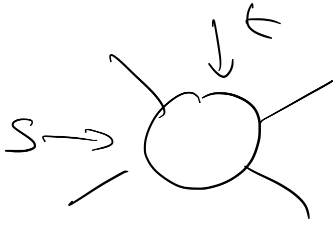
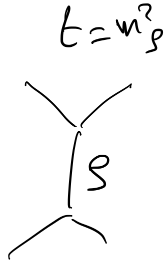
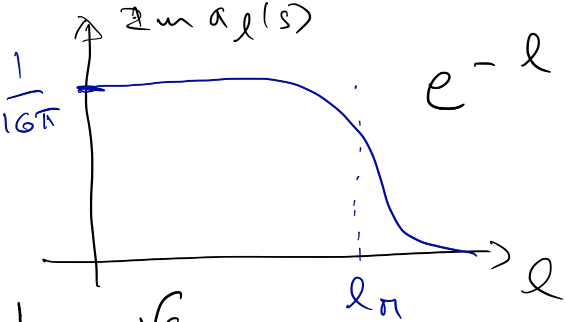
Day 8

Froissart Bound

$$A(s,t) = \sum_{l=0}^{\cancel{\infty}\; l_M} (2l+1)\, a_l(s)\, P_l\Big(\overbrace{1+\frac{2t}{s-4m^2}}^{z_s}\Big)$$







$$e^{-l \underbrace{2\sqrt{\frac{t_0}{s}}}_{\mathrm{arch}(z_0)}}$$

$$k = \frac{\sqrt{s}}{2}$$

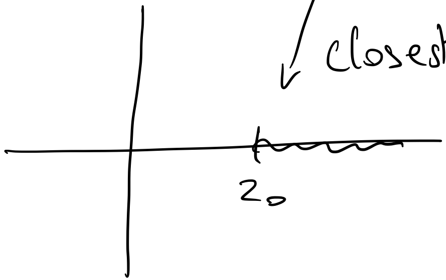
$$r = \frac{l_M}{k} = \frac{l_M 2}{\sqrt{s}}$$

$$P_l \sim e^{l\, \mathrm{arch}(z_s)}$$

$$a_l P_l \; : \; t < t_0$$

$$a_\ell(s) = \frac{1}{\pi}\int_{z_0(s)}^{\infty}\left[D_t A(z_s,s) + (-)^\ell D_u A(z_s,s)\right] Q_\ell(z_s)\, dz_s$$

closest singularity in the t-channel



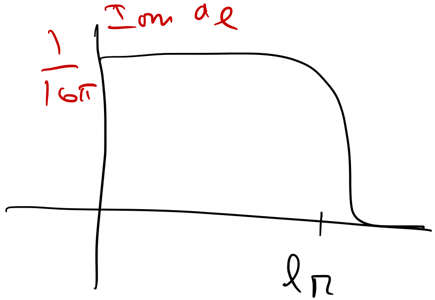
$$\xi(z_0) = \text{arch}(z_0) \sim 2\sqrt{t_0/s}$$

t_0 closest sing in t

$$a_\ell(z) \sim e^{-\ell\,\xi(z_0)}$$

$$\sim e^{-\ell\, 2\sqrt{\frac{t_0}{s}}}$$

$$a_\ell(s) \underset{\ell\to\infty}{\sim} \underbrace{P(s)\, e^{-\ell\, 2\sqrt{\frac{t_0}{s}}}}_{e^{-\ell\, 2\sqrt{\frac{t_0}{s}} + n\log(s)}}$$



$$\ell_M \sim \sqrt{s/t_0}\,\log(s)$$

$$\mathrm{Im}A(s,t) = \sum_{\ell=0}^{\ell_M=} (2\ell+1)\, \underbrace{\mathrm{Im}\, a_\ell(s)}_{1/16\pi}\; \underbrace{P_\ell\left(1+\cancel{\frac{2t}{s-4m^2}}\right)}_{1}$$

$$\sim \frac{1}{16\pi}\,\ell_M^2 = s/t_0\,\log^2(s)$$

$$\ell_M \sim k\rho_0 \sim \frac{\sqrt{s}}{2}\rho_0 \to \rho_0 \sim \log(s)$$

$$|A(s,t)| \le s^N \qquad N = 1$$

$$\sigma_{tot} = \frac{1}{s}\,\mathrm{Im}\,A \sim cst \quad \text{at hig energy}$$

$f^+(\ell,t)$ matches $f_\ell(t)$ for even ℓ

$f^-(\ell,t)$ " " " odd ℓ

$$A^\pm(s,t) = \sum_{\ell=0}^{\infty} (2\ell+1)\, f_\ell^\pm(t)\, P_\ell(z_t) \qquad z_t = 1 + \frac{2s}{t-4m^2}$$

$$f_\ell(t) = \frac{1}{2}\Big[\underset{e^{-i\pi\ell}}{(-)^\ell} + 1\Big] f^+(\ell,t) - \frac{1}{2}\Big[(-)^\ell - 1\Big] f^-(\ell,t)$$

$$A^+(s,t) = \sum_{\alpha > -1/2} \beta(t)\, \frac{s^{\alpha(t)}}{\sin \pi\alpha} + O(s^{-1/2})$$

$$A(s,t) = \sum_{\alpha^\pm > -1/2} \beta^\pm(t)\, \frac{\pm 1 - e^{-i\pi\alpha^\pm}}{\sin \pi\alpha^\pm}\, (s/s_0)^{\alpha^\pm} + O(s^{-1/2})$$

$$P_\alpha\left(1+\frac{2s}{t-4m^2}\right) \underset{s\to\infty}{\sim} \left(\frac{2s}{t-4m^2}\right)^\alpha$$

$$\left(s/s_0\right)^\alpha$$

$$A(s,t) \sim \bar{\beta}(t)\left(\frac{s}{t-4m^2}\right)^\alpha$$

$$\beta_1(t)\left(s/s_1\right)^\alpha = \beta_2(t)\left(s/s_2\right)^\alpha$$

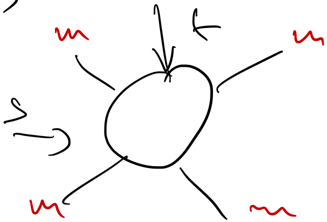
$$\sim \beta(t)\left(s/s_0\right)^{\alpha(t)}$$

$$s_0$$

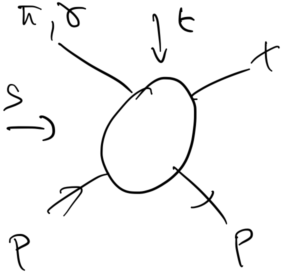
$$s^{\alpha(t)} \quad \underbrace{s_0^{-\alpha(t)}}$$

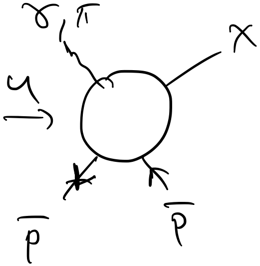
$$e^{-\alpha(t)\log s_0} \sim e^{-b_0 t}$$

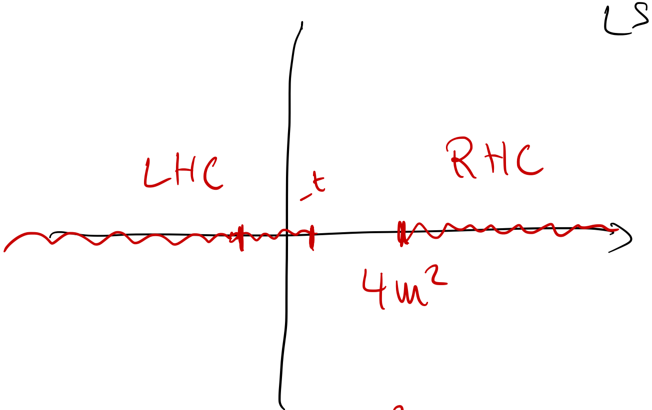
$$\mathrm{Im}\, A^{Reg}(s,t) = \beta(t)\,(s/s_0)^{\alpha(t)}$$



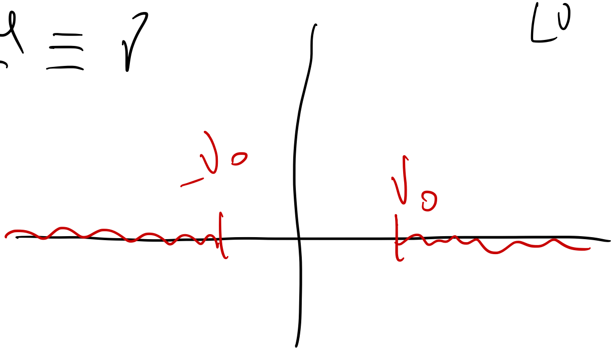
Finite Energy Sum Rules







$$\frac{s-u}{2} \equiv \nu$$



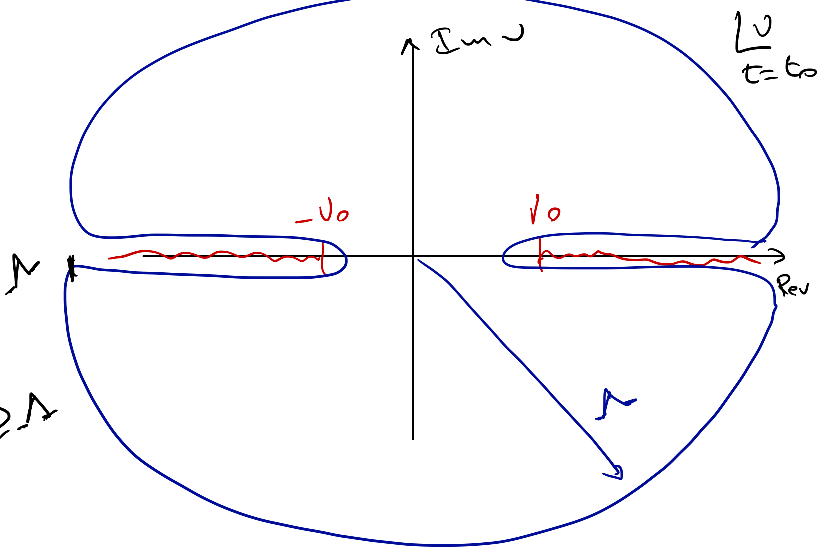
$$u = -s - t + 4m^2$$

$$s = -u - t + 4m^2 \qquad u = 4m^2$$

$$= -t$$

$$\oint_C A(\nu,t)\,\nu^k\,d\nu = 0$$

$k \in \mathbb{N}$



$$\mathrm{Im}\,A^{\pm}(\nu,t) = \bar{\beta}(t)\,(s/s_0)^{\alpha} \qquad \nu \gtrsim \Lambda$$

$$= \beta(t)\, s^{\alpha}$$

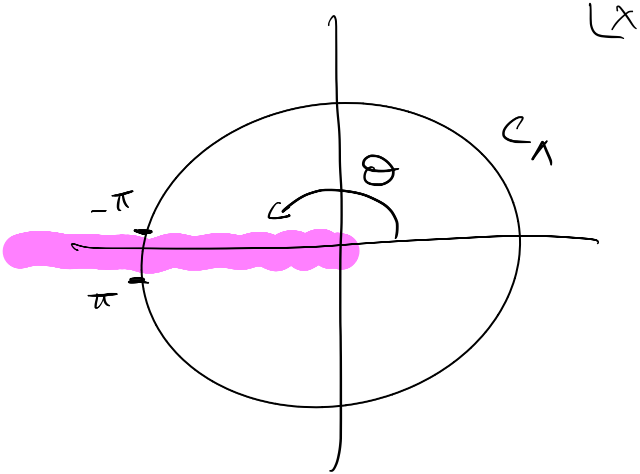
$$\int_{\nu_0}^{\Lambda} \left[D_s A(\nu,t) \mp (-)^k D_u A(\nu,t) \right] \nu^k \, d\nu = -\frac{1}{2i} \oint_{C_\Lambda} \nu^k A^{\pm}(\nu,t)\, d\nu$$

$\tau = \pm$

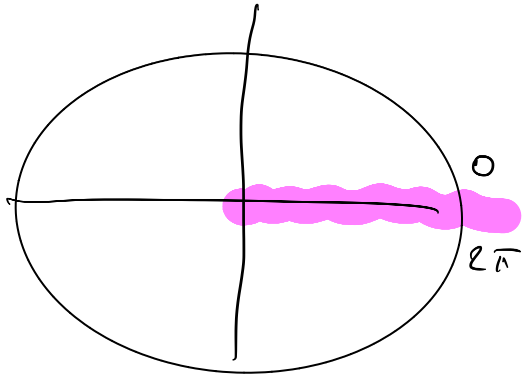
$$A^{\tau}(\nu,t) = -\beta\, \frac{\tau + \overbrace{e^{-i\pi\alpha}}^{(-)^{\alpha}}}{\sin \pi\alpha}\, \nu^{\alpha} = -\beta\, \frac{\tau\,\nu^{\alpha} + (-\nu)^{\alpha}}{\sin \pi\alpha} \qquad \nu = \frac{s-u}{2}$$

$$\nu = \Lambda\, e^{i\theta}$$

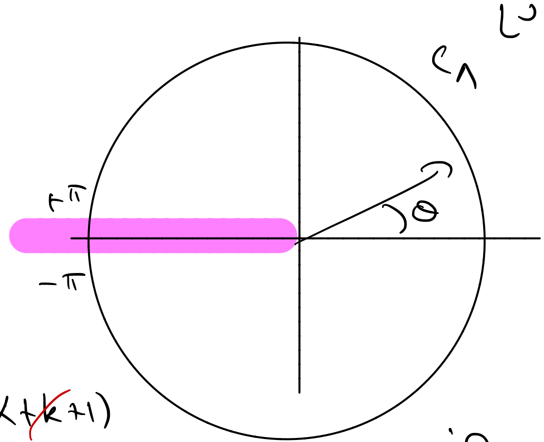
$$\oint_{C_R} \sqrt{x}\,dx =$$



$\sqrt{-x}$



$$-\frac{1}{2i}\oint_{C_\Lambda} \frac{-\beta\tau\,\nu^{\alpha}\,\nu^{k}}{\sin\pi\alpha}\,d\nu$$



$$= \frac{\beta\tau}{2i}\int_{-\pi}^{\pi} \frac{\Lambda i}{\sin\pi\alpha}\, e^{i\theta(\alpha+k+1)}\,\Lambda^{k+\alpha}\,d\theta$$

$$\nu = \Lambda e^{i\theta}$$

$$= \frac{\beta\tau}{2}\,\frac{\Lambda^{\alpha+k+1}}{\sin\pi\alpha}\,\frac{e^{i\pi(\alpha+\cancel{k}+1)} - e^{-i\pi(\alpha+\cancel{k}+1)}}{(\alpha+k+1)\,i}$$

$$\underbrace{e^{i\pi k}}\qquad\underbrace{e^{-i\pi k}}_{(-)^k}$$

$$= -\tau\beta\,\underline{(-)^k}\,\frac{\Lambda^{k+\alpha+1}}{\alpha+k+1}$$

$$\frac{1}{2i}\beta\int_0^{2\pi} i\Lambda^{\alpha+k+1} e^{i\theta(k+1+\alpha)} \frac{e^{-i\pi\alpha}\, d\theta}{\sin\pi\alpha} \qquad (-\upsilon)^{\alpha}$$

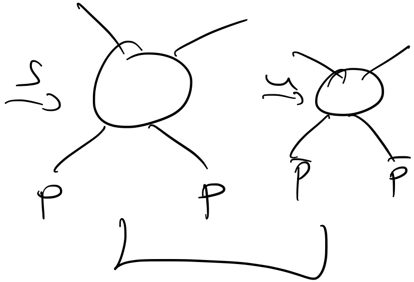
$$= \frac{\beta}{2}\,\frac{\Lambda^{\alpha+k+1}}{\sin\pi\alpha}\int_0^{2\pi} e^{i\phi(\alpha+k+1)} e^{-i\pi\alpha}\, d\phi$$

$$= \frac{\beta}{2}\,\frac{\Lambda^{\alpha+k+1}}{\sin\pi\alpha}\, e^{-i\pi\alpha}\,\frac{\left[e^{i2\pi(\alpha+k+1)}-1\right]}{i(\alpha+k+1)} = \beta\,\frac{\Lambda^{\alpha+k+1}}{\alpha+k+1}$$

$$\boxed{\int_{\nu_0}^{\Lambda}\left[D_s A(\nu,t) - \tau(-)^k D_u A(\nu,t)\right]\frac{\nu^k\, d\nu}{\Lambda^{k+1}} = \left[1-\tau(-)^k\right]\beta\,\frac{\Lambda^{\alpha}}{\alpha+k+1}}$$

$$\int_{\nu_0}^{\Lambda} \Big[D_s A^{\tau}(\nu,t) - \tau(-)^k \underbrace{D_u A^{\tau}(\nu,t)}_{D_s A = \mathrm{Im}\, A} \Big] \frac{\nu^k \, d\nu}{\Lambda^{k+1}} = \underbrace{[1-\tau(-)^k]}\, \beta \frac{\Lambda^{\alpha}}{\alpha+k+1}$$

$$\int_{\nu_0}^{\Lambda} \mathrm{Im}\, A^{\tau}(\nu,t) \frac{\nu^k \, d\nu}{\Lambda^{k+1}} = \beta(t) \frac{\Lambda^{\alpha(t)}}{\alpha(t)+k+1}$$

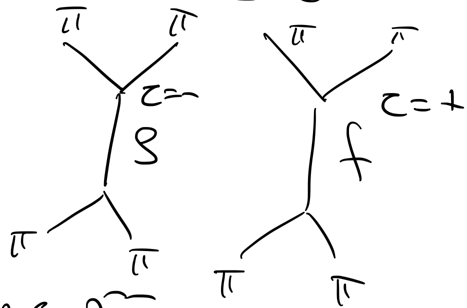


C invariance

$$[1-\tau(-)^k] \neq 0$$

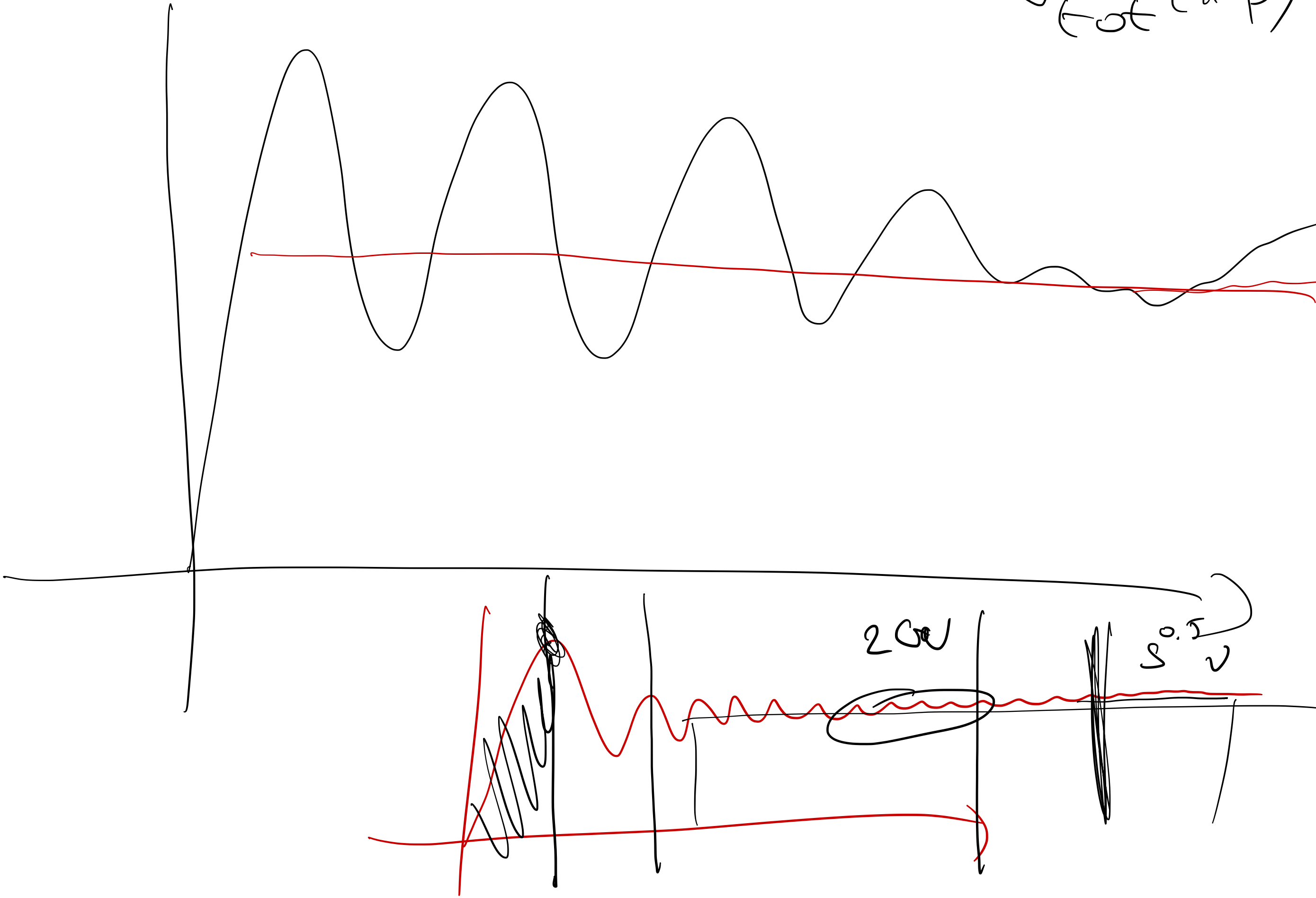
$\tau = -$ $\quad k = \text{even}$

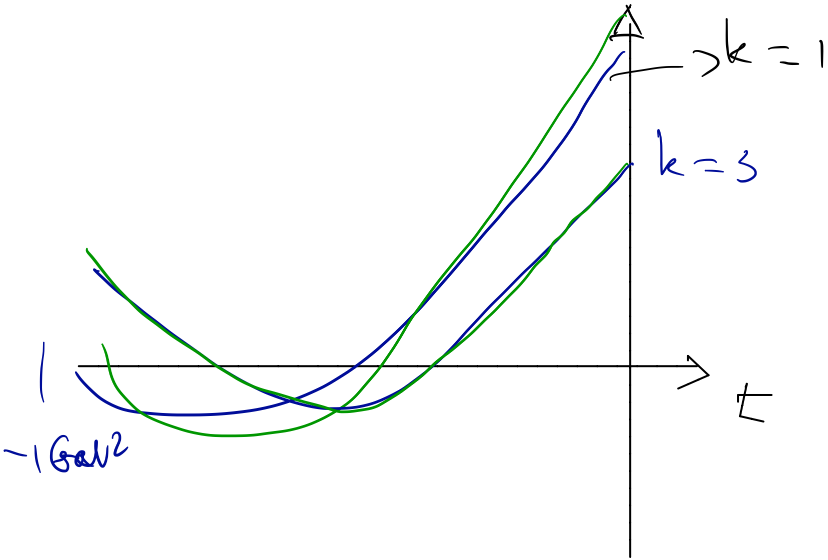
$\tau = +$ $\quad k = \text{odd}$



$\rho \; (1,3,5,\ldots)^{--}$ $\qquad f(0,2,4\ldots)^{++}$

$\mathrm{Im}\, A \approx \sigma_{tot}(\pi^{+}p)$





$$\underbrace{\int \mathrm{Im}\, A(\nu, t)\, \frac{\nu^k\, d\nu}{\Lambda^{k+1}}} = \underbrace{\beta(t)\, \frac{\Lambda^{\alpha(t)}}{\alpha(t)+k+1}}$$

$$S_k(t) = \int_{\nu_0}^{\Lambda} \mathrm{Im}\, A(\nu, t) \frac{\nu^k \, d\nu}{\Lambda^{k+1}} = \beta \frac{\Lambda^{\alpha}}{\alpha+k+1} + \cancel{\beta_P \frac{\Lambda^{\alpha_P}}{\alpha_P+k+1}}$$

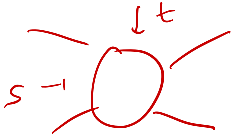
fitted to low E data

fitted to high E data

$$\boxed{\beta(t) = \frac{\alpha+k+1}{\Lambda^{\alpha}} S_k(t)}$$

I R+f

Reggeons

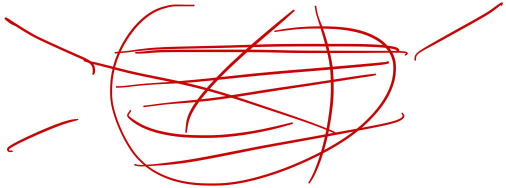


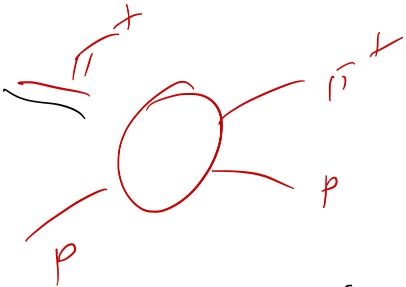
$A(s,t)$

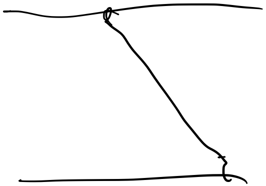
large s, small t

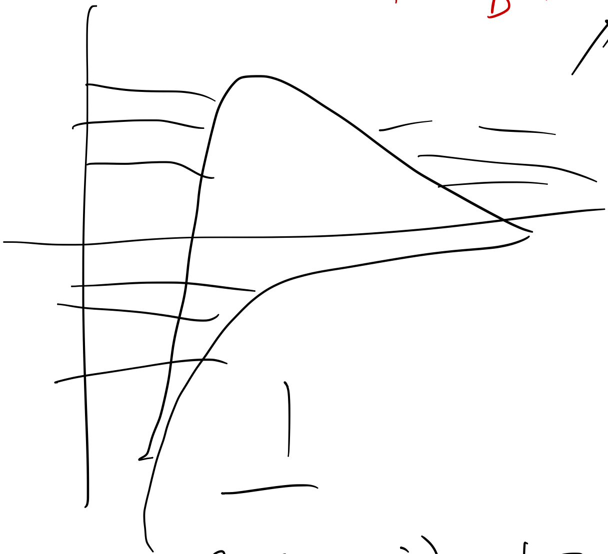
$$\beta(t) = \beta_{ab}(t)\,\beta_{cd}(t)$$

$$s^{\alpha(t)} \quad \beta(t)$$



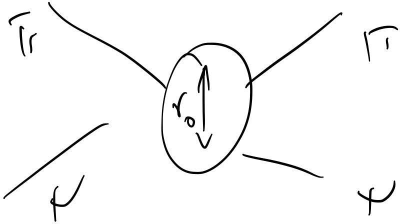


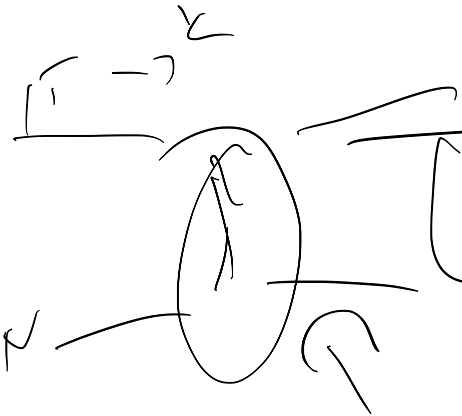




$$\frac{1}{m_{\pi}^2 - t} = \frac{1}{m_{\pi}^2 + (\vec{p} - \vec{p}\,')}$$

$$\frac{1}{m_{\pi}^2 - (E - E')^2 + (\vec{p} - \vec{p}\,')^2}$$



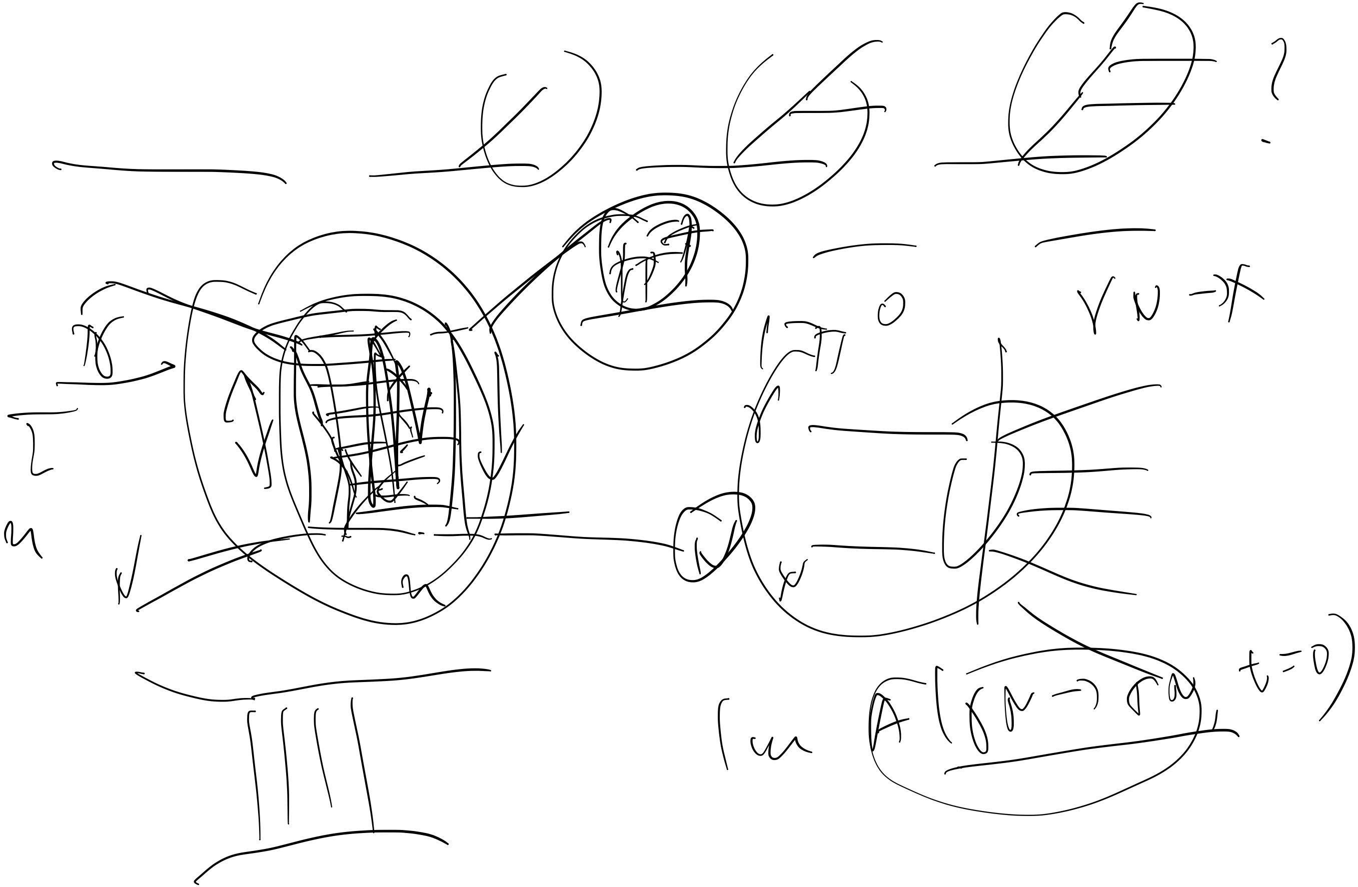


$$\boxed{\ell_{max} \sim k r_0}$$

$$\sigma \sim const$$

$$A(s,0) \sim s$$

$$\sigma \sim \frac{\operatorname{Im} A(s,0)}{s} \; !$$



$$A(s,t) = s^{\alpha(t)}\,\beta(t)$$

$$\alpha(t) \Rightarrow \underline{\alpha_P(t)}$$

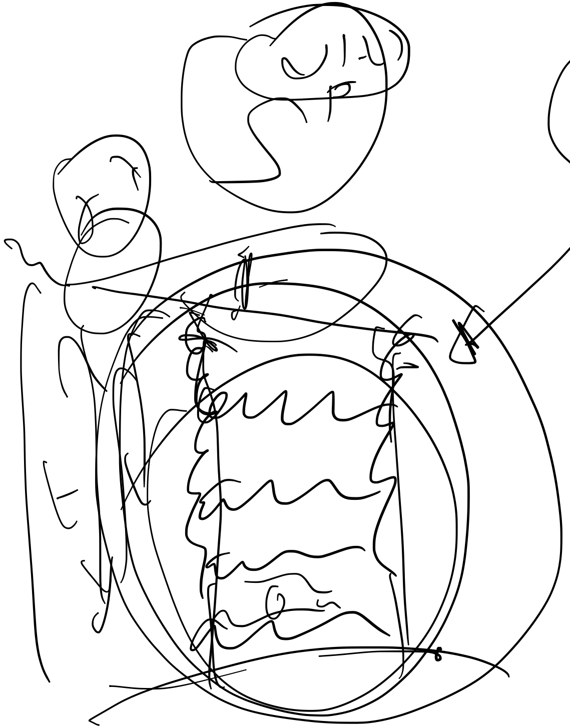
$$\alpha_P(t) \approx 1$$

$$\alpha(t) \Rightarrow \alpha_R(t)$$

$$\alpha_R(t) \approx \frac{1}{2}$$

$$\sigma \sim s$$

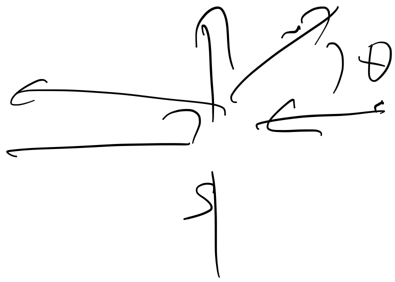
$$\sigma \sim s \log s$$



$\alpha(t) = 1 + \varepsilon + \alpha' t$

$\varepsilon = 0.08$

$1 + 0.25$



$$F(t) = \int_{4m_\pi^2}^{\infty} \frac{dt'}{\pi} \frac{\operatorname{Im} F(t')}{t'-t+i0}$$

Dispersion relation

Let $\mathcal{M}$ be the invariant amplitude

Dispersion relation

DISPERSION RELATION

rest your hand!