

$$\Rightarrow A(s,t,u) = \sum_{I} \sum_{\ell=0}^{+\infty} c_{\ell,I} (2\ell+1) P_\ell(\cos\theta) f_\ell^I(s)$$

$\ell_{max} = 1$

$\ell = 0, I = 2$

$l = 0 \qquad P_0(\cos\theta) = 1$

$l = 1 \qquad P_1(\cos\theta) = \cos\theta$