The electromagnetic $\Sigma^0 - \Lambda$ Transition Form Factor at low energies

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UPPSALA UNIVERSITET

2017 International Summer Workshop on Reaction Theory

Bloomington, Indiana, USA

June 12-22

Motivation

To further investigate the structure of matter one can try the phenomenal combo:

Electromagnetism
Form Factors (FF)

+

Strangeness
Hyperons (Y)

Hyperons are not stable!

Experimental information about hyperon form factors is rather limited

- Hyperon FFs are more easily accessible in the time-like region ($q^2 > 0$) for high and low energies via:
  - $e^+e^- \rightarrow Y_1 \bar{Y}_2$ reactions (BESIII)
  - $Y_1 \rightarrow Y_2\, e^+e^-$ Dalitz decays (PANDA, HADES)

Focus of this work:

Electric and magnetic transition form factor Σ^0 to Λ

- accessible by high-precision measurement of the decay $\Sigma^0 \to \Lambda e^+ e^-$ (possible @FAIR)

About experimental feasibility:

- FFs are functions of dilepton invariant mass q^2

  $\rightarrow$ not very large range available, $q^2 < (m_{\Sigma^0} - m_\Lambda)^2 \approx (77\ \text{MeV})^2$

  $\rightarrow$ high experimental precision required

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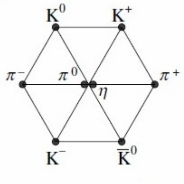
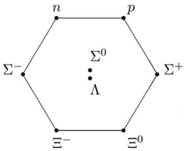
- Chiral Perturbation Theory (EFT) $\rightarrow$ pseudo-Goldstone bosons



$\rightarrow$ systematically improvable, reliable uncertainty estimate

Ingredients

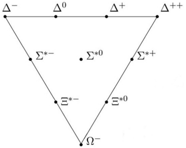
- Chiral Perturbation Theory (EFT) $\rightarrow$ pseudo-Goldstone bosons
  - Include baryon octet



Kubis/Meißner, Eur. Phys. J. C 18, 747 (2001)

Ingredients

- Chiral Perturbation Theory (EFT) $\rightarrow$ pseudo-Goldstone bosons
  - Include baryon octet
  - Include baryon decuplet



Jenkins/Manohar, Phys.Lett. B259, 353 (1991)

Pascalutsa/Vanderhaeghen/Yang, Phys.Rept. 437, 125 (2007)

Ledwig/Camalich/Geng/Vacas, Phys.Rev. D 90, 054502 (2014)

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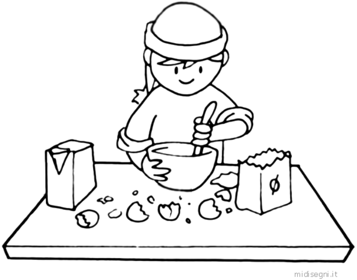
Q: What about other hadronic states, e.g. vector mesons?

A: It's unclear how to treat them in a systematic, model-independent way.

- ρ meson experimentally shows up in pion form factor and p-wave pion phase shift (well-known quantities)

Dispersion theory allows to combine these ingredients:



EFT + data

i.e. ChPT and extension thereof + F_π^V

A few words about dispersion relations

Consider the S-matrix $S = \mathbb{1} + \mathrm{i}\,\mathrm{T}$

Unitarity requires

$$SS^\dagger = \mathbb{1} + i(T - T^\dagger) + |T|^2 = \mathbb{1}$$

which implies that

$$2\,\mathrm{Im}\,T = |T|^2 \quad \longrightarrow \quad \boxed{\mathrm{Im}\,T_{A\to B} = \frac{1}{2}\sum_X T_{A\to X}\,T^\dagger_{X\to B}} \quad \text{Optical theorem}$$

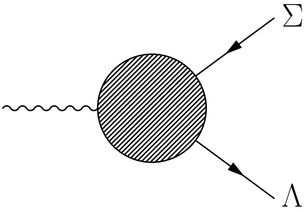
$\rightarrow$ consider only most relevant intermediate states X

Analyticity requires

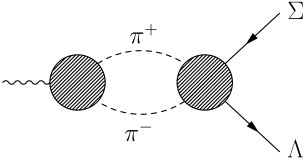
$$\boxed{T(q^2) = T(0) + \frac{q^2}{\pi}\int_{-\infty}^{+\infty} \mathrm{d}s\, \frac{\mathrm{Im}\,T(s)}{s\,(s - q^2 - i\epsilon)}}$$

$\rightarrow$ get the whole amplitude T from its imaginary part

Our paper in figures...



$\Sigma - \Lambda$ transition form factor



Two-pion exchange: dominant contribution at low energies

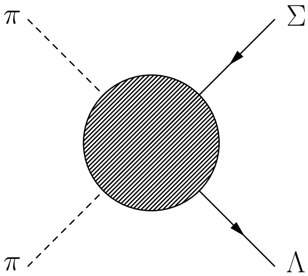
Need:

- pion vector form factor F_π^V ✓
- $\Sigma\Lambda\pi\pi$ scattering amplitude $A_{\Sigma\Lambda\pi\pi}$

Granados/Leupold/Perotti, Eur.Phys.J. A53, 117 (2017)

Alarcon/Blin/Vacas/Weiss, Nucl.Phys. A964, 18 (2017)

Let's take a closer look at $A_{\Sigma\Lambda\pi\pi}$



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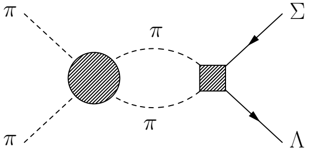
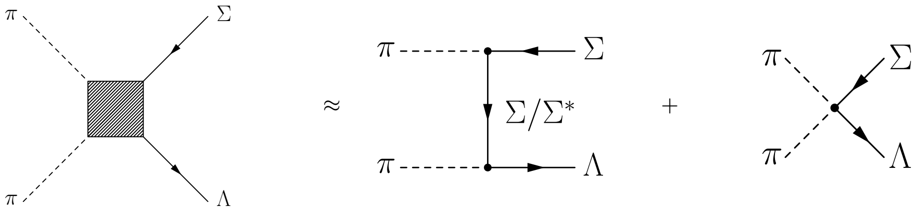


pion rescattering (circle) + a part containing left-hand cuts and polynomials (box)



- no data available for pion-hyperon :(

  → use three-flavor baryon ChPT at LO and NLO

  → include decuplet states

Parameters

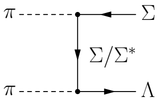
Baryon exchange diagrams from LO BChPT:

- octet baryon $\rightarrow$ Born diagrams

  vertices: $\Sigma\Lambda\pi$ and $\Sigma\Sigma\pi$ (F and D parameters)

- decuplet baryon

  vertices: $\Sigma^*\Lambda\pi$ and $\Sigma^*\Sigma\pi$ (h_A parameter, $2.2 < h_A < 2.4$)

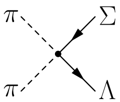


Four-point diagram from NLO BChPT:

- vertex $\Sigma\Lambda\pi\pi$ (b_{10} parameter, $0.85 < b_{10} < 1.35\,\mathrm{GeV}^{-1}$)

  $\rightarrow$ b_{10} is not very well known!

  $\rightarrow$ b_{10} is directly related to magnetic transition radius of $\Sigma - \Lambda$



Results: TFF at photon point

Electric charge, magnetic moment and electric(magnetic) radius of $\Sigma - \Lambda$ transition

Λ [GeV]	quantity	Born	NLO	NLO+res	χPT
1	$G_M(0)$	-0.438	5.55	2.58	1.98 (exp.)
2		-0.65	5.98	2.66	
1	$\langle r_M^2 \rangle$ [GeV^{-2}]	0.453	33.7	17.9	18.6
2		0.613	35.2	18.8	
1	$G_E(0)$	-0.432	-	0.0026	0
2		-0.562	-	-0.031	
1	$\langle r_E^2 \rangle$ [GeV^{-2}]	-3.13	-	0.866	0.773
2		-2.91	-	1.044	

Comparison to χPT (Kubis, Meißner 2001), using $h_A = 2.3$, $b_{10} = 1.1\ \text{GeV}^{-1}$

- Born terms alone are insufficient to produce reasonable results

  $\rightarrow$ need NLO and decuplet-resonance exchange

- varying the cut off Λ has rather small impact (10% at most)

However...

- uncertainty related to h_A moderate

quantity	$h_A = 2.2$	$h_A = 2.4$	χPT
$G_M(0)$	2.94	2.36	1.98 (exp.)
$\langle r_M^2 \rangle$ [GeV^{-2}]	20.2	17.3	18.6
$G_E(0)$	-0.076	0.016	0
$\langle r_E^2 \rangle$ [GeV^{-2}]	0.708	1.40	0.773

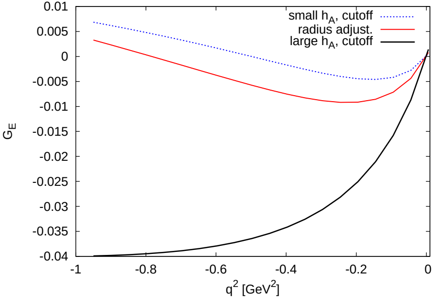
Comparison to χPT using $\Lambda = 2$ GeV and $b_{10} = 1.1$ GeV^{-1}

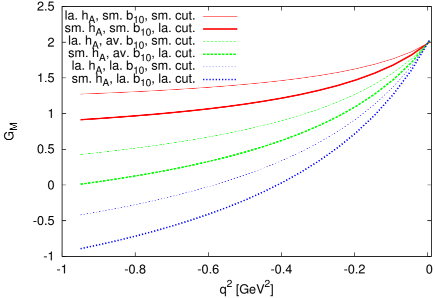
- uncertainty related to b_{10} sizable

b_{10}	quantity	NLO	NLO+res	χPT
0.85	$G_M(0)$	4.47	1.15	1.98 (exp.)
1.35		7.49	4.17	
0.85	$\langle r_M^2 \rangle$ [GeV^{-2}]	27.4	10.9	18.6
1.35		43.1	26.7	

Comparison to χPT using $\Lambda = 2$ GeV and $h_A = 2.3$

Results: TFF





- G_E close to zero at low energies
- G_M is very sensitive to variations of b_{10}
  $\rightarrow$ need input from experiment

Summary and Outlook

So...What have we learned so far?

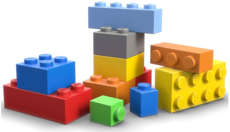
- Dispersion theory relates the low-energy electromagnetic $\Sigma\Lambda$ TFF with F_π^V
- Relativistic NLO BChPT determines $A_{\Sigma\Lambda\pi\pi}$
  - Inclusion of decuplet baryons essential to obtain reasonable results!
- Electric TFF very small in the whole low-energy region
- Magnetic TFF depends strongly on a poorly known LEC of the NLO Lagrangian (b_{10})

$\rightarrow$ can be determined from measurement of the magnetic transition radius (@FAIR)

$\rightarrow$ obtain predictive power

And...What are we going to do next?

- NNLO corrections
- decuplet – octet TFF: $\Sigma(J^P = \frac{3}{2}^+)$ to Λ, Δ to nucleon



Thank you for the attention!

Lagrangians

Relevant interaction part of the LO chiral Lagrangian:

- including only octect baryons

$$\mathcal{L}_8^{(1)} = i\langle \bar{B}\gamma_\mu D^\mu B\rangle + \frac{D}{2}\,\langle \bar{B}\,\gamma^\mu\,\gamma_5\,\{u_\mu, B\}\rangle + \frac{F}{2}\,\langle \bar{B}\,\gamma^\mu\,\gamma_5\,[u_\mu, B]\rangle$$

- including also decuplet resonances

$$\mathcal{L}_{8+10}^{(1)} = \mathcal{L}_8^{(1)} + \frac{1}{2\sqrt{2}}\, h_A\, \epsilon_{ade}\, g_{\mu\nu}\,(\bar{T}^\mu_{abc}\, u^\nu_{bd}\, B_{ce} + \bar{B}_{ec}\, u^\nu_{db}\, T^\mu_{abc})$$

Relevant interaction part of the NLO chiral Lagrangian:

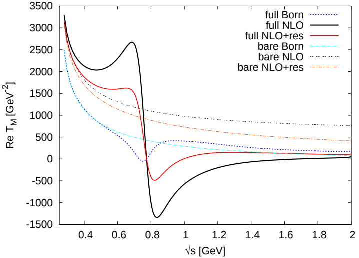
- including only octect baryons

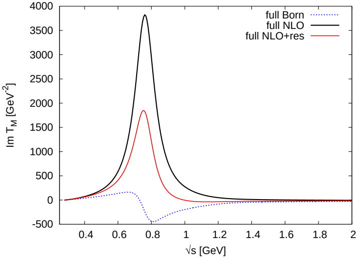
$$\begin{aligned}\mathcal{L}_8^{(2)} \quad = \quad & b_D\langle \bar{B}\{\chi_+, B\}\rangle + b_3\langle \bar{B}\{u^\mu, [u_\mu, B]\}\rangle + ib_6\big(\langle \bar{B}[u^\mu, \{u^\nu, \gamma_\mu D_\nu B\}]\rangle \\ & - \langle \bar{B}\overleftarrow{D}_\nu\{u^\nu, [u^\mu, \gamma_\mu B]\}\rangle\big) + \frac{i}{2}\, b_{10}\,\langle \bar{B}\{[u^\mu, u^\nu], \sigma_{\mu\nu} B\}\rangle\end{aligned}$$

Results: Helicity Amplitudes

Real and Imaginary part of $\Sigma\bar{\Lambda} \to \pi^+\pi^-$ helicity amplitudes

- Magnetic part





- full amplitudes have both real and imaginary part
- full amplitudes include ρ meson (via π-π phase shift)
- decuplet resonance exchange modifies considerably the amplitudes

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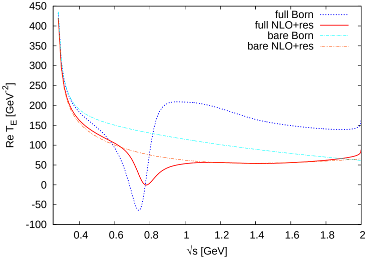
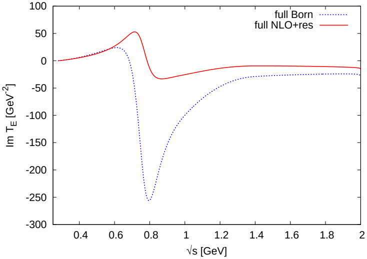
Real and Imaginary part of $\Sigma\bar{\Lambda} \to \pi^+\pi^-$ helicity amplitudes

- Electric part





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