Joint Physics Analysis Center

Summer Workshop on the Reaction Theory
Exercise sheet 1

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Classwork

Fourier transform of a charge distribution

Consider a spherical symmetric exponential (rest frame) charge distribution in 3 dimensions:

$$\rho_{ch}(r) = \rho_0 e^{-\Lambda r}, \tag{1}$$

where ρ_0 is a constant and Λ is a mass scale.

(1) Determine ρ_0 in terms of Λ from the normalization condition for the total charge.

(2) Determine the form factor through the Fourier transform:

$$\tilde{\rho}_{ch}(Q) = \frac{1}{4\pi}\int d^3\vec{r}\, e^{i\vec{q}\cdot\vec{r}}\rho_{ch}(\vec{r}), \tag{2}$$

where we define $Q \equiv |\vec{q}|$.

(3) Determine the rms radius $\langle r^2\rangle$ in terms of Λ from a Taylor expansion:

$$\tilde{\rho}_{ch}(Q) = 1 - \frac{1}{6}\langle r^2\rangle Q^2 + \mathcal{O}(Q^4). \tag{3}$$

(4) Discuss the two limiting cases of very small and very large value of Λ (relative to the mass of the system). How does the form factor looks in both cases?