2017 International Summer
Workshop on Reaction Theory

Effects Beyond the Born Approximation for the Elastic Scattering of Leptons by Nuclei

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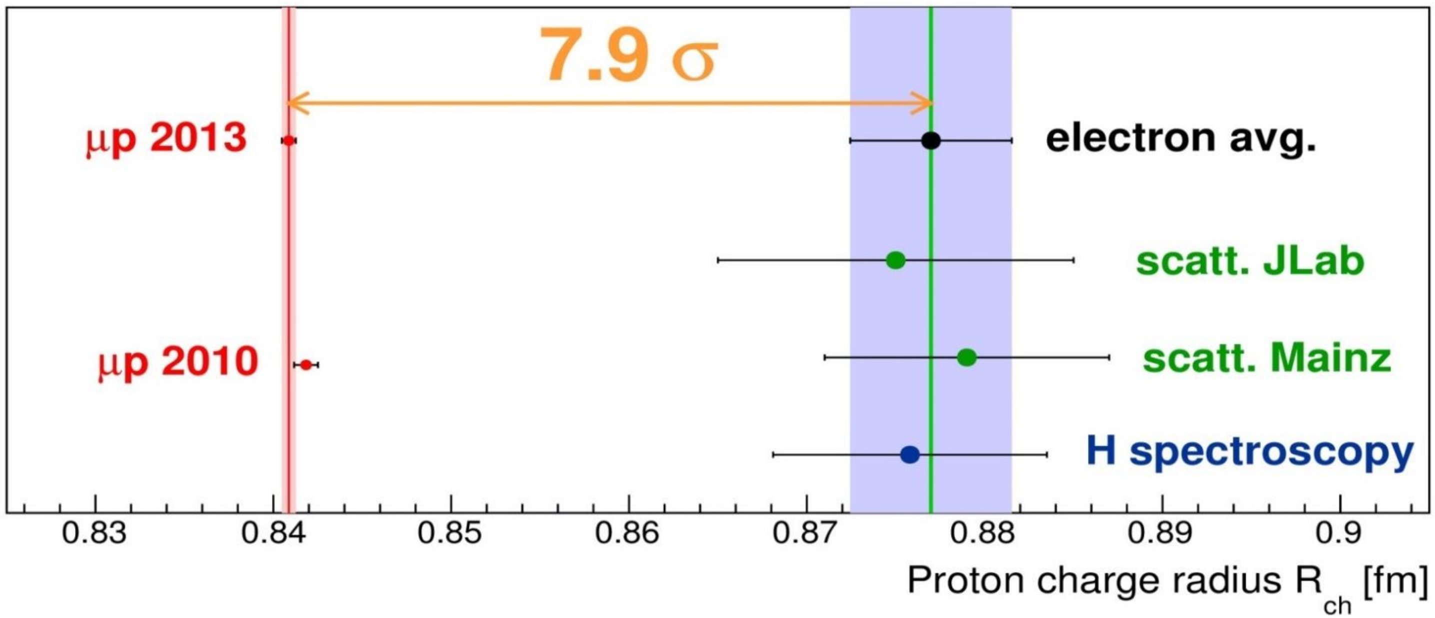
Outline

- The Proton Radius Puzzle and MUSE experiment
- Theoretical background on the elastic lp scattering
- Elradgen: MC generator for MUSE
- Recent update to Elradgen: influence of lepton mass on charge asymmetry contribution
- Helicity-flip transitions in MUSE: σ-meson exchange in the t-channel
- Conclusion

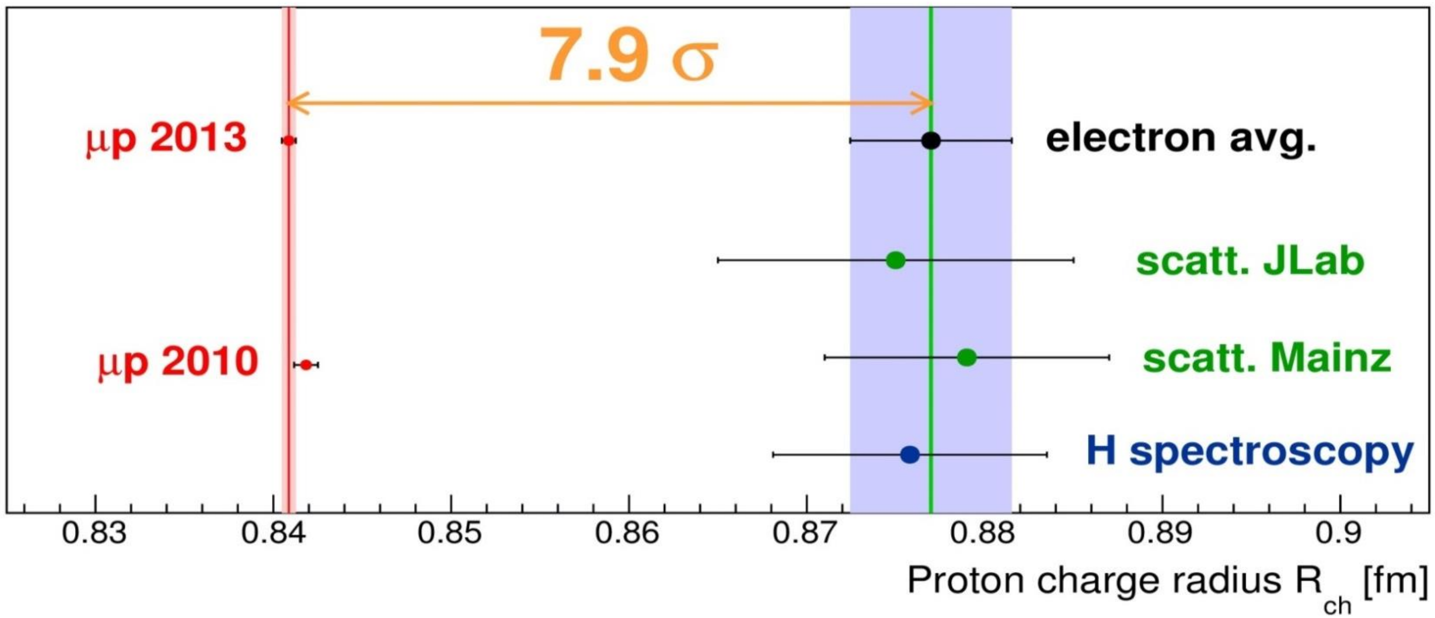
The Proton Radius Puzzle



[https://www.psi.ch/muonic-atoms/]

	Muon	Electron
Spectroscopy	0.8409(4)	0.8758(77)
Scattering	???	0.8770(60)

The Proton Radius Puzzle



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Muon Scattering Experiment (MUSE) at PSI, Switzerland

MUSE at PSI

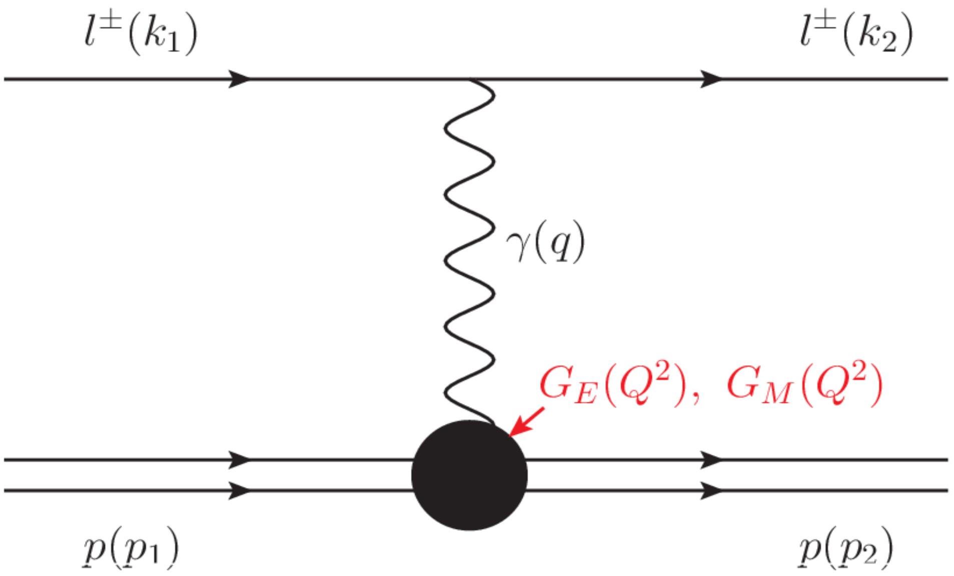
- Will measure simultaneously elastic $e^{\pm}p$ and $\mu^{\pm}p$ scattering:

  - Direct Access to TPE Corrections
  - Test Lepton Universality

- First signicant μp scattering radius determination, at roughly the same level as done in previous scattering experiments:

  - Theoretical estimations beyond the Born approximation are required (ultrarelativistic limit ($\varepsilon \gg m$) cannot be used for scattering of muons!)

Theoretical Background: Born Approximation



Lab Frame :

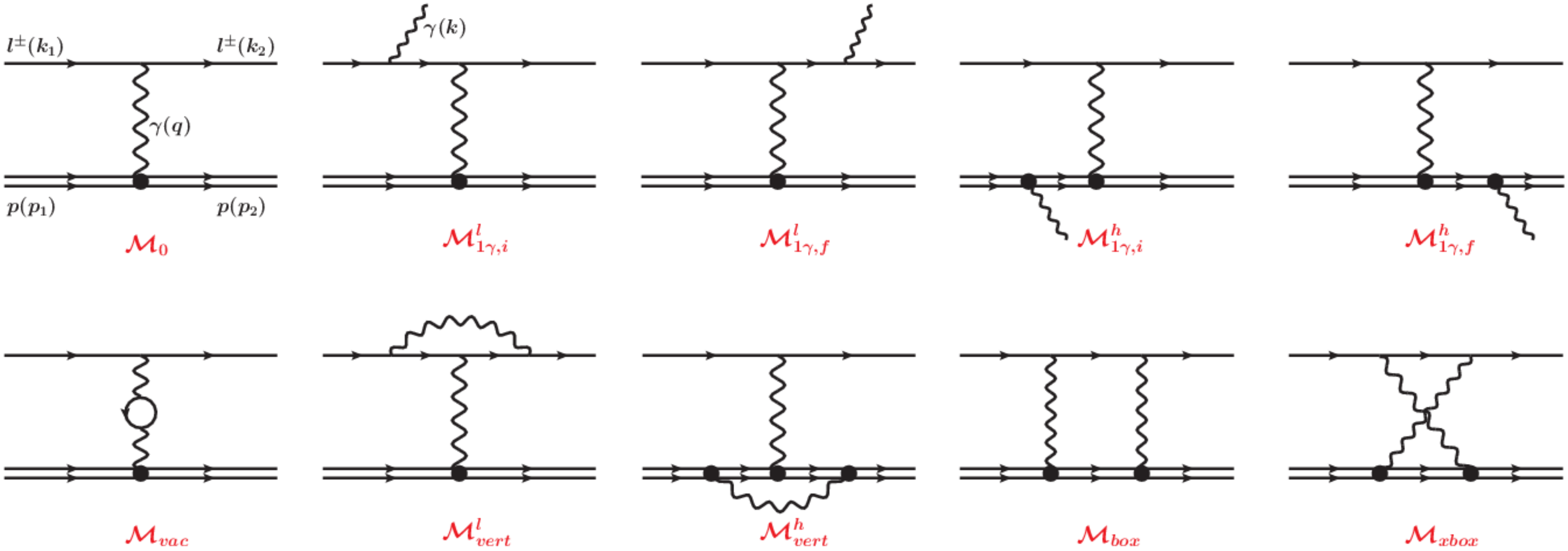
$$k_1 = (\varepsilon_1, \vec{k}_1),$$
$$k_2 = (\varepsilon_2, \vec{k}_2),$$
$$p_1 = (M, 0),$$
$$p_2 = (E_2, \vec{p}_2),$$
$$Q^2 = -q^2 = -(k_1 - k_2)^2 > 0.$$

$G_E(Q^2),\ G_M(Q^2)$ → **Electric and Magnetic form factors**

Rosenbluth separation: $$\frac{d\sigma}{d\Omega} \propto G_E^2(Q^2) + \frac{\tau}{\in} G_M^2(Q^2)$$

Charge radius definition: $$\langle r^2 \rangle = -6 \frac{dG_E(Q^2)}{dQ^2}\Big|_{Q^2=0}$$

Theoretical Background: Higher Order Corrections



$$|M|^2 = \sum_i |M_i|^2 = |M_0|^2 (1+\delta)$$

$$\sigma^{\exp} = \sigma_0 (1+\delta)$$

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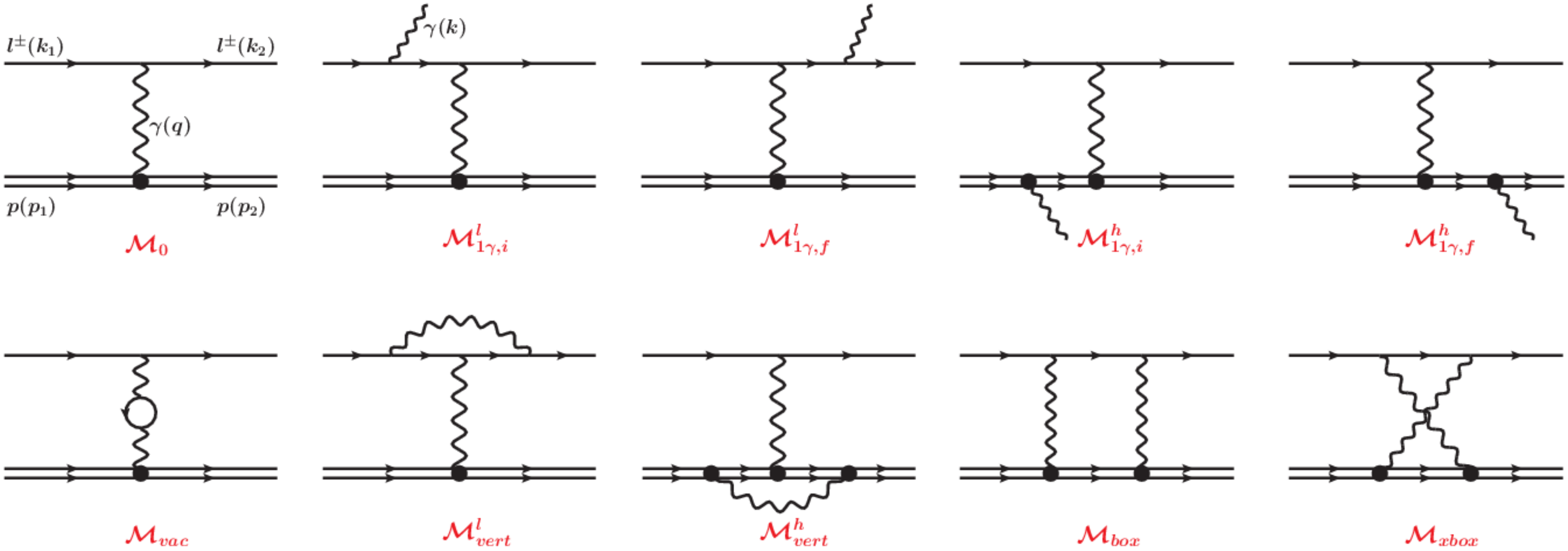

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Need to know this value to extract the radius!

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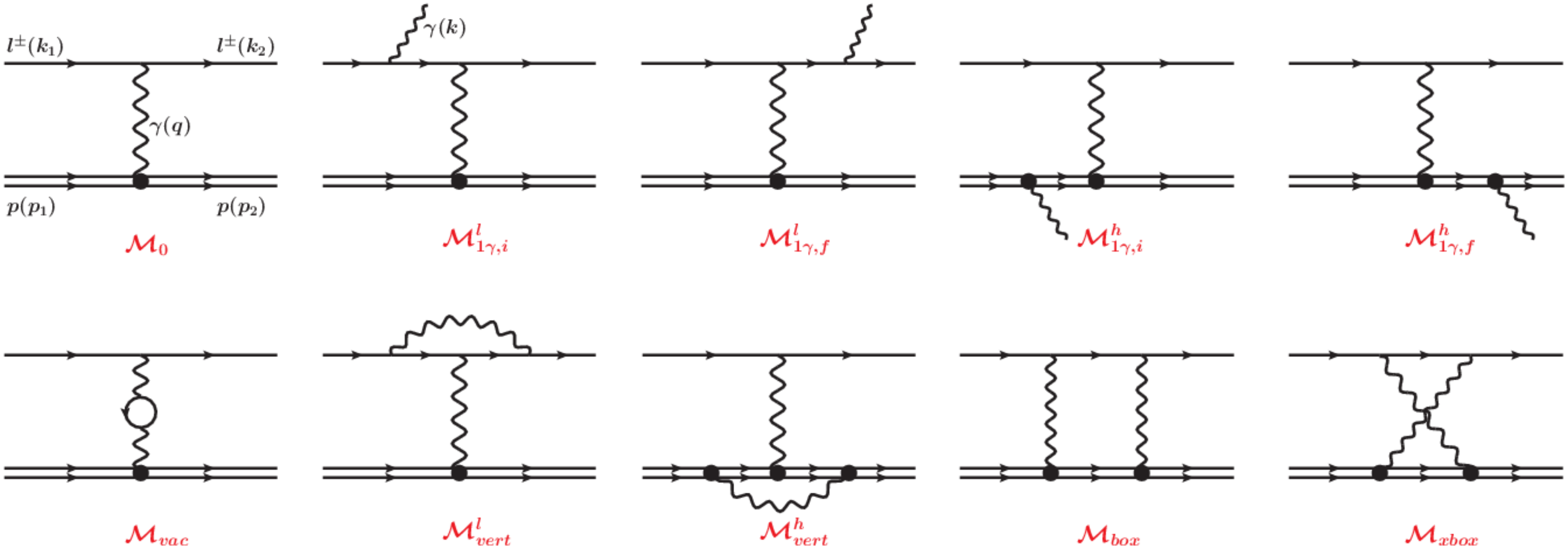

$$|M|^2 = \sum_i |M_i|^2 = |M_0|^2 (1+\delta)$$

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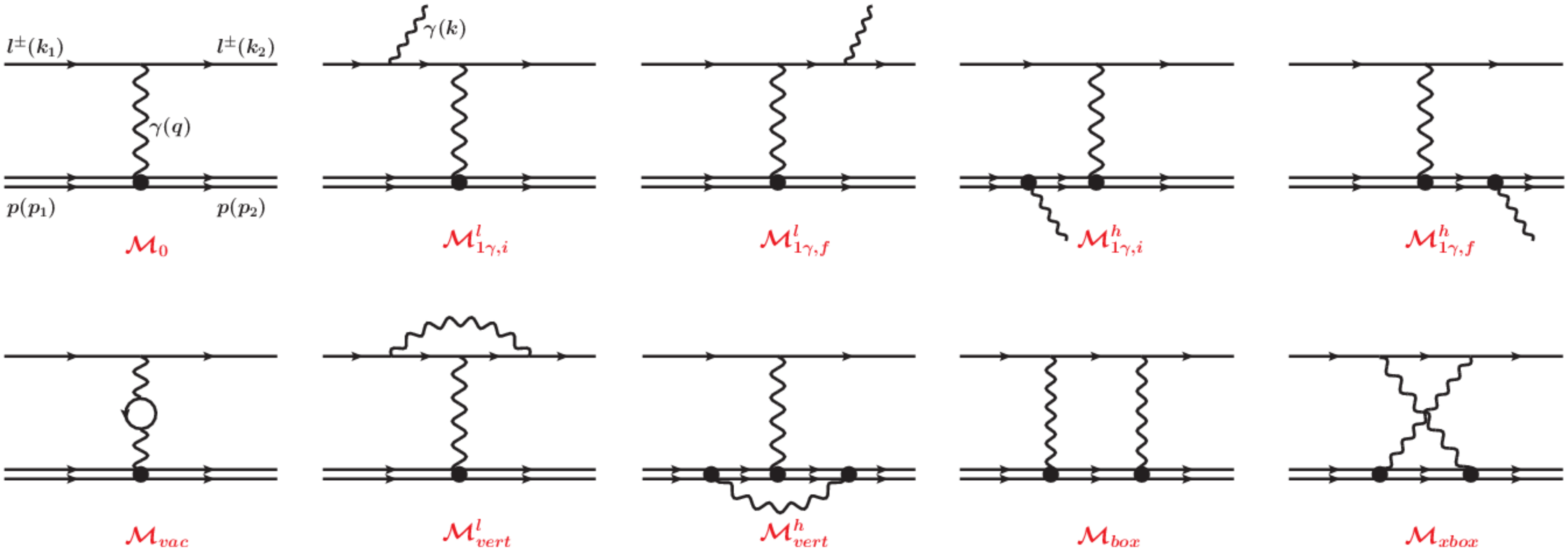
Need estimations of δ!

Born and Higher Order Relevant Diagrams

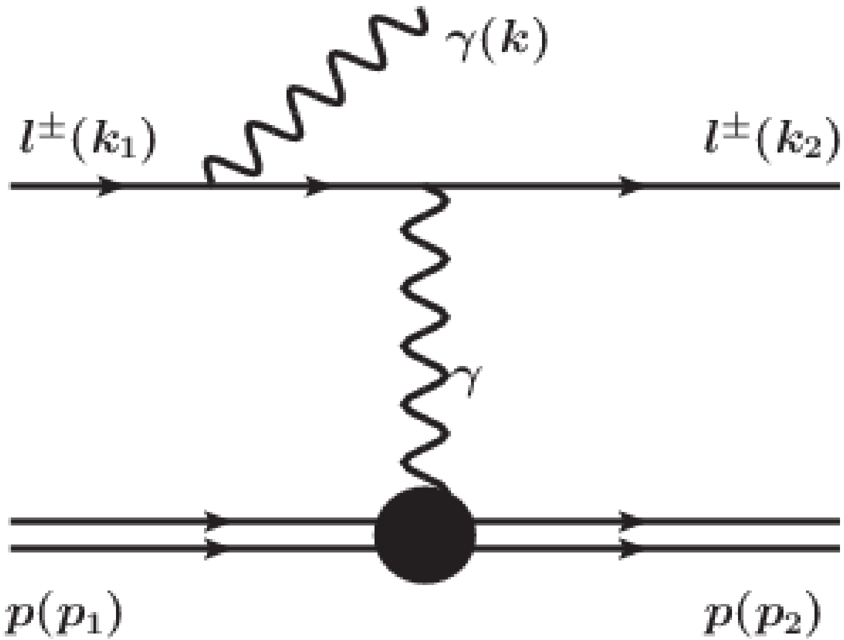


- Leading and next-to-leading order contributions:

$$|M|^2 = |M_0|^2 + 2\,\mathrm{Re}\left[M_0^* M_{vac}\right] + 2\,\mathrm{Re}\left[\left(M_{1\gamma}^l\right)^* M_{1\gamma}^h\right] + 2\,\mathrm{Re}\left[M_0^* M_{2\gamma}\right]$$

$$+\left|M_{1\gamma}^l\right|^2 + 2\,\mathrm{Re}\left[M_0^* M_{vert}^l\right] + \left|M_{1\gamma}^h\right|^2 + 2\,\mathrm{Re}\left[M_0^* M_{vert}^h\right] + O\left(\alpha^4\right)$$

How We Calculate Bremsstrahlung



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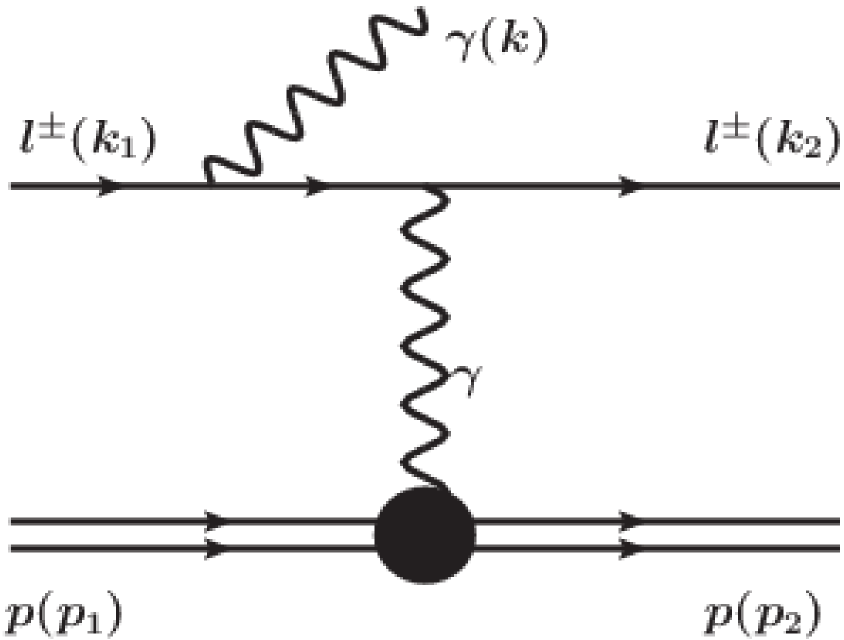

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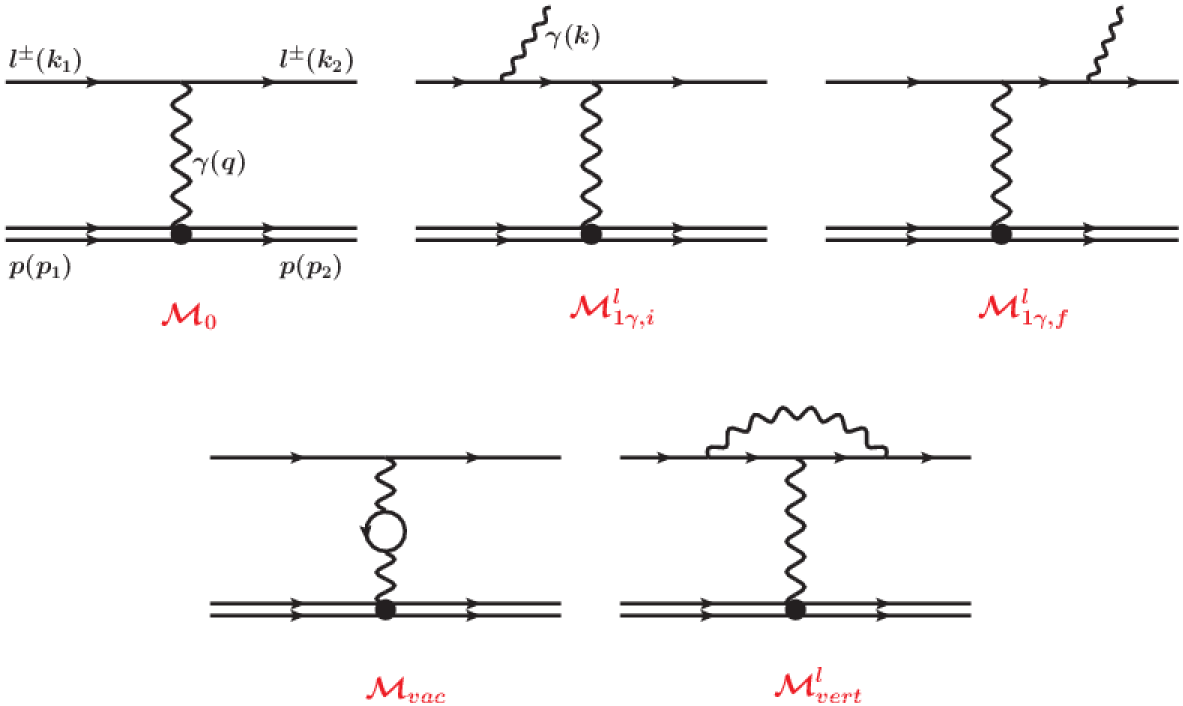
Need to account for hard photon radiation!

Use Elradgen!

[Akushevich et.al. Comp.Phys.Comm, 2012]

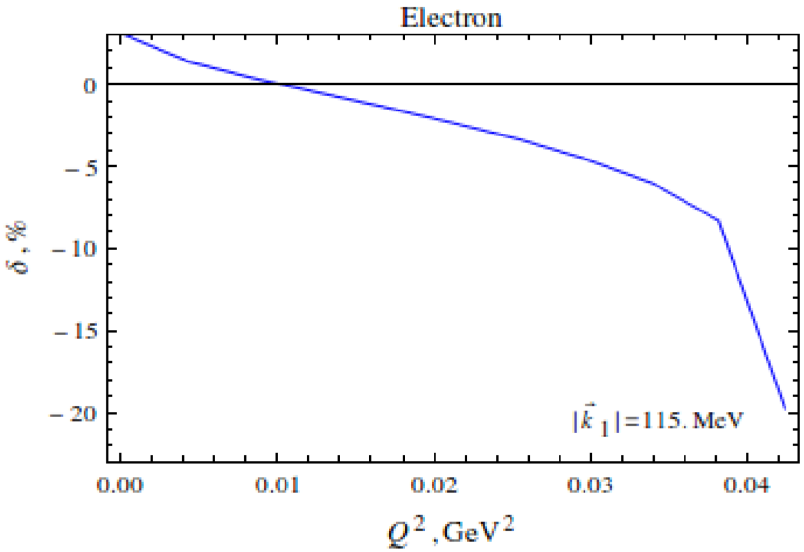
Why Elradgen 2.1?

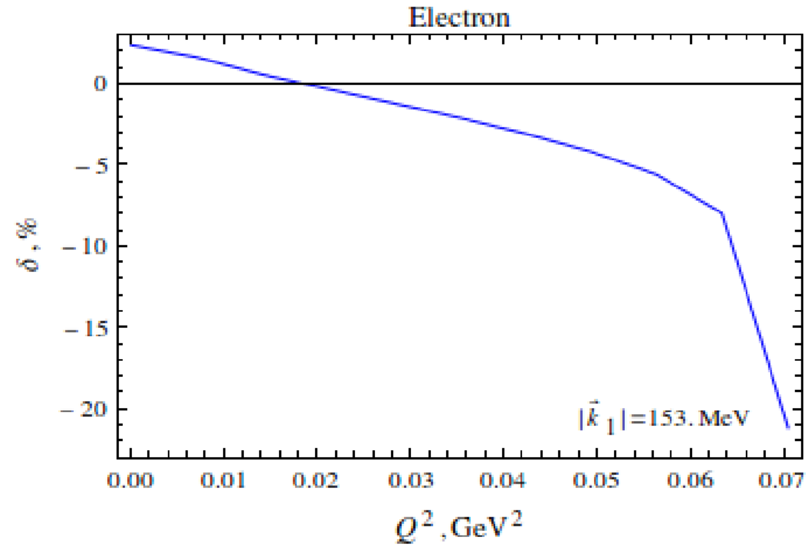
- Takes into account the mass of the lepton

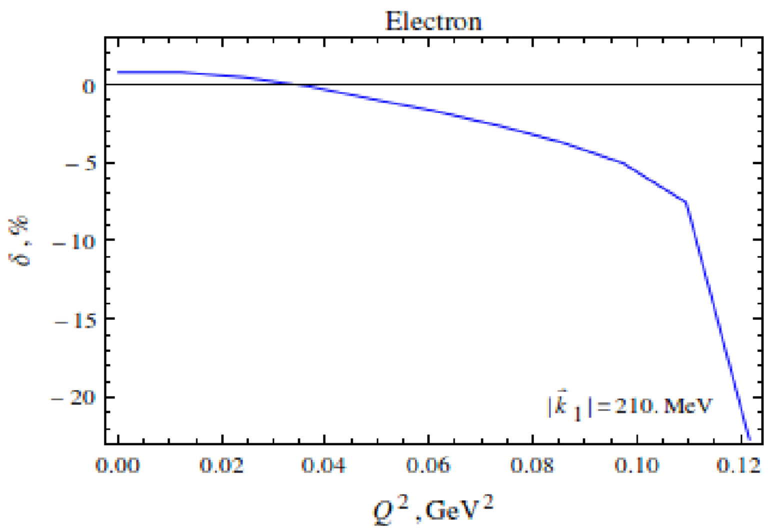


$$|M|^2 = \sum_i |M_i|^2 = |M_0|^2 (1+\delta)$$

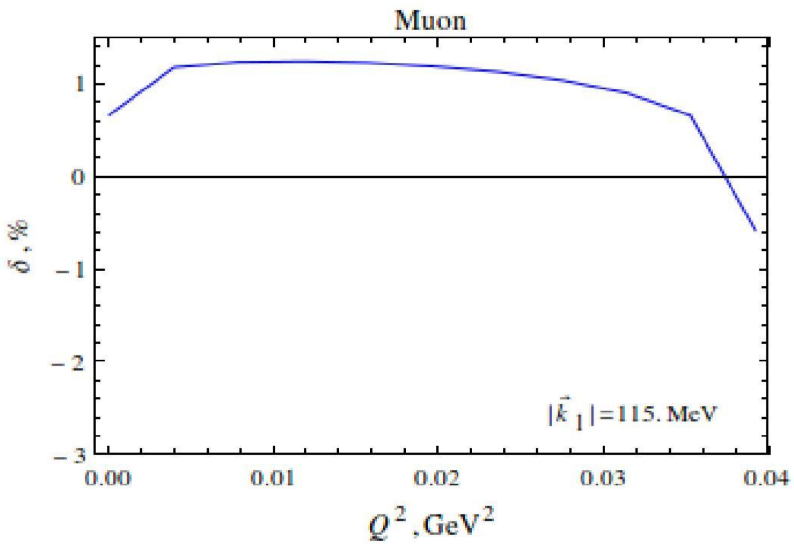
Radiative Corrections: Electron

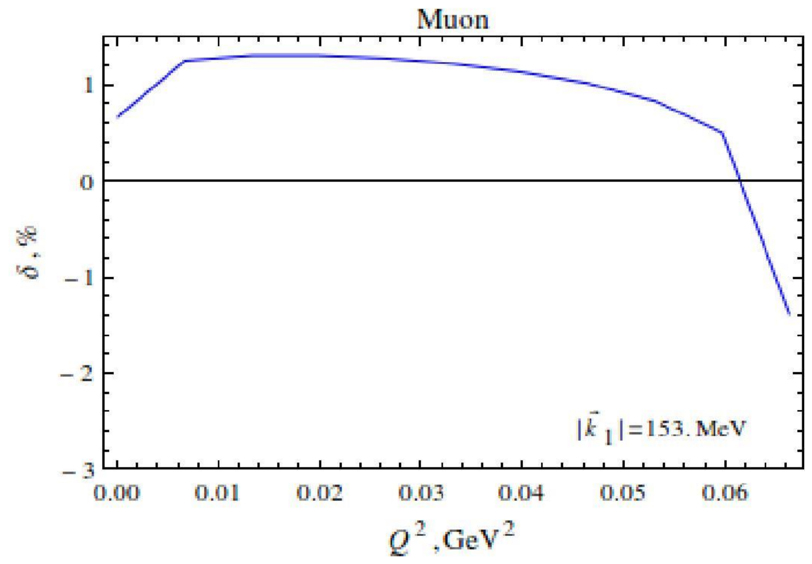


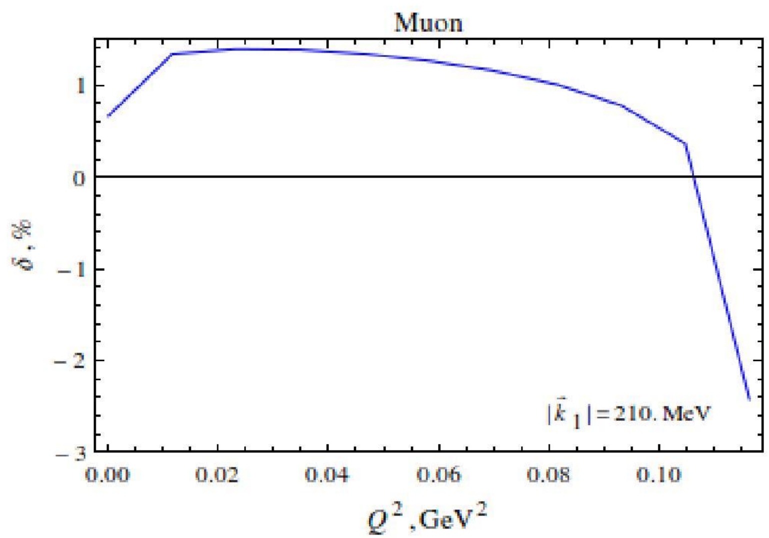




Radiative Corrections: Muon







Extra contributions to include

- Terms of our interest:

$$|M|^2 = |M_0|^2 + 2\,\mathrm{Re}\left[M_0^* M_{vac}\right] + \left|M_{1\gamma}^l\right|^2 + 2\,\mathrm{Re}\left[M_0^* M_{vert}^l\right]$$
$$+\, 2\,\mathrm{Re}\left[\left(M_{1\gamma}^l\right)^* M_{1\gamma}^h\right] + 2\,\mathrm{Re}\left[M_0^* M_{2\gamma}\right]$$

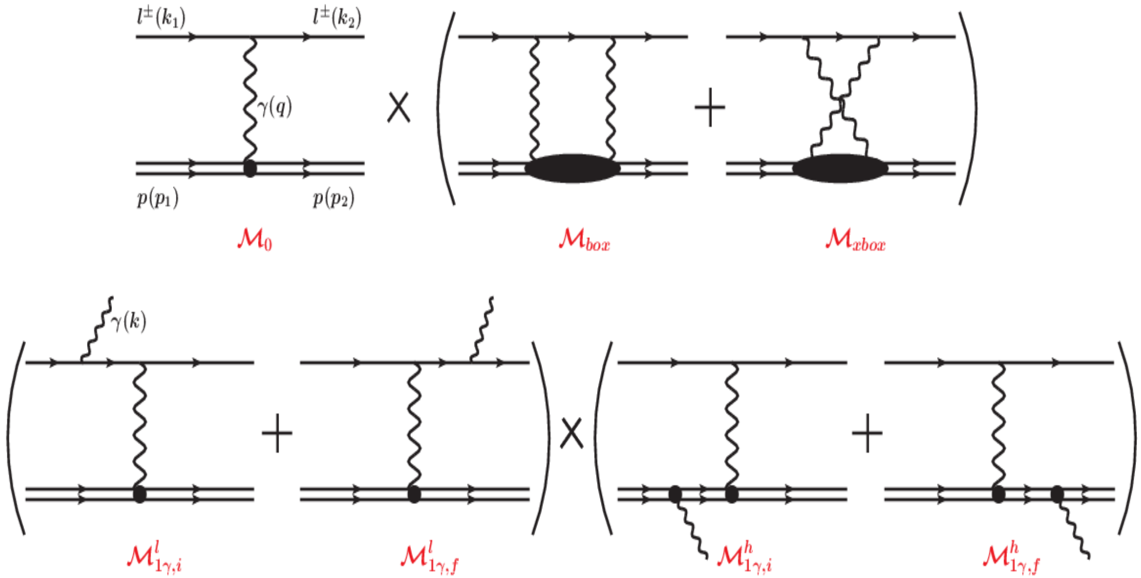
- Elradgen 2.1 terms:

$$|M_0|^2 + 2\,\mathrm{Re}\left[M_0^* M_{vac}\right] + \left|M_{1\gamma}^l\right|^2 + 2\,\mathrm{Re}\left[M_0^* M_{vert}^l\right]$$

- Recent calculation:

$$2\,\mathrm{Re}\left[\left(M_{1\gamma}^l\right)^* M_{1\gamma}^h\right] + 2\,\mathrm{Re}\left[M_0^* M_{2\gamma}\right]$$

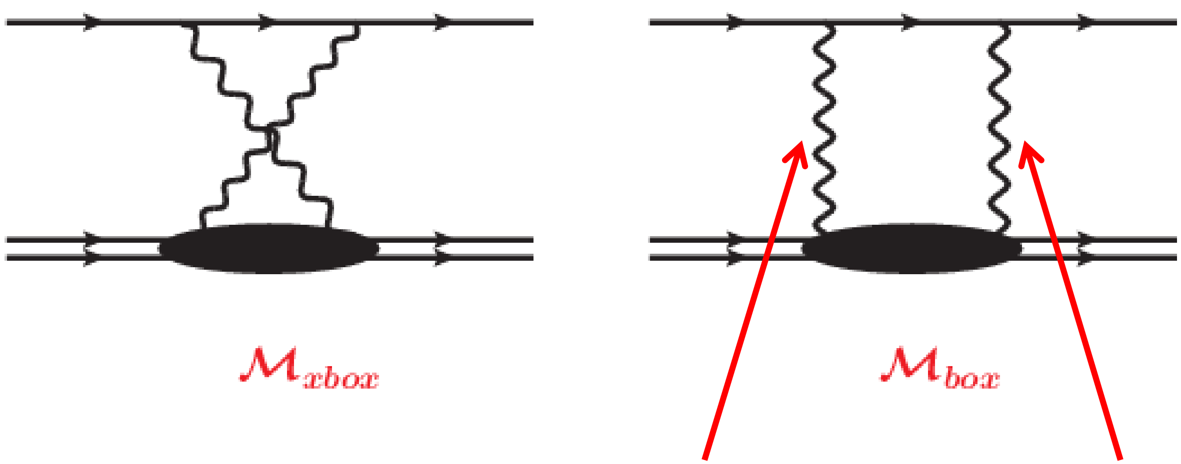
My contribution to Elradgen



- Pros: the only charge-dependent contribution to order α^3 $\rightarrow$ <u>direct access in MUSE!</u>

- Cons: various intermediate hadronic states in the TPE loop are possible

Model-independent TPE calculation



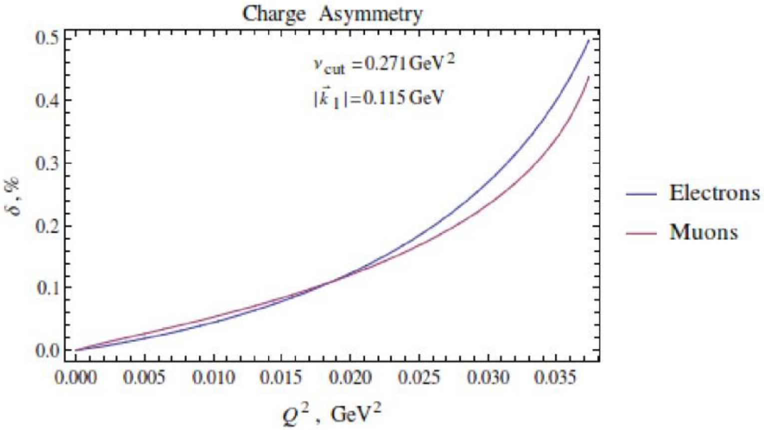
Soft Photon Approximation: $q_1 \to q$ $q_2 \to 0$

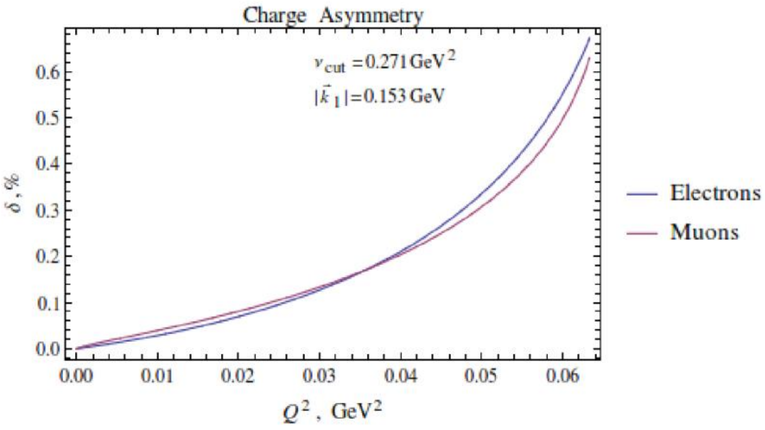
Two Approaches:

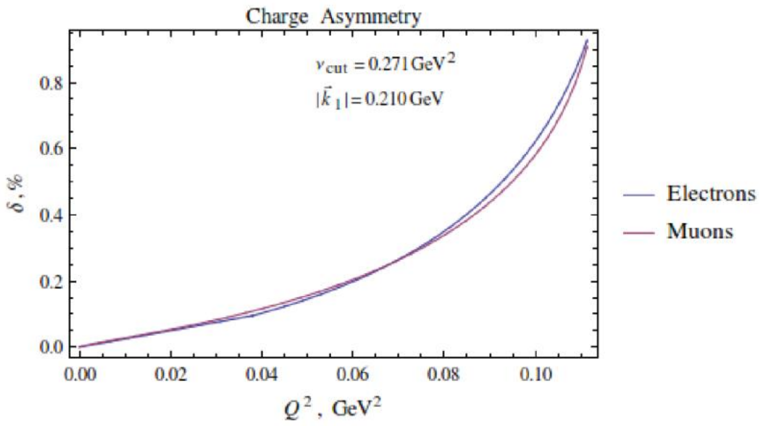
1. [Yung-Su Tsai, Phys Rev 1961]

2. [Maximon, Tjon, Phys Rev C 2000]

Asymmetry Comparison

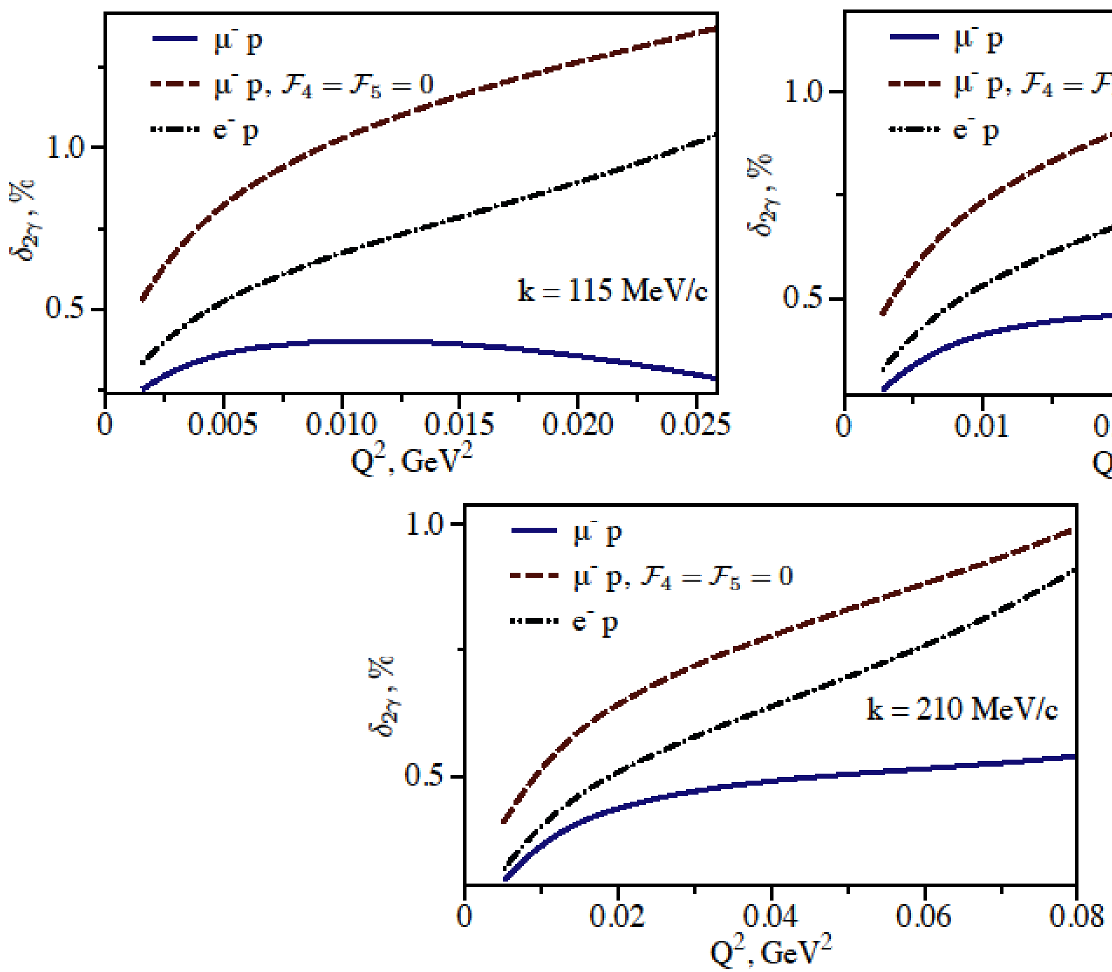






[Koshchii, Afanasev arXiv:1705.00338]

Alternative Calculation

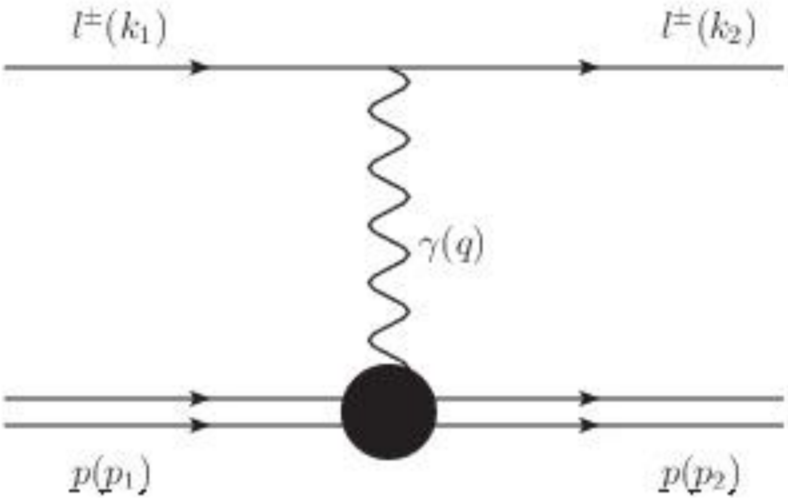


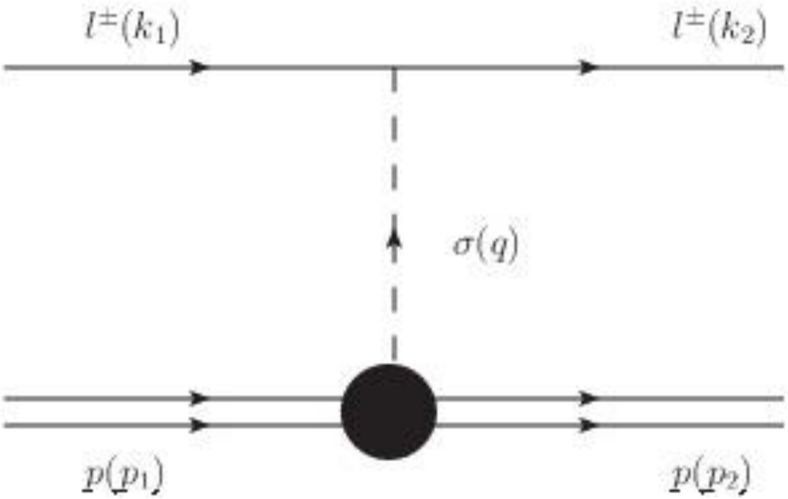
[Tomalak, Vangerhaeghen Phys Rev D 2014]

Extra contribution to be considered: helicity-flip transitions

σ-meson exchange in t-channel

Consider the interference between following diagrams:





$$j_\mu^v = \bar{u}(k_2)\gamma_\mu u(k_1)$$

$$J_\mu^v = \bar{U}(p_2)\left(\gamma_\mu F_1(Q^2) + \frac{i\sigma_{\mu\nu}q_\nu}{2M}F_2(Q^2)\right)U(p_1)$$

$$j_\mu^s = f_s\ \bar{u}(k_2)u(k_1)$$

$$J_\mu^s = \bar{U}(p_2)U(p_1)$$

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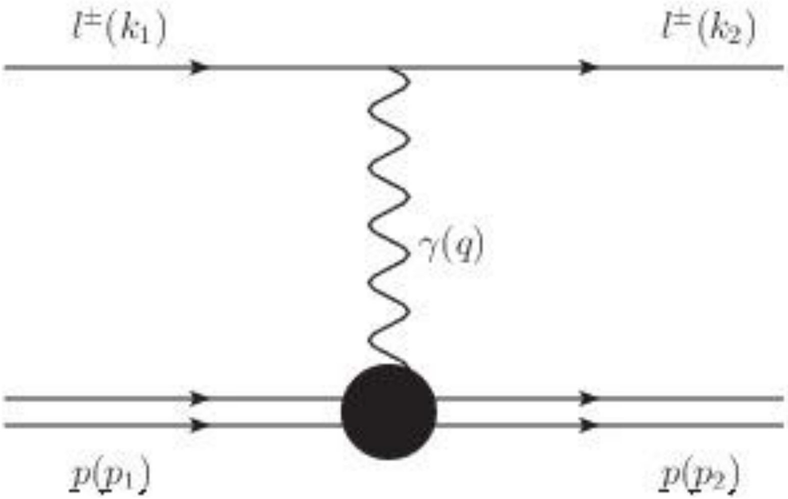

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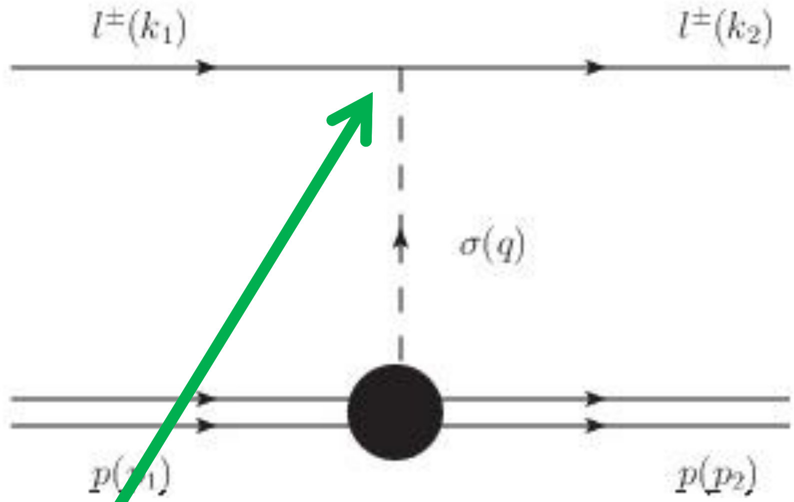

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$$j_\mu^s = f_s\ \bar{u}(k_2)u(k_1)$$

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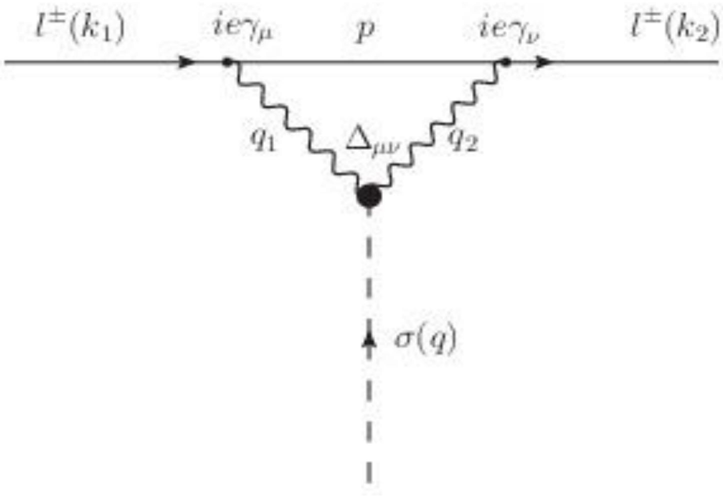
The coupling of σ to lepton is described via form factor f_s

Model to calculate f_s

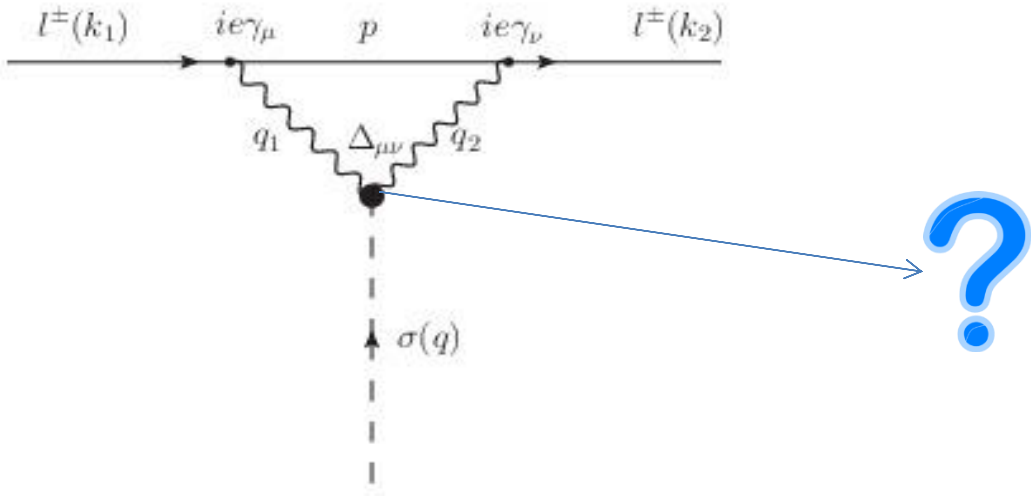


Corresponding amplitude

$$T = ie^4 \int \frac{d^4 p}{(2\pi)^4}\ \bar{u}(k_2)\ \frac{\gamma_\nu (\not{p} + m)\gamma_\nu}{p^2 - m^2}\ u(k_1) \frac{1}{q_1^2} \Delta_{\mu\nu} \frac{1}{q_2^2}$$

Everything that is sandwiched between spinors is the form factor!

Model to calculate f_s



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Everything that is sandwiched between spinors is the form factor!

Vertex description

The most general form to describe the vertex:

$$\Delta_{\mu\nu} = A_s(q^2;q_1^2,q_2^2)\left(g_{\mu\nu}(q_1\cdot q_2)-q_1^{\nu}q_2^{\mu}\right)+B_s(q^2;q_1^2,q_2^2)\left(q_1^2q_2^{\mu}-(q_1\cdot q_2)q_1^{\mu}\right)\left(q_2^2q_1^{\nu}-(q_1\cdot q_2)q_2^{\nu}\right)$$

[A.E. Dorokhov et. al. Eur. Phys. J. C (2012)]

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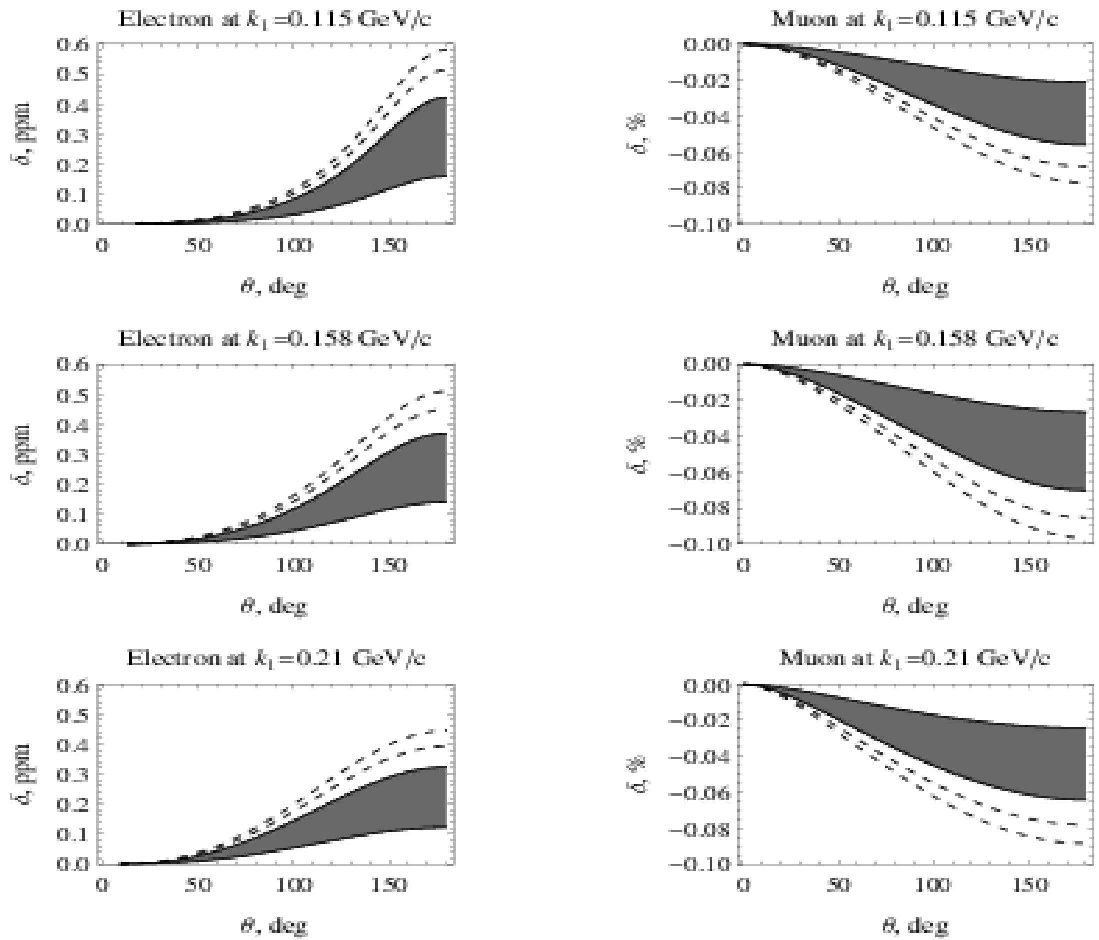
[A.E. Dorokhov et. al. Eur. Phys. J. C (2012)]

Vector meson dominance (VMD) model for transverse photons:

$$A_s(q^2;q_1^2,q_2^2) = \frac{g_{\sigma\gamma\gamma}\ m_\rho^4}{\left(m_\rho^2 - q_1^2\right)\left(m_\rho^2 - q_2^2\right)}$$

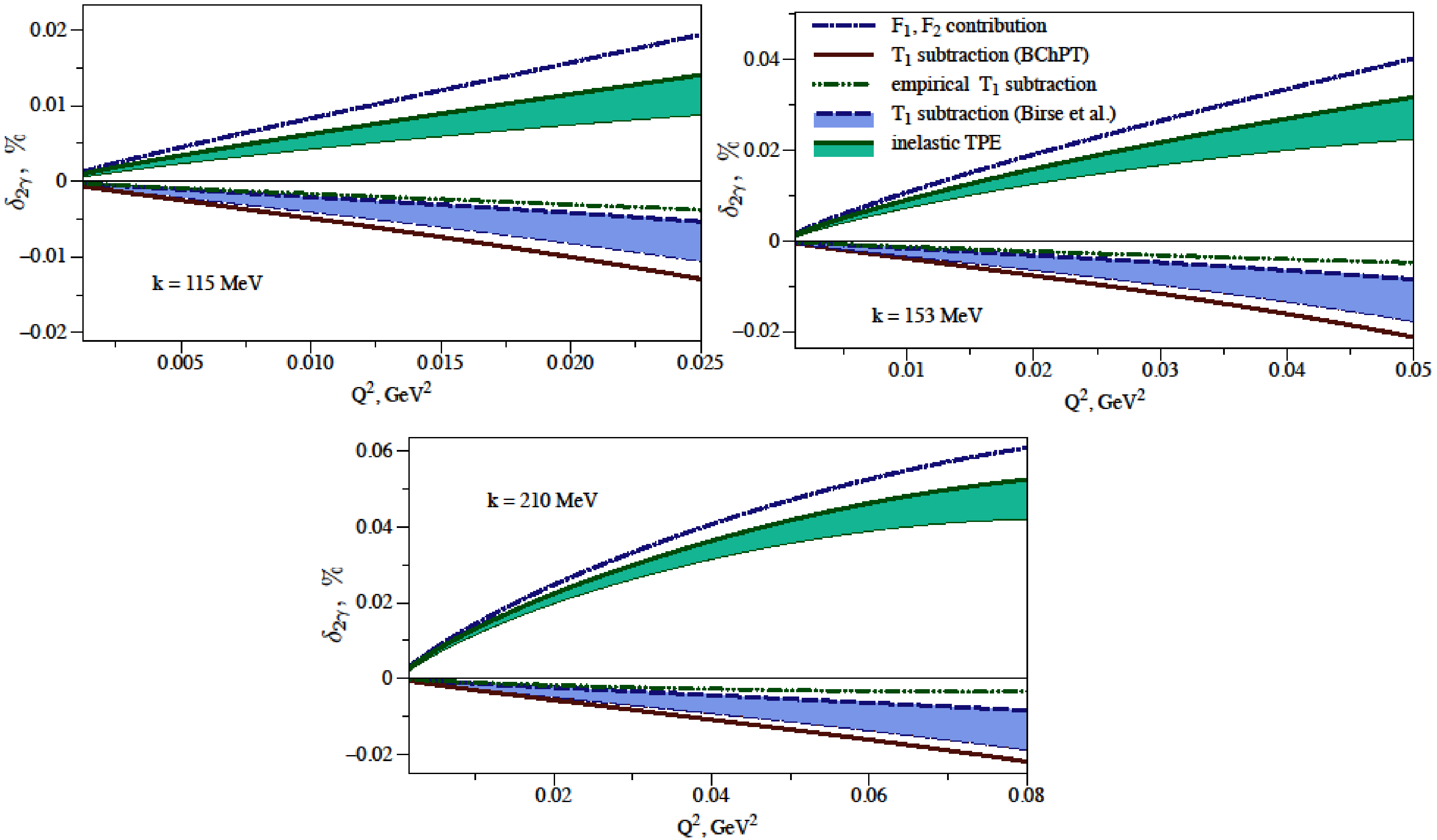
Obtained experimentally

Results



[Koshchii, Afanasev PRD, 2016]

Other Estimations



[Tomalak, Vanderhaeghen EPJC, 2016]

Conclusion

- Monte Carlo generator Elradgen 2.1 was developed to include mass effects in elastic $l^{\pm}p$ scattering
- Charge asymmetry contributions were recently added to Elradgen
- The estimates of major helicity-flip contribution were performed

Thank you!