Light hadrons at e+e- colliders

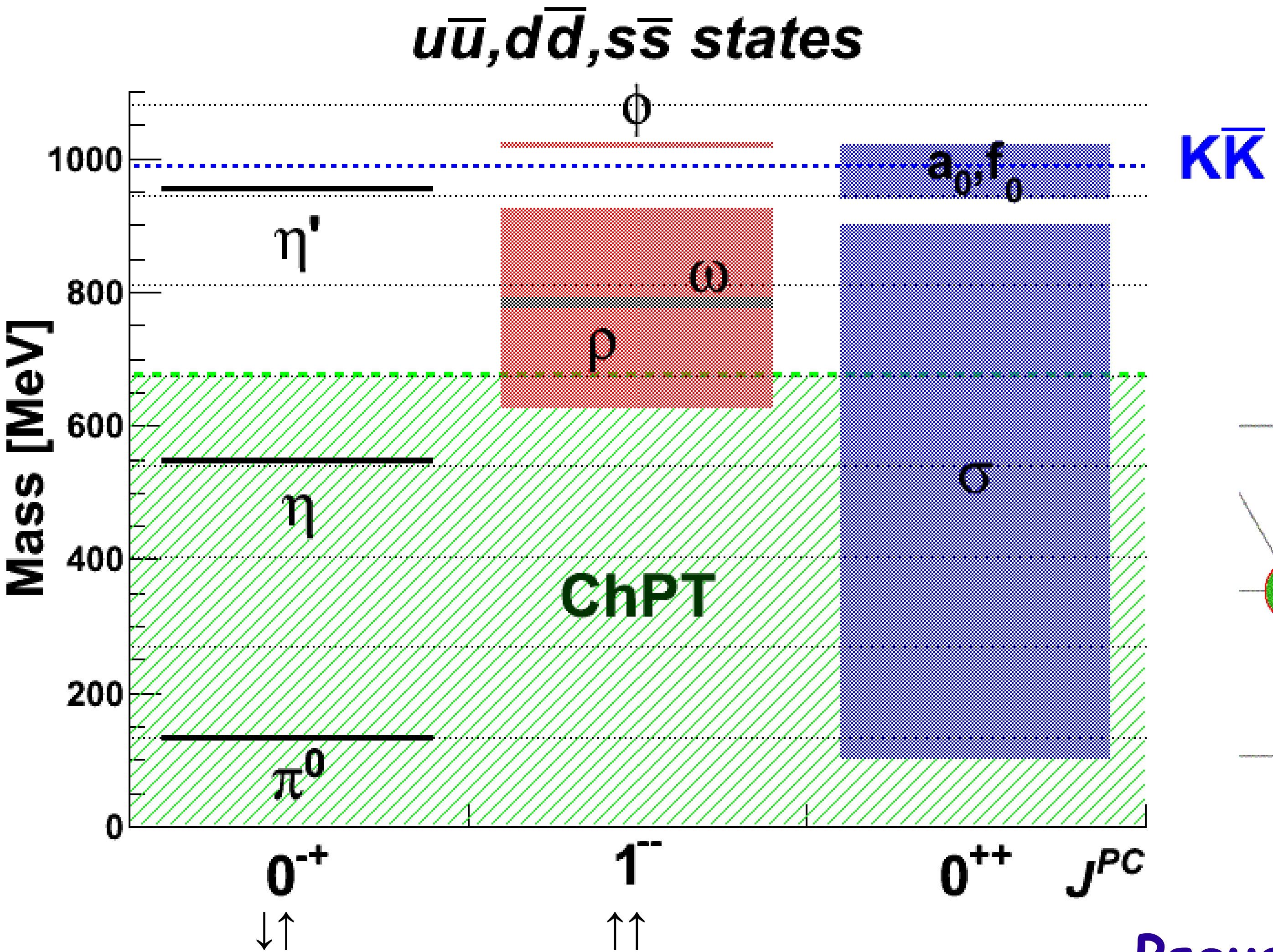
Andrzej Kupsc

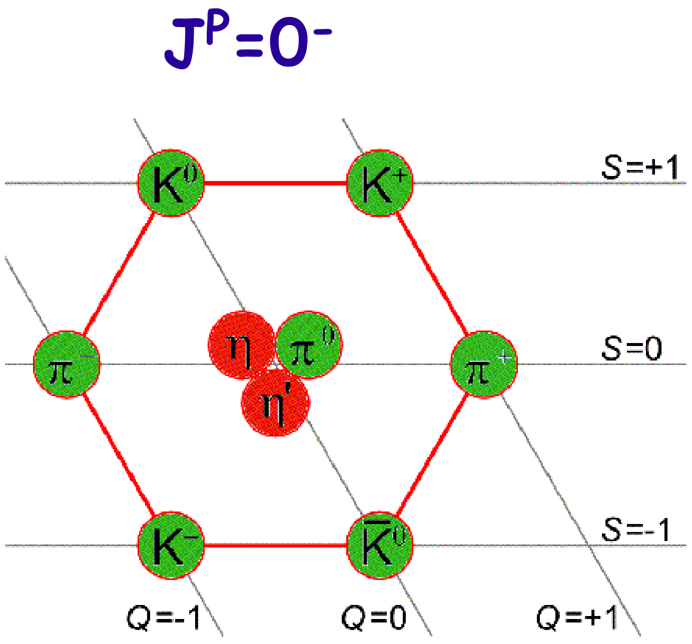
- Experiment: e+e- colliders
- Dispersive methods for hadronic contribution to muon g-2
- Two hadrons: Pion form factor / $\eta,\eta' \to \pi^+\pi^-\gamma$
- Three hadrons: Dalitz Plot / $\eta,\omega,\eta' \to \pi^+\pi^-\pi^0$
- ...

UPPSALA UNIVERSITET

IU, June 13th, 2017

Lightest neutral mesons





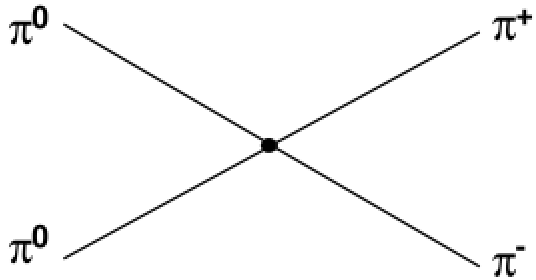
π^0 – lightest hadron

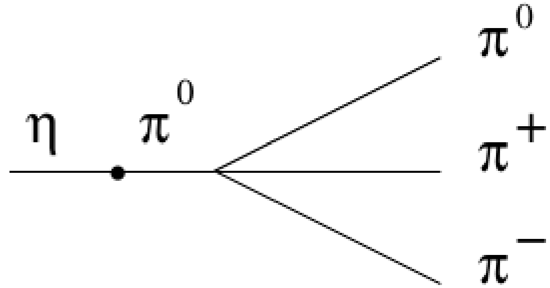
Pseudo Goldstone bosons: "stable"
Low energy QCD degrees of freedom

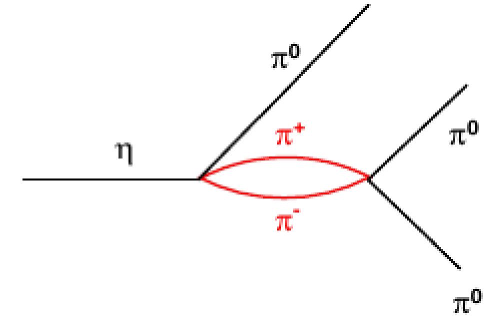
Vector Mesons Dominance: $V^0 \leftrightarrow \gamma$

Low Energy QCD processes

Even # pseudoscalars PPPP



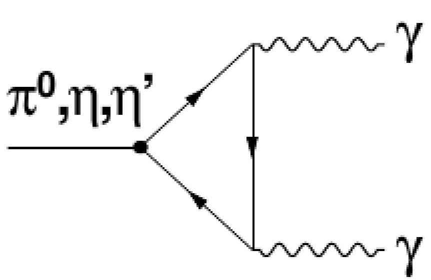


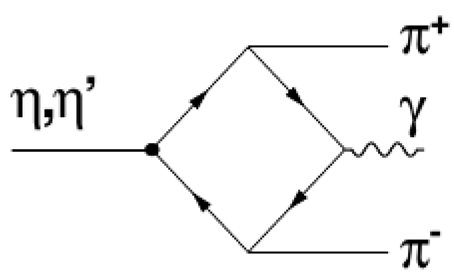


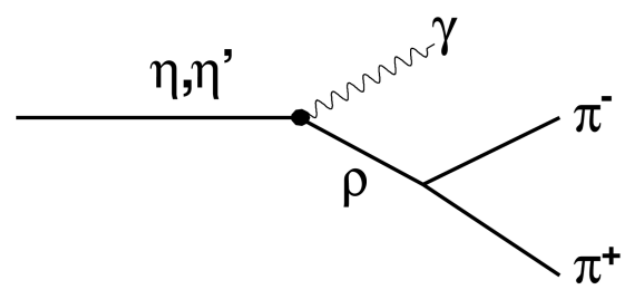
u-d quark masses

π-π, π-η (re-)scattering

Odd intrinsic parity: PVV/Pγγ, PPPV → TFF







Vector Meson Dominance

Content

Light hadrons at e+e- colliders

Hadronic contribution to muon g-2

(using dispersive methods)

Pion vector form factor

- $e^+e^- \to \pi^+\pi^-$ ($\tau \to \pi^+\pi^-\nu_\tau$)

Anomalous processes/transition form factors

- $\eta,\eta' \to \pi^+\pi^-\gamma$
- Dalitz decays $\eta,\eta' \to e^+e^-\gamma$
- $\omega \to \pi^+\pi^-\pi^0$ ($e^+e^- \to \pi^+\pi^-\pi^0$)

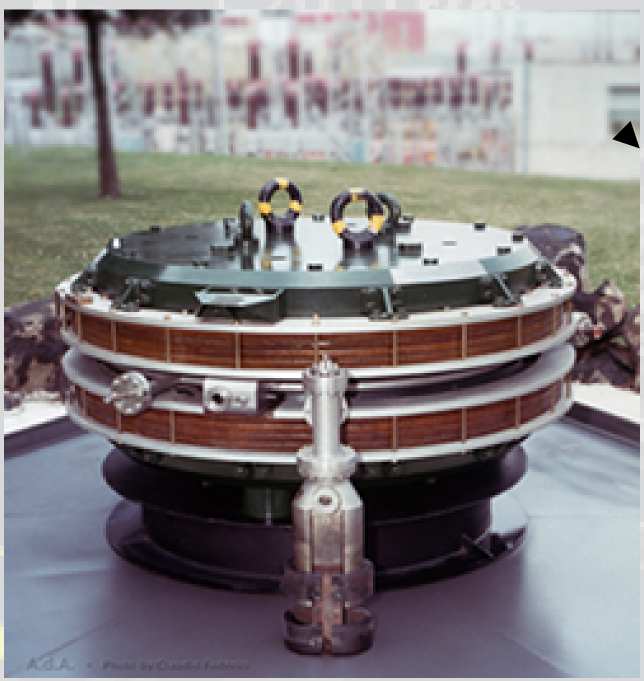
Even P processes

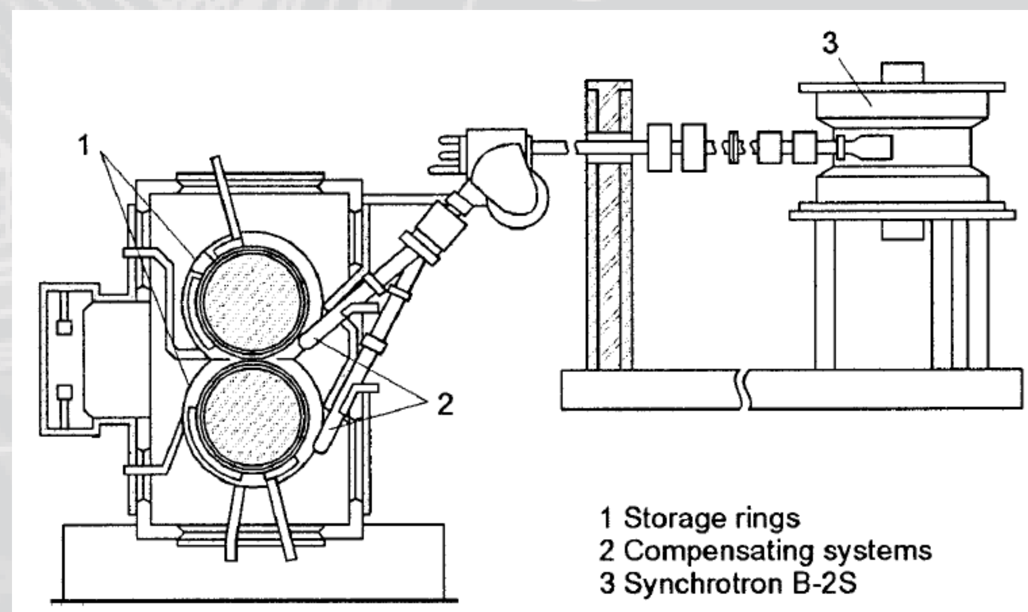
- $\eta,\eta' \to \pi^+\pi^-\pi^0$

...an example of amplitude analysis...

e+e- colliders

AdA 1961, LNF Frascati



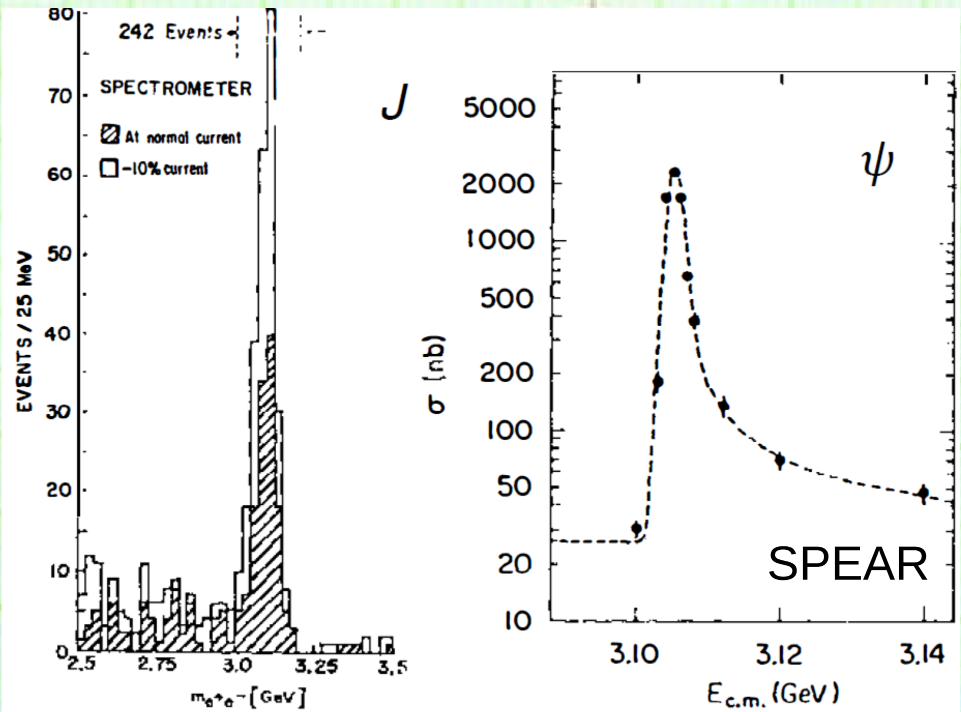


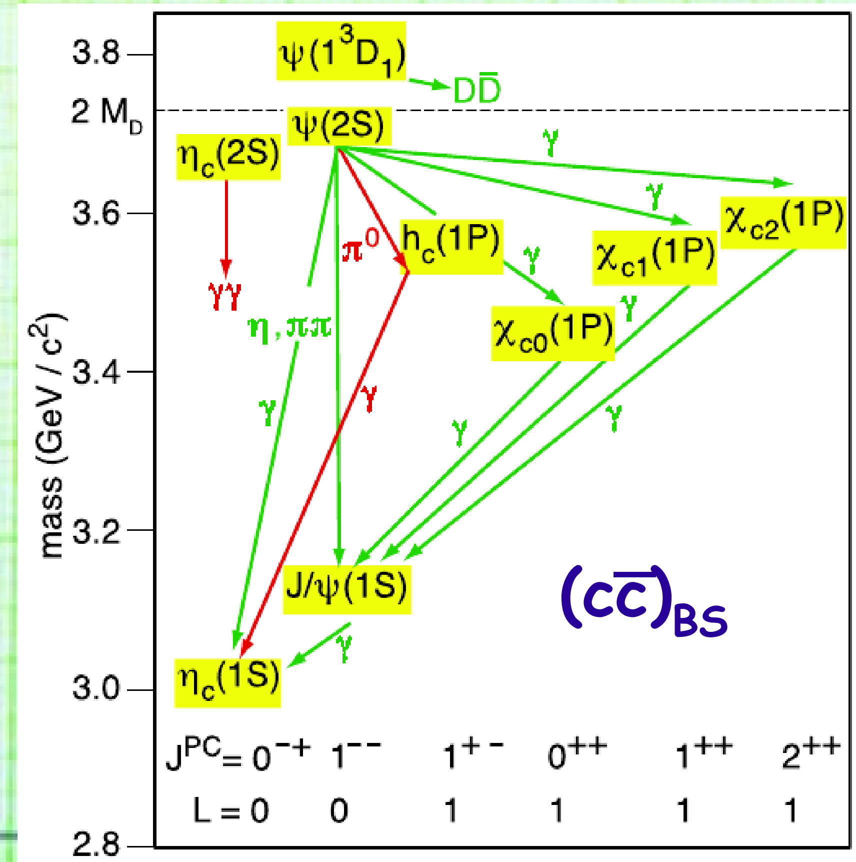
VEP-1 1965-1967, Novosibirsk

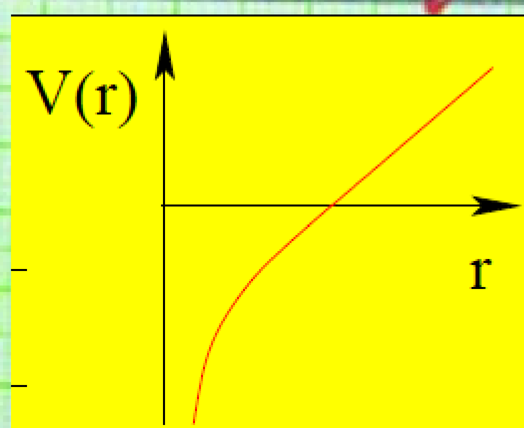
November revolution 1974

11/10/74

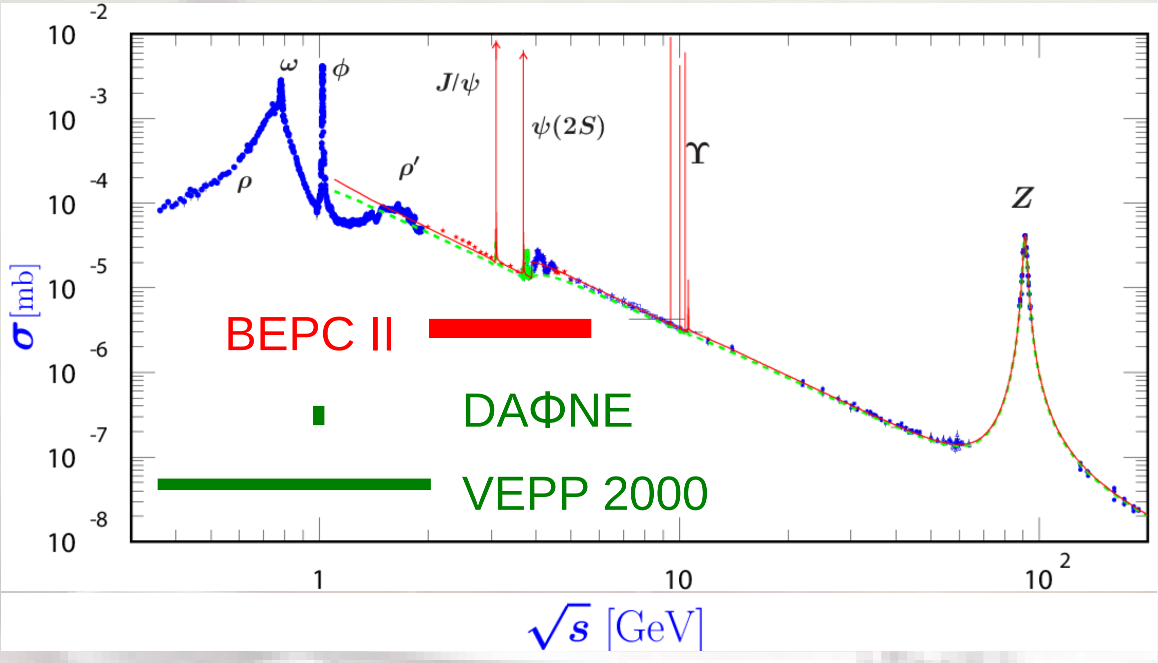
Charmonia

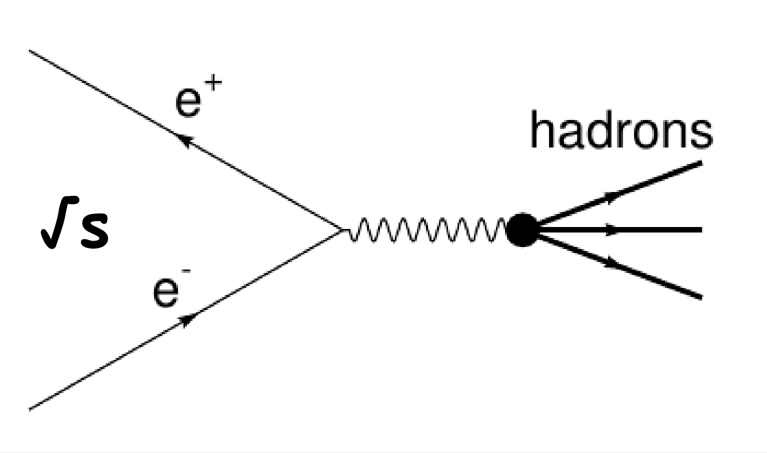


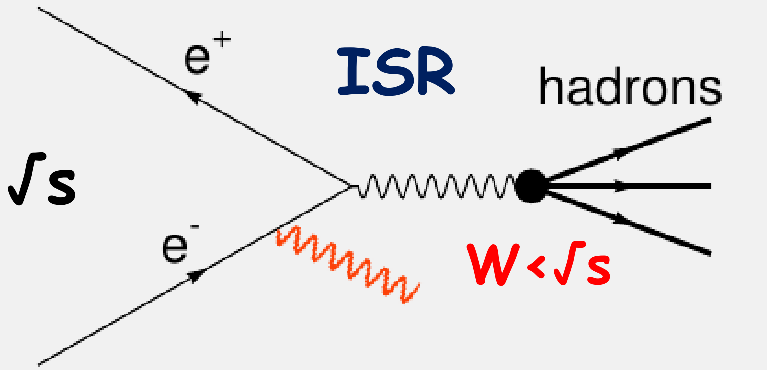


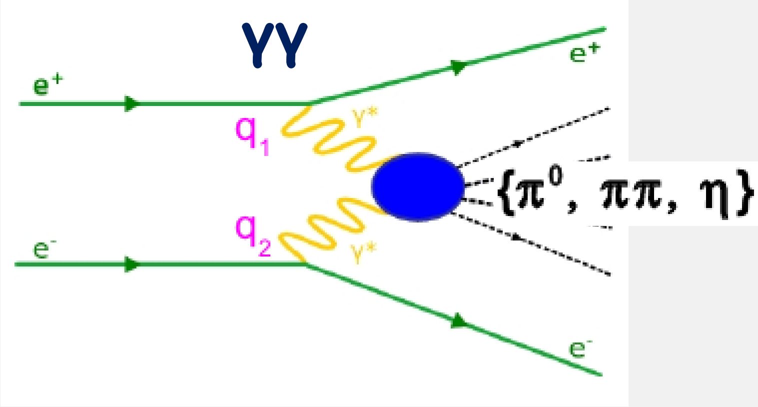


e+e- colliders









e+e- colliders in operation:

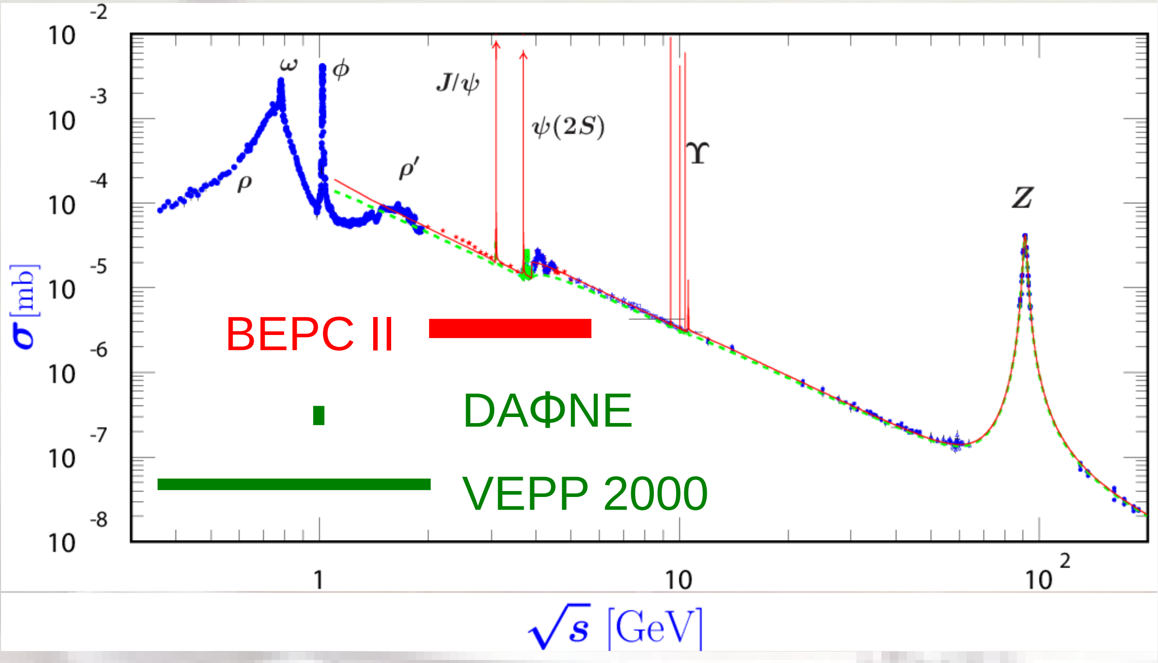
BEPCII L= 10^{33} cm^{-2}s^{-1} at Ψ(3.77) BESIII

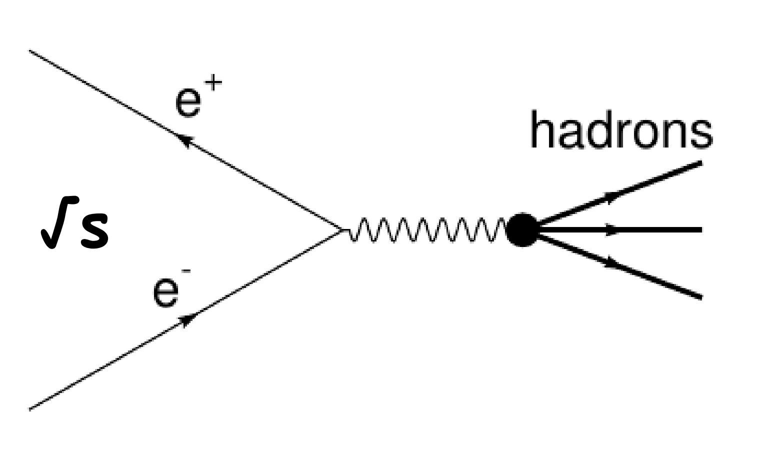
DAΦNE L= $2 \cdot 10^{32}$ cm^{-2}s^{-1} at Φ **KLOE-2**

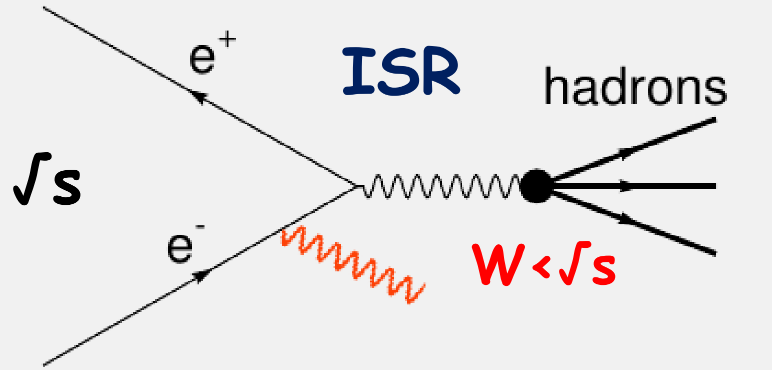
VEPP2000 L= 10^{32} cm^{-2}s^{-1} at 2GeV **CMD-3,SND**

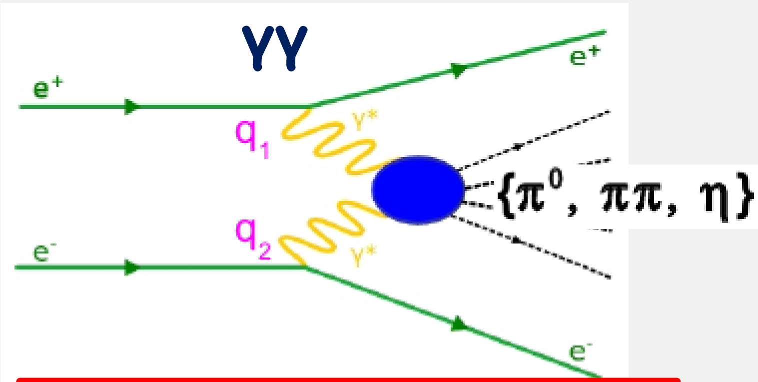
BaBar: ISR √s ≈ 3.0 GeV,
Belle/BelleII

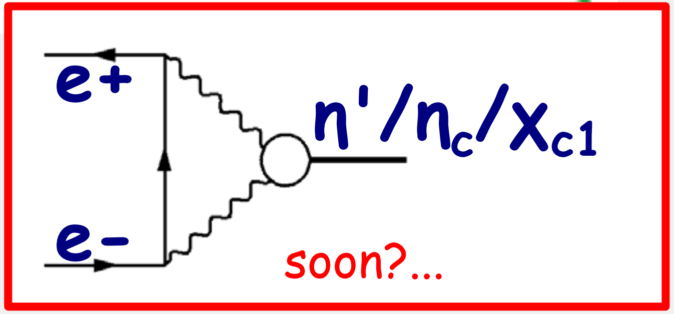
e+e- colliders











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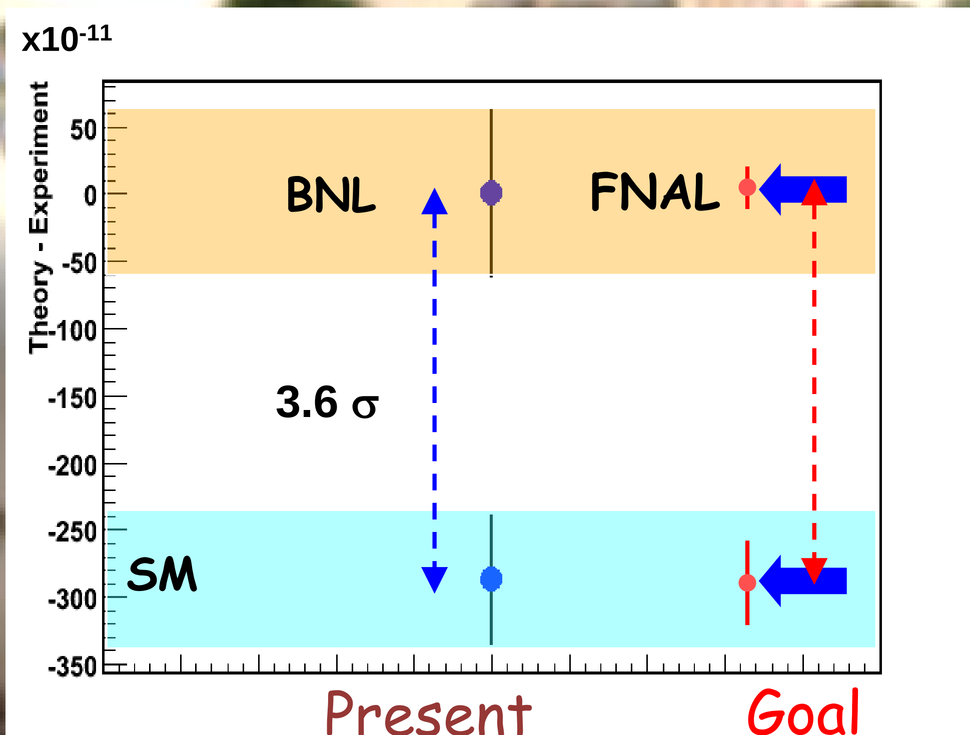
Muon anomalous magnetic moment puzzle



D.Hertzog, arXiv:1512.00928

Anomalous magnetic moment of elementary fermions

Single non trivial parameter coming from loops in QFT

$$a = \frac{g-2}{2}$$

QED:

$$a = \frac{\alpha}{2\pi} \approx 0.0011614$$

a_e = 1159652180.73(28) × 10^{-12} (0.24 × 10^{-9})

PRL 100, 120801 (2008)

QED test or α_{em} determination

a_μ = 116592091(63) × 10^{-11} (0.54 × 10^{-6})

E821, PRD 73, 072003 (2006)

Sensitive test of the Standard Model

a_τ = −0.018(17) or − 0.052 < a_τ < 0.013 95%CL

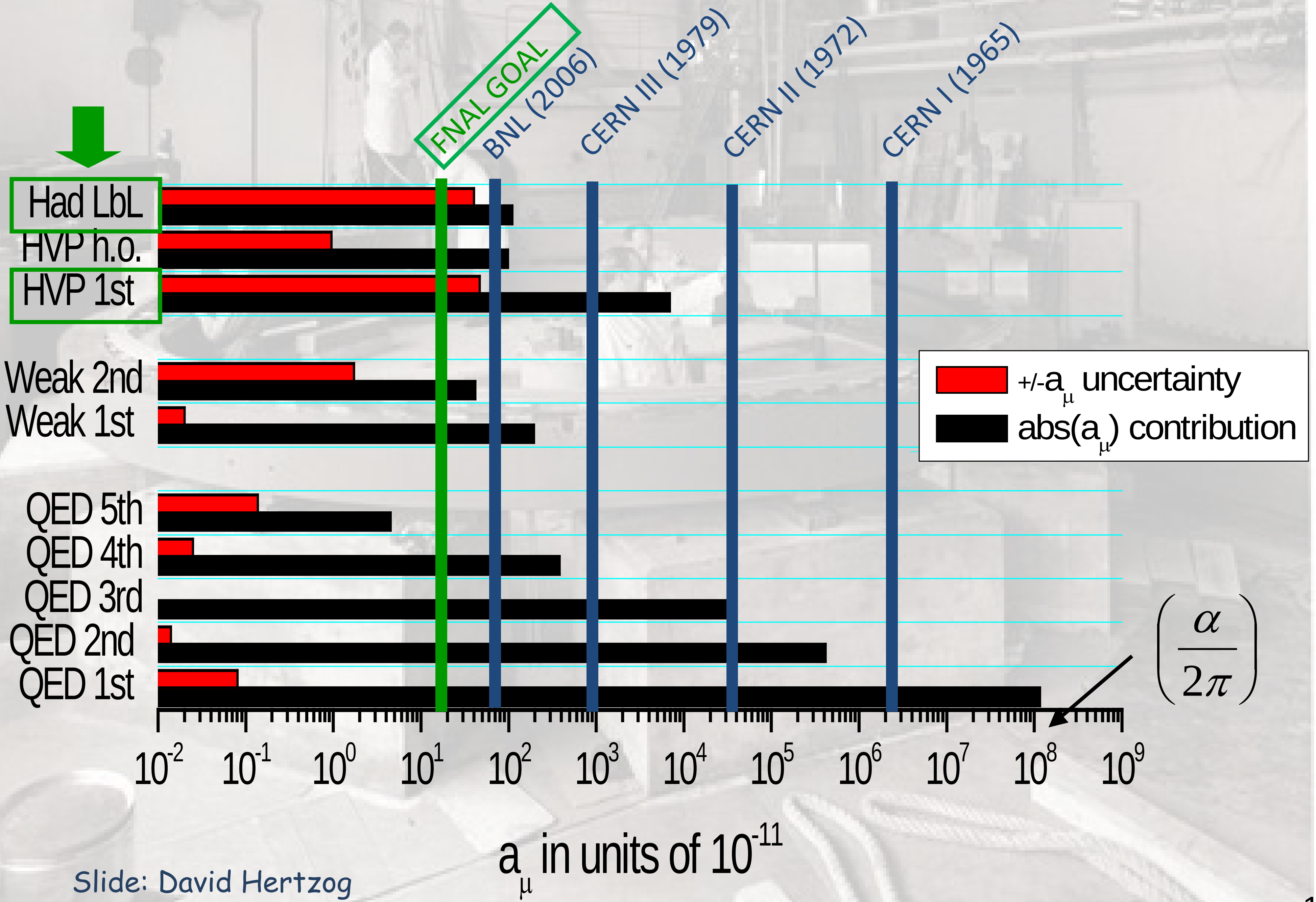
(DELPHI), EPJC 35, 159 (2004)

Theory: 117721(5) × 10^{-8},

Eidelman, Passera, MPL A 22, 159 (2007)

a_μ much more sensitive to NP than a_e ~ $(m_\mu/m_e)^2 \approx 4.3 \cdot 10^4$

Muon g-2 measurements sensitivity



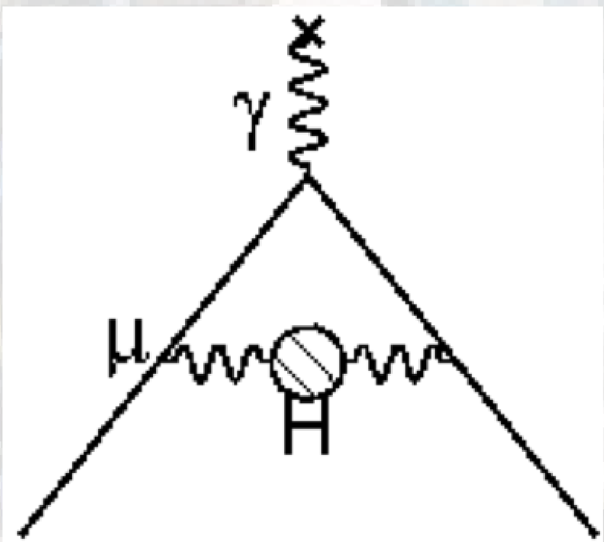
Slide: David Hertzog

BNL result

$$a_\mu^{BNL} = (116\ 592\ 091 \pm 63) \cdot 10^{-11}$$

$$a_\mu^{exp} - a_\mu^{SM} = (249 \pm 87) \cdot 10^{-11} \quad (3\sigma)$$

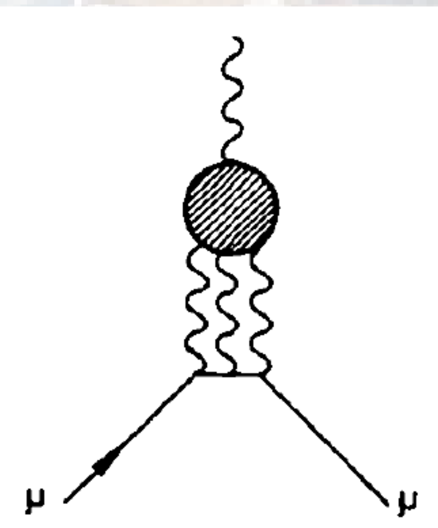
hadronic vacuum polarization
(HVP)



$$a_\mu^{HVP} = (6\ 923 \pm 42) \cdot 10^{-11}$$

$a_\mu^{exp} - a_\mu^{SM}$: 4% HVP

hadronic light-by-light scattering
(HLbL)



$$a_\mu^{HLbL} = (116 \pm 40) \cdot 10^{-11}$$

215% HLbL
(1% of leptonic LbL)

HVP

dispersive approach: precision predictions from precision data

$$a_\mu^{\mathrm{HLO}} = \frac{\alpha}{\pi^2} \int_0^\infty \frac{ds}{s} K(s)\, \mathrm{Im}\Pi_{\mathrm{had}}(s+i\epsilon)$$

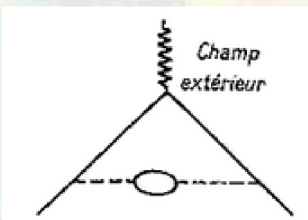


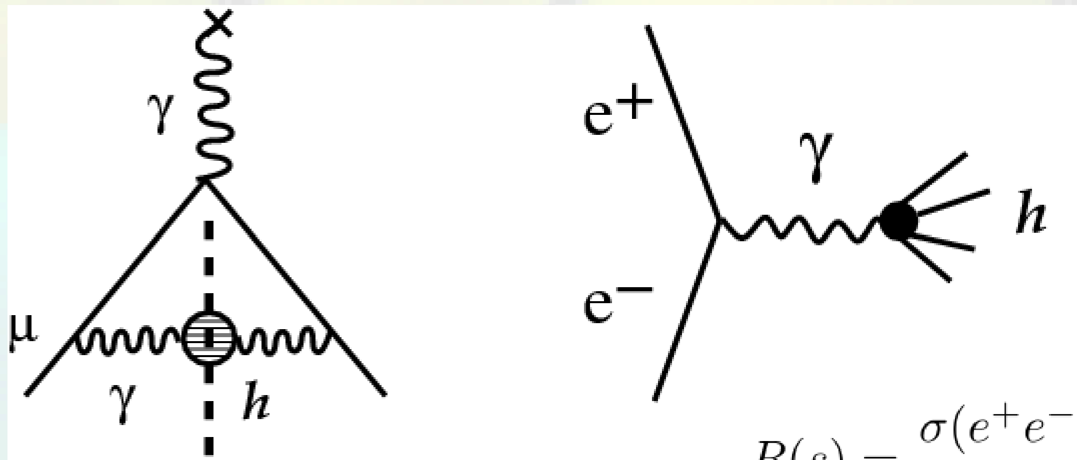
FIG. 1b.

LA RÉSONANCE
DANS LA DIFFUSION MÉSON π — MÉSON π
ET LE MOMENT MAGNÉTIQUE ANORMAL
DU MÉSON μ

Par Claude BOUCHIAT et Louis MICHEL,

J. Phys. Radium 22,121 (1961)

K(s): kernel



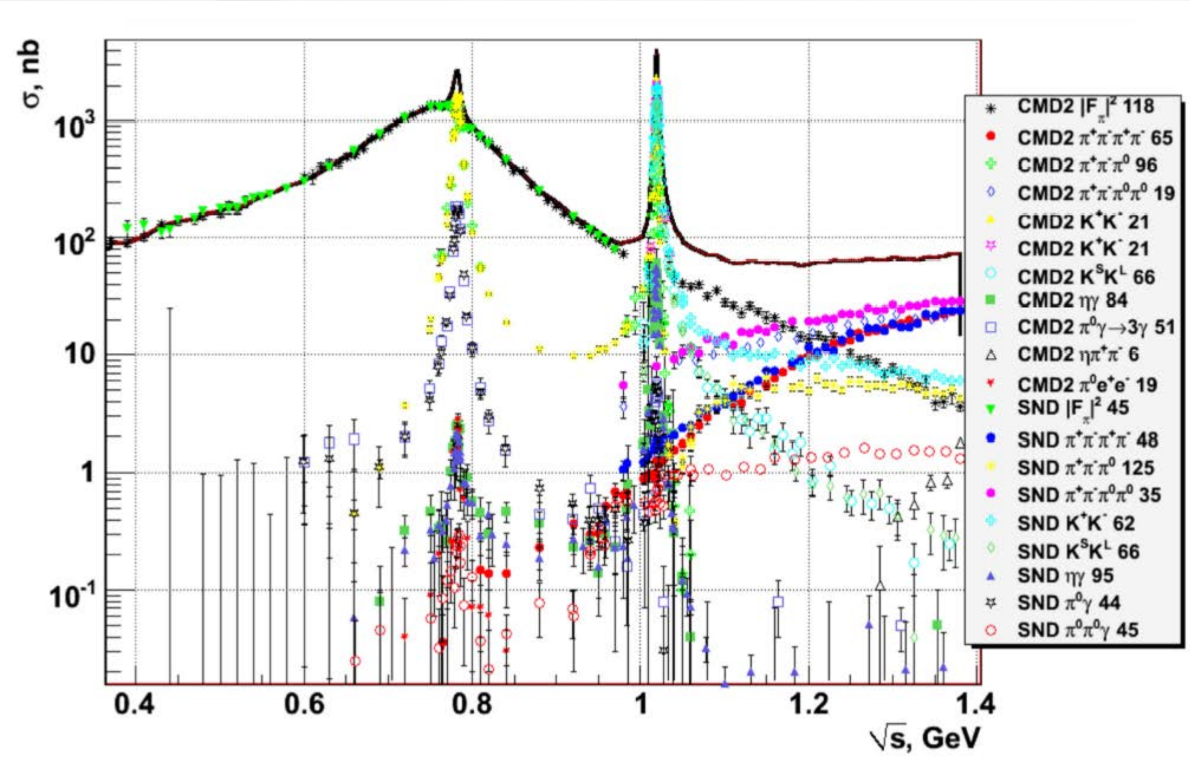
$$R(s) = \frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

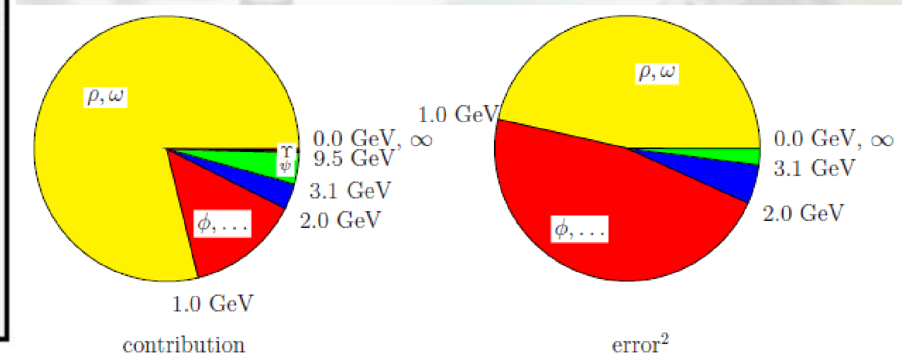
$$\left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int_{m^2_{\pi^0}}^{E^2_{\mathrm{cut}}} \mathrm{d}s\, \frac{R^{\mathrm{data}}_{\mathrm{had}}(s)\,\hat{K}(s)}{s^2} + \int_{E^2_{\mathrm{cut}}}^{\infty} \mathrm{d}s\, \frac{R^{\mathrm{pQCD}}_{\mathrm{had}}(s)\,\hat{K}(s)}{s^2} \right)$$

$0.63<\hat{K}(s)<1$

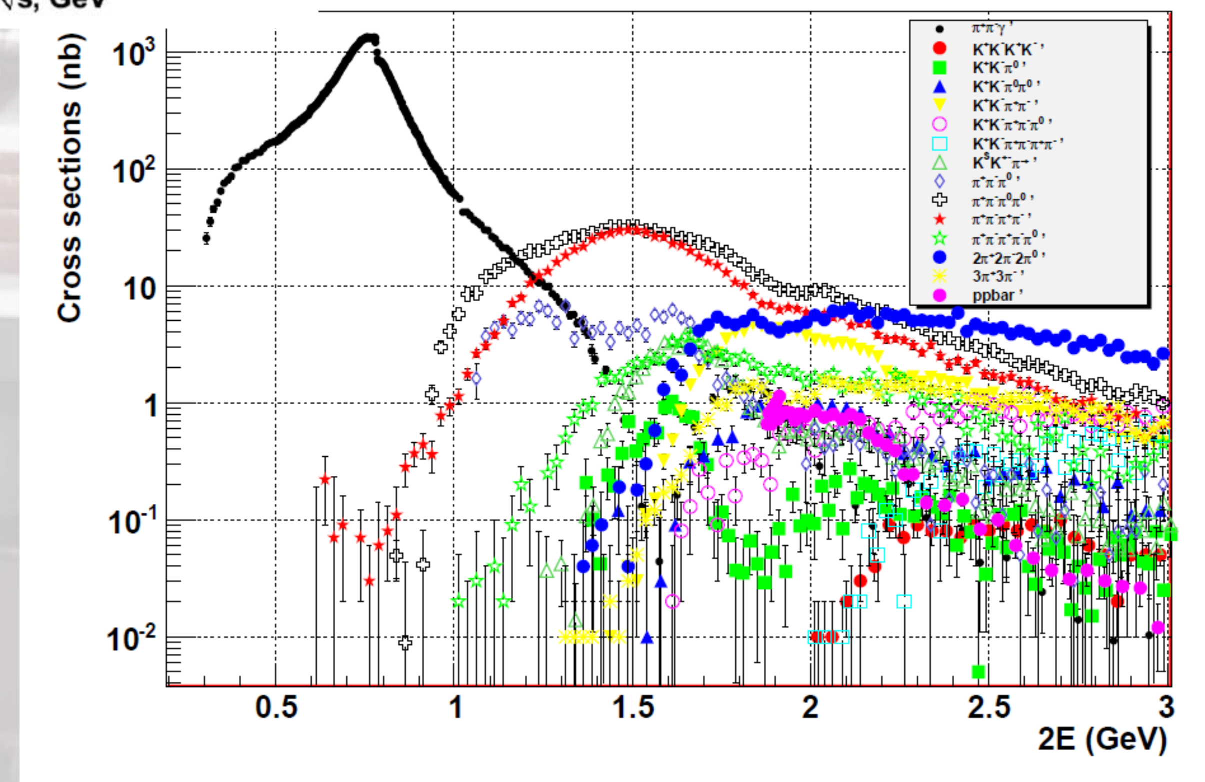
Phys.Rept. 477, 1

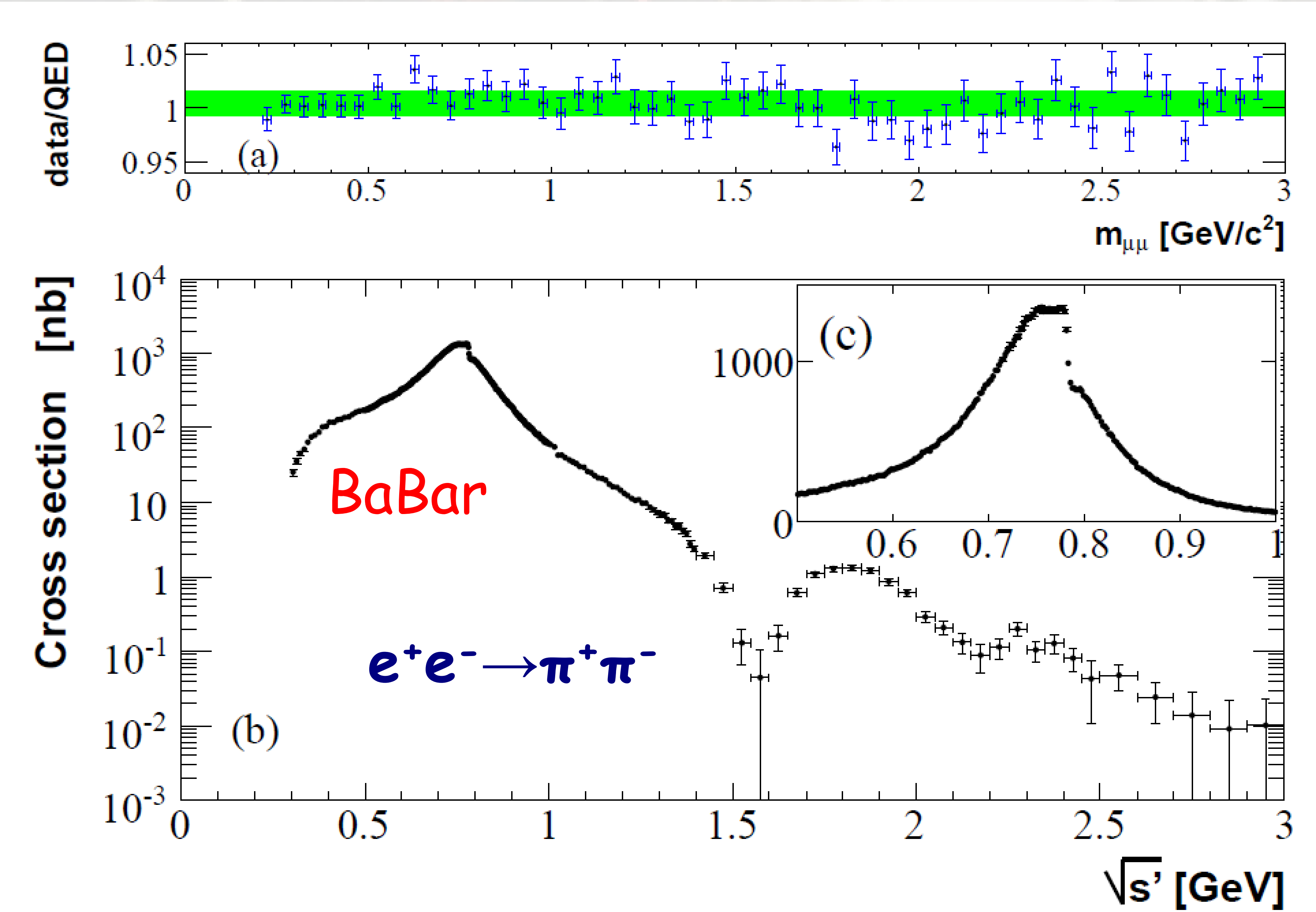
CMD-2, SND (scan at √s < 1.4 GeV)
KLOE (ISR at √s < 1.0 GeV)
S.Eidelman,F.Jegerlehner



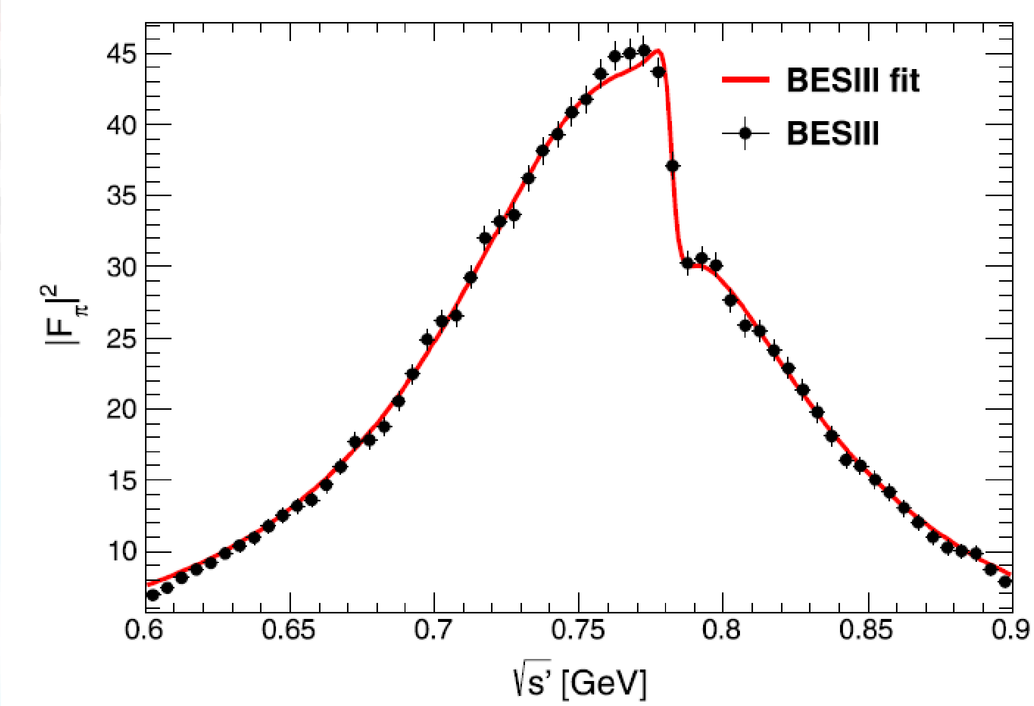


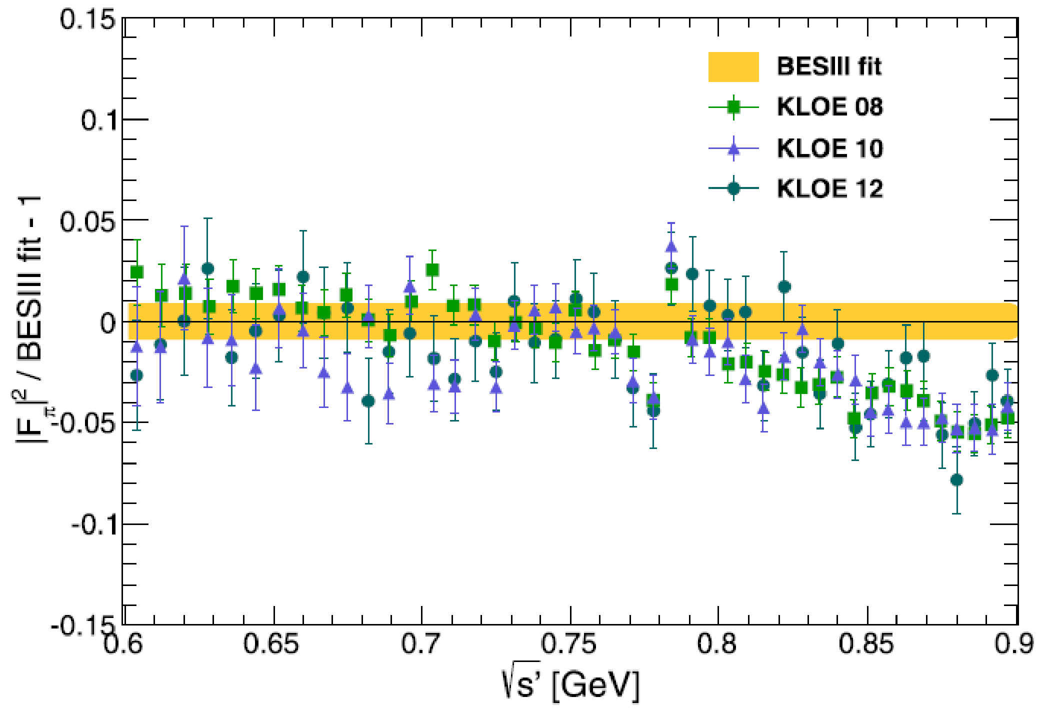
Channel	HLMNT 11
$\eta\pi^+\pi^-$	0.88 ± 0.10
K^+K^-	22.09 ± 0.46
$K_S^0 K_L^0$	13.32 ± 0.16
$\omega\pi^0$	0.76 ± 0.03
$\pi^+\pi^-$	505.65 ± 3.09
$2\pi^+2\pi^-$	13.50 ± 0.44
$3\pi^+3\pi^-$	0.11 ± 0.01
$\pi^+\pi^-\pi^0$	47.38 ± 0.99
$\pi^+\pi^-2\pi^0$	18.62 ± 1.15
$\pi^0\gamma$	4.54 ± 0.14



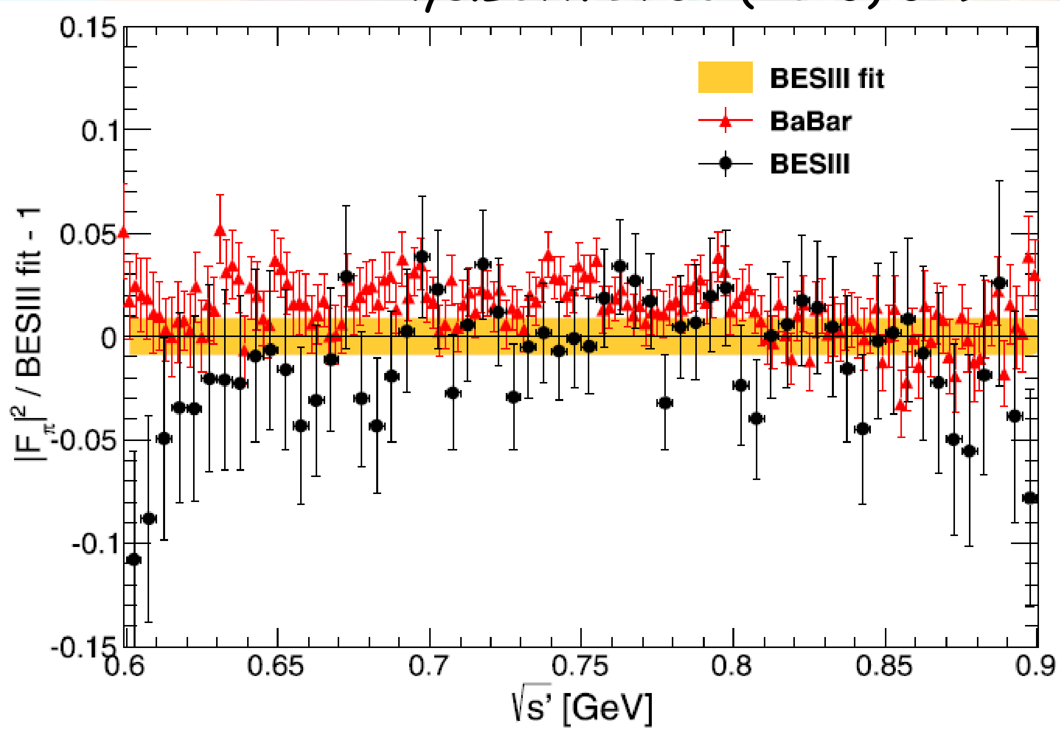


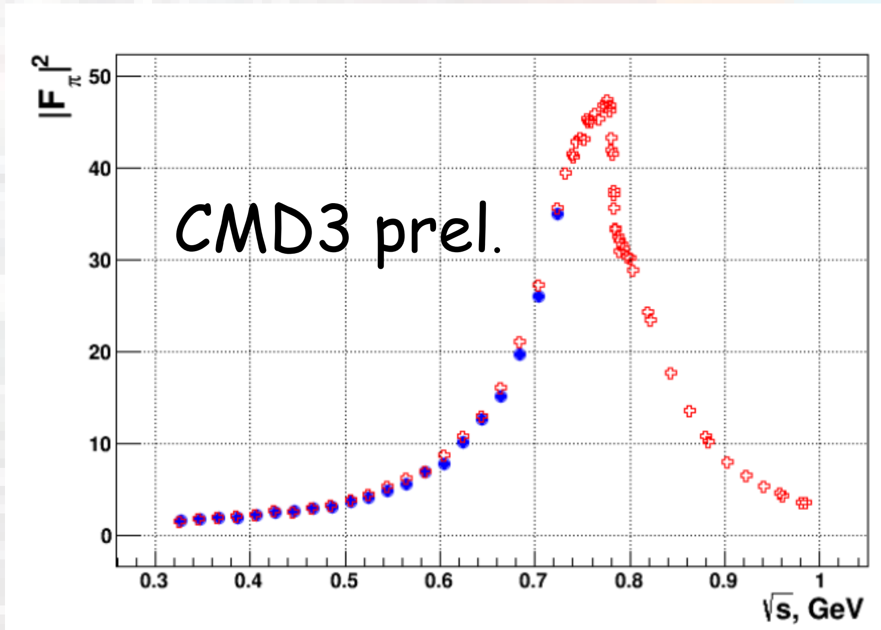
J.P. Lees et al., Phys. Rev. D86 (2012) 032013





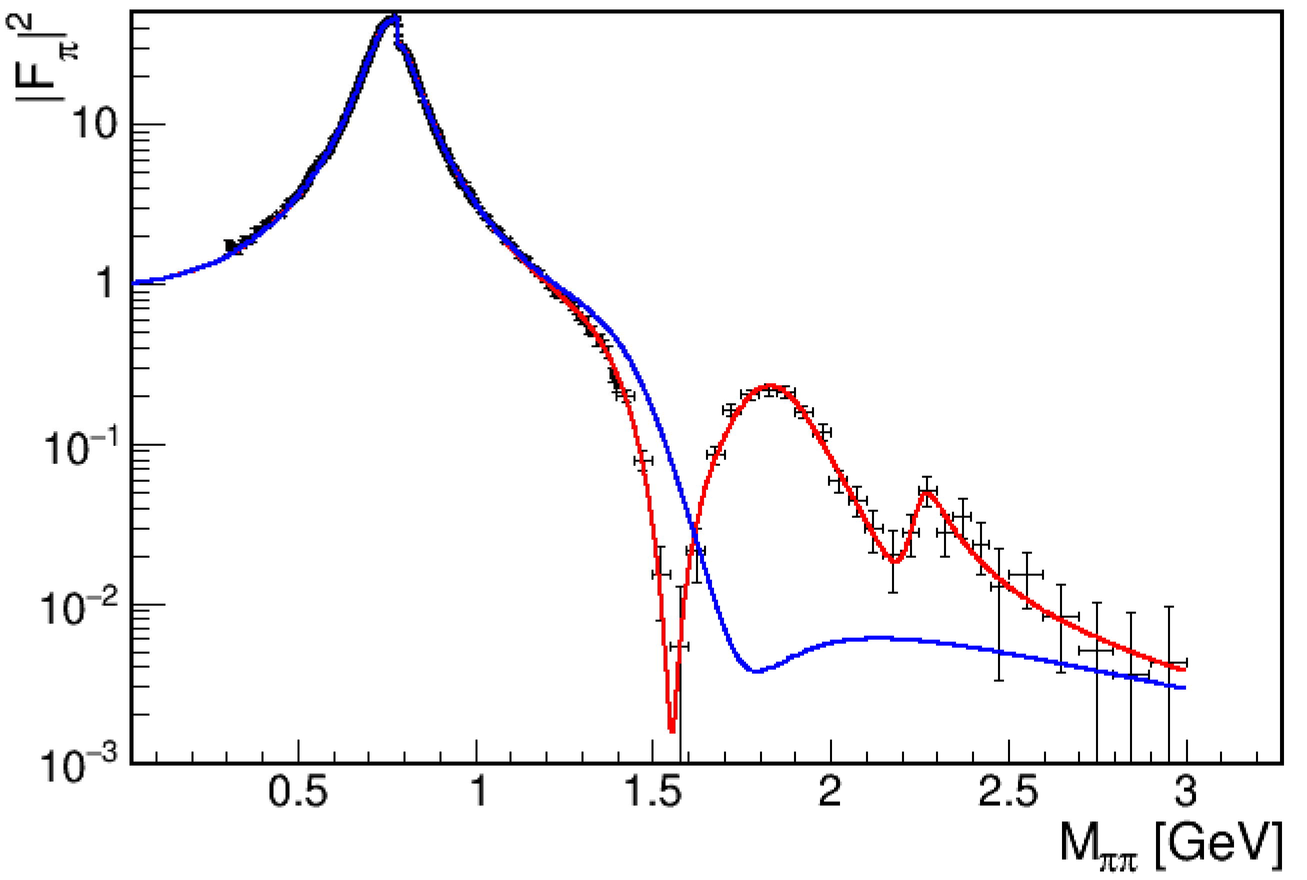
Phys.Lett. B753 (2016) 629





$e^+e^- \rightarrow \pi^+\pi^-$

Excercise: pion vector form factor fits



$$\sigma(e^+e^- \to \pi^+\pi^-) = \frac{\pi\alpha_{\rm em}^2}{3s}\ \beta_\pi^3\ |\mathbf{F_\pi^V(s)}|^2$$

$$F_\pi(s) = \frac{\mathrm{BW}^{\rm GS}_{\rho(770)}(s)\cdot\left(1+\delta\frac{s}{M_\omega^2}\mathrm{BW}_\omega(s)\right)+\beta\,\mathrm{BW}^{\rm GS}_{\rho(1450)}(s)+\gamma\,\mathrm{BW}^{\rm GS}_{\rho(1700)}(s)}{1+\beta+\gamma}$$

Gounaris-Sakurai PRL,21,244 (1968)

FINITE-WIDTH CORRECTIONS TO THE VECTOR-MESON-DOMINANCE PREDICTION FOR $\rho \to e^+e^-$ *

G. J. Gounaris and J. J. Sakurai

P-wave I=1 ππ scattering phase-shift generalized effective-range Chew-Mandelstam formula

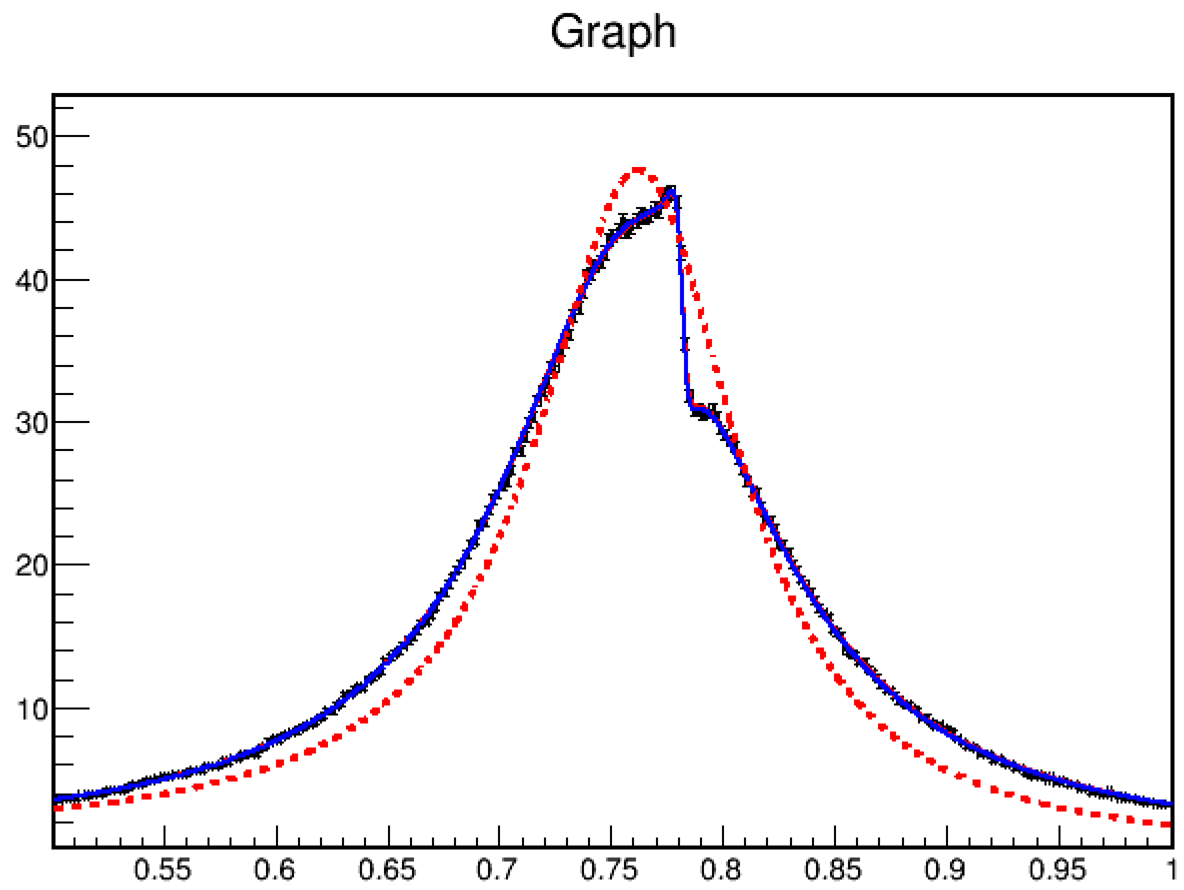
$$BW^{\mathrm{GS}}(s,m,\Gamma) = \frac{m^2(1+d(m)\Gamma/m)}{m^2 - s + f(s,m,\Gamma) - im\Gamma(s,m,\Gamma)}$$

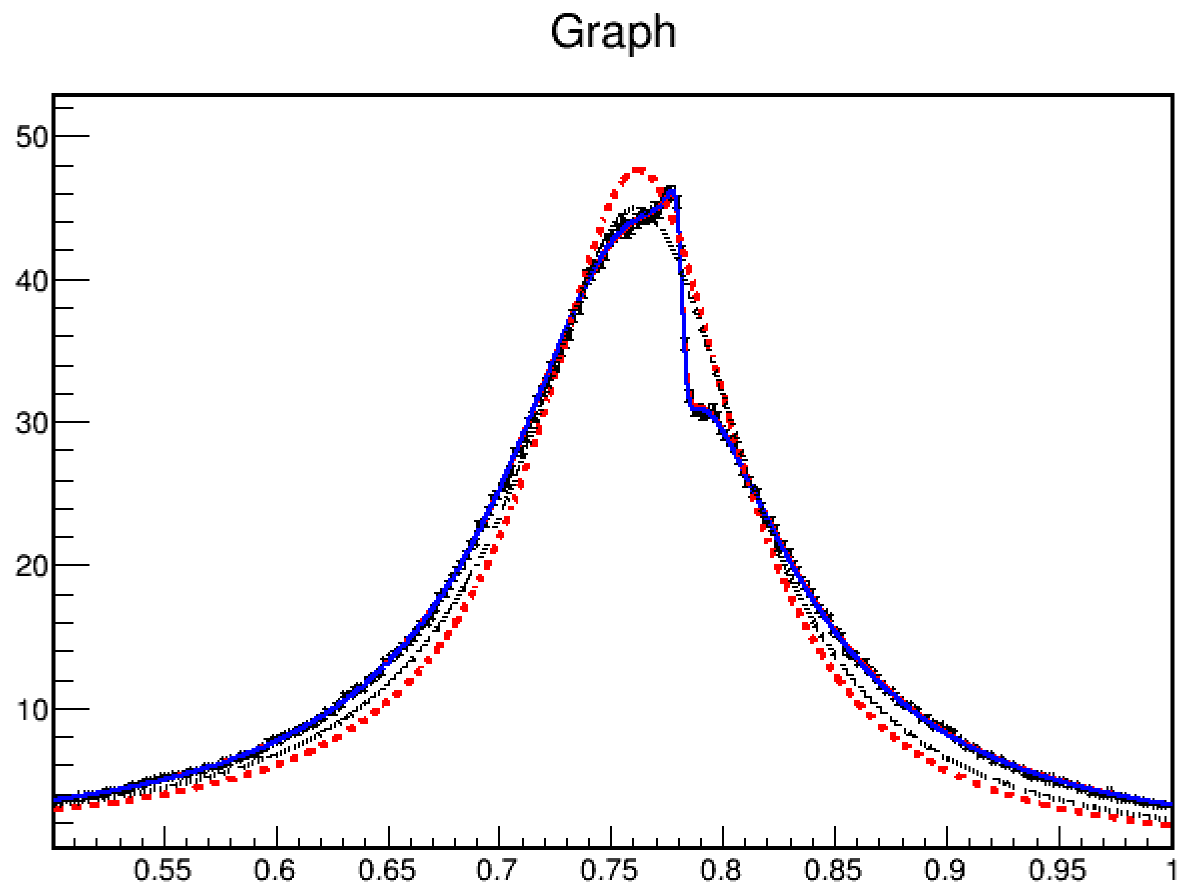
$$\Gamma(s,m,\Gamma) = \Gamma \frac{s}{m^2}\left(\frac{\beta_\pi(s)}{\beta_\pi(m^2)}\right)^3$$

$$(k^3/\sqrt{s})\cot\delta_1 = k^2 h(s) + a + bk^2,$$

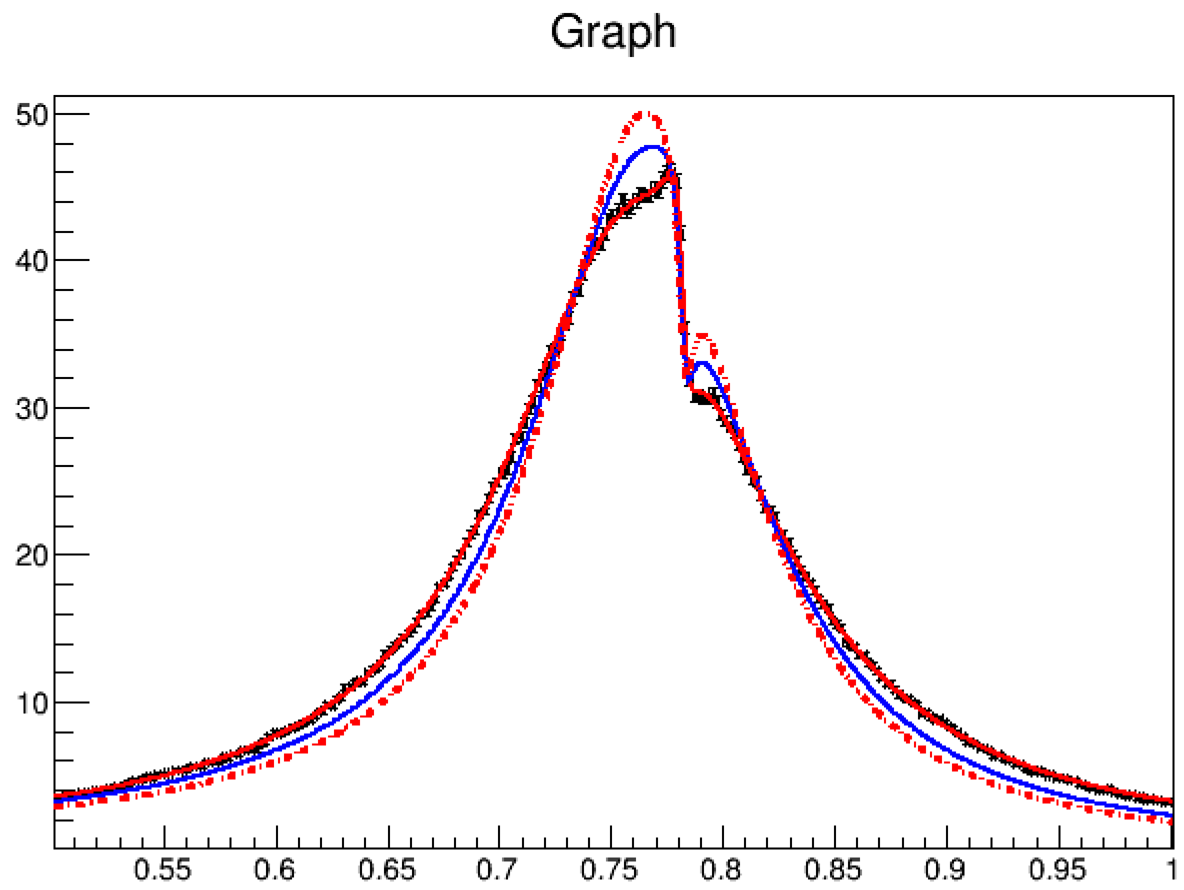
$$h(s) = \frac{2}{\pi}\frac{k}{\sqrt{s}}\ln\left(\frac{\sqrt{s}+2k}{2m_\pi}\right) \qquad k = (\tfrac{1}{4}s - m_\pi^{\ 2})^{1/2}$$

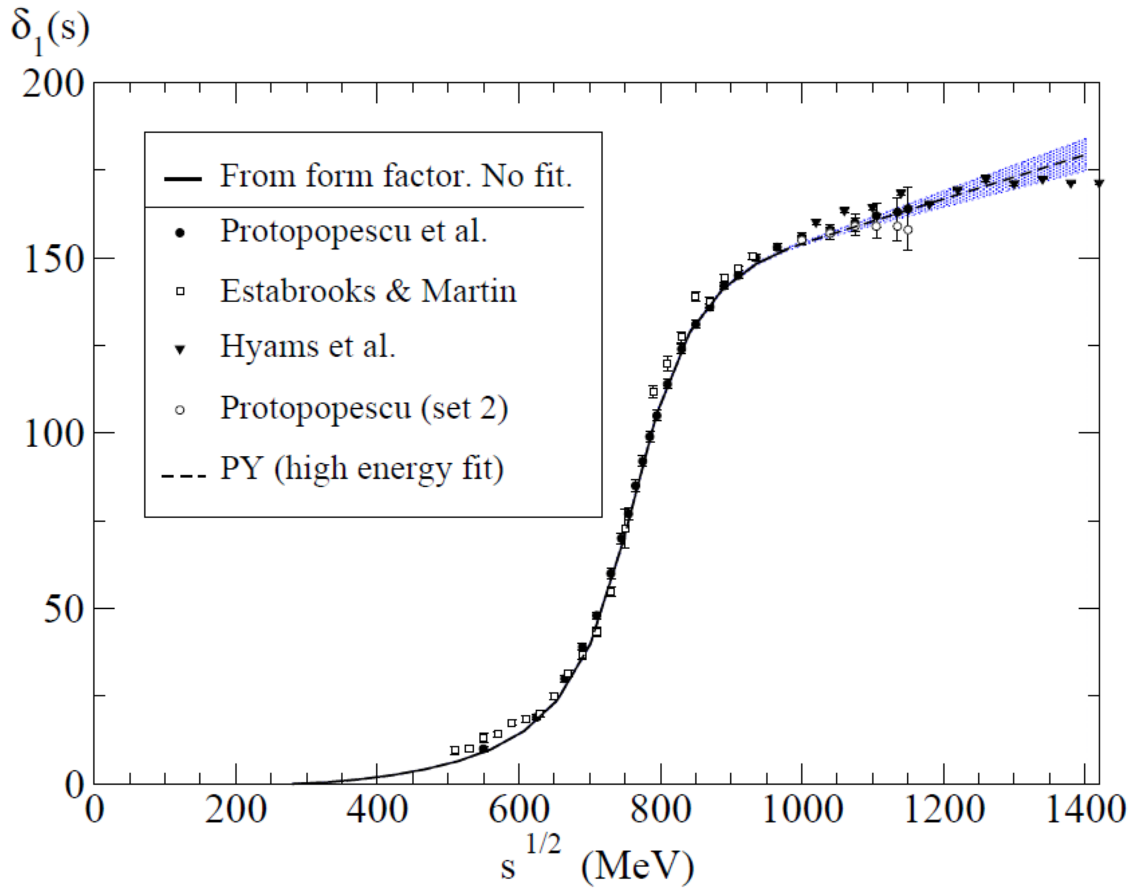
$$\cot\delta_l(E) = \cot\delta_l(E_R) + (E-E_R)\left.\frac{d\cot\delta_l}{dE}\right|_{E=E_R} + \cdots$$


Graph
50
40
30
20
10
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1



Graph
50
40
30
20
10
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1

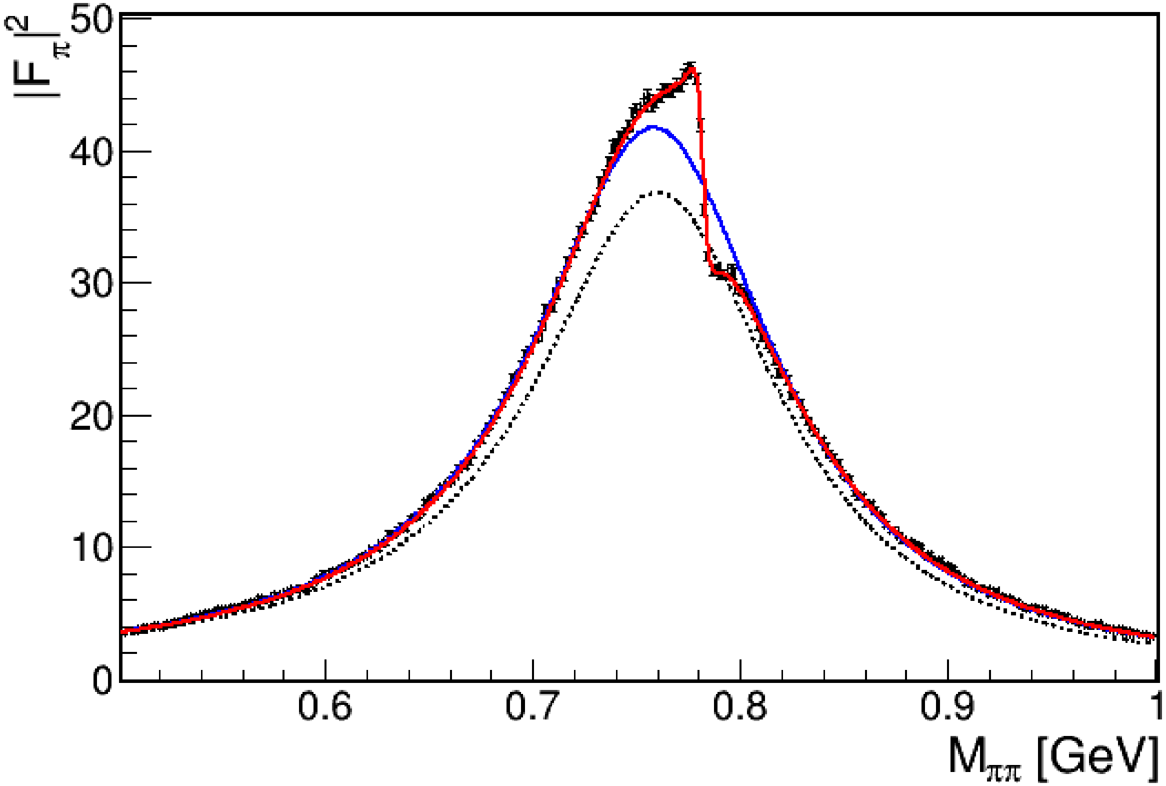

Graph
50
40
30
20
10
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1



π-π phase shifts
I=1 (P wave)

$$\Omega(s) = \exp\left\{ \frac{s}{\pi} \int_{4m_\pi^2}^{\infty} \mathrm{d}x \frac{\delta_1(x)}{x(x-s-i\varepsilon)} \right\}$$

Phys.Rev. D71 (2005) 074016



Omnes function fits

$$F_V(s) = R(s)\Omega(s).$$

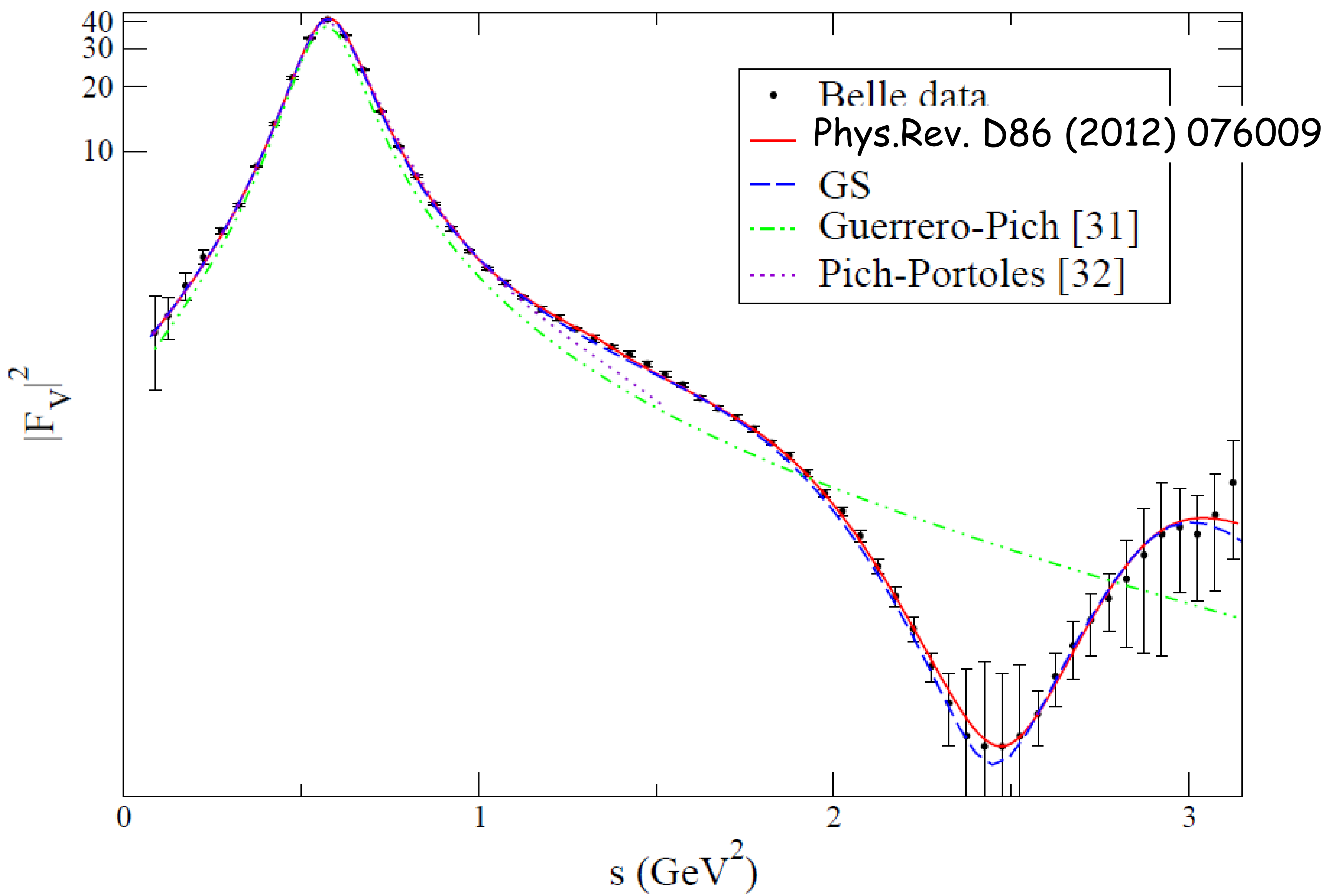
$$R(s) = 1 + \alpha_V\, s + \frac{\kappa_1\, s}{m_\omega^2 - s - i m_\omega \Gamma_\omega^{\text{tot}}}$$

isospin violation
I=0 (ρ-ω mixing)

$$F_\pi^V(s) \approx (1 + 0.1\,\text{GeV}^{-2} s)\Omega(s)$$

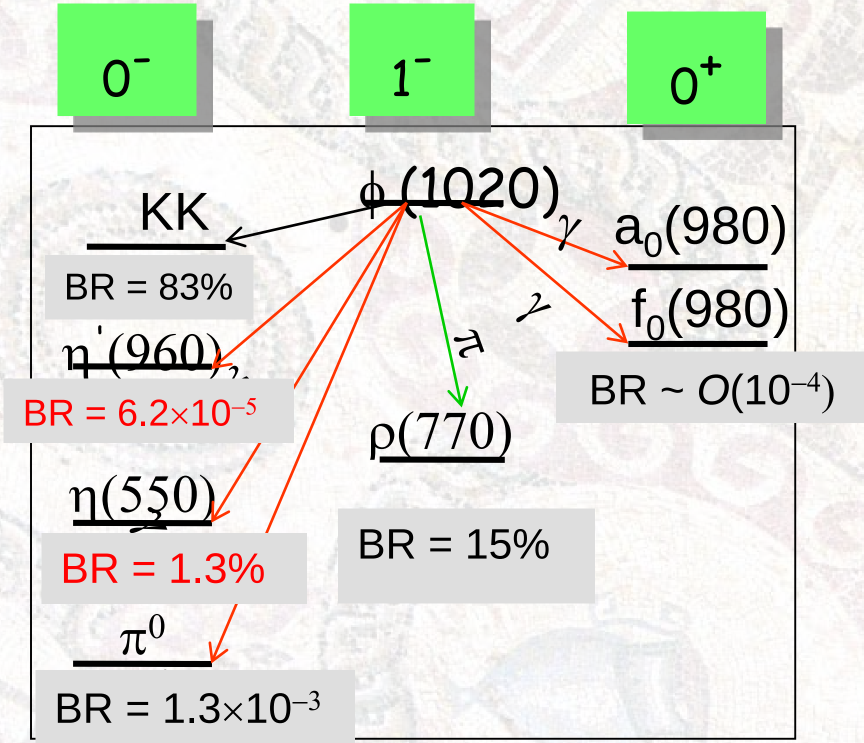
EPJC77 (2017) 98

$\tau^- \to \pi^- \pi^0 \nu_\tau$



Belle, Phys.Rev. D78 (2008) 072006

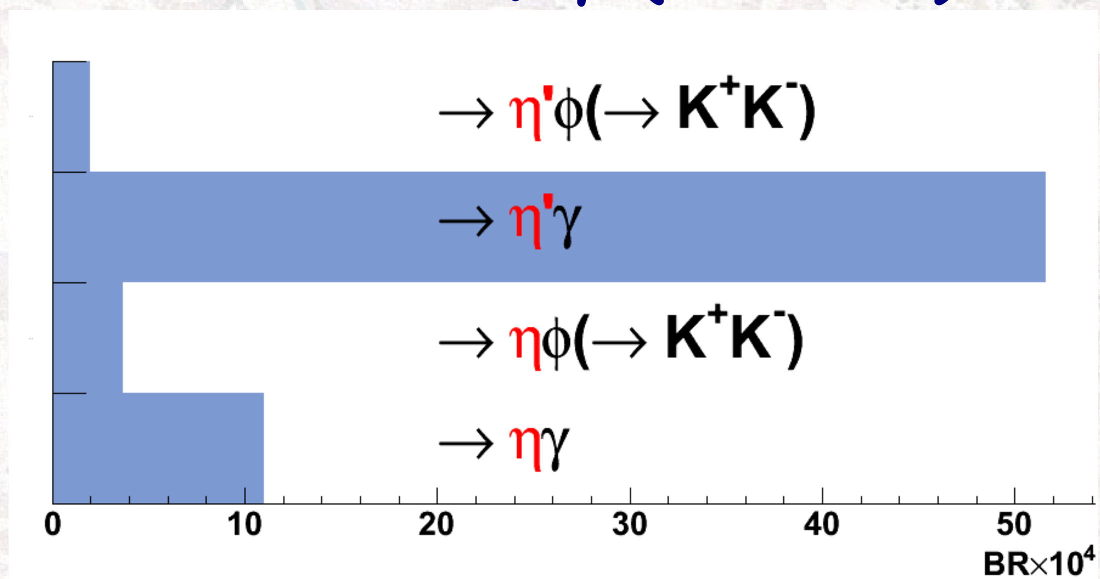
η/η' decays at e^+e^- colliders



$\phi \to \eta^{(\prime)}\gamma$ (KLOE)

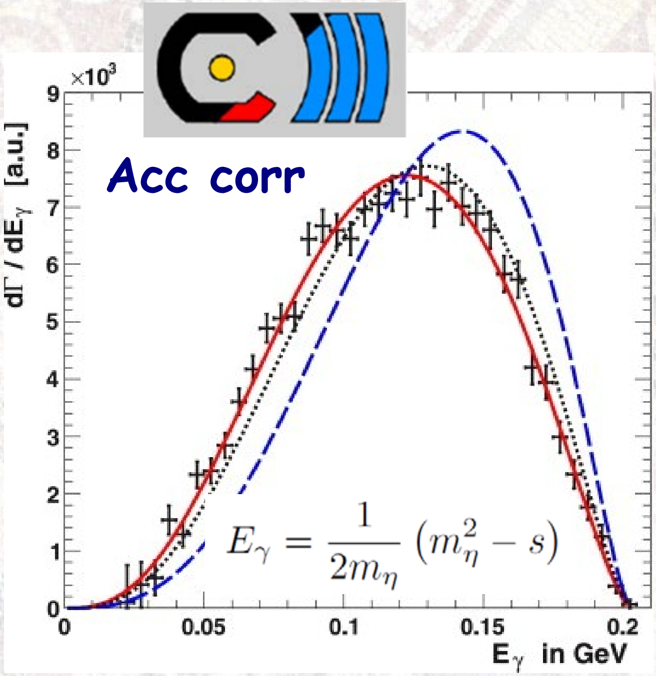
• 548 MeV, $\Gamma = 1.3$ keV		• 958 MeV, $\Gamma = 200$ keV	
Hadronic Decays			
$\eta \to \pi^0\pi^0\pi^0$	32%	$\eta' \to \pi^+\pi^-\eta$	44%
$\eta \to \pi^+\pi^-\pi^0$	23%	$\eta' \to \pi^0\pi^0\eta$	21%
Radiative Decays			
$\eta \to \gamma\gamma$	39%	$\eta' \to \rho^0\gamma$	29%
$\eta \to \pi^+\pi^-\gamma$	5%	$\eta' \to \omega\gamma$	3%
		$\eta' \to \gamma\gamma$	2%
	Σ 99%		Σ 99%

$J/\Psi \to \eta^{(\prime)}\gamma$ (BESIII)

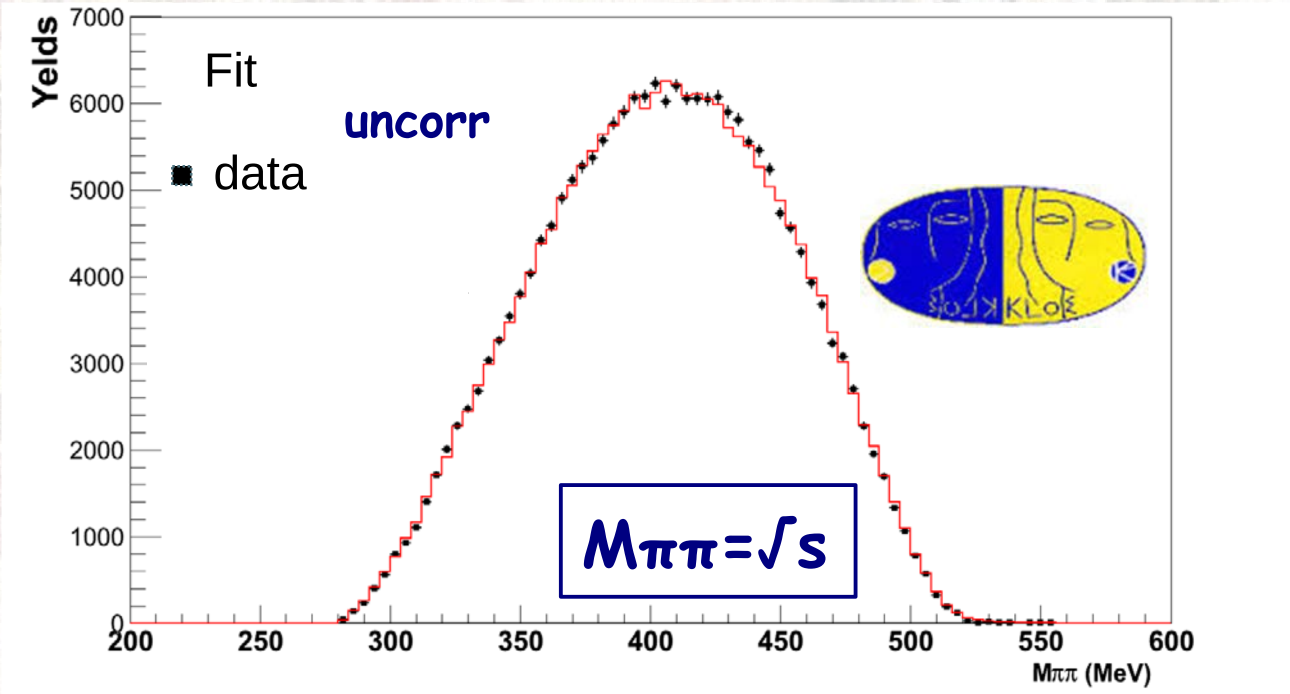


1.31×10^9 J/Ψ events ➔ 6×10^6 η'

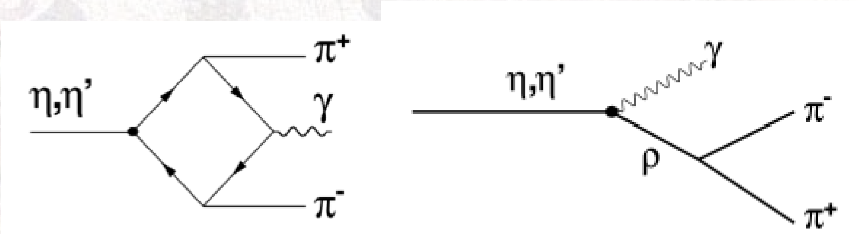
$\eta \rightarrow \pi^+\pi^-\gamma$



WASA PLB707 (2012) 243



KLOE PLB718 (2013) 910



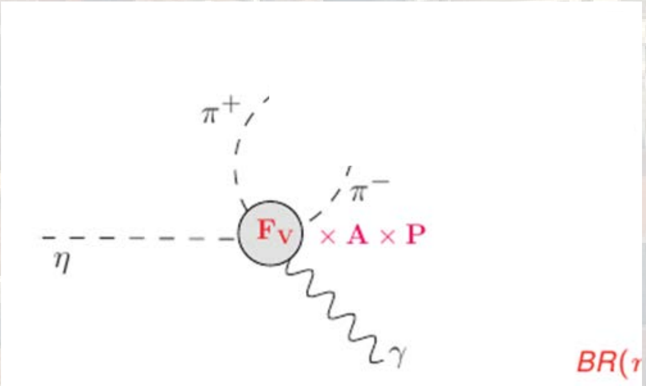
$$\frac{d\Gamma_{\eta(\eta')}}{ds_{\pi\pi}} \propto \left| C + \frac{1}{s_{\pi\pi} - m_\rho^2 - i m_\rho \Gamma_\rho} \right|^2$$

$\eta/\eta' \to \pi^+\pi^-\gamma$

$$\frac{d\Gamma}{ds} = |A(1+\alpha s+\ldots)F_V(s)|^2 K_P(s)$$

PLB707 (2012) 184

$e^+e^- \to \pi^+\pi^-$



$\eta \to \pi^+\pi^-\gamma$

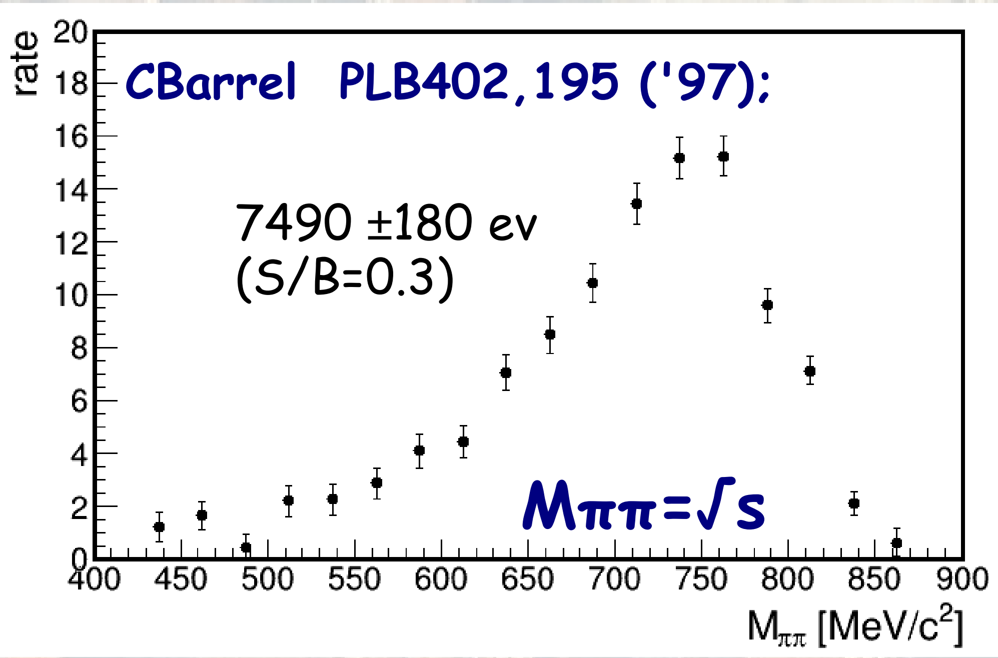
$\alpha = 1.89 \pm 0.25_{stat} \pm 0.59_{syst}$ GeV^{-2}

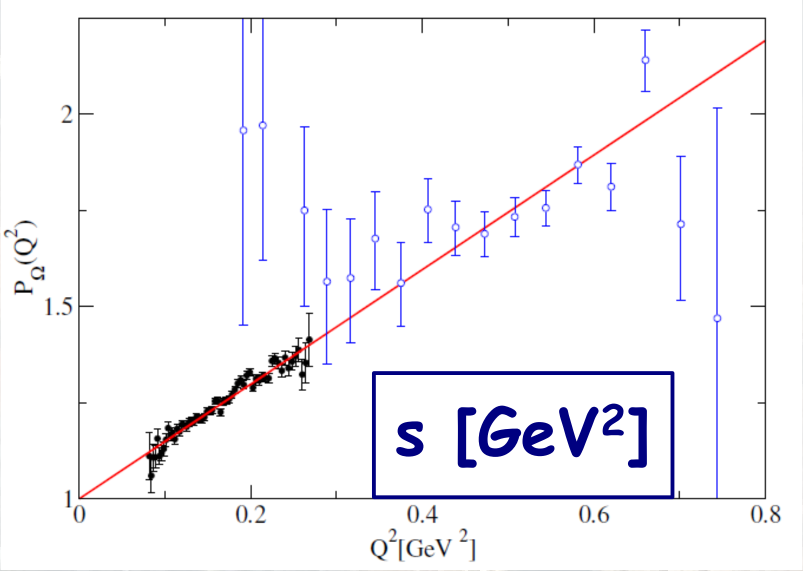
WASA PLB707 (2012) 243

$\alpha = 1.31 \pm 0.08_{stat} \pm 0.40_{syst}$ GeV^{-2}

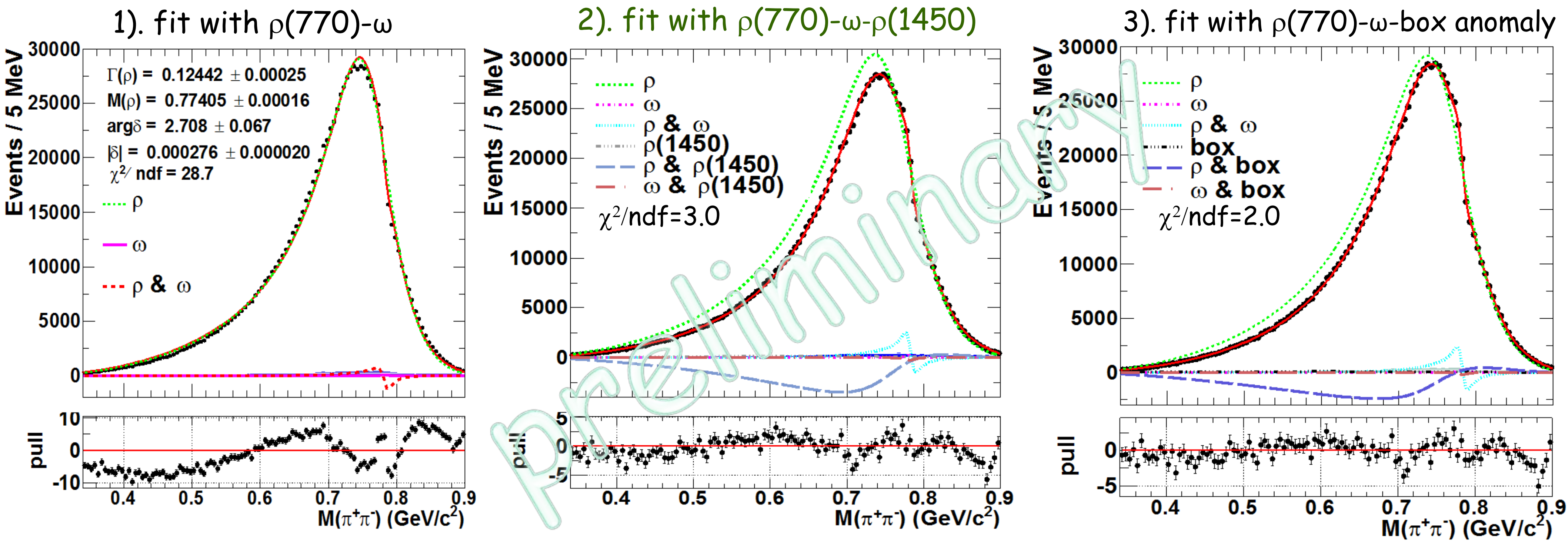
KLOE PLB718 (2013) 910

$\eta' \to \pi^+\pi^-\gamma$





Model-dependent fit

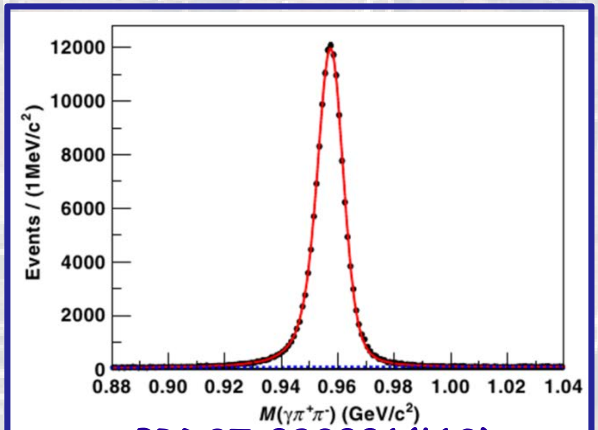


$$\frac{d\Gamma}{dm} \propto k_\gamma^3 q_\pi^3(m) \left| \frac{BW_\rho^{GS}(1+\delta\frac{m^2}{m_\rho^2}BW_\omega) + \beta \cdot BW_{\rho'}^{GS}}{1+\beta} + \frac{E_{\eta'}}{F_{\eta'}} \right|^2$$

- ✓ Beside ρ(770) resonance, ω is needed
- ✓ ρ(770)-ω is not enough;
- ✓ Extra contribution (maybe ρ(1450) or box-anomaly, maybe both of them) is also necessary

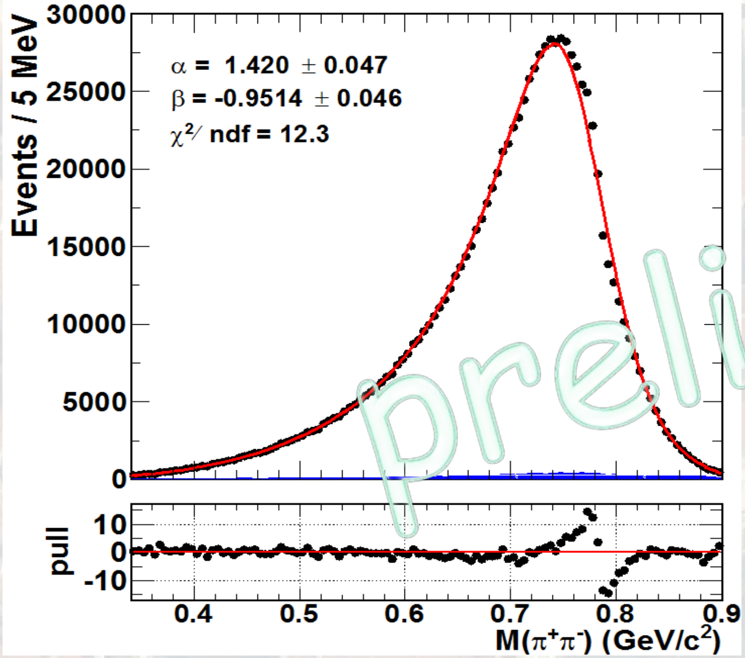
Prel. analysis based on 9.7 × 10⁵ η → π+π-γ

$$\frac{d\Gamma}{ds_{\pi\pi}} = \left|AP(s_{\pi\pi})F_V(s_{\pi\pi})\right|^2 \Gamma_0(s_{\pi\pi})$$

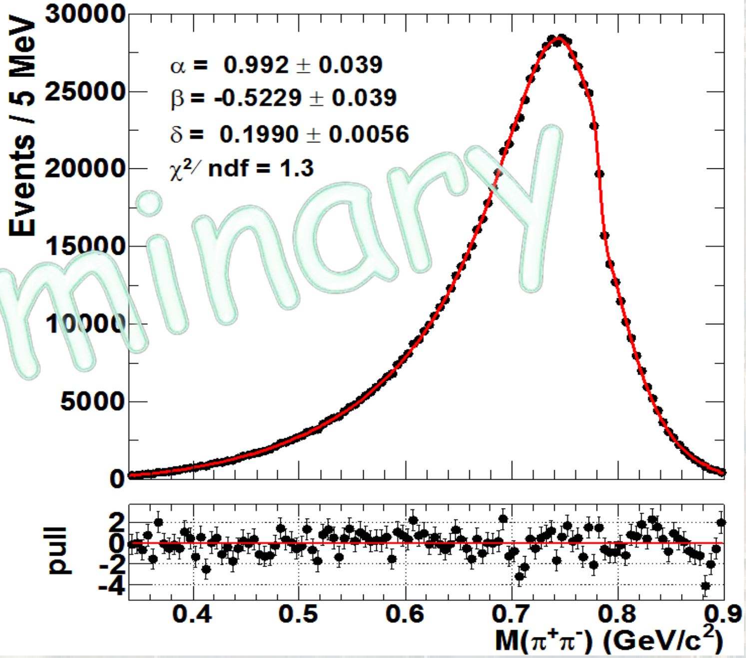


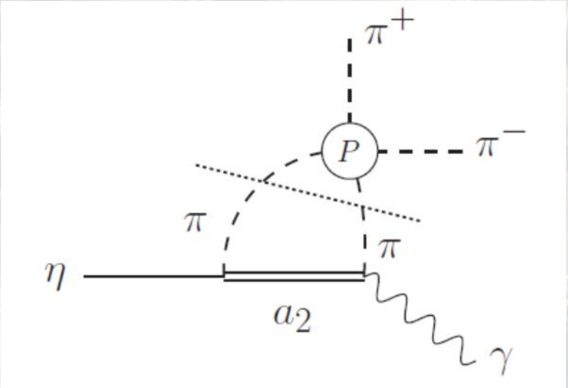
PRD87,092001('13)

$P(s_{\pi\pi}) = 1 + \alpha\, s_{\pi\pi} + \beta\, s^2_{\pi\pi}$



$P(s_{\pi\pi}) = 1 + \alpha\, s_{\pi\pi} + \beta\, s^2_{\pi\pi} + \delta\, BW_\omega$



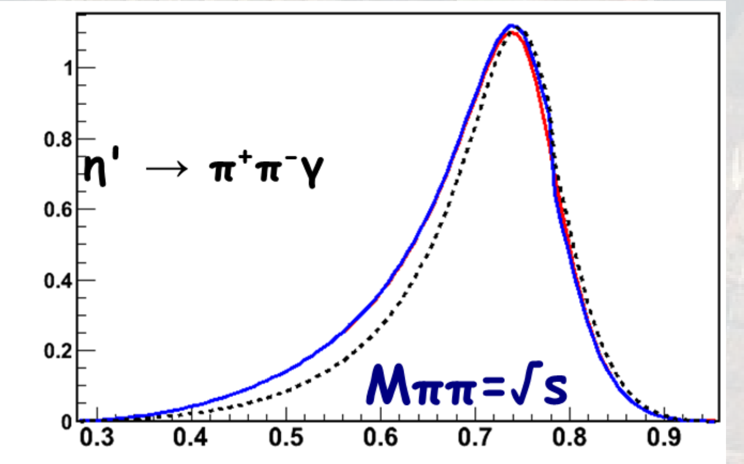


ω contribution necessary

Linear polynomial is insufficient…

Crystal Barrel: $\alpha = (1.80 \pm 0.49 \pm 0.04)\,\mathrm{GeV}^{-2}$
$\beta = (0.04 \pm 0.36 \pm 0.03)\,\mathrm{GeV}^{-4}$

GAMS-2000: $\alpha = (2.7 \pm 1.0)\,\mathrm{GeV}^{-2}$



Analysis: Liqing QIN SDU,IHEP

π^0,η,η' Transition Form Factors (TFF)

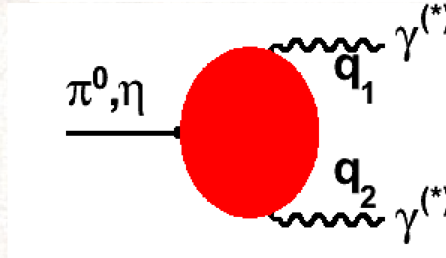
Low energy QCD

l^+l^- spectra

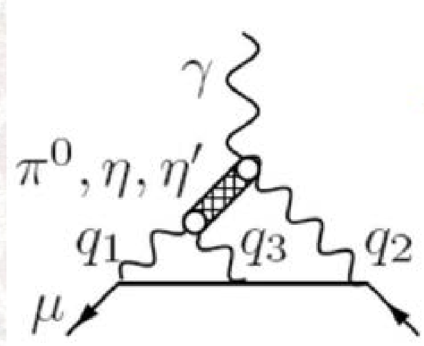
dark photon (U boson)

$F\pi^0$ $q^2\rightarrow\infty$ puzzle: BaBar

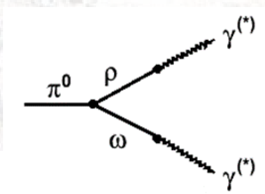
$$\Gamma(P \rightarrow \gamma\gamma)$$

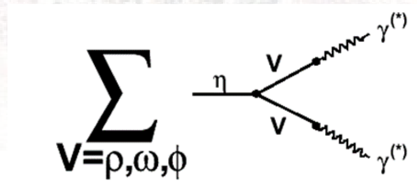


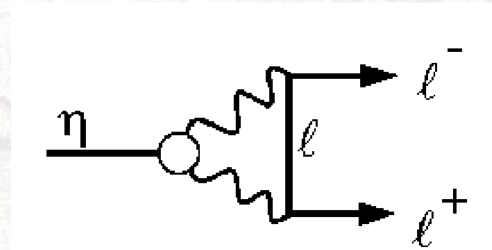
$$F_P(q_1^2, q_2^2)$$



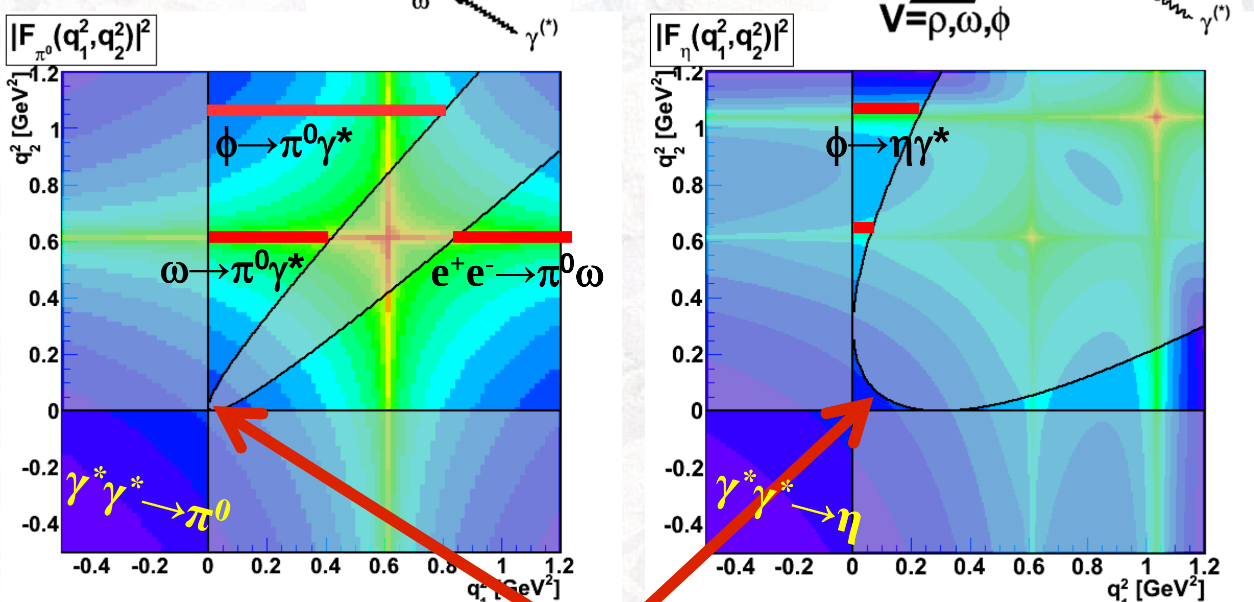
HLbL for $a\mu$



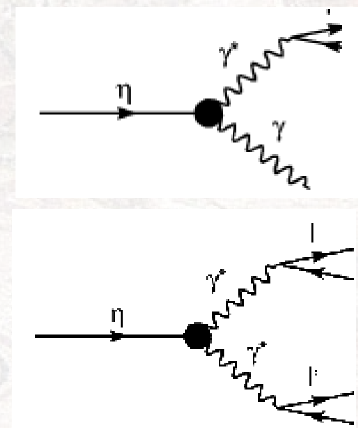




Access to $q^2<0$

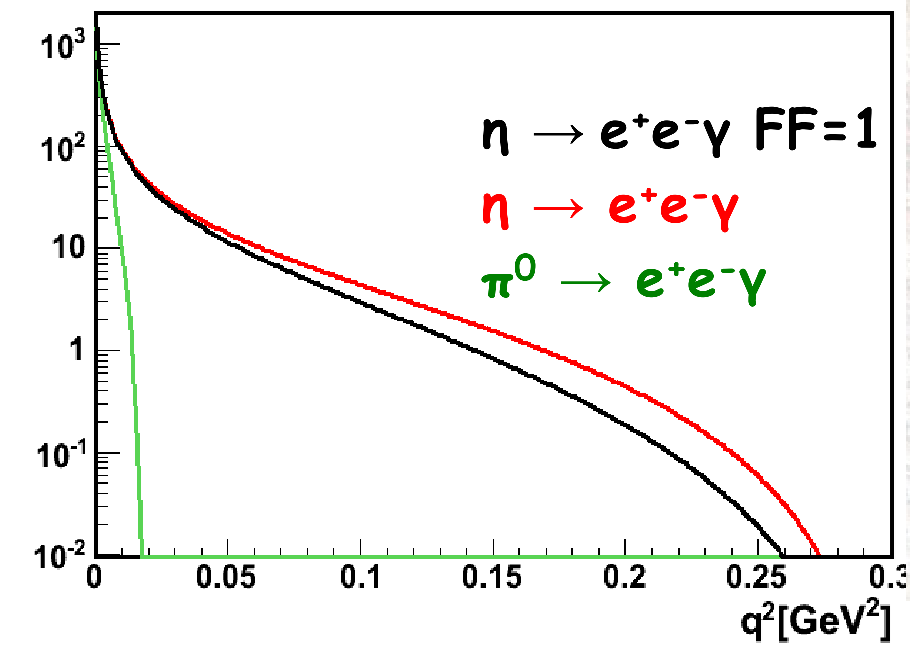


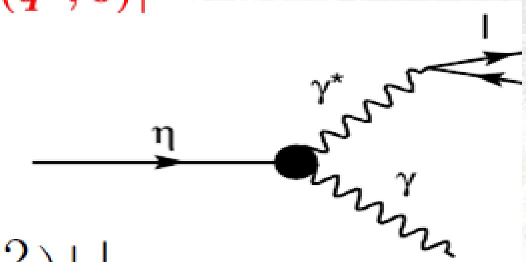
$P\rightarrow\gamma^*\gamma^*$



Single Dalitz decays

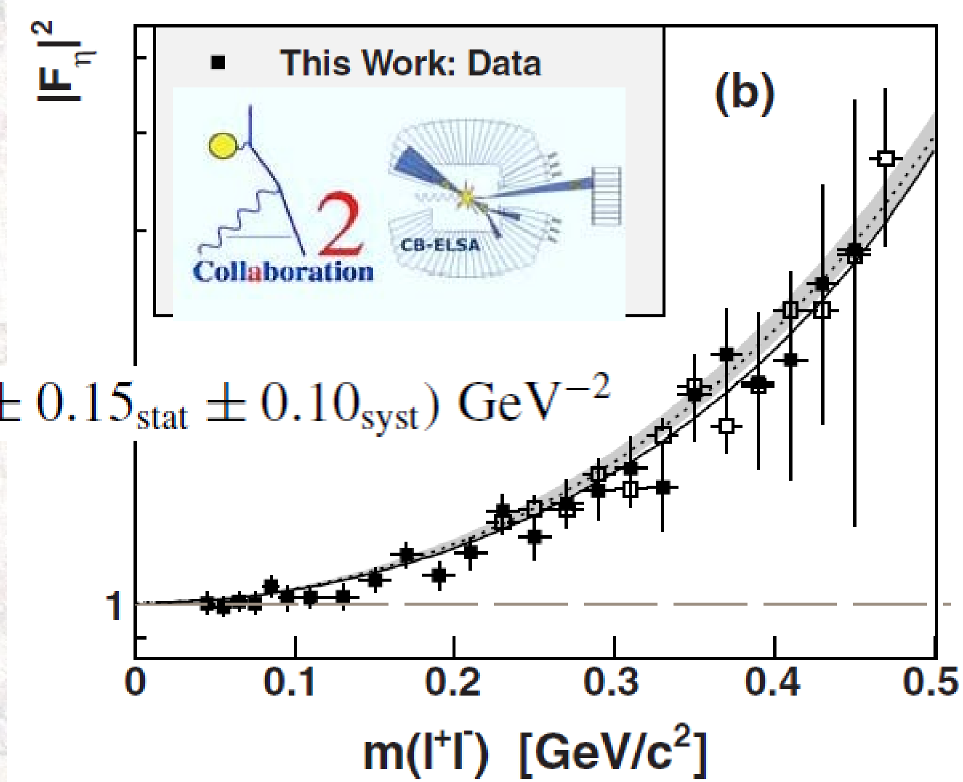
$$\frac{d\Gamma(P \to \ell^+\ell^-\gamma)}{dq^2\Gamma_{\gamma\gamma}} = \frac{2\alpha}{3\pi}\frac{1}{q^2}\sqrt{1-\frac{4m_\ell^2}{q^2}}\left(1+\frac{2m_\ell^2}{q^2}\right)\left(1-\frac{q^2}{M_P^2}\right)^3 |F_P(q^2,0)|^2$$



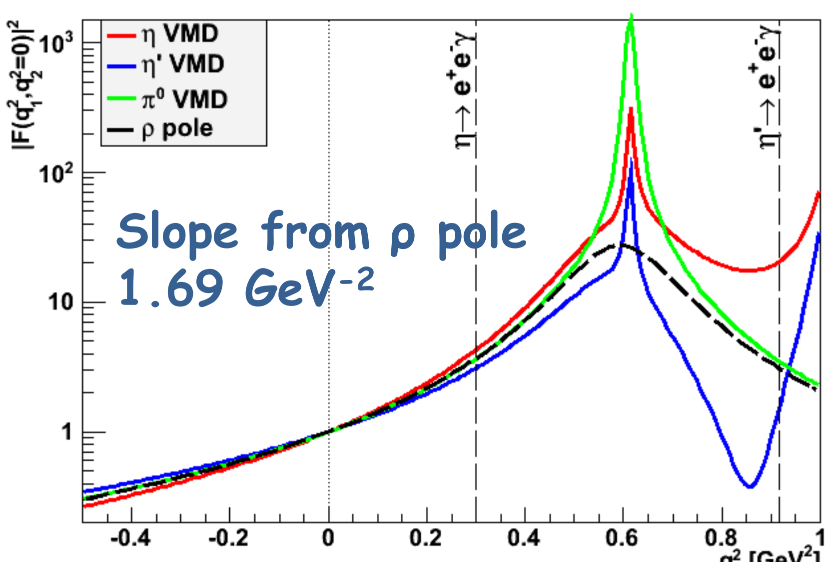


$$b_P = \left.\frac{d\ln|F_P(q^2)|}{dq^2}\right|_{q^2=0}$$

CB/TAPS: PRC89, 044608 (2014)

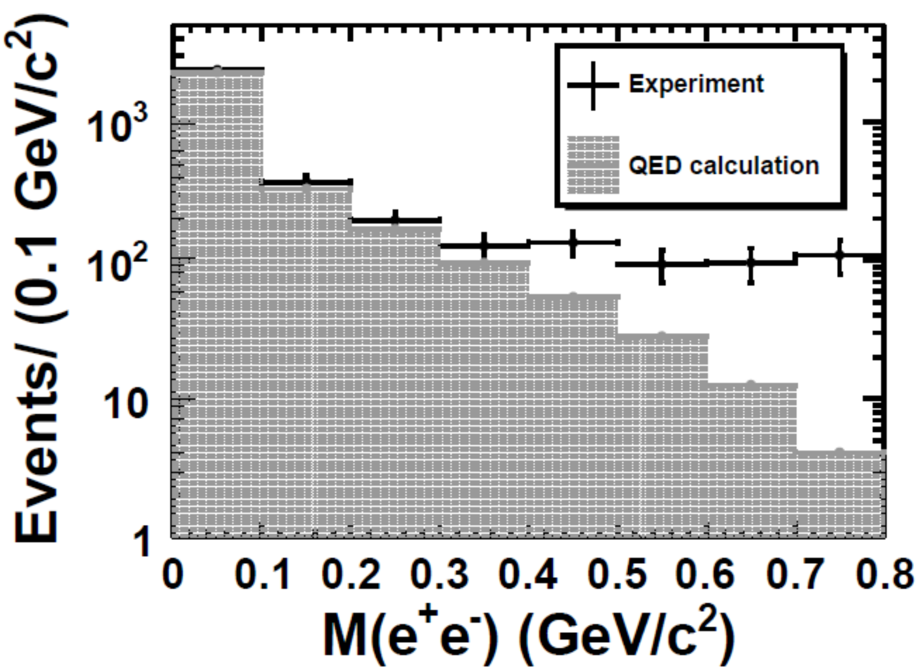


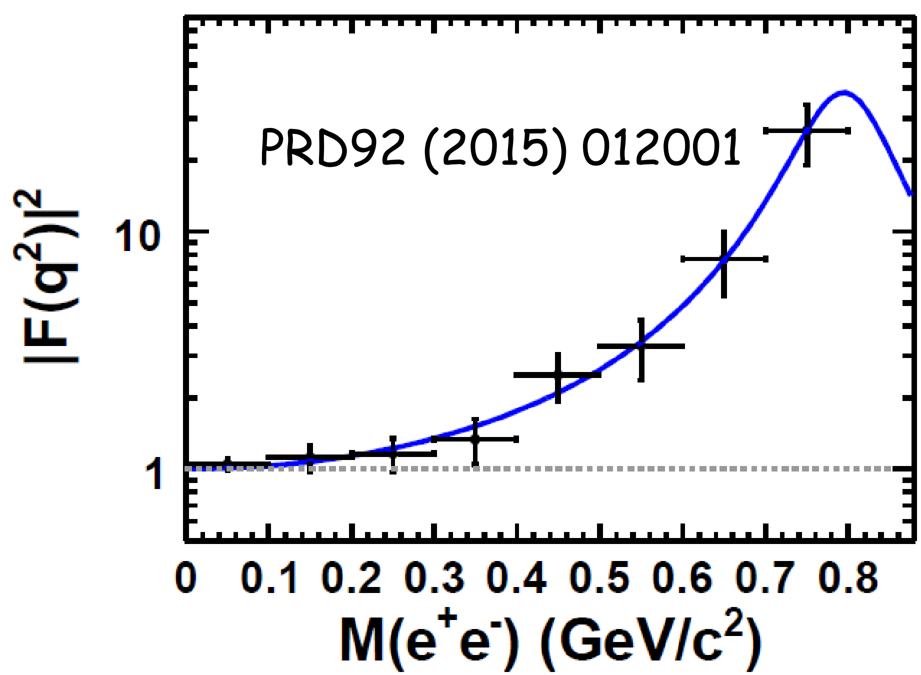
$(1.95 \pm 0.15_{\rm stat} \pm 0.10_{\rm syst})\ {\rm GeV}^{-2}$

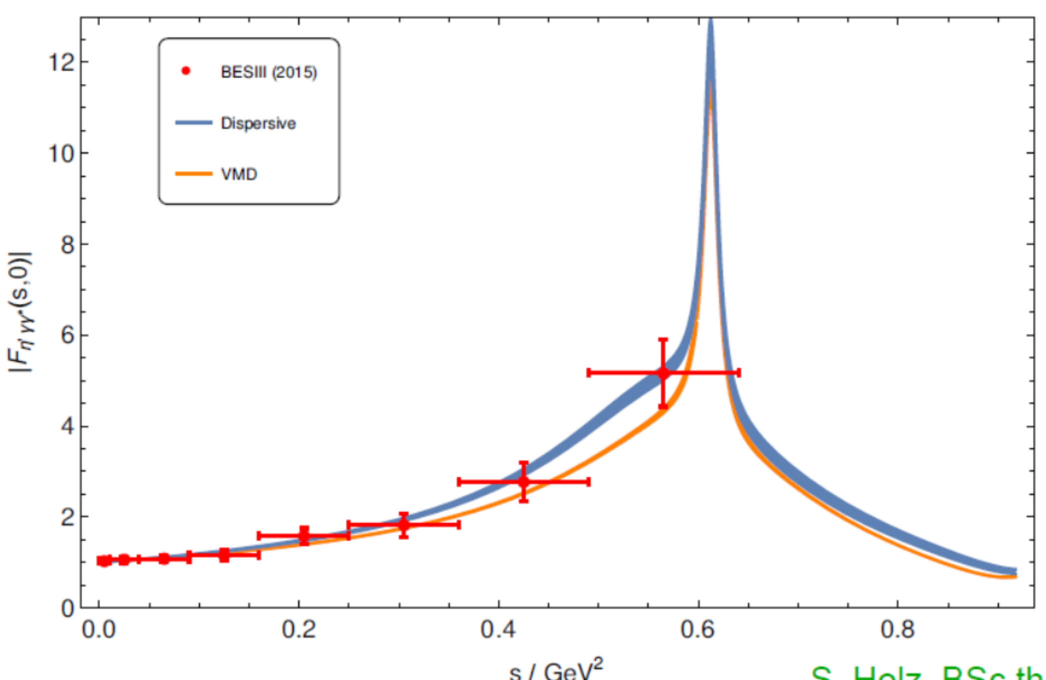


Slope from ρ pole
1.69 GeV^{-2}

Transition form factor η′→γe+e-

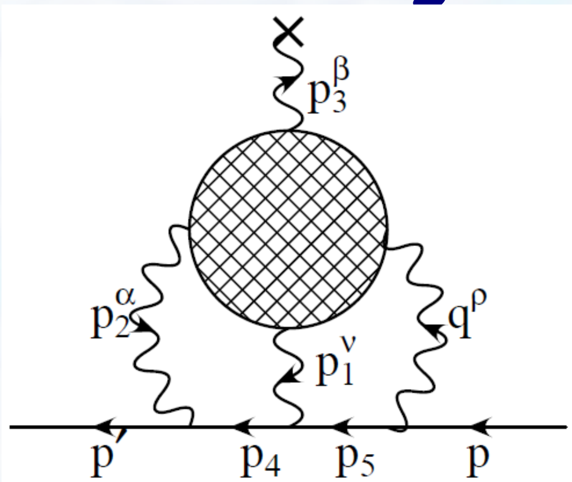




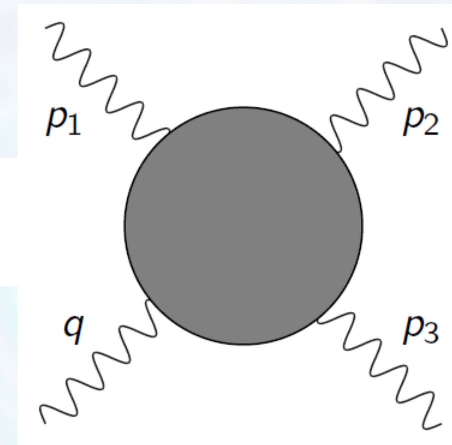


S. Holz, BSc thesis 2016

g-2 Hadronic Light by Light



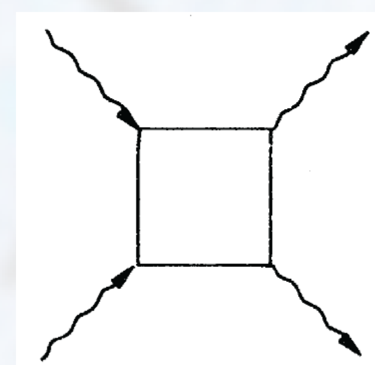
$$\Pi^{\rho\nu\alpha\beta}(p_1, p_2, p_3) =$$



In general 138 Lorentz structures
(only 28 contribute to a_μ)
vs HVP: one function, one variable

Low and high energy mixed
Hadrons vs quarks

γγ measurement?
-- dominated by lepton contribution

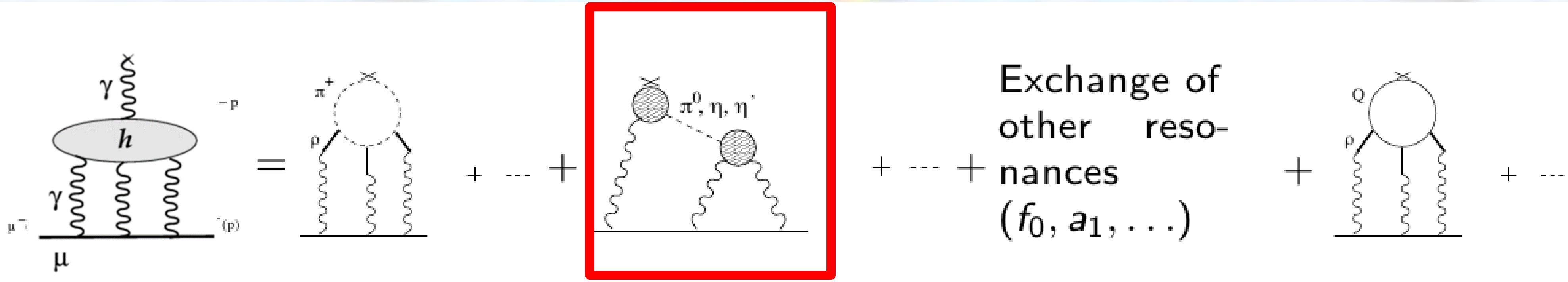


Slide: J. Bijnens

Hadronic Light by Light

"must be calculated using **hadronic models** that correctly reproduce properties of QCD"

$$a_\mu^{\mathrm{HLbL}} = 10.5(2.6) \times 10^{-10}$$



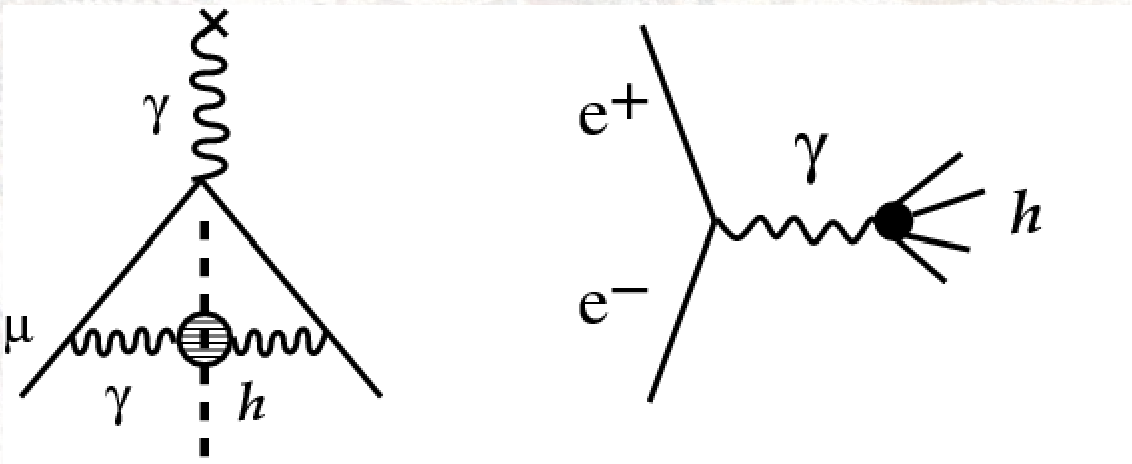
Chiral counting:	p^4	p^6	p^8	p^8
N_C-counting:	1	N_C	N_C	N_C
	$\pi^\pm$,K-loop	**π^0, η,η' exchange**		**quark-loop**
PdRV	$(-1.9\pm1.9)\cdot10^{-10}$	$(11.4\pm1.3)\cdot10^{-10}$	$(0.8\pm1.1)\cdot10^{-10}$	$0.23\cdot10^{-10}$

E. de Rafael, "Hadronic contributions to the muon g-2 and low-energy QCD," Phys. Lett. **B322** (1994) 239-246. [hep-ph/9311316].

Slide: J. Bijnens, A.Nyffeler

Hadronic contribution to a_μ

HVP

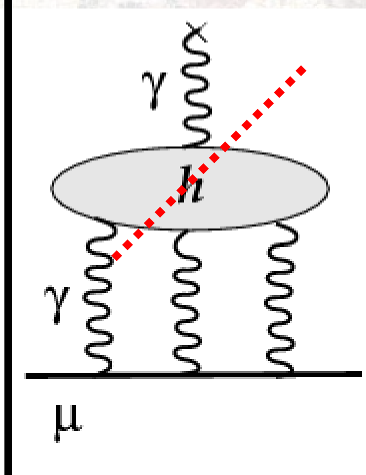


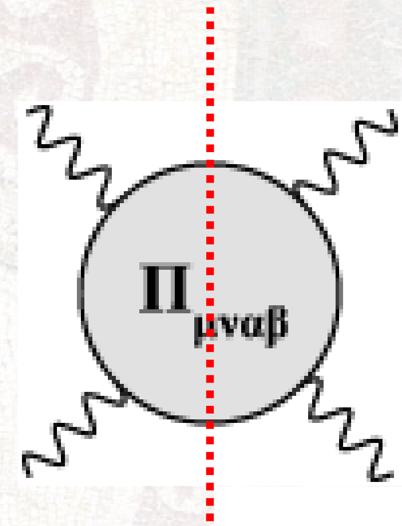
$e^+e^- \rightarrow \gamma^* \rightarrow h$

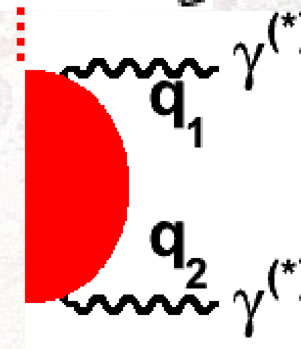
KLOE-2
CMD3/SND
BESIII
BelleII

Goal: reduce Δa_μ (10^{-11})			
HVP	42	→	11
HLbL	39/26	→	10

HLbL







Hadrons

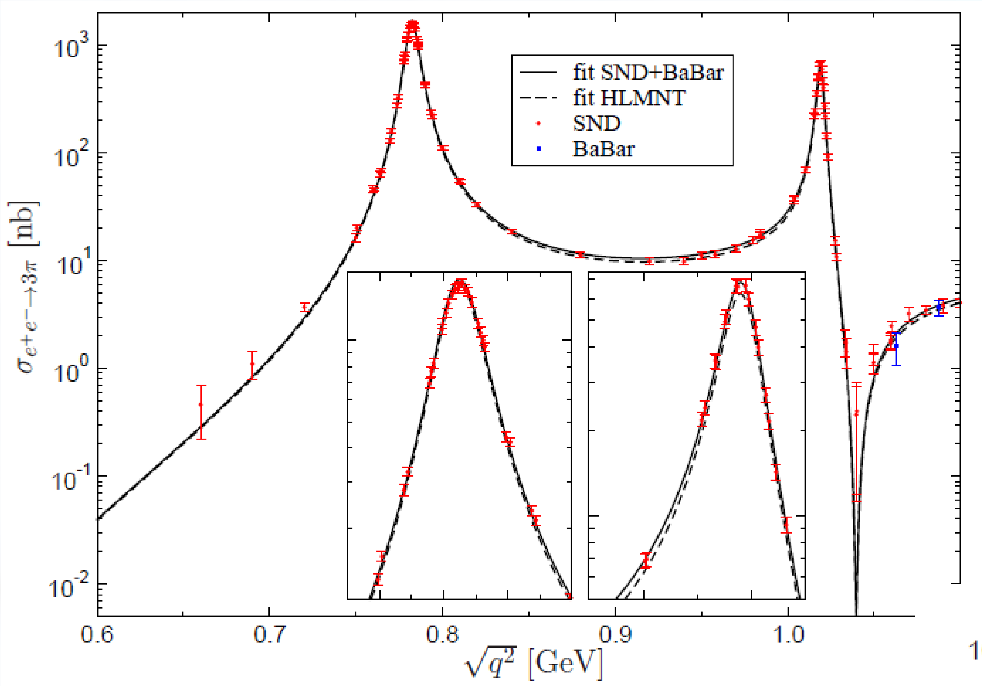
$\gamma^{(*)}\gamma^{(*)} \leftrightarrow h$

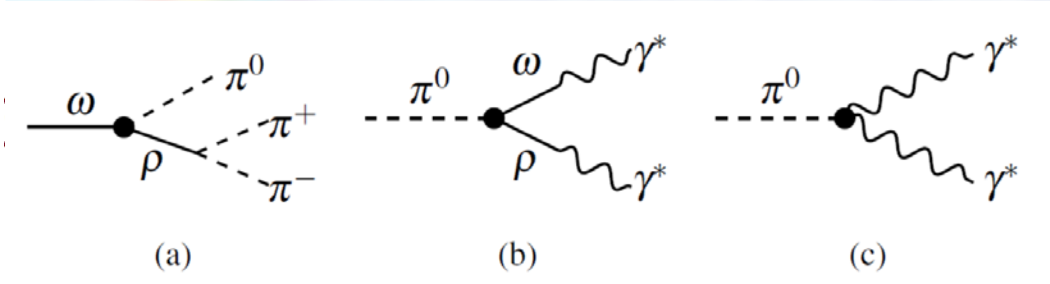
$\gamma^{(*)} \rightarrow h\gamma^{(*)}$

+ hadro- photo-production exp

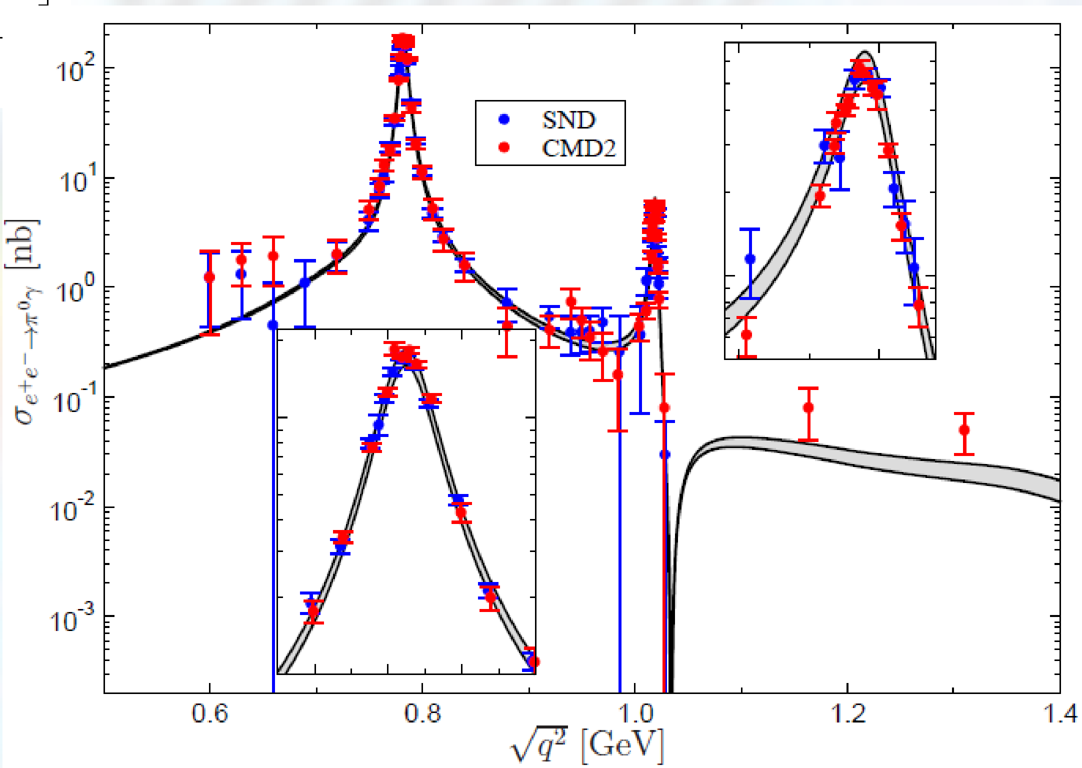
$m_h < 1-2$ GeV

From e+e- → π+π-π^0 to π^0 TFF





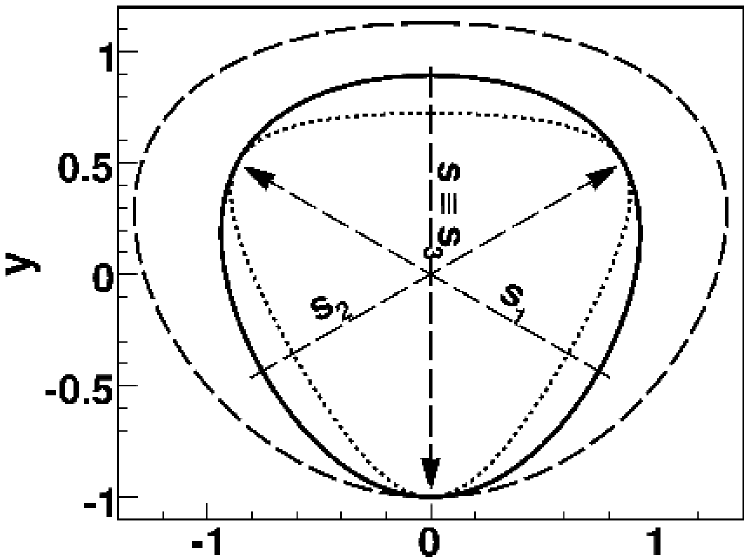
ππ phase shifts + e+e- → 3 π data
Eur.Phys.J. C74 (2014) 3180



Similar strategy for η
From e+e- → π+π-η to η TFF

arXiv:1509.02194

Three body decays, Dalitz plot



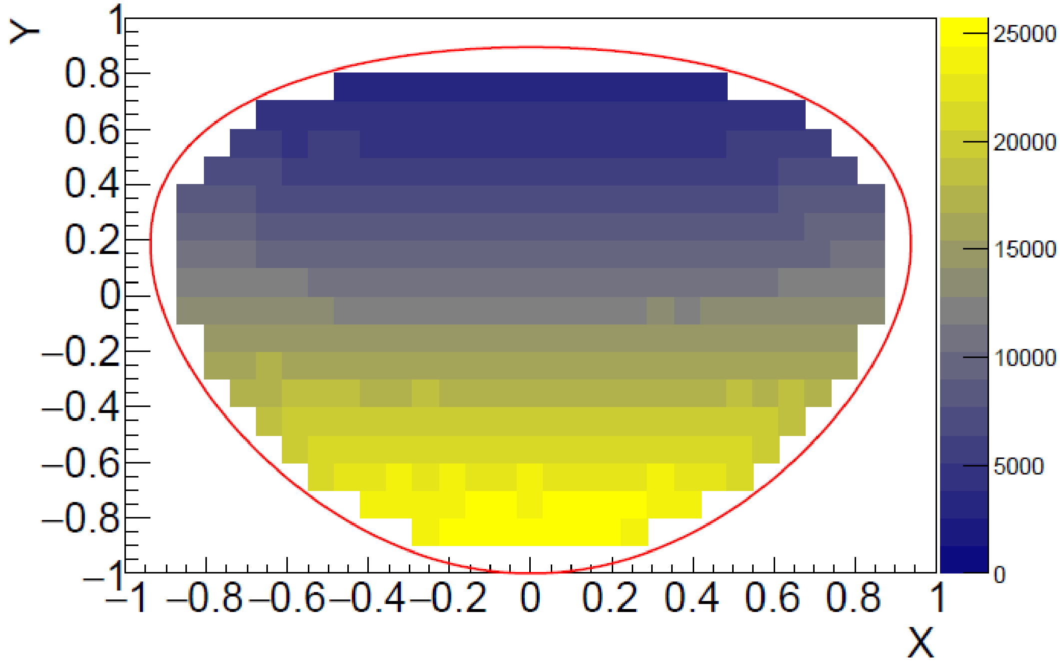
$$x \equiv \frac{1}{\sqrt{3}} \frac{T_1 - T_2}{\langle T \rangle}; \quad y \equiv \frac{1}{3} \left(\sum_{i=1}^{3} \frac{m_i}{m} \right) \frac{T_3}{\langle T \rangle} - 1$$

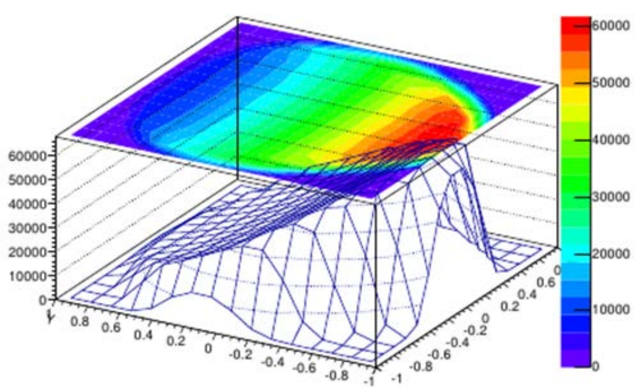
3 body decay: $0 \rightarrow 1 + 2 + 3$
$s_i \equiv (p_0 - p_i)^2 = (m_0 - m_i)^2 - 2T_i m_0$

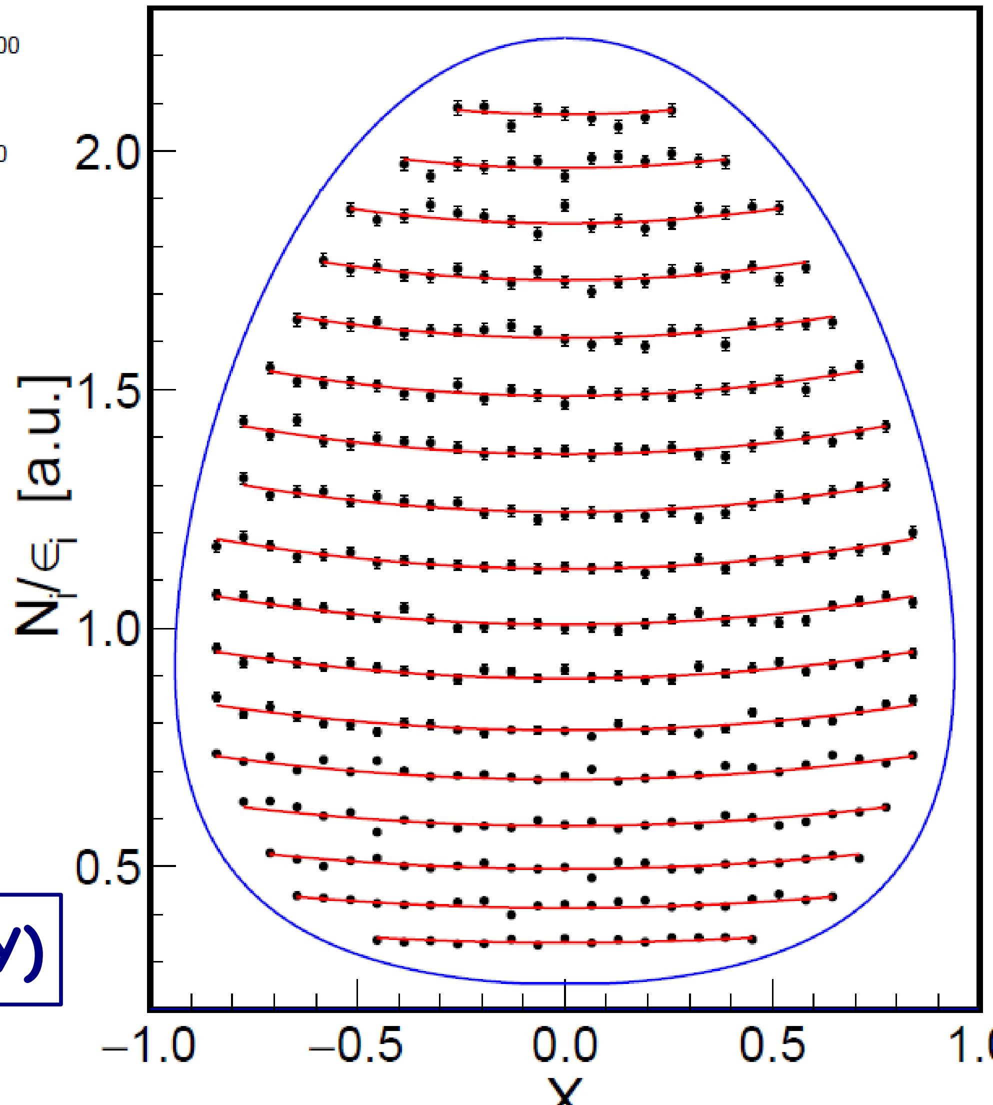
$\eta' \rightarrow \pi^+ \pi^- \eta$

$$\frac{d\Gamma}{ds_{12}\, ds_{23}} = \frac{1}{(2\pi)^3} \frac{1}{32 m^3} |\mathcal{M}|^2$$

$\eta \rightarrow \pi\pi\pi$



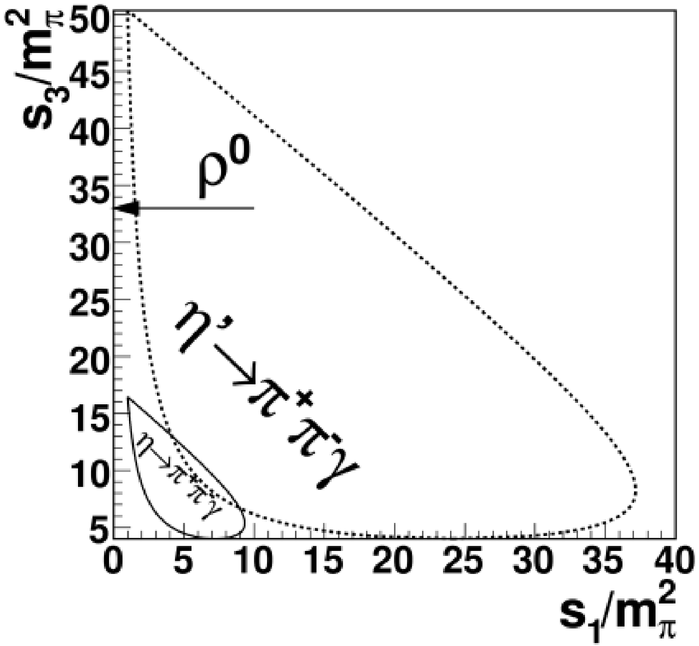




$$|A(X,Y)|^2 = N(1 + aY + bY^2 + dX^2 + fY^3 + gX^2Y)$$

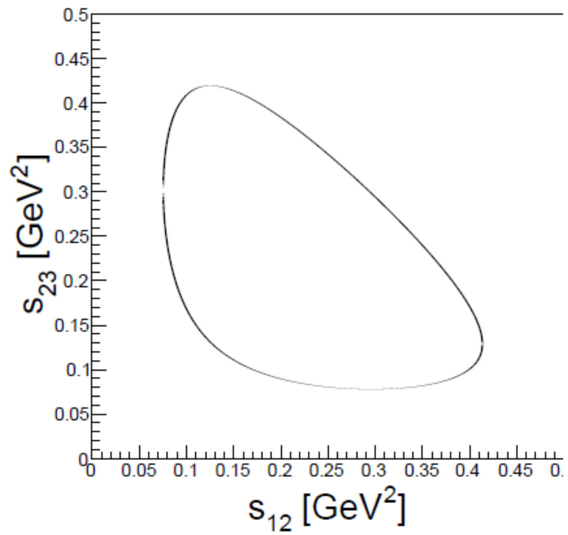
KLOE-2 JHEP 1605 (2016) 019

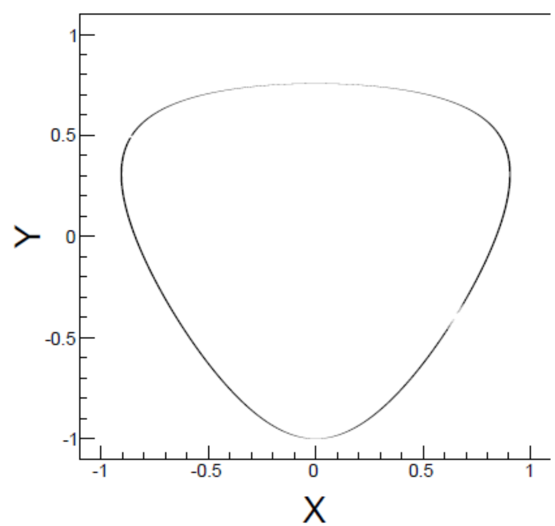
$\eta/\eta' \rightarrow \pi^+\pi^-\gamma$

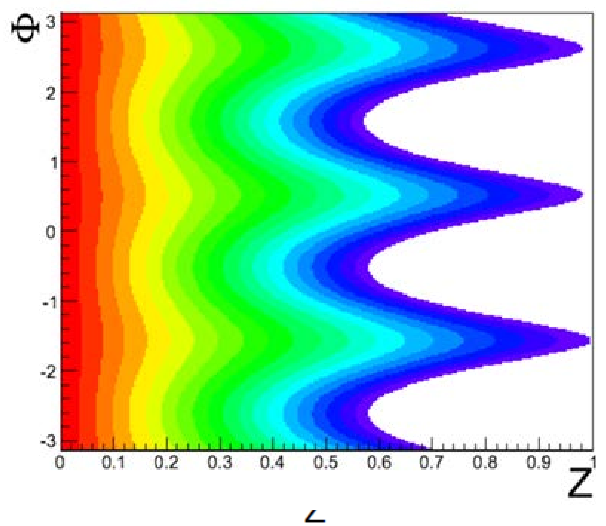


$$\Gamma_1(s) = \frac{4}{3}\left(\frac{m_{\eta'}^2 - s}{16\pi m_{\eta'}}\sqrt{s - 4m_\pi^2}\right)^3$$

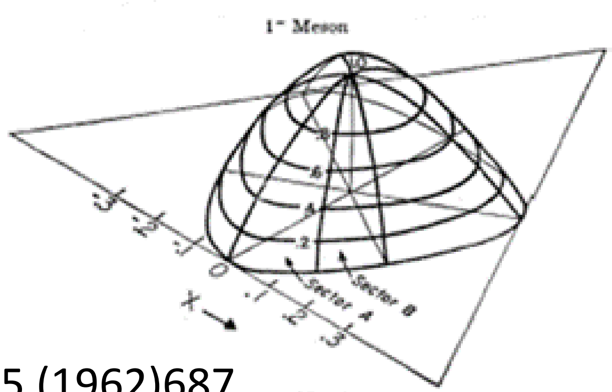
Dalitz plot for $\omega \to \pi^+\pi^-\pi^0$







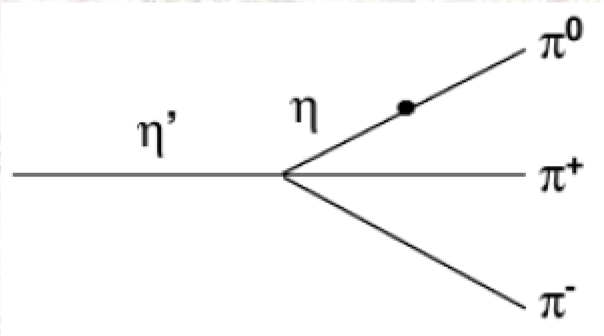
$$F(Z,\Phi) = \mathscr{P}\cdot\left\{1+2\alpha Z+2\beta Z^{3/2}\sin 3\Phi+2\gamma Z^2+\mathcal{O}(Z^{5/2})\right\}$$



PR,125 (1962)687

(c)

$\eta' \to \pi\pi\pi$



d-u quark masses

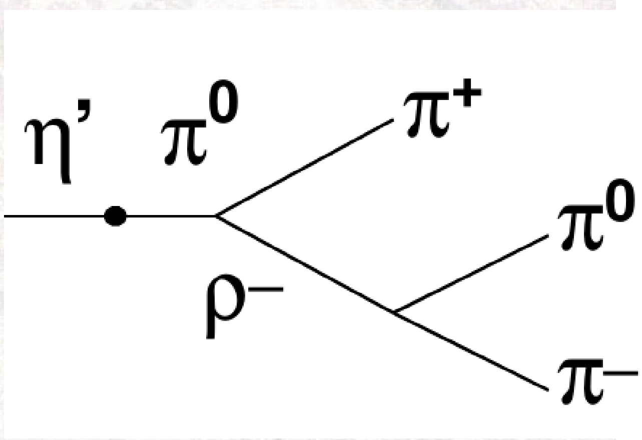
$$\frac{BR(\eta' \to \pi^+\pi^-\pi^0)}{BR(\eta' \to \pi^+\pi^-\eta)} \text{ and } \frac{BR(\eta' \to \pi^0\pi^0\pi^0)}{BR(\eta' \to \pi^0\pi^0\eta)}$$

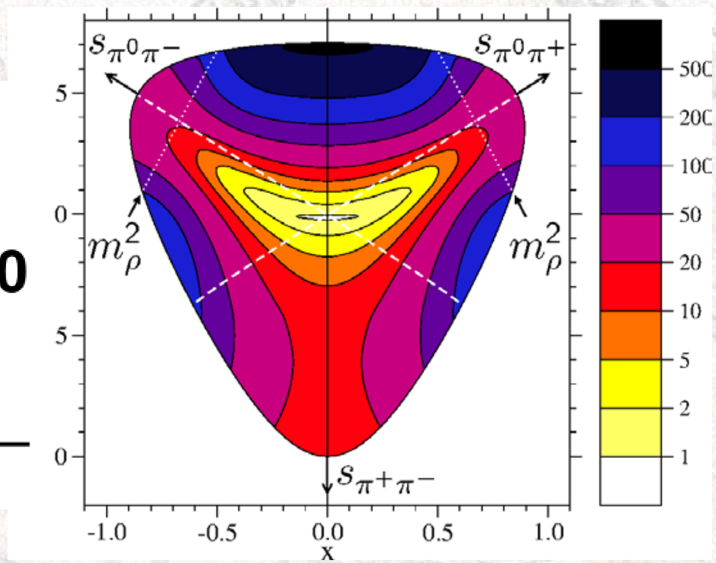
Gross,Treiman,Wilczek PRD19,2188(1979)

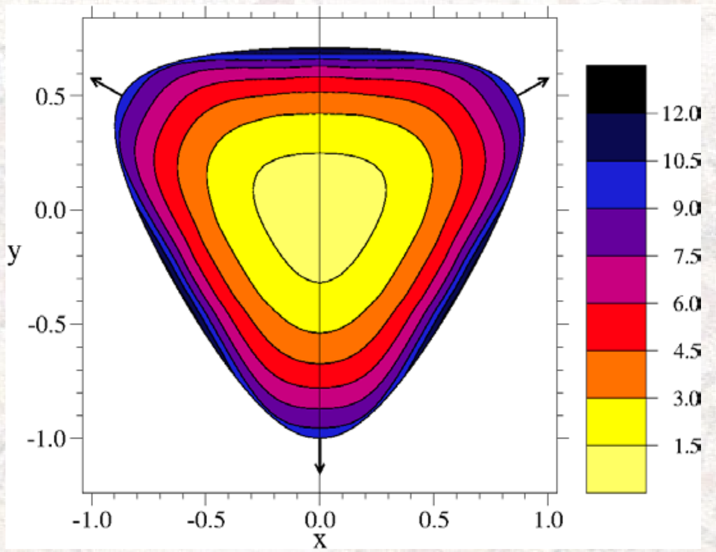
Difficult/dubious:
- other tree diagrams
- rescattering

U(3) CHPT, Borasoy, Nißler 2005:
$BR(\eta \to \pi^+\pi^-\pi^0) \approx 1.8\%$ large $\rho^+\pi^-$ + cc

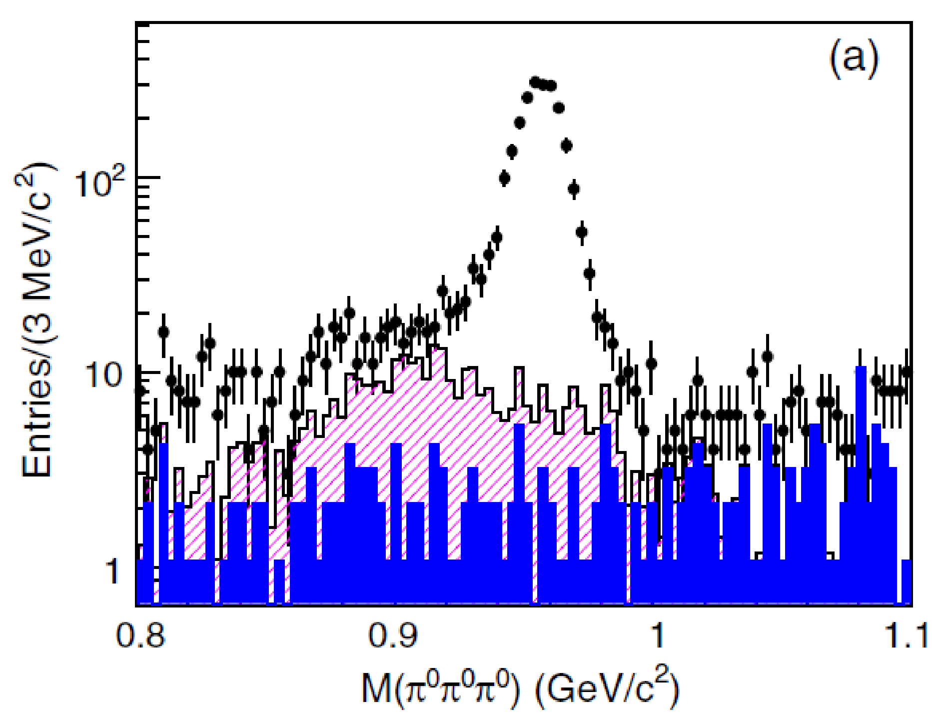
Borasoy, Nißler, Meißner 06

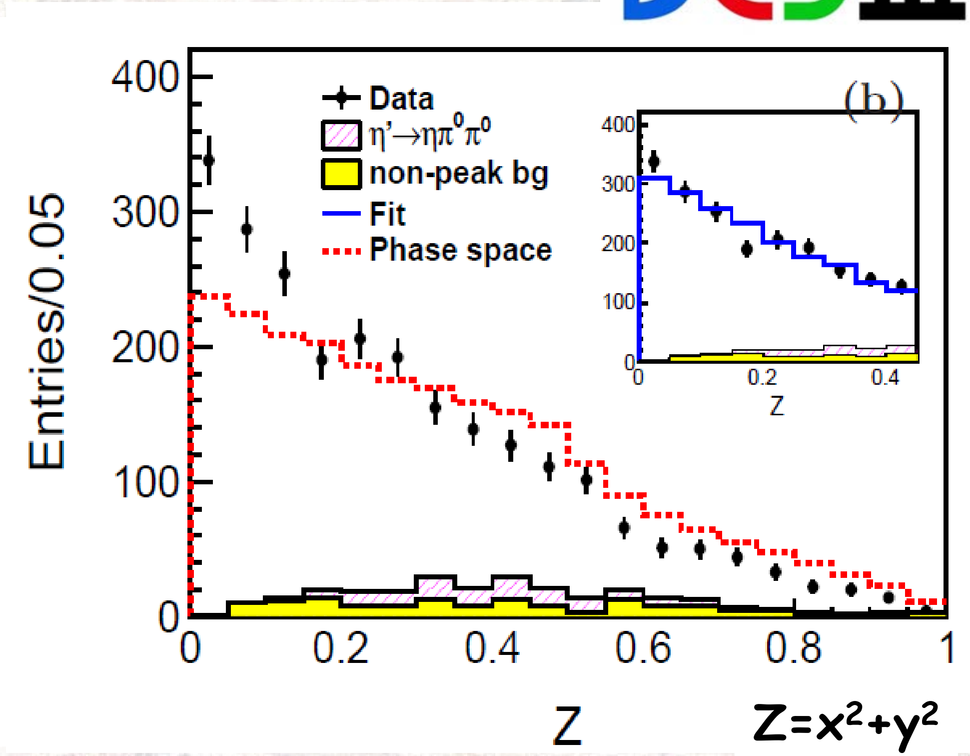






$\eta' \to \pi^0\pi^0\pi^0$

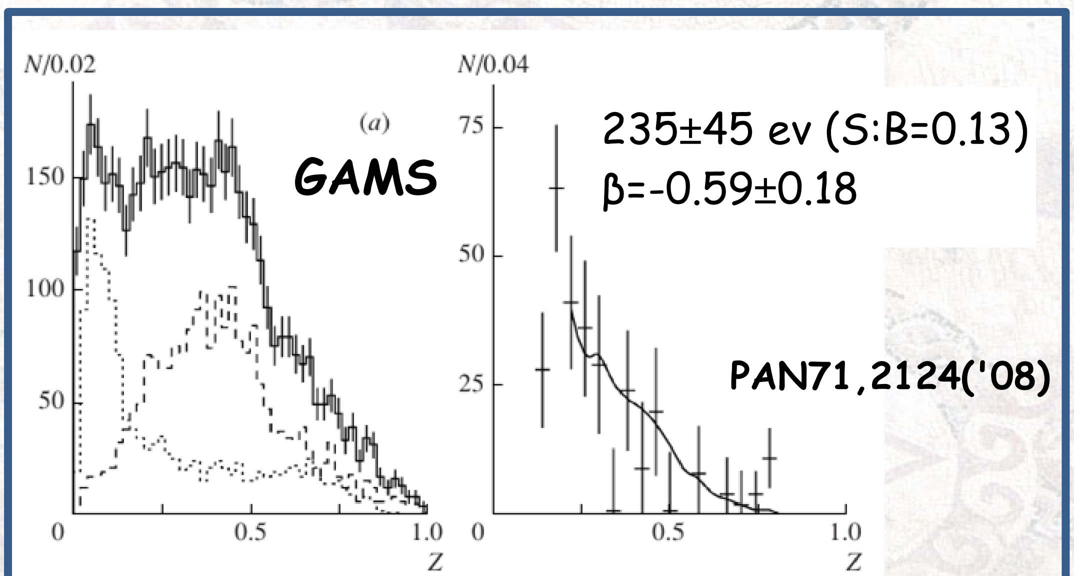




1900 ev

PRD92 ('15) 012014

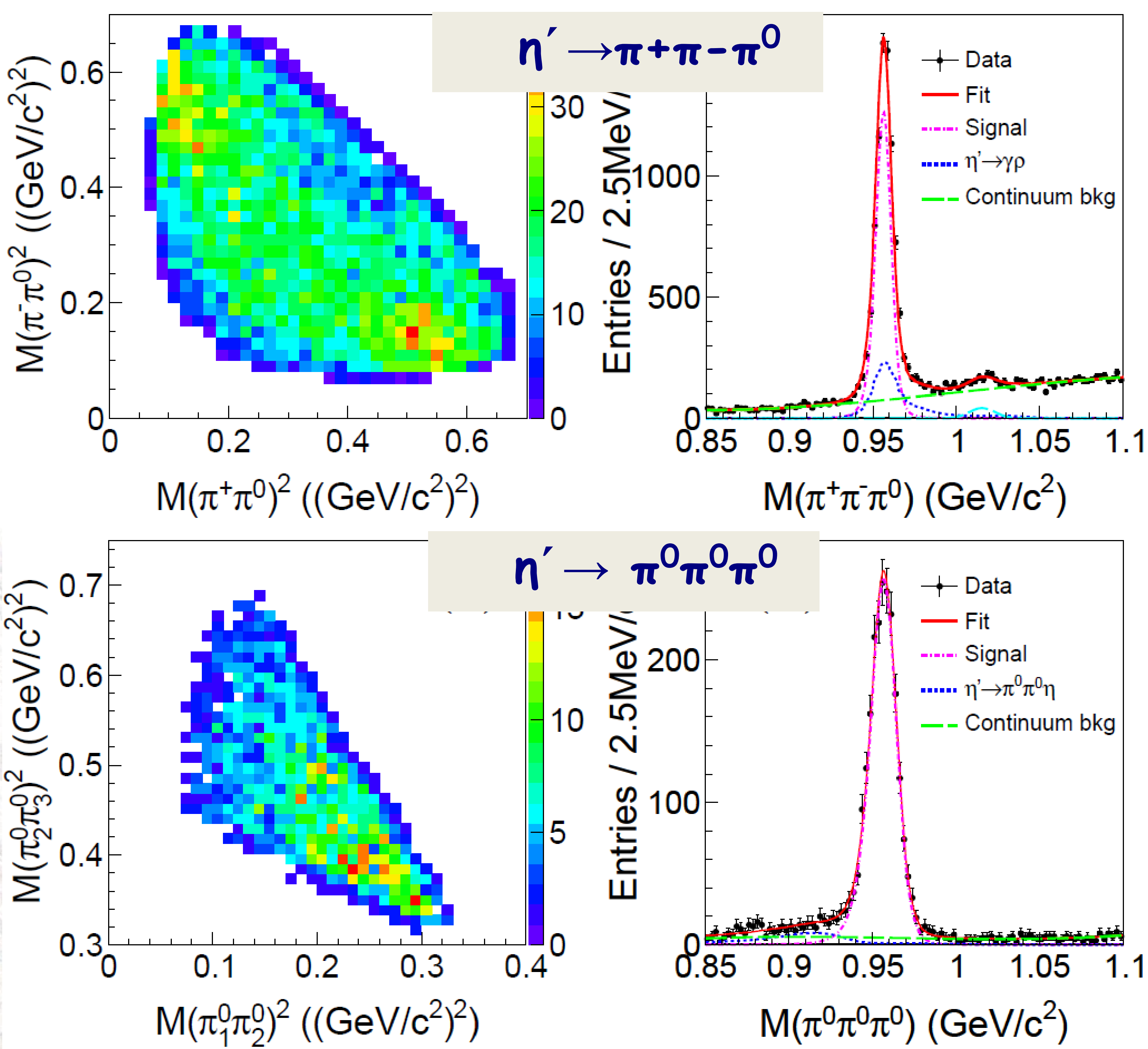
β=−0.640± 0.046±0.047



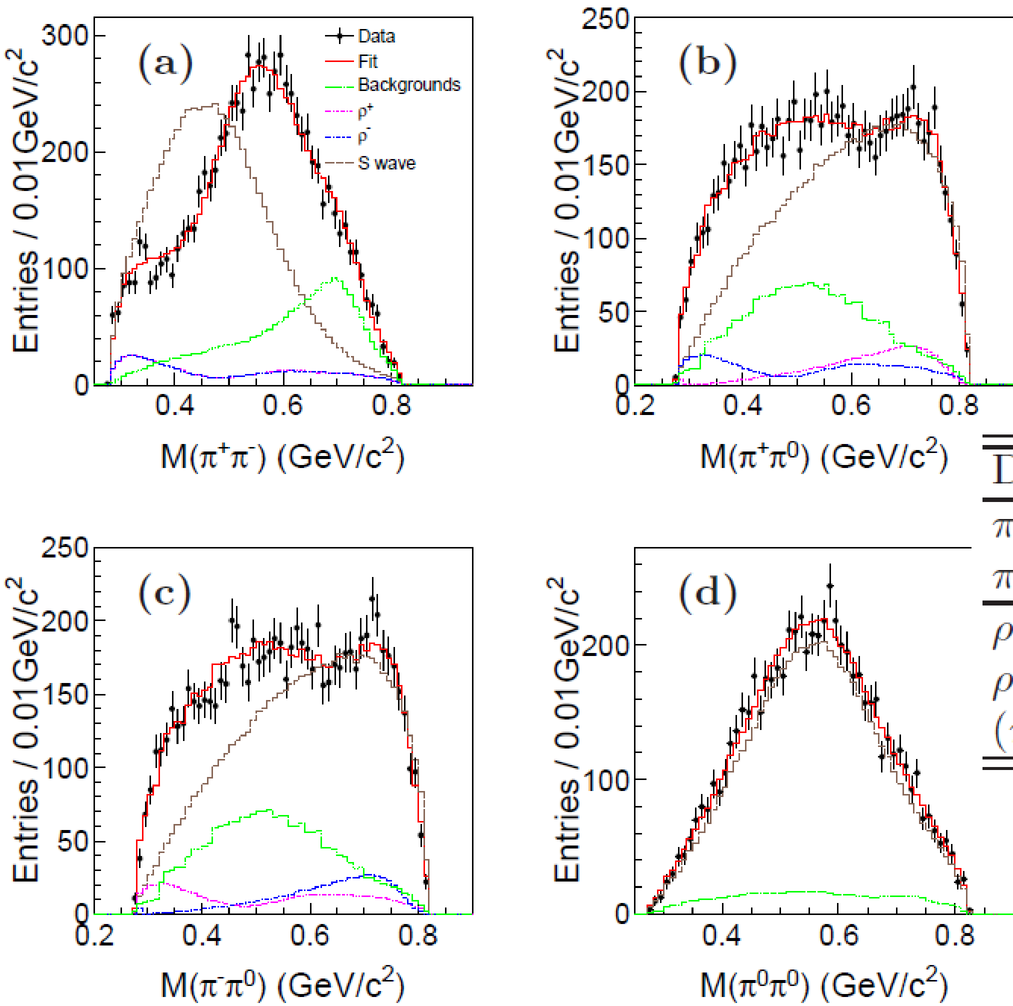
$$|M|^2 \propto 1 + 2\beta Z$$

Analysis: Xiaolin KANG, IHEP

BESIII, arXiv 1606.03847



PWA $\eta' \to \pi\pi\pi$



BESIII

arXiv 1606.03847

Decay Mode	Yield	ε (%)	$\mathcal{B}$ $(\times 10^{-4})$
$\pi^+\pi^-\pi^0$	6067 ± 91	25.3	$35.91 \pm 0.54 \pm 1.74$
$\pi^0\pi^0\pi^0$	2015 ± 47	8.8	$35.22 \pm 0.82 \pm 2.60$
$\rho^+\pi^-$	616 ± 49	24.8	$3.72 \pm 0.30 \pm 0.63 \pm 0.92$
$\rho^-\pi^+$	615 ± 49	24.7	$3.72 \pm 0.30 \pm 0.63 \pm 0.92$
$(\pi^+\pi^-\pi^0)_S$	6580 ± 134	26.2	$37.63 \pm 0.77 \pm 2.22 \pm 4.48$

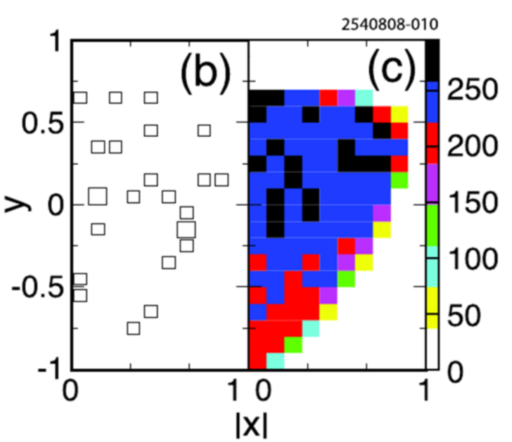
$B(\eta' \to \rho^+\pi^- + cc) = (7.44 \pm 0.60 \pm 1.26 \pm 1.84)\times 10^{-4}$

$B(\eta' \to \pi^0\pi^0\pi^0)$ puzzle

$B(\eta' \to \pi^0\pi^0\pi^0) / B(\eta' \to \eta\pi^0\pi^0)$
from GAMS ('84,'87,'08)
$(78\pm10) \times 10^{-4}$

vs

BESIII $(159\pm12) \times 10^{-4}$



CLEO

PRL 102,061801('09)

20±5 ev.

BR= 3.7 ± 1.0 10^{-3}

e+e- ->J/ψ-> $\Lambda\bar{\Lambda}$

Use spin correlations and polarization to extract hyperon decay parameters and test CP for baryons

Revise assumption that hyperons from decays are unpolarized

Göran Fäldt, AK arXiv:1702.07288