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INTERACTION OF GAMMA QUANTA AND ELECTRONS WITH NUCLEI AT HIGH ENERGIES

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It is shown that if the longitudinal distances which are important in the electromagnetic interactions of hadrons increase linearly with increase of energy, then only the nucleons on the surface of a nucleus participate in the interaction with a high energy γ quantum, and the total cross section σ_γ of hadronic processes for heavy nuclei is equal to $2\pi R^2(1 - Z_3)$, where Z_3 is the renormalization constant of the photon Green's function, and $1 - Z_3$ is the probability for the virtual transition of a γ quantum into hadrons. The corrections associated with volume absorption of γ quanta are discussed in detail, and also the situation in the case when the longitudinal distances increase slowly with increase of energy.

1. INTRODUCTION

IN an article by Ioffe, Pomeranchuk, and the author[1] the question was raised about the feasibility of an experimental determination of what distances are important in the strong interactions at high energies. It was shown that if the amplitude for the scattering of particle a by a certain target b (Fig. 1) essentially depends on the square of the 4-momentum p_a^2 (the mass), then longitudinal distances which increase with energy are important in the interaction; these distances are of the order of $|p|/\mu^2$ ($\hbar = c = 1$), where p is the momentum of the incoming particle in the laboratory coordinate system and μ is a certain characteristic mass.

Unfortunately, it was shown that the method proposed in [1] for an experimental investigation of the dependence of the amplitude on the "mass" of the particle with the aid of an analysis of the bremsstrahlung cannot provide an answer to the question about the important longitudinal distances because of the cancellations that are due to conservation of charge.[2] In the present article we wish to point out that investigations of the interaction of γ quanta and electrons with nuclei yield the possibility to experimentally clarify what longitudinal distances are important in the electromagnetic interactions of hadrons.

An interesting effect was observed in an article by Bell,[3] consisting in the fact that if the interaction of γ quanta with nucleons is dominated by vector mesons, but the neutrino interaction is dominated by π mesons, then surface effects appear in connection with the interaction of γ quanta and neutrinos with nuclei, i.e., together with the volume terms which are proportional to the number A of nucleons in the nucleus, the amplitudes also contain surface terms proportional to $A^{2/3}$. At high energies the surface terms turned out to be decisive, and this was regarded as a specific property of the ρ- or π-dominance model.

In the present article we show that the nature of the interaction of γ quanta and neutrino with nuclei and the appearance of surface effects at high energies is not connected with ρ meson or π meson dominance, but is

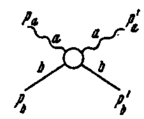

FIG. 1

only determined by what distances are important in these interactions. We show that if large longitudinal distances of the order of $\delta = |p|/\mu^2$ are important, then the total cross section, for example, of γ quanta with heavy nuclei, including only hadronic processes, is given by

$$\sigma_\gamma = 2\pi R^2(1 - Z_3), \qquad (1)$$

where R is the nuclear radius, and Z_3 is the charge renormalization constant due to the hadrons. One can express the quantity $1 - Z_3$ in terms of the hadronic part of the Lehman density of the photon Green's function or in terms of the cross section $\sigma_{e^+e^-}(\varkappa^2)$ for the annihilation of electron-positron pairs into hadrons:

$$1 - Z_3 = e^2 \int \rho(\varkappa^2)\frac{d\varkappa^2}{\varkappa^2} = \frac{1}{8\pi^2 e^2}\int \sigma_{e^+e^-}(\varkappa^2)\, d\varkappa^2. \qquad (2)$$

Formula (1) has a simple physical meaning: $2\pi R^2$ is the total cross section for the interaction of hadrons with the nucleus, and $1 - Z_3$ is the fraction of the time which the γ quantum spends in the hadronic state.

The assumption that large distances are important in the interaction is equivalent to an assumption about the convergence of the integrals (2). The condition for the applicability of (1) is $\delta^2 \gg Rl$ where l is the mean free path of the hadrons in the nucleus. If the characteristic mass μ is of the order of the ρ-meson mass, and the mean free path is of the order of $1/m_\pi$ (m_π denotes the mass of the π meson), then surface effects should appear at energies greater than 10 GeV.

One can understand the origin of the surface effects and formula (1) almost without making any calculations. Let us assume that a γ quantum interacts with the nucleons in the nucleus in the following way: it first decays into virtual hadrons, and then these hadrons interact with the nucleons inside the nucleus. Let this fluctuation last a time δ. Then the total cross section for the interaction of γ quanta with a nucleus will be determined by the probability πR^2 for the γ quantum to hit the nucleus, the probability $(R/137\delta)$ that a fluctuation arises inside the nucleus, and the probability δ/l that the hadrons which are produced are able to interact with some kind of nucleon inside the nucleus. Therefore σ_γ is of the order of $\pi R^2(R/137\delta)(\delta/l) \sim \frac{1}{137}\pi R^2(R/l) \sim A\sigma_{\gamma N}$.

This argument is valid, however, only if $\delta \lesssim l$. If in the coordinate system in which the quantum has a small energy the duration of the fluctuation is of the order of $1/\mu$, then in the laboratory system where the quantum has momentum p the duration of the fluctuation is given by $\delta \approx p/\mu^2$, i.e., it increases with the energy of the quantum. If $R > \delta > l$, then the cross section for the interaction of a quantum will be of the order of $\pi R^2 \times (R/137\delta)$, i.e., it decreases with increase of energy. In actual fact, as was essentially noted by Bell,[3] the possibility of the formation of fluctuations of size δ larger than the mean free path l is suppressed by a factor δ/l due to quantum-mechanical interference, and the cross section is of the order of $\pi R^2(Rl/137\delta^2)$, i.e., it decreases even more rapidly with increase of energy. Under these conditions it is now impossible to neglect the probability that a fluctuation may arise outside of the nucleus. When δ becomes larger than the nuclear radius, then essentially all fluctuations will arise outside of the nucleus, and the created hadrons will collide with the nucleus with a cross-section πR^2, i.e., the interaction cross section of a quantum will be of the order of $\frac{1}{137}\pi R^2$. Thus we arrive at a cross section of the type (1).

It is also easy to explain the appearance of the factor $1 - Z_3$ in expression (1) if the amplitude for forward elastic scattering of a γ quantum by a nucleus, which determines the total cross section, is represented with the aid of the diagram shown in Fig. 2a. The amplitude F for the scattering of a beam of hadrons by a nucleus of radius R changes substantially during a change of the transverse momenta of the particles by an amount of the order of 1/R and falls off rapidly for large changes of the momenta. Since 1/R is appreciably smaller than the scale of the momenta which are important in the diagram of Fig. 2a, the momenta k_i' of the particles almost do not differ from k_i. The ordinary amplitude for the elastic scattering of a single particle may be written under similar conditions in the form $2\pi i R^2\delta(q)$. The corresponding amplitude for the scattering of a group of particles is proportional to $2\pi i R^2 \Pi\delta(k_i - k_i')$. As a result the diagram of Fig. 2a is equivalent to the diagram of Fig. 2b multiplied by $2\pi i R^2$. The diagram shown in Fig. 2b determines the charge renormalization.

Such a picture of the interaction arises under the assumption that a quantum of small energy undergoes virtual decay into small masses of the order of μ. However, it is possible that a quantum will comparatively frequently decay into very large masses. This corresponds to a divergence of the integral for $1 - Z_3$. The existence of such fluctuations with large masses may not develop in any way for a quantum with small energy if the path length l for such states with large masses is large. However, with an increase of the energy, when the length of the fluctuations δ even for very large masses is comparable with the path length l inside a nucleus, such masses begin to participate in the interaction with a cross section of the order of πR^2. Only masses for which the length δ of the fluctuations for arbitrary energy is smaller than the path length may, for arbitrary energy, give a contribution proportional to the volume of the nucleus to the cross section.

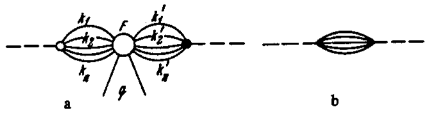

FIG. 2

In the case when the integral over the mass for $1 - Z_3$ diverges, one can write the cross section for the interaction of a γ quantum with a nucleus in the form

$$\sigma_\gamma = 2\pi R^2[1 - Z_3(\varkappa_0^2)] + \sigma_\gamma^V, \tag{3}$$

$$1 - Z_3(\varkappa_0^2) = e^2 \int_0^{\varkappa_0^2} \rho(\varkappa^2)\frac{d\varkappa^2}{\varkappa^2}. \tag{4}$$

The mass κ_0 at which the integral (4) is cut-off is determined by the condition

$$\delta(\varkappa_0^2) = 2p/\varkappa_0^2 \approx l(\varkappa_0^2, p), \tag{5}$$

where $l(\kappa^2, p)$ is the mean free path of a group of particles with total mass κ^2 and momentum p. The correct definition of $l(\kappa^2, p)$ will be given in the text.

If the integral (2) diverges logarithmically, i.e., if the cross section for the annihilation of e^+ and e^- into hadrons is of the same order of magnitude as the cross section for the annihilation into leptons,[4,5] then

$$1 - Z_3(\varkappa_0^2) \approx e^2\rho(\infty)\ln(\varkappa_0^2/\mu^2), \tag{6}$$

where μ is a certain constant.

The second term in Eq. (3) is proportional to the volume of the nucleus and is of order

$$\sigma_\gamma^V \sim e^2\pi R^2 \frac{R}{l(\varkappa_0^2, p)}. \tag{7}$$

If $l(\kappa_0^2, p)$ does not depend on the energy, which is possible if $l(\kappa^2, p) = l(\kappa^2/p)$, then $\kappa_0^2 \sim p$ and σ_γ^V does not depend on the energy, but the surface term increases logarithmically. If $l(\kappa_0^2, p)$ increases with an increase of the energy, then condition (5) holds only up to energies for which $l(\kappa_0^2, p) < R$. For $l(\kappa_0^2, p) > R$ the cut-off κ_0^2 is determined by the condition

$$l(\varkappa_0^2, p) = R. \tag{8}$$

In this connection the volume term is of the order $e^2\pi R^2$, but the surface term, which is the major term, either tends to a constant if $l(\kappa_0^2, p)$ does not depend on p or else continues to increase logarithmically.

By experimentally studying the dependence of σ_γ on the energy and on A, one can distinguish both terms and determine the dependence of κ_0^2 on p. The dependence of κ_0^2 on p reflects the energy dependence of the longitudinal distances $\delta = 2p/\kappa^2$ which are important in the interaction of γ quanta with nucleons. If κ_0^2 increases with increase of p but more slowly than p ($\kappa_0^2/2p \to 0$), then large longitudinal distances, which increase with energy but are smaller than for finite $1 - Z_3$, are important. If $\kappa_0^2 - p\mu$ then longitudinal distances up to $1/\mu$ are important. The growth of σ_γ with energy may continue up to those energies at which $\pi^{-1}e^2 \ln(\kappa_0^2/\mu^2) \sim 1$ and perturbation theory becomes invalid for the electromagnetic interaction.

If the integral (2) were to diverge more rapidly than logarithmically, then the cross section would increase according to a power law. Perturbation theory in electrodynamics would become invalid at energies which are appreciably smaller than is usually assumed. We shall not consider this possibility.

The cross section for the interaction of electrons with nuclei which is described by the diagram shown in Fig. 3 will have the same properties for given values of p^2 and p_0. The only difference consists in the fact that instead of $1 - Z_3$ it will be determined by the value of the polarization operator (Fig. 2b) for $p^2 \neq 0$. We show that

$$d\sigma^S = 2\pi R^2 \frac{e^4}{p^4}\left\{\left[4m^2 + p^2 + \frac{p^2}{p_0^2}(k_0 + k_0')^2\right]\Pi_1 \times p^2[4m^2 + 2p^2]\Pi_2\right\}\frac{p_0}{2k_0 k_0'}\frac{d^3k'}{(2\pi)^3}, \quad (9)$$

$$d\sigma_\gamma = d\sigma^3 + d\sigma^V, \quad (10)$$

where

$$\Pi_1(p^2, \varkappa_0^2) = \int^{\varkappa_0^2} \frac{\rho(\varkappa^2)\,d\varkappa^2}{\varkappa^2 - p^2}, \quad \Pi_2(p^2) = \int^{\varkappa_0^2} \frac{\rho(\varkappa^2)\,d\varkappa^2}{(\varkappa^2 - p^2)^2}. \quad (11)$$

It is of interest to note that in the case when distances smaller than $2p/\mu^2$ are important, i.e., when the integral for Π_1 diverges, $\Pi_1(p^2, \kappa_0^2)$ does not depend on p^2, and consequently the surface term in the cross section does not depend on p^2 for $p^2 \ll \kappa_0^2$.

The results cited above are obtained under the assumption that one can regard the interaction of fast hadrons with a nucleus as the result of successive interactions with the nucleons inside the nucleus and the interaction of the nucleons inside the nucleus can be described with the aid of pair correlations. The latter assumption apparently is not essential, but its rejection would only complicate the investigation.

2. INTERACTION OF HADRONS WITH A NUCLEUS AT HIGH ENERGIES

As noted above the interation of a γ quantum with a nucleus at high energies takes place in such a way that the γ quantum first turns into hadrons, and then the hadrons interact with the nucleus. Therefore, before going on to an examination of the interaction of a γ quantum with a nucleus, let us discuss how the description of the interaction of hadrons with a nucleus is changed upon transition to high energies in comparison with the description at low energies.

Total cross sections and the elastic interaction of hadrons with a nucleus at not very high energies are usually described either with the aid of the optical

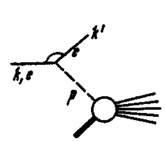

FIG. 3

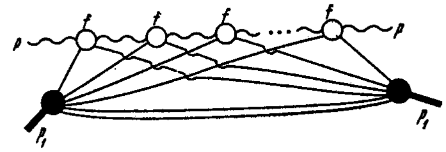

FIG. 4

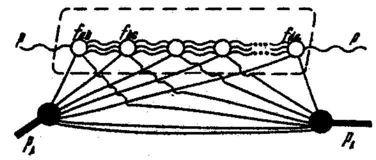

FIG. 5

model or with the aid of Glauber's theory of successive collisions. These two methods are similar, provided the pair correlations of the nucleons inside the nucleus are taken into account, and reduce to an investigation of Feynman diagrams of the type shown in Fig. 4, which describe successive elastic scatterings by the nucleons in the nucleus. Under the assumption that the average momenta of the nucleons in the nucleus is much smaller than the momenta which are important in the hadronic interactions, at low energies one can consider only elastic scattering processes since the inelastic processes require large momentum transfers, leading to a collapse of the nucleus.

As shown in [5], during an increase of the energy, when the momentum transfers necessary for the creation of particles decrease and become of the order of the momenta of the nucleons in the nucleus, it is necessary to take inelastic processes and the diagrams of Fig. 5 into account.

Before going on to an investigation of the diagrams of Fig. 5 and their influence on the character of the total cross sections, let us briefly consider how the evaluation of the total cross sections is carried out at high energies, but such that inelastic processes are still not important. Everywhere below we shall not take into account the shrinkage of the diffraction peak in hadronic processes.

Evaluation of the diagrams of Fig. 4 at these energies gives the following result. The forward scattering amplitude $F^{(n)}$, corresponding to n-fold rescattering (see, for example, the Appendix), has the form

$$F^{(n)} = \left(\frac{iN}{4pmV}\right)^{n-1}\frac{N^2}{V}\int d^2\rho_1\,dz_1\ldots dz_n\,f\varkappa(z_1 - z_2)f\ldots\varkappa(z_{n-1} - z_n)f, \quad (12)$$

where N is the number of nucleons in the nucleus, V is the volume of the nucleus, p is the momentum of the incoming particle, ρ_i and z_i are the coordinates of the nucleons: z_i is the coordinate in the direction of the incoming particle's momentum, ρ_i is the coordinate perpendicular to p, $\kappa(z_i - z_{i+1})$ is the correlation function of two nucleons in the nucleus, $\kappa(\infty) = 1$, f denotes the scattering amplitude, m is the mass of a nucleon, and $z_1 > z_2 \ldots > z_n$. If the amplitude F which is given by

$$F = \sum_n F^{(n)}, \quad (13)$$

is written in the form

$$F = \frac{N^2}{V}\int F(\rho, z)\,dV, \quad (14)$$

then one can easily verify that $F(\rho, z)$ satisfies the equation

$$F(\rho, z) = f + \frac{iN}{4pmV} \int_{-z_0(\rho)}^{z} dz' f\varkappa(z - z') F(\rho, z'), \quad z_0(\rho) = \sqrt{R^2 - \rho^2}, \tag{15}$$

which is the analog of the equations for the optical model with the scattering amplitude f playing the role of a potential. If correlations are neglected, i.e., if one sets $\kappa = 1$, then the following trivial result follows from Eq. (15):

$$F(\rho, z) = f \exp\{-l^{-1}[z + z_0(\rho)]\}, \tag{16}$$

$$l^{-1} = (-iN/4pmV) f \tag{17}$$

and

$$F = \frac{N^2}{V} \int F(\rho, z)\, dV = 2pmN \cdot 2\pi R^2 i, \quad \sigma_t = 2\pi R^2. \tag{18}$$

The idea of the following investigation consists in the fact that an equation of the type (15) remains valid with only a small change if, by the amplitudes f and $F(\rho, z)$ one understands the amplitudes for the interaction of groups of hadrons with a nucleon with a transition to the other groups of hadrons which enter into the diagrams shown in Fig. 5. An evaluation of the diagram shown in Fig. 5 is given in the Appendix under the assumption that the nucleons in the nucleus are nonrelativistic, and their momenta are appreciably smaller than the momentum transfers which are important in the strong interactions at high energies. The latter is equivalent to the assumption that the mean free path of the hadrons inside the nucleus is larger than the distances between the nucleons. It is assumed that under these conditions we can confine ourselves only to the correlations between nucleons which participate in two successive collisions.

One can write the result for the amplitude of a process, including the interaction with n-nucleons, in the form

$$F_{aa}^{(n)} = \left(\frac{iN}{4pmV}\right)^{n-1} \frac{N^2}{V} \sum_{b, c, d, \ldots} \int d^2\rho_1\, dz_1 \ldots dz_n\, f_{ab}$$

$$\times \exp\{-iq_z{}^b(z_1 - z_2)\} \varkappa(z_1 - z_2) f_{bc} \exp\{-iq_z{}^c(z_2 - z_3)\} \ldots f_{da}, \tag{19}$$

where f_{bc} is the amplitude for the process corresponding to the diagram shown in Fig. 6. A summation is carried out over the real intermediate states

$$q_z{}^b = (m_b{}^2 - \mu^2)/2p, \tag{20}$$

where m^b denotes the mass of the intermediate state, and μ is the mass of the incoming particle.

Let us introduce the operator $F(\rho, z)$ whose matrix elements between arbitrary states are defined by the equation

$$F_{ab}(\rho, z) = \sum_n \left(\frac{iN}{4pmV}\right)^{n-1} \sum_{c, d, e, \ldots} \int dz_2 \ldots dz_n\, f_{ac} \exp\{-iq_z{}^c(z_2 - z_3)\}$$

$$\times \varkappa(z_2 - z_3) f_{cd} \ldots \exp\{-iq_z{}^e(z_{n-1} - z_n)\} \varkappa(z_{n-1} - z_n) f_{eb}. \tag{21}$$

The operator $F(\rho, z)$ satisfies the equation

$$F(\rho, z) = f + i\zeta \int_{-z_0(\rho)}^{z} dz' f \exp\{-iq_z(z - z')\} \varkappa(z - z') F(\rho, z'),$$

$$\zeta = \frac{N}{4pmV}, \quad (q_z)_{cd} = \delta_{cd} \frac{m_c{}^2 - \mu^2}{2p}. \tag{22}$$

This equation describes all possible transformations in the beam of hadrons associated with the interaction with

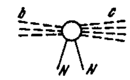

FIG. 6

the nucleons of the nucleus. It goes over into Eq. (15) if only one intermediate state with mass m equal to the mass μ of the incoming particle is possible.

One can easily find a symbolic solution of this equation if $F(\rho, z)$ is written in the form

$$F(\rho, z) = \frac{1}{2i\pi} \int_{\alpha - i\infty}^{\alpha + i\infty} d\xi\, e^{\xi(z + z_0)} F(\xi), \tag{23}$$

$$F(\xi) = \frac{1}{1 - i\zeta f \varkappa(\xi + iq_z)} \frac{f}{\xi}, \tag{24}$$

$$\varkappa(\xi) = \int_0^\infty e^{-\xi z} \varkappa(z)\, dz. \tag{25}$$

The scattering amplitude is given by

$$F = \frac{N^2}{V} \int F(\rho, z)\, dV = \frac{N^2}{V} \int d^2\rho \int d\xi \frac{e^{2z_0\xi} - 1}{2\pi i \xi} F(\xi). \tag{26}$$

Let us write Eq. (24) in the form

$$F(\xi) = \frac{1}{1 - i\zeta f \varkappa(iq)} \frac{f}{\xi} + \frac{1}{1 - i\zeta f \varkappa(iq)} i\zeta f$$

$$\times \varkappa'(iq) \frac{1}{1 - i\zeta f \varkappa(iq)} f + \tilde{F}(\xi), \quad \tilde{F}(0) = 0. \tag{27}$$

Substituting (27) into (26) and integrating, we obtain

$$F = N^2 \frac{f}{1 - i\zeta f \varkappa(iq)} + \frac{N^2}{V} \pi R^2 \frac{1}{1 - i\zeta f \varkappa(iq)} i\zeta f$$

$$\times \varkappa'(iq) \frac{1}{1 - i\zeta f \varkappa(iq)} f + \tilde{F}, \tag{28}$$

$$\tilde{F} = \frac{N^2}{V} \int d^2\rho \int_{-\alpha - i\infty}^{-\alpha + i\infty} \frac{d\xi}{2\pi i} \frac{e^{2z_0\xi}}{\xi^2} \frac{1}{1 - i\zeta f \varkappa(\xi + iq)} f. \tag{29}$$

The first term, which is proportional to the number of nucleons in the nucleus, is actually equal to zero in application to a real state with mass μ^2 since here $q = 0$ ($\kappa(iq) \sim 1/iq$, $q \to 0$). For the same reason the second term is equal to $2pmN \cdot 2\pi i R^2$. The last term in (28), $\tilde{F}$, is determined by the poles of the integrand in (29). These poles are located at negative values $\xi = -\bar{\xi}$ and determine the attenuation of $F(z, \rho)$. In this connection $\tilde{F}$ is of the order of $N^2 f l^3/V$, where $l = 1/\bar{\xi}$ is the mean free path. Thus

$$F = 2pmN[2\pi R^2 i + O(N\sigma l^3/V)], \quad f \sim 2pmi\sigma. \tag{30}$$

Consequently the amplitude for the scattering of a group of particles by a sufficiently large nucleus is a diagonal operator and the total cross section is equal to $2\pi R^2$, just like at lower energies.

In conclusion we emphasize that volume absorption is equal to zero only for a real state of the incident particle, i.e., for the scattering amplitude on the mass shell. If p_μ^2 for the incident particle does not coincide with μ^2 of the intermediate state, then $q_z \neq 0$, and we obtain volume absorption proportional to $[(p^2 - \mu^2) l/2p]^2 N$. This means that at small energies the amplitude for the scattering of a virtual particle differs substantially from the scattering amplitude on the mass shell.

3. INTERACTION OF GAMMA QUANTA WITH NUCLEI. LARGE DISTANCES

In this section we present a derivation of formula (1) in which the concept of the distances at which the interaction takes place will explicitly figure. For this purpose, let us write the forward scattering amplitude $F_{\nu\nu}(s, \kappa^2)$ for virtual Compton scattering in the form of an integral of the T-product of the electromagnetic currents:

$$F_{\nu\nu}(s, \varkappa^2) = ie^2 \int e^{ipx} \langle A | Tj_\nu(-x_1) j_\nu(x_2) | A \rangle d^4x_1 d^4x_2; \qquad (31)$$

here $\kappa^2 = p_\mu^2$ denotes the "mass" of a quantum.

As discussed in article [1], for large energies s the amplitude $F_{\nu\nu}(s, \kappa^2)$ depends on κ^2 only in the case when large longitudinal distances of the order of p/μ^2 are important in the integral (31); here μ is a certain characteristic mass, p is the quantum's momentum in the laboratory system, $s \sim 2pM$, and M is the mass of the nucleus. In fact, by writing the argument of the exponential in (31) in the form

$$px = p_0 t - pz \approx p_0(t - z) + (\varkappa^2 / 2p_0) z,$$

we see that values of $t - z \sim 1/p_0$ are essential in expression (31) and $F_{\nu\nu}(s, \kappa^2)$ depends on κ^2 only if values $z \sim t \sim p/\mu^2$ are important. We shall assume that this holds, and by using the reduction formulas we write the quantity $\langle A | Tj_\nu(x_1) j_\nu(x_2) | A \rangle$ in the form

$$\langle A | Tj_\nu(x_1) j_\nu(x_2) | A \rangle = i \int \exp\{-ip_A(y - y')\} d^4y d^4y' \langle 0 | Tj_\nu(x_1) j_\nu(x_2) u_A(y') \bar{u}_A(y) | 0 \rangle, \qquad (32)$$

where $u_A(y')$ and $\bar{u}_A(y)$ are the operator sources of the field of the nucleus. Substituting (32) into (31) and changing variables, we obtain

$$F_{\nu\nu}(s, \varkappa^2) = -e^2 \int \exp\{-ip(x_1 - x_2) + ip_A\xi\} \times \langle 0 | Tj_\nu(x_1) u(0) \bar{u}(\xi) j_\nu(x_2) | 0 \rangle d^4x_1 d^4x_2 d^4\xi. \qquad (33)$$

Taking into consideration that $x_{10} - x_{20} \to \pm\infty$ and $\xi_0 \sim 1/M$, we may assume that in Eq. (33) the points 0, $\xi_0 \sim 1/M$ are located between the points x_{10} and x_{20}, and instead of (33) we may write

$$F_{\nu\nu}(s, \varkappa^2) = -e^2 \int \exp\{ip(x_2 - x_1) + ip_A\xi\} \times \langle 0 | j_\nu(x_2) T(u(0) \bar{u}(\xi)) j_\nu(x_1) | 0 \rangle d^4x_1 d^4x_2 d^4\xi + x_1 \to x_2 \qquad (34)$$

or after an expansion of the product of the operators over the intermediate states we obtain

$$F_{\nu\nu}(s, \varkappa^2) = e^2 \sum_{n,m} \frac{\langle 0 | j_\nu | n \rangle}{p_{n0} - p_0} \left\langle n \left| \int e^{ip_A\xi} Tu(0) \bar{u}(\xi) d^4\xi \right| m \right\rangle \times \frac{\langle m | j_\nu | 0 \rangle}{p_{m0} - p_0} + p \to -p, \qquad (35)$$

where $p_n = p_m = p$.

The expression $\langle n | \int \exp(ip_A\xi) \times Tu(0)\bar{u}(\xi) d^4\xi | m \rangle$ is equal to F_{nm} where F_{nm} is the amplitude for the forward scattering of a group of particles with momentum p by a nucleus, which was discussed in the previous Section. Taking into consideration that

$$p_0 \approx p + \frac{\varkappa^2}{2p}, \quad p_{n0} \approx p + \frac{M_n^2}{2p}, \quad p_{m0} \approx p + \frac{M_m^2}{2p},$$

we obtain

$$F_{\nu\nu}(s, \varkappa^2) = e^2 \sum_{n,m} \frac{\langle 0 | j_\nu | n \rangle}{M_n^2 - \varkappa^2} F_{nm} \frac{\langle m | j_\nu | 0 \rangle}{M_m^2 - \varkappa^2} (2p)^2. \qquad (36)$$

The second term in (35), corresponding to the substitution $p \to -p$, is small since the denominator $p_n + p_0 \approx 2p$ instead of $(M_n^2 - \kappa^2)/2p$. For a similar reason the contributions of the other regions to (33), with a time relation differing from (34), are small if the integrals over the mass of the type (35) converge. As was shown,

$$F_{nm} = 2\pi R^2 i \cdot 2pM\delta(n - m). \qquad (37)$$

Substituting (37) into (36), we obtain

$$F_{\nu\nu}(s, \varkappa^2) = 2\pi R^2 i \cdot 2pMe^2 \int \frac{dM_n^2}{(M_n^2 - \varkappa^2)^2} \rho_{\nu\nu}(M_n^2), \qquad (38)$$

where

$$\rho_{\alpha\beta}(p_n) = \sum \langle 0 | j_\alpha(0) | n \rangle \langle n | j_\beta(0) | 0 \rangle (2\pi)^4 \delta\left(\sum k_i - p_n\right) = (-\delta_{\alpha\beta} p_n^2 + p_{n\alpha} p_{n\beta}) \rho(p_n^2),$$
$$\rho_{\nu\nu} = e_\alpha^\nu e_\beta^\nu \rho_{\alpha\beta}(p_n) = p_n^2 \rho(p_n^2), \qquad (39)$$

e_α^ν is the polarization vector of a quantum, and $\rho(M_n^2)/M_n^2$ is the spectral density of the photon Green's function. Hence the total cross section for the interaction of a real photon with a nucleus is given by

$$\sigma_t = 2\pi R^2 (Z_3^{-1} - 1) \approx 2\pi R^2 (1 - Z_3), \qquad (40)$$

$$1 - Z_3 = e^2 \int \frac{dM^2}{M^2} \rho(M^2). \qquad (40a)$$

In connection with the derivation of Eq. (40) we assumed that the integral for $1 - Z_3$ converges. In order to estimate the accuracy to which expression (40) is valid, and in order to consider the case when the integral for $1 - Z_3$ diverges, it turns out to be more convenient to first express the amplitude for the scattering of a quantum by a nucleus in terms of the amplitude for the interaction of the quantum and hadrons with the nucleons in the nucleus, and then to use formulas of the type (33) or dispersion relations with respect to the mass, but now for the amplitudes characterizing the interaction of a quantum with a nucleon. We do this in the following section.

4. INTERACTION OF A GAMMA QUANTUM WITH A NUCLEUS. NOT VERY LARGE DISTANCES

Under the assumptions formulated in Sec. 2, one can represent the amplitude $F_{\gamma\gamma}(s)$ for the forward scattering of a γ-quantum by a nucleus in the form of a set of diagrams of the type shown in Fig. 7, which are similar to the diagrams shown in Fig. 5. The amplitude $F_{\gamma\gamma}$ may be written down in the form (19), the only difference being that state a is a γ-quantum and f_{ab} and f_{da} are replaced by $f_{\gamma b}$ and $f_{d\gamma}$. As before, the remaining amplitudes represent the amplitudes for hadronic processes.

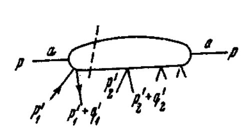

FIG. 7

For a real quantum the parameters q_z are equal to $m_b^2/2p$. In analogy with (21), let us introduce the amplitudes $F_{\gamma\gamma}(\rho, z)$ and $F_{\gamma a}(\rho, z)$ where a denotes the hadronic state. These amplitudes satisfy equations similar to (22):

$$F_{\gamma\gamma}(\rho, z) = f_{\gamma\gamma} + i\zeta \int_{-z_0(\rho)}^{z} dz' f_\gamma \exp\{-iq_z(z-z')\}\varkappa(z-z')F_\gamma(z', \rho), \quad (41a)$$

$$F_\gamma(\rho, z) = f_\gamma + i\zeta \int_{-z_0(\rho)}^{z} dz' f \exp\{-iq_z(z-z')\}\varkappa(z-z')F_\gamma(\rho, z'); \quad (41b)$$

here $f_{\gamma\gamma}$ is the amplitude of the Compton effect per nucleon, and f_γ is the amplitude for photoproduction of the hadronic state.

Equations (41) are written in operator form with respect to the hadronic states. Solving Eq. (41) by changing to the momentum representation (24), we obtain

$$F_\gamma(\xi) = \frac{1}{1 - i\zeta f\varkappa(\xi + iq_z)} f_\gamma \frac{1}{\xi}, \quad (42a)$$

$$F_{\gamma\gamma}(\xi) = \frac{f_{\gamma\gamma}}{\xi} + i\zeta f_\gamma \varkappa(\xi + iq_z)\frac{1}{1 - i\zeta f\varkappa(\xi + iq_z)}\frac{f_\gamma}{\xi}. \quad (42b)$$

Hence, computing

$$F_{\gamma\gamma} = \frac{N^2}{V}\int F_{\gamma\gamma}(\rho, z)\, dV,$$

instead of expression (28) we may write down

$$F_{\gamma\gamma} = N^2\Big[f_{\gamma\gamma} + i\zeta f_\gamma \varkappa(iq_z)\frac{1}{1 - i\zeta f\varkappa(iq_z)} f_\gamma\Big] + \frac{N^2}{V}\pi R^2 i\zeta f_\gamma \varkappa(iq_z)\frac{1}{1 - i\zeta f\varkappa(iq_z)}\frac{\varkappa'(iq_z)}{\varkappa(iq_z)} \times \frac{1}{1 - i\zeta f\varkappa(iq_z)} f_\gamma + F_{\gamma\gamma}, \quad (43)$$

where

$$(g_z)_{ab} = \delta_{ab} m_b^2 / 2p.$$

The first term in (43) represents the volume interaction $F_{\gamma\gamma}^V$, the second represents the surface interaction $F_{\gamma\gamma}^S$, and the third term is small in analogy to (30). In contrast to (30) the volume term does not vanish, and the surface term is not equal to $2\pi R^2 i \cdot 2pmN$ since f_γ differs from f and $q_z \neq 0$ in application to a real state.

Let us estimate the order of magnitude of the individual terms in (42). We note that in order of magnitude

$$-i\zeta f \sim -i\frac{N}{V4pm}\cdot 2ipm\sigma \approx \frac{1}{2}\frac{N}{V}\sigma \sim \frac{1}{2l},$$

where σ is the cross section for the hadronic process, and l is the mean free path. The correlation function $\varkappa(iq_z)$ depends on iq_z and on the average distance r_0 between the particles. For $q_z r_0 \ll 1$ we have $\varkappa(iq_z) = 1/iq_z$, and for $qr_0 \gg 1$ the function $\varkappa(iq_z)$ falls off. The characteristic denominators which determine the dependence of $F_{\gamma\gamma}^V$ and $F_{\gamma\gamma}^S$ on the value of q in the intermediate states have the form $2l/[\varkappa(iq_z) + 1]$, and are equal to $2ilq_z + 1$ for small values of q_z. These denominators are of the order of unity for $q_z \lesssim 1/l$ and are large for $q_z > 1/l$. One can write the contributions to $F_{\gamma\gamma}^V$ and $F_{\gamma\gamma}^S$ from the regions $q_z \ll 1/l$ and $q_z \gg 1/l$, respectively, in the form

$$F_{\gamma\gamma}^V = N^2\Big[f_{\gamma\gamma} - f_\gamma \frac{1}{f} f_\gamma\Big], \quad F_{\gamma\gamma}^S = i\frac{N^2}{V}\pi R^2 f_\gamma \frac{1}{\zeta f^2} f_\gamma \quad (44)$$

for $q_z \ll 1/l$ and

$$F_{\gamma\gamma}^V = N^2\Big[f_{\gamma\gamma} + \zeta f_\gamma \frac{1}{q_z}\Big(1 - \frac{1}{2iq_z l}\Big) f_\gamma\Big],$$

$$F_{\gamma\gamma}^S = \frac{N^2}{V}\pi R^2 i\zeta f_\gamma \frac{1}{q_z^2} f_\gamma \quad (45)$$

for $q_z \gg 1/l$. Here $1/2l \equiv -i\xi f$.

If, as is true for the ρ-dominance model,[3] $f_\gamma = g_\gamma f$ and $f_{\gamma\gamma} = g_\gamma f g_\gamma$ and $q_z \to 0$ since $q_z = m_\rho^2/2p$, then according to Eq. (44)

$$F_{\gamma\gamma}^V = 0, \quad F_{\gamma\gamma}^S = 2pMi \cdot 2\pi R^2 g_\gamma^2. \quad (46)$$

In order to evaluate $F_{\gamma\gamma}$ without making any assumption about ρ-dominance, one can use formulas of the type (31) but for the amplitudes characterizing the interaction of a γ quantum with a nucleon, or else by a dispersion relation with respect to the masses of the quanta for those same amplitudes. In this section we shall use dispersion relations. Let us assume that an unsubtracted dispersion relation in the masses of the quanta of the following form holds for the amplitude for forward scattering of a virtual quantum of mass p_1^2 which is changing into a quantum with mass p_2^2:

$$f_{\gamma\gamma}(s, p_1^2, p_2^2) = \frac{1}{\pi^2}\int \frac{d\varkappa_1^2 d\varkappa_2^2 \tilde{f}_{\gamma\gamma}(s, \varkappa_1^2, \varkappa_2^2)}{(\varkappa_1^2 - p_1^2)(\varkappa_2^2 - p_2^2)}, \quad (47)$$

$$\tilde{f}_{\gamma\gamma}(s, p_1^2, p_2^2) = \sum_{n,m}\Gamma_\gamma(k_1, \ldots, k_n)\delta^4\Big(p_1 - \sum k_i\Big) \times f_{nm}(k_1, \ldots, k_n, k_1', \ldots k_m', q)\delta^4\Big(p_2 - \sum k_j'\Big)\Gamma_\gamma(k_1', \ldots, k_m'). \quad (48)$$

Here $f_{nm}(k_1, \ldots, k_n, k_1', \ldots, k_m', q)$ is the amplitude for the hadronic processes per nucleon with momentum transfer $q(q_0, q_\perp, q_z)$ to the nucleon, $q_0^2 - q_\perp^2 = 0$, $q_z = (p_1^2 - p_2^2)/2p$; $\Gamma_\gamma(k_1, \ldots, k_n)$ is the vertex part for the transition of a photon into n hadrons. In similar fashion, with the aid of a single dispersion relation one can write the amplitude $f_{\gamma a}(s, p_1^2, k_1, \ldots, k_a)$ in the form

$$f_{\gamma a}(s, p_1^2, \ldots) = \frac{1}{\pi}\int \frac{d\varkappa_1^2}{\varkappa_1^2 - p_1^2}\tilde{f}_{\gamma a}(s, \varkappa_1^2, \ldots), \quad (49)$$

$$\tilde{f}_{\gamma a}(s, p_1^2, \ldots) = \sum_n \Gamma_\gamma(k_1, \ldots, k_n)\delta^4\Big(p_1 - \sum k_i\Big) \times f_{na}(k_1, \ldots, k_n, k_1', \ldots, k_m', q). \quad (50)$$

With regard to the possibility of using these dispersion relations, two questions naturally arise: whether the dispersion relations (47) and (49) (especially (49)) are violated because of the presence of more complicated complex singularities than the threshold singularities, and whether the unsubtracted dispersion relations are valid? Even if complex singularities exist, they are not important at high energies. One can verify this, for example, with the aid of formula (36), which is nothing other than a double dispersion relation over the masses of the quanta, and which is obtained only from assumptions that the integrals over the intermediate states converge. An attempt to investigate the analytic properties of the Feynman diagrams leads to the same result.

The use of unsubtracted dispersion relations is obviously an hypothesis which cannot be proved. However, it is necessary to emphasize that there is an important distinction between the dispersion relations over the energy which are usually used and dispersion relations with respect to the mass. The growth of the invariant amplitudes at large energies s is a normal phenomenon both in the strong as well as in the weak interactions. In contrast to this it is natural to assume that at large masses, i.e., in the case of strongly virtual processes, the amplitudes decrease because of the cut-off due to the strong interactions, at all events for masses $\kappa^2 \gtrsim s$.

If the dispersion relations (47)–(50) are assumed, then by substituting them into (43) we obtain for $\chi = 1/iq_z$

$$\frac{1}{2pM}F_{\gamma\gamma}{}^{S} = 2\pi i R^2 \frac{1}{\pi^2}\int \frac{d\varkappa_1{}^2\, d\varkappa_2{}^2}{\varkappa_1{}^2\varkappa_2{}^2} \times \Gamma_\gamma\left[\frac{1}{2iq_z l+1}\frac{1}{q_z}\frac{1}{2iq_z l+1}q_z\right]\Gamma_\gamma, \quad (51)$$

$$F_{\gamma\gamma}{}^{V} = \frac{N^2}{\pi^2}\int \frac{d\varkappa_1{}^2\, d\varkappa_2{}^2}{\varkappa_1{}^2\varkappa_2{}^2}\Gamma_\gamma \frac{2iq_z lf}{2iq_z l+1}\Gamma_\gamma. \quad (52)$$

If the integrals over κ_1^2 and κ_2^2 converge for finite masses of the order of μ^2 (large distances), then $ql \sim \mu^2 l/2p \to 0$ and

$$\frac{1}{2pm}F_{\gamma\gamma}{}^{S} = 2\pi i R^2 \frac{1}{\pi^2}\int \frac{d\varkappa_1{}^2\, d\varkappa_2{}^2}{\varkappa_1{}^2\varkappa_2{}^2}\Gamma_\gamma[1 - 2i(q_z l + lq_z) + O(q_z{}^2 l^2)]\Gamma_\gamma, \quad (53)$$

$$F_{\gamma\gamma}{}^{V} = N^2\frac{1}{\pi^2}\int \frac{d\varkappa_1{}^2\, d\varkappa_2{}^2}{\varkappa_1{}^2\varkappa_2{}^2}\Gamma_\gamma\left[-\frac{q_z}{\zeta} + 4(q_z l)^2 f\right]\Gamma_\gamma. \quad (54)$$

Hence, by using the optical theorem we obtain, in analogy to Eqs. (36)–(40), the total cross sections in the form

$$\sigma_\gamma{}^{S} = 2\pi R^2\left[1 + O\left(\frac{\mu^4}{p^2}l^2\right)\right](Z_3{}^{-1} - 1), \quad (55)$$

$$\sigma_\gamma{}^{V} = \frac{N}{2pm\pi^2}\int \frac{d\varkappa_1{}^2\, d\varkappa_2{}^2}{\varkappa_1{}^2\varkappa_2{}^2}\Gamma_\gamma \cdot 4(q_z l)^2 f\Gamma_\gamma \approx N\sigma_\gamma{}^{N} O\left(\frac{\mu^4}{\mu^2}l^2\right), \quad (56)$$

i.e., Eq. (40) holds with good accuracy of the order of $\sigma_\gamma^V/\sigma_\gamma^S = Rl\mu^2/p^2$.

If the integral (40a) for $1 - Z_3$ diverges, i.e., if masses much larger than μ^2 are important, then according to Eq. (51) $F_{\gamma\gamma}^S$ is determined by masses which satisfy the condition $q_z l \lesssim 1$. The contribution from large q_z is small due to the rapid decrease of the expression inside the square brackets. In this connection, if the integral (40a) over κ^2 for $1 - Z_3$ diverges logarithmically, then it is determined by the region $q_z l \ll 1$, and σ_γ^S is given by

$$\sigma_\gamma{}^{S} = 2\pi R^2 \int^{\varkappa_0^2} \frac{d\varkappa^2}{\varkappa^2}\rho(\varkappa^2), \quad \varkappa_0{}^2 \sim \frac{2p}{l}. \quad (57)$$

The volume term on the other hand is determined by the region $ql > 1$ and, in order of magnitude, is given by

$$\sigma_\gamma{}^{V} = \sigma_\gamma{}^{N}(\varkappa_0{}^2)N, \quad (58)$$

where $\sigma_\gamma^N(\kappa_0^2)$ is that part of the cross section for the interaction of a γ quantum with a nucleon which is due, from the viewpoint of the dispersion integral (47), to masses κ_1^2 and κ_2^2 which are larger than κ_0^2. The question of the energy dependence of κ_0^2 was discussed in the Introduction. The assertions made there are obvious from the point of view of formulas (51) and (52).

5. INTERACTION OF ELECTRONS WITH NUCLEI

The interaction of an electron with a nucleus reduces to the interaction of a virtual γ quantum with a nucleus (see the diagram shown in Fig. 4). The differential cross-section for the scattering of an electron accompanied by the creation of an arbitrary number of hadrons may be written in the form

$$d\sigma = \frac{e^2}{kM}\frac{1}{p^4}[K_\mu K_\nu F_{\mu\nu} + p_\nu{}^2 F_{\mu\mu}]\frac{d^3k'}{2k_0'(2\pi)^3}, \quad (59)$$

where k denotes the incoming-electron momentum in the laboratory system, $K_\mu = k_\mu + k'_\mu$, $p_\mu = k'_\mu - k_\mu$ denotes the momentum of the virtual quantum, and M is the mass of the nucleus. The quantity $F_{\mu\nu}$ denotes the imaginary part of the forward scattering amplitude for the virtual γ-quantum. In the preceding sections the quantity $F_{\mu\nu}$ was calculated for a real quantum, i.e., for $p^2 = 0$ and polarization vectors perpendicular to the momentum of the quantum. As is clear from the preceding discussion, a generalization to the case $p^2 \neq 0$ does not present any difficulties since only at the last stage did we assume the photon mass equal to zero. A small difficulty arises only in connection with calculations of the contribution of the longitudinally polarized quanta. This is associated with the fact that expressions (38) and (39) are approximate, and with the fact that the longitudinal polarization vector depends on the energy. One can write expression (39) for $\rho_{\mu\nu}$, written in the laboratory system, in the form

$$\rho_{\mu\nu} = [-\delta_{\mu\nu}M_n{}^2 + p_\mu p_\nu + \alpha_\mu p_\nu + \alpha_\nu p_\mu + \alpha_\mu\alpha_\nu]\rho(M_n{}^2), \quad (60)$$

$$\alpha_\mu = \frac{p^2 - M_n{}^2}{2|\mathbf{p}|}p_\mu{}^A,$$

where p_μ^A is the momentum of the nucleus. This expression is not gauge invariant. At first glance it may appear that only the principal term $p_\mu p_\nu$ has meaning in this expression, and the remaining terms, which are of order $1/|\mathbf{p}|^2$ in comparison with it, should be discarded. This same term does not, in virtue of current conservation, give a contribution to any process, and therefore we have evaluated an uninteresting quantity. In actual fact, if one assumes $j_\lambda = j_\mu e_\mu^\perp$ from the very beginning, as was done above, then these questions do not arise, and the principal term is $\rho_{\lambda\lambda}^\perp = M_n^2\rho(M_n^2)$. The correction terms coming from intermediate states of the other type are of order $1/|\mathbf{p}|$ in comparison with the term written down since no additional energy dependence can arise from $e_\mu^\perp$.

The situation is different with $e_\mu^\parallel$. In this case if $(e_\mu^\parallel e_\mu^\parallel) = -1$ enters in the principal term but the quantity $(e_\mu^\parallel p_\mu^A)^2 p^{-2} \approx 1$ in the correction term, then we obtain a contribution of the same order. This means that the calculation of the longitudinal polarization is not valid. In order to avoid this difficulty, let us write down a general expression for $F_{\mu\nu}$ which satisfies the conditions $p_\mu F_{\mu\nu} = 0$ and $p_\nu F_{\mu\nu} = 0$. It has the form

$$F_{\mu\nu} = -A\left[\delta_{\mu\nu} + \frac{p^2}{(p^A p)^2}p_\mu{}^A p_\nu{}^A - \frac{p_\mu{}^A p_\nu + p_\nu{}^A p_\mu}{(p^A p)}\right] \times B(p_\mu p_\nu - p^2\delta_{\mu\nu}). \quad (61)$$

On the other hand, in analogy to (38) we may write

$$F_{\mu\nu} = 2\pi R^2 i \cdot 2(pp^A) e^2 \int \frac{dM_n^2}{(M_n^2 - p^2)^2} \rho_{\mu\nu}. \qquad (62)$$

Comparing the principal term, which is proportional to $p_\mu p_\nu$, in the right-hand side of expressions (62) and (60) with expression (61) we obtain

$$B = 2\pi R^2 i \cdot 2p_0 M e^2 \int \frac{dM_n^2}{(M_n^2 - p^2)^2} \rho(M_n^2). \qquad (63)$$

Calculating $F_{\mu\nu} e_\mu^\perp e_\nu^\perp$ with the aid of Eqs. (62) and (61) we will have

$$A + Bp_\nu^2 = 2\pi R^2 i \cdot 2p_0 M e^2 \int \frac{M_n^2 dM_n^2}{(M_n^2 - p^2)^2} \rho(M_n^2). \qquad (64)$$

Substituting (63) and (64) into (61) and (59) we obtain formulas (9)–(11) which were mentioned in the Introduction.

In conclusion I wish to express my deep gratitude to I. T. Dyatlov, B. L. Ioffe, L. B. Okun', and K. A. Ter-Martirosyan for numerous helpful discussions.

APPENDIX

Here we derive formula (21) for the amplitude for the scattering of a particle by a nucleus in terms of the amplitude of the hadronic processes per nucleon. Formula (12), which is valid at small energies, is obtained from (21) as a special case.

Let us consider the diagram shown in Fig. 5, and we shall consider the part of the diagram separated by the dashed line as the unique amplitude for the scattering of a hadron by n nucleons, and let us denote this amplitude by $F_{aa}(p, p_1', p_1' + q_1', p_2', p_2' + q_2', \ldots)$.

Assuming that the nucleons in the nucleus are nonrelativistic and introducing the relative momenta of the nucleons

$$p_{i\mu}' = p_{1\mu}/n + k_{i\mu}',$$

it is easy to see (see, for example, [6]) that $k_{i0}' \sim q_{i0}' \sim k_i^2/2m$ is much smaller than the energies entering into $F_{aa}(p, p_1', p_1' + q_1', \ldots)$. In this connection, neglecting k_{i0}' and q_{i0}' in the amplitude F_{aa} we obtain the result that

$$F_{aa}(p, p_1', p_1' + q_1', \ldots) = F(p, q_1', q_2', \ldots, q_{n-1}', pp_A).$$

This makes it possible to integrate over k_{i0}' and q_{i0}' and to write the integral corresponding to the diagram of Fig. 5 in the following nonrelativistic form:

$$F_{aa}^{(n)} = \frac{1}{n!(N-n)!} \int \frac{\Gamma(k_i') F_{aa}(p, q_i', p_\mu p_{A\mu}) \Gamma(k_i' + q_i')}{D_1 D_2} \cdot \prod_{i=1}^{N-1} \frac{d^3 k_i'}{(2\pi)^3 2m} \prod_{i=1}^{n-1} \frac{d^3 q_i'}{(2\pi)^3 2m}. \qquad (A.1)$$

Here

$$D_1 = N\Delta^2 + \Big(\sum_i k_i'\Big)^2 + \sum_i k_i'^2, \quad D_2 = N\Delta^2 + \Big(\sum_i (k_i' + q_i')\Big)^2 + \sum_i (k_i' + q_i')^2.$$

$$\Delta^2 = m^2 - M^2/N^2.$$

Or, introducing the wave function of the nucleons in the nucleus in the coordinate representation, we have

$$\frac{\Gamma(k_i')}{D_1} = \sqrt{N!(2m)^{N-1}} \int \exp(i k_i r_i) \psi(r_1, \ldots, r_{N-1}) dV_1 \ldots dV_{N-1}. \qquad (A.2)$$

For such a choice of the factor in (A.2) we obtain

$$\int |\psi(r_1, \ldots, r_{N-1})|^2 dV_1 \ldots dV_{N-1} = N. \qquad (A.2a)$$

Hence for $n \ll N$

$$F_{aa}^{(n)} = \frac{N^n}{n!} \frac{1}{(2m)^{n-1}} \int \chi(r_1, \ldots, r_n) \times \exp(i q_i' r_i) F_{aa}(p, q_i', pp_A) \prod_1^n dV_i \prod_1^{n-1} \frac{d^3 q_i'}{(2\pi)^3}, \qquad (A.3)$$

$$\chi(r_1, \ldots, r_n) = \int \psi^2(r_1, \ldots, r_{N-1}) dV_{n+1} \ldots dV_{N-1}.$$

Let us represent the vectors q_i' in the form $q_i' = q_{i\perp}' + q_{iz}'$, where $q_{i\perp}'$ lies in a plane perpendicular to the momentum of the incoming particle in the laboratory system, and similarly $r_i = \rho_{i\perp} + z_i$. Then the integral

$$\int \exp(i q_{i\perp}' \rho_{i\perp}) \chi(r_1, \ldots, r_n) d^2\rho_1 \ldots d^2\rho_{n-1}$$

will change substantially during a change of $q_{i\perp}'$ by an amount of the order of $1/R_n$, where R_n is the average distance between the nucleons $r_1, \ldots, r_N$. We assume that the mean free path inside the nucleus is larger than the transverse distances which are important in the strong interactions at high energies. Since R_n is of the order of the mean free path, then $1/R_n$ is significantly smaller than the transverse momenta which are important in the strong interactions and which enter into the amplitude $F_{aa}(p, q_i', pp_A)$. This means that we may carry out the integration over $q_{i\perp}'$, having set $q_{i\perp}' = 0$ in $F_{aa}(p, q_i', pp_A)$. Then integration over $d_2 q_{i\perp}'$ gives $\delta(\rho_1 - \rho_2)\delta(\rho_2 - \rho_3) \ldots$ and

$$F_{aa}^{(n)} = \frac{N^n}{n!} \frac{1}{(2m)^{n-1}} \int d^2\rho_1 dz_1 \ldots dz_n \chi(\rho_1, z_1, \ldots, z_n) \times \exp(i q_{zi}' z_i) F_{aa}(p, q_{zi}', pp_A) \prod_{i=1}^{n-1} \frac{dq_{zi}}{2\pi}. \qquad (A.4)$$

Let us go on to the most important integration over z_i. Since $\chi(\rho_1, z_1, \ldots, z_n)$ is symmetric with respect to $z_1, \ldots, z_n$ (taking the difference between neutrons and protons into account does not change the result), we may omit the $1/n!$ in front of the integral (A.4) and assume

$$z_1 < z_2 < z_3 \ldots < z_{n-1}. \qquad (A.5)$$

One can write the expression $\sum_i q_{iz}' z_i$ in the form

$$\sum_i q_{iz}' z_i = q_{1z}(z_1 - z_2) + q_{2z}(z_2 - z_3) + \ldots + q_{n-1,z}(z_{n-1} - z_n), \qquad (A.6)$$

where $q_{1z} = q_{1z}'$, $q_{2z} = q_{2z}' + q_{1z}'$, $q_{3z} = q_{1z}' + q_{2z}' + q_{3z}'$, ...

We note that

$$s_1 = (p - q_1')_\mu^2 = (p - q_1)_\mu^2 \approx p_\mu^2 + 2pq_{z1}$$

is equal to the square of the mass of the intermediate state which appears after scattering by the first nucleon,

$$s_2 = (p - q_1' - q_2')_\mu^2 = (p - q_2)_\mu^2 \approx p_\mu^2 + 2pq_{z2}$$

is the square of the mass of the intermediate state which appears after scattering by two nucleons, etc.

integration over q_{z1}, q_{z2} ... is integration over masses s_1, s_2, ... of the intermediate states. These integrations have the usual Feynman character. Since $\exp[iq_{zi}(z_i - z_{i+1})]$ decreases in the lower half-plane, one can close the contour of integration over s_i in the lower half-plane and reduce the integral of $F_{aa}(\ )$ to an integral of the absorptive part, i.e., to an integration over real intermediate states, and one can substitute $q_{zi} = (s_i - p^2)/2p$.

As a result one can write $F_{aa}^{(n)}$ in the form

$$F_{aa}^{(n)} = \frac{N^n i^{n-1}}{(4pm)^{n-1}} \int d^2\rho_1\, dz_1 \ldots dz_{n-1} \sum_{b,c,d\ldots} f_{ab} \exp[iq_z^b(z_1 - z_2)] f_{bc} \times \exp[iq_z^c(z_2 - z_3)] \ldots \exp[iq_z^d(z_{n-1} - z_n)] f_{ea} \chi(\rho_1, z_1, \ldots, z_n), \quad \text{(A.7)}$$

where the sum over b, c, and d is a summation over the possible intermediate states, and f_{bc} denotes the amplitude for the conversion of the group of particles b into the group of particles c by a nucleon (see Fig. 6).

In contrast to the amplitudes for the interaction of individual particles (not groups of particles) which are usually considered, these amplitudes are not matrix elements of the S-matrix. In fact, when we calculated the absorptive part, for example, with respect to the variable s_3, it was determined by the product $f_{ac}f_{ca}^*$; in other words, the contribution of real intermediate states to f_{ac} is determined in the usual way by the substitution $s_3 \to s_3 + i\epsilon$ and in f_{ca}^* by the substitution $s_3 \to s_3 - i\epsilon$. If after this one evaluates the absorptive part with respect to s_2, then f_{ac} goes over into $f_{ab}f_{bc}^*$, where f_{bc}^* is determined by the replacement of s_2 by $s_2 - i\epsilon$. Thus, f_{bc}^* is determined as $f_{bc}(s_2 - i\epsilon, s_3 + i\epsilon)$. At first glance it may appear that the introduction of such quantities may lead to difficulties. In actual fact this is not so, and quantities of this type represent a natural generalization of the ordinary amplitudes to a case involving the interaction of groups of particles. One can verify this if we represent the amplitude $F_{aa}^{(n)}$ in the form of an integral of the T-product of the nucleon operators:

$$\langle a | TA(x_1, x_1')A(x_2, x_2') \ldots A(x_n, x_n') | a \rangle,$$

where $A(x_1, x_1') \sim \bar{\psi}(x_1)\psi(x_1')$, and if we decompose the product with respect to the intermediate states

$$\langle a | A(x_1, x_1') | n \rangle \langle n | A(x_2, x_2') | m \rangle \ldots$$

The amplitudes $\langle n | A(x_2, x_2') | m \rangle$ are not matrix elements of the S-matrix provided none of the states $\langle n |$, $| m \rangle$ are single-particle states, and they coincide with the quantities discussed above.

In order to complete the final step in order to obtain formula (21), we shall utilize the assumption that the mean free path is large in comparison with the distance between nucleons. This means that the points r_1, r_2, and r_3 are located, on the average, in the integral (A.4) at distances from each other which exceed the distance between particles, and therefore under condition (A.5) one can confine one's attention to only the correlations of the nearest nucleons, i.e., one can write

$$\chi(\rho_1, z_1, z_1 - z_2, z_2 - z_3, \ldots, z_{n-1} - z_n) = \varphi(\rho_1, z_1)\chi(z_1 - z_2)\chi(z_2 - z_3) \ldots \chi(z_{n-1} - z_n), \quad \text{(A.8)}$$

$$\chi(z_{i-1} - z_i) \to 1 \text{ for } z_i - z_{i-1} \to \infty,$$

where $\varphi(\rho_1, z_1)$ is constant inside the nucleus and equal to zero outside of it. By virtue of the normalization condition (A.2a)

$$\varphi(\rho_1, z_1) = N / V^n, \quad \text{(A.9)}$$

where V is the volume of the nucleus. Substituting (A.8) and (A.9) into (A.7), we obtain formulas (21) which were used in the text.

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