1) Elementary complex algebra

find solutions of $z^8 = 1$

simplify $\frac{1+i}{2-i}, \quad \sqrt{1+\sqrt{i}}$

show that maximum absolute value of z^2+1 on a unit disk $|z|\leq 1$ is 2

show that

$$1 + \cos\phi + \cos 2\phi + \cdots \cos n\phi = \frac{1}{2} + \frac{\sin(n+\frac{1}{2})\phi}{2\sin\frac{\phi}{2}}$$

solve $\frac{d^2x(t)}{dt^2} + \omega^2 x^2(t) = 0$

2) Complex functions

show that $\cos z = \frac{1}{2}$ has only real solutions

Find all values of i^i

show that $\sin(z_1 + z_2) = \sin z_1 \cos z_2 + \sin z_2 \cos z_1$

Show that under $z \rightarrow \sin(z)$ lines parallel to the real axis are mapped to ellipses and that lines parallel the the imaginary axis are mapped to hyperbolas

3) Complex integrals

Integrals:

$$\int_\gamma dz \qquad \int_\gamma z^n dz$$

$$\int_\gamma \frac{dz}{z} \qquad \int_{\gamma'} \frac{dz}{z}$$

γ = unit circle

γ' = unit square

$$\int_\gamma \frac{dz}{z^2}$$

4) More integrals

Examples to consider

$$\int_{-1}^{1} dx \frac{1}{\sqrt{1-x^2}}$$

$$\int_{1}^{\infty} dx \frac{1}{x\sqrt{x^2-1}}$$