Joint Physics Analysis Center

Summer Workshop on the Reaction Theory
Exercise sheet 3

Andrew Jackura / Marc Vanderhaeghen

Contact: http://www.indiana.edu/~ssrt/index.html

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Classwork

DIS hadronic tensor in parton model

The general hadronic tensor for inelastic electron scattering on a hadron (e.g proton) target is given by

$$W^{\mu\nu} = \frac{1}{2\pi}\sum_X (2\pi)^4\delta^4(p+q-p_X)\langle p|J^{\mu\dagger}(0)|X\rangle\langle X|J^{\nu}(0)|p\rangle \tag{1}$$

In the Bjorken limit, $Q^2 \gg M^2$, $p\cdot q \gg M^2$, with $Q^2/(2p\cdot q) \equiv x_B$ a constant ($0 < x_B < 1$), this process is described at leading order (LO) in QCD by the incoherent scattering from individual quarks.

To calculate this LO contribution, we want to evaluate the tensor of Eq. (1) for the situation where the initial state is a massless quark (with momentum p_q and spin projection s), and the final state X is also a quark (with momentum p'_q and spin projection s'). The initial quark carries a fraction x of the target momentum, i.e. $p_q = xp$.

(1) How does the sum $\sum_X$ looks like for this physical situation ?

(2) Show that the energy-momentum conserving δ-function yields

$$\int \frac{d^3\vec{p}'_q}{(2\pi)^3 2E'_q}(2\pi)^4\delta^4(q+p_q-p'_q)... = (2\pi)\frac{x_B}{Q^2}\delta(x-x_B)... \tag{2}$$

(3) Using the Dirac current for a quark

$$\langle X|J^{\nu}(0)|p\rangle = \bar{u}(p'_q,s')\gamma^{\nu}u(p_q,s), \tag{3}$$

show that the partonic DIS tensor $w^{\mu\nu}$ is given by

$$w^{\mu\nu} = \frac{x_B}{Q^2}\delta(x-x_B)\mathrm{Tr}\left\{\left(\frac{1+(2s)\gamma_5}{2}\right)\gamma\cdot p_q\,\gamma^{\mu}\,\gamma\cdot p'_q\,\gamma^{\nu}\right\}, \tag{4}$$

(4) By working out the trace in Eq. (4), show that this leads to :

$$\begin{aligned} f_1 &= \frac{x_B}{2}\delta(x-x_B), \\ f_2 &= 2x_B f_1, \\ g_1 &= (2s) f_1, \\ g_2 &= 0. \end{aligned} \tag{5}$$