Day 2

2) Polyakov

1) Rajaraman

3) Greensite

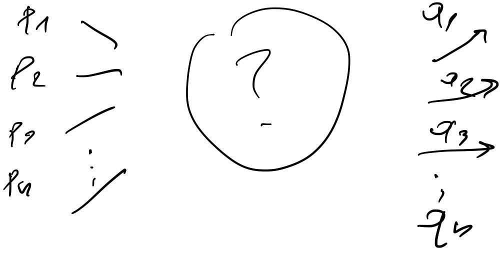
on monopoles

vortices

solitons etc

$$|p_1, \mu_1\rangle \otimes |p_2, \mu_2\rangle \dots = |p_1 \mu_1 ; p_2 \mu_2 \dots\rangle$$

$t \to -\infty$



$$\langle \text{out}_{t \to +\infty} | \qquad | \text{in}_{t \to -\infty} \rangle$$

$$\langle q_1 \lambda_1 q_2 \lambda_2 \dots | S | p_1 \mu_1 \, p_2 \mu_2 \dots \rangle$$

$$S = \mathbb{1} + i\,\mathcal{T} \qquad \mathcal{T} = (2\pi)^4 \delta^4(p_f - p_i)\, T$$

$$P = |\langle S \rangle|^2 \qquad d\sigma = \frac{|\langle S \rangle|^2}{T \, \rho v_{rel}} =$$

$$d\sigma = \frac{1}{4 E_1^i E_2^i v_{rel}} \sum_{\lambda_f} |\langle T \rangle|^2 \frac{d^3 q_1}{(2\pi)^3 2E_1^f} \frac{d^3 q_2}{(2\pi)^3 2E_2^f} \frac{d^3 q_3}{(2\pi)^3 2E_3^f} \dots d^3 q_n \quad (2\pi)^4 \delta^4(p_f - p_i)$$

$$\xrightarrow{p_1} \xleftarrow{p_2}$$

$$A\,B \to A\,B$$
$$p_1\ q_1 \quad p_2\ q_2$$

"LSZ"

$$\langle p_2 q_2 | S | p_1 q_1 \rangle = \# \int d^4x\, d^4y \; e^{-ip_1 x}\, e^{+ip_2 y} \left(\langle q_2 | T\{\varphi(y)\varphi^\dagger(x)\} | q_1 \rangle \right)$$

$$T\{A(x)B(y)\} = \theta(x^0-y^0)\,A(x)B(y) + \theta(y^0-x^0)\,B(y)A(x)$$

$$T\{\varphi(y)\varphi^\dagger(x)\} = \theta(y^0-x^0)\,[\varphi(y),\varphi^\dagger(x)] + \cancel{\varphi^\dagger(x)\varphi(y)}$$

$$\to \theta((y-x)^2)$$

$$= \# \int d^4x\, d^4y\; e^{-i(p_1-p_2+q_1-q_2)x}\, e^{+ip_2(y-x)}$$
$$\langle q_2 | \theta(y^0-x^0)\,\theta((y-x)^2)\,[\varphi(y),\varphi^\dagger(x)] | 0 \rangle$$

$$\delta^4(p_1-p_2+q_1-q_2) \int d^4z\; e^{ip_2 z}\; \langle q_2 | \underbrace{\theta(z^0)\,\theta(z^2)} [\varphi(z),\varphi^\dagger(0)] | 0 \rangle$$

$$\int d^3z \int_{|\vec z|}^{\infty} dz^0 \qquad e^{i\underline{E_2} z^0}$$

$$\to E_2 + i F_2$$

- CAUSALITY $\Longrightarrow$ ANALYTICITY IN THE UPPER ENERGY PLANE

$$\langle \overline{p_1}, q_2 | S | \overline{p_2}, q_1 \rangle = \# \int e^{ip_1 x} e^{-ip_2 y}$$

- CROSSING SYMMETRY

$$\pi^- p \to \pi^- p$$

$$\pi^+ p \to \pi^+ p$$

$$p_1 \to -p_1$$
$$p_3 \to -p_3$$

$$s = (-p_1 + p_2)^2$$

$$t = (p_1 + p_3)^2$$

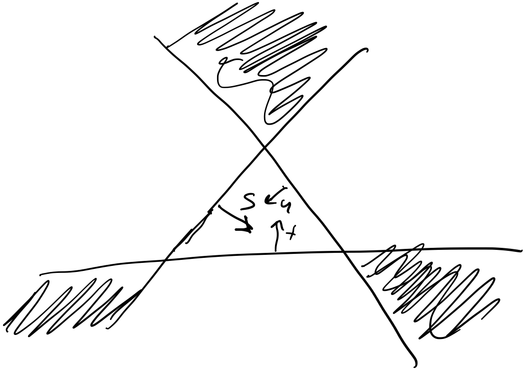
$$u = (-p_1 - p_4)^2$$

$$\cos\theta = \frac{t-u}{4p^2}$$

$$p = \frac{\sqrt{s - 4m^2}}{2}$$

$$E = \frac{\sqrt{s}}{2}$$

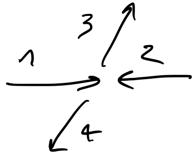
(EQUAL MASSES)



S-CHANNEL

$12 \to 34$

$\pi^- p \to \pi^0 n$



$s = 4E_s^2$

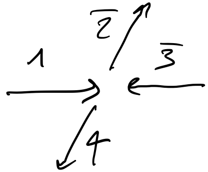
$t = -4p_s^2 \sin^2\frac{\theta_s}{2}$

$u = -4p_s^2 \cos^2\frac{\theta_s}{2}$

T-CHANNEL

$1\bar{3} \to \bar{2}4$

$\pi^-\pi^0 \to \bar{p}n$



$t = 4E_t^2$

$s = -4p_t^2 \sin^2\frac{\theta_t}{2}$

$u = -4p_t^2 \cos^2\frac{\theta_t}{2}$

DECAY CHANNEL

(DALITZ REGION)

u-channel

$1\bar{4} \to \bar{2}3$

$\pi^- \bar{n} \to \pi^0 \bar{p}$

- UNITARITY

$$\sum_f |\langle f | S | i \rangle|^2 = 1 \qquad SS^\dagger = 1$$

$$S = 1 + iT$$

$$(1+iT)(1-iT^\dagger) = 1 = 1 + iT - iT^\dagger + TT^\dagger$$

$$\frac{T - T^\dagger}{i} = TT^\dagger$$

$$2\,\mathrm{Im}\,T = TT^\dagger$$

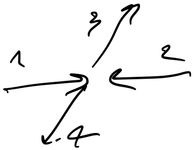
$$2\,\mathrm{Im}\,T_{fi}\,\cancel{(2\pi)^4\delta^4(P_f - P_i)} = \sum_n (2\pi)^4 T_{fn}\,\delta^4(P_f - P_n)\,\cancel{(2\pi)^4 T^\dagger_{ni}\,\delta^4(P_f - P_i)}$$

$$2\,\mathrm{Im}\,T_{fi} = \left\{\sum_n (2\pi)^4\delta^4(P_f - P_n)\right\} T_{fn}\,T^\dagger_{ni}$$

$$\int \frac{d^3 p_n \dots}{2E_n (2\pi)^3}$$

$$2\,\mathrm{Im}\,T_{fi} = \frac{k}{32\pi^2\sqrt{s}}\int d\Omega\, T_{fn}\,T^\dagger_{ni}$$

$$\frac{d^3p_1}{(2\pi)^3 2E_1}\,\frac{d^3p_2}{(2\pi)^3 2E_2}\,(2\pi)^4\,\delta^4(P_f-p_1-p_2)$$



$$\operatorname{Im} T = T(12\to \underline{1'2'})\, T^\dagger(\underline{1'2'\to 34})$$

$$\operatorname{Im} T = \rho\, T T^\dagger$$

PHYSICAL SCATTERING

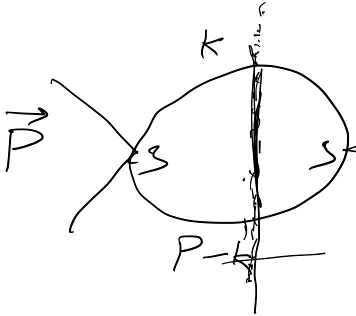
S-CHANNEL

$$T(s)$$

$$T(s^*) = T(s)^*$$

$$\frac{T(s) - T^*(s)}{2i} = \rho\, T(s)\, T^*(s)$$

$$\frac{T(s+i\varepsilon) - T(s-i\varepsilon)}{2i} = \rho\, \underline{T(s)}\, T^*(s)$$



$$\Sigma = \frac{g^2}{2i}\int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2-m^2+i\varepsilon}\,\frac{1}{(P-k)^2-m^2+i\varepsilon}$$

$$\mathrm{Im}\,\Sigma = \ell\,\Sigma\Sigma^\dagger = \ell\, g^2$$

$$\boxed{\Sigma_F\, \cancel{g}}$$

$$p^2 < 0$$

$$P = (0, \vec{P})$$

$$\Sigma = g\Sigma^0 + g^2\Sigma^1 + g^3\Sigma^2 + \dots$$

$$\mathrm{Im}\,\Sigma^1 = \Sigma^0\Sigma^{0\dagger}$$

$$\frac{g^2}{2i}\int \frac{d^4k}{(2\pi)^4}\,\frac{1}{k^{0\,2}-\underbrace{\vec{k}^2-m^2}_{\omega_k^2}+i\varepsilon}\,\frac{1}{k^{0\,2}-\underbrace{(p-k)^2-m^2}_{\omega_{pk}^2}+i\varepsilon}$$

$$\frac{g^2}{2i}\int \frac{dk^0\, d^3k}{(2\pi)^4}\,\frac{1}{(k^0-\omega_k+i\varepsilon)(k^0+\omega_k-i\varepsilon)(k^0-\omega_{pk}+i\varepsilon)(k^0+\omega_{pk}-i\varepsilon)}$$

k^0

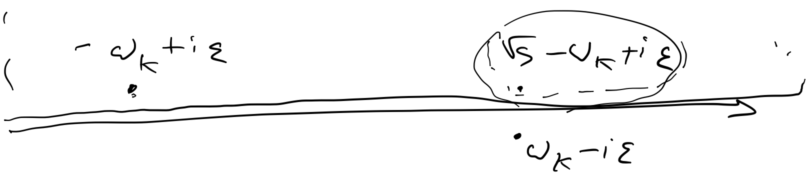
$-\omega_{pk}+i\varepsilon \quad -\omega_k+i\varepsilon$

$\omega_k-i\varepsilon \quad \omega_{pk}-i\varepsilon$

$p^2 > 0 \qquad P = (\sqrt{s}, \vec{0})$

$$\Sigma(p) = \frac{g^2}{2i} \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^{0\,2} - \underbrace{\vec{k}^2 - m^2}_{\omega_k^2} + i\varepsilon} \frac{1}{(k^0 - \sqrt{s})^2 - \underbrace{\vec{k}^2 - m^2}_{\omega_k^2} + i\varepsilon}$$

$\sqrt{s} + \omega_k - i\varepsilon$



$\sqrt{s} - \omega_k = \omega_k$

$s = 4m^2 + \vec{k}^2$

$k^0 = \sqrt{s} - \omega_k + i\varepsilon$

$$\Sigma(p) = \frac{g^2}{2i} \cancel{2\pi i} \int \frac{d^3k}{(2\pi)^3} \frac{1}{(k^0 - \omega_k + i\varepsilon)(k^0 + \omega_k - i\varepsilon)(k^0 - \sqrt{s} - \omega_k + i\varepsilon)}$$

$$= -\frac{g^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{(\sqrt{s} - 2\omega_k + i\varepsilon)\sqrt{s}(+2\omega_k - i\varepsilon)} = \qquad \frac{1}{x + i\varepsilon} \sim -i\pi\delta(x) + \cancel{P\frac{1}{x}}$$

$$-\frac{g^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{s}(\sqrt{s} - 2\omega_k) 2\omega_k + i\varepsilon} = \frac{g^2}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\sqrt{s}\, 2\omega_k} i\pi\delta(\sqrt{s} - 2\omega_k)$$

$$= \frac{g^2 i}{32\pi^2} \int \frac{d\Omega\, k\, \cancel{\omega_k}\, d\omega_k}{\sqrt{s}\, \cancel{\omega_k}} \delta(\sqrt{s} - 2\omega_k) = \frac{g^2 i}{64\pi^2} \int d\Omega \frac{k}{\sqrt{s}} = i g^2 \underline{\rho}$$

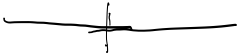
$$\int ds \frac{1}{(s-s_1)(s-s_2)(s-s_3)(s-s_4)}$$

$$= 2\pi i \left[\frac{s=s_3}{(s-s_1)(s-s_2)(s-s_4)} + \frac{s=s_4}{(s - \;-\;-\;\cdot\;)} \right]$$

$$\frac{T(s+i\varepsilon) - T^{\dagger}(s+i\varepsilon)}{2i} = \rho(s+i\varepsilon)\, T(s+i\varepsilon)\, T^{\dagger}(s+i\varepsilon)$$

$$\frac{\operatorname{disc} T(s)}{2i} = \rho(s+i\varepsilon)\, T(s+i\varepsilon)\, T^{\dagger}(s+i\varepsilon)$$

$$\rho(s)\, T(s)\, T^{\dagger}(s)$$



$$\rho(s)\, T(s+i\varepsilon)\, T(s-i\varepsilon)$$

$$\int ds \, \frac{f(s)}{(s-s_0)\cdots} = 2\pi i \, f(s_0)$$