Introduction to reaction theory

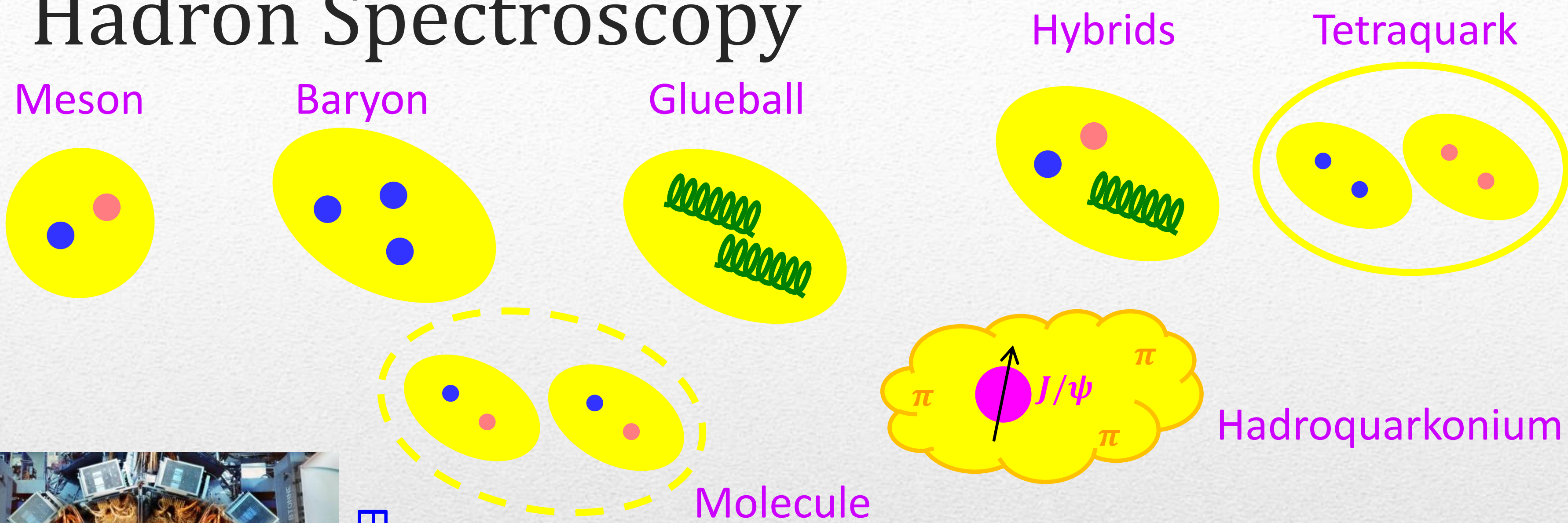
Alessandro Pilloni

SWRT, Bloomington, June 12th, 2017

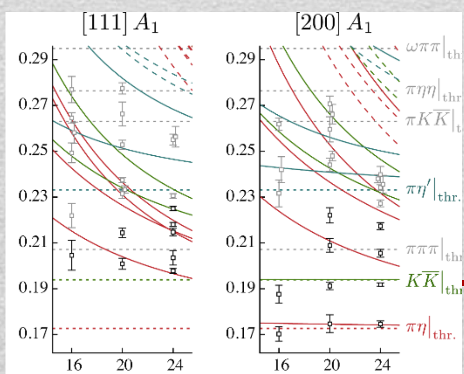
Jefferson Lab

INDIANA UNIVERSITY

Hadron Spectroscopy



Experiment

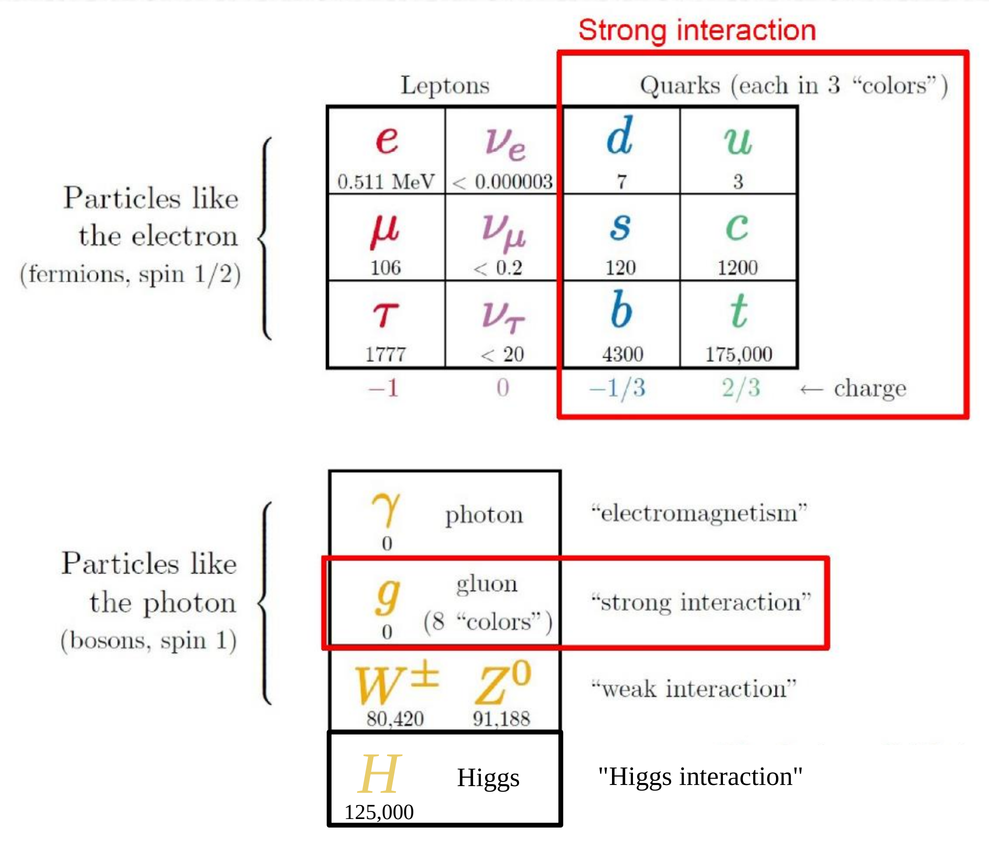


Lattice QCD

Data → Amplitude analysis → Properties, Model building

Interpretations on the spectrum leads to understanding fundamental laws of nature

Standard Model Constituents



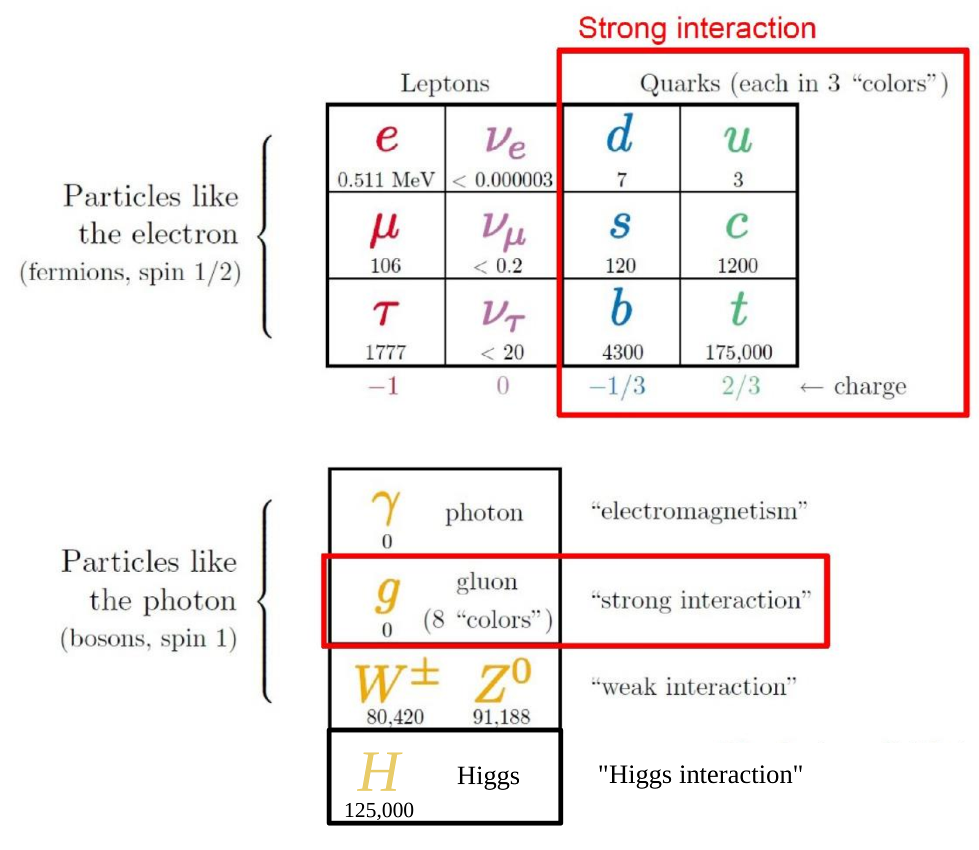
Standard model is a remarkable simple* theory

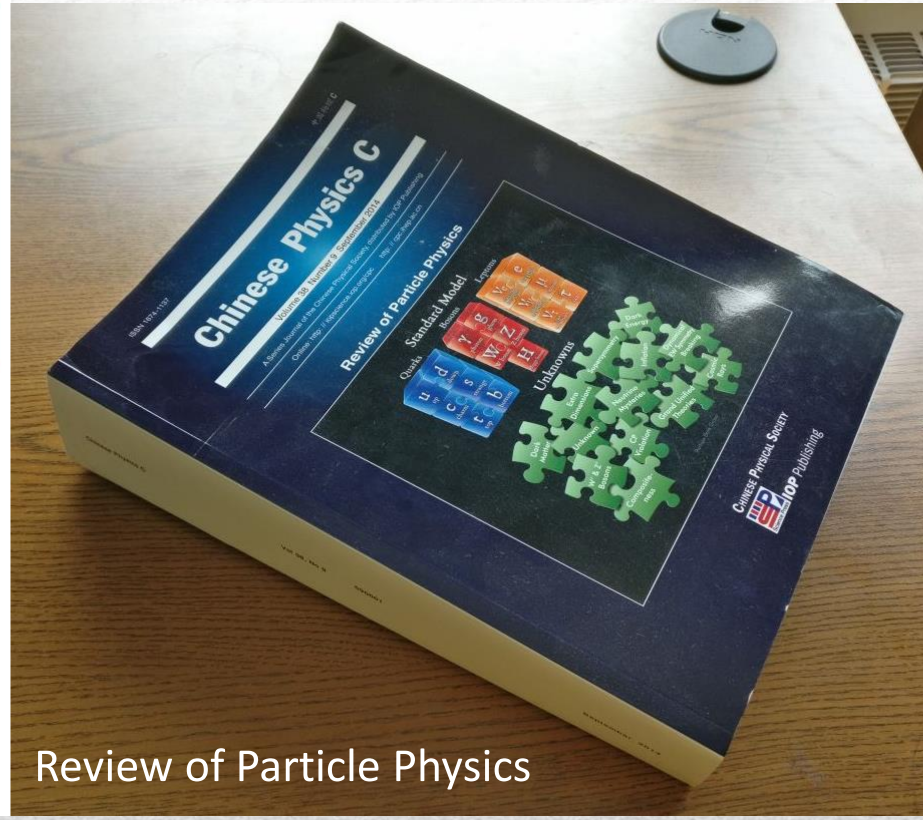
The particle in the spectrum can easily fit in a table

*this concept is scheme-dependent: $SU(3)_C \otimes SU(2)_W \otimes U(1)_Y$ chiral gauge theory, spontaneously broken via scalar field to $SU(3)_C \otimes U(1)_Q$, where anomalies cancel because we are lucky

Heaviest particle $\sim$ 175 GeV

Standard Model Constituents

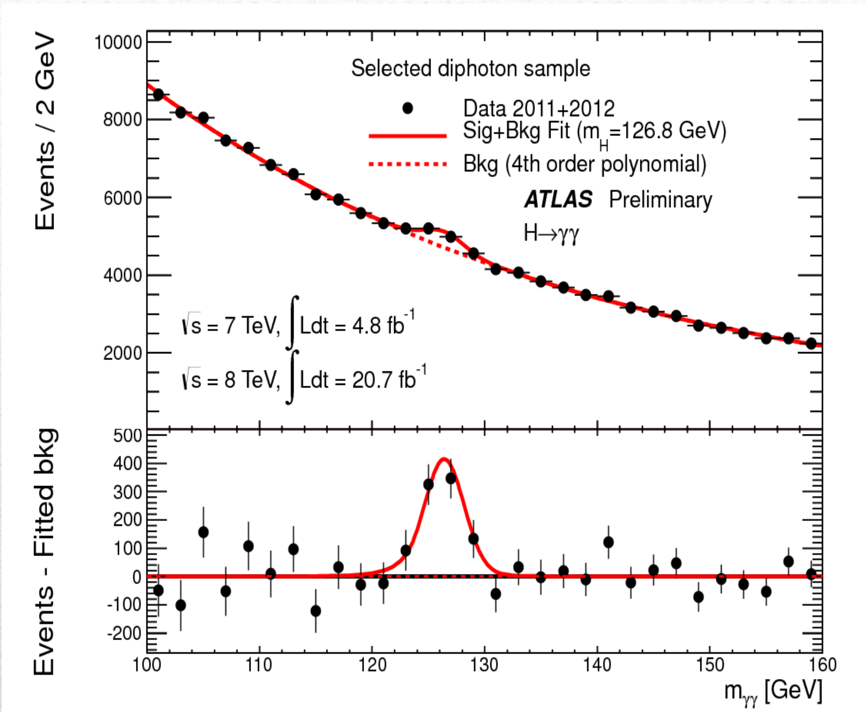


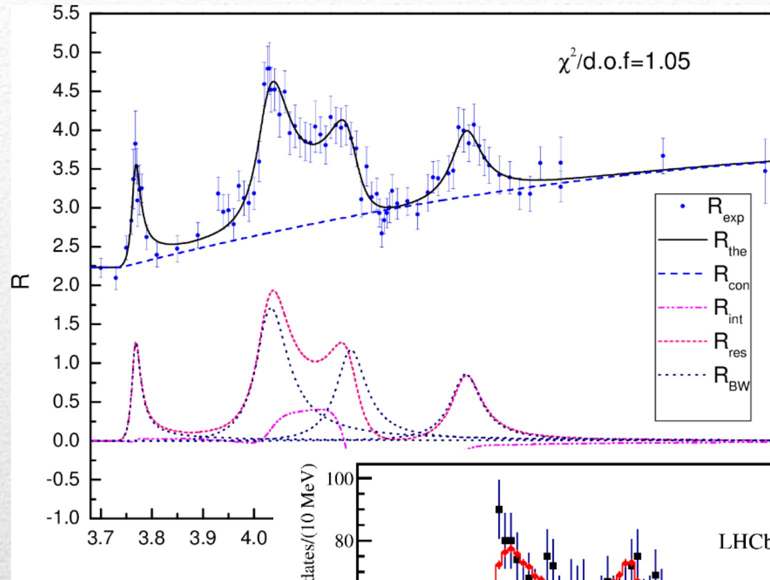


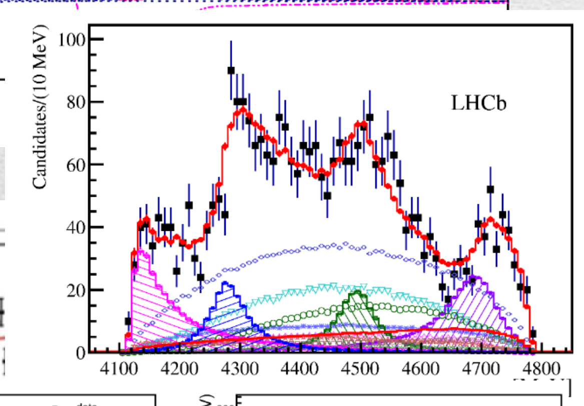
Heaviest particle ~ 175 GeV

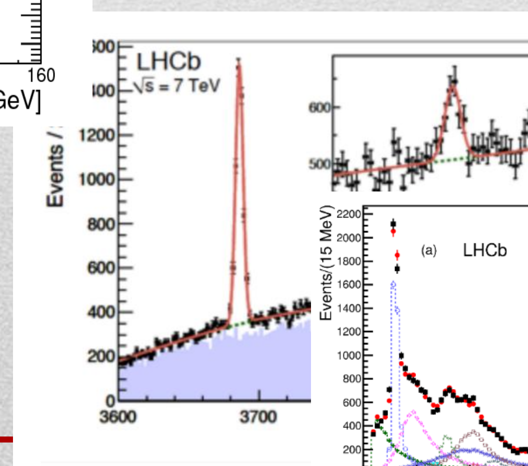
How to fill a $\sim 2 \times 10^{27}$ GeV book with just that?

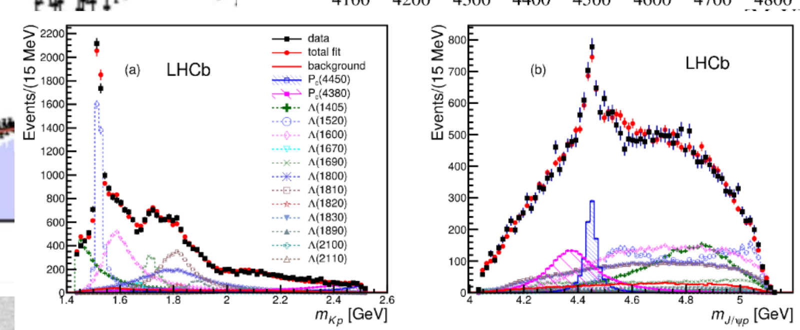
Standard Model Constituents





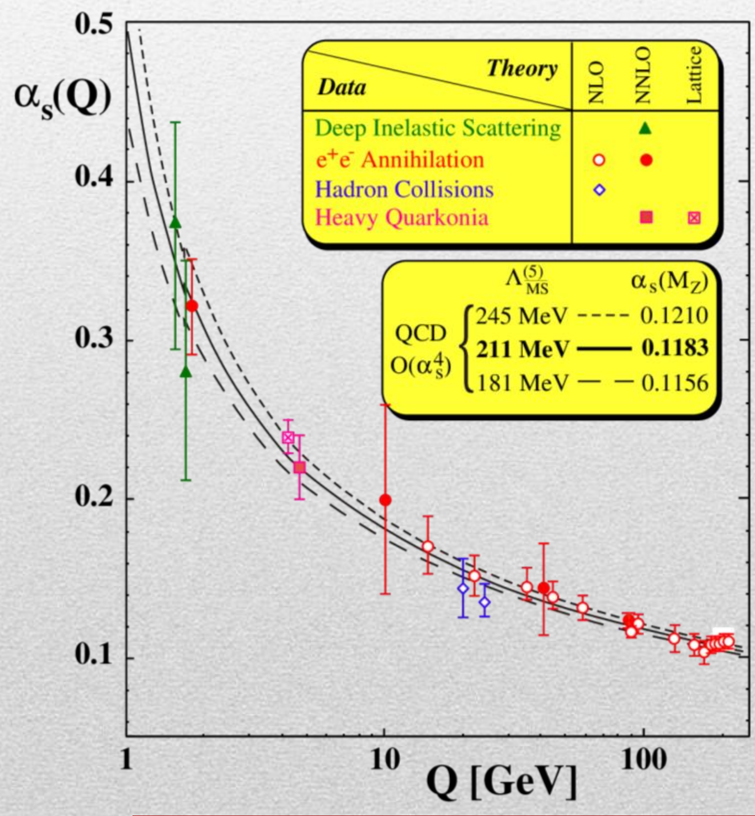






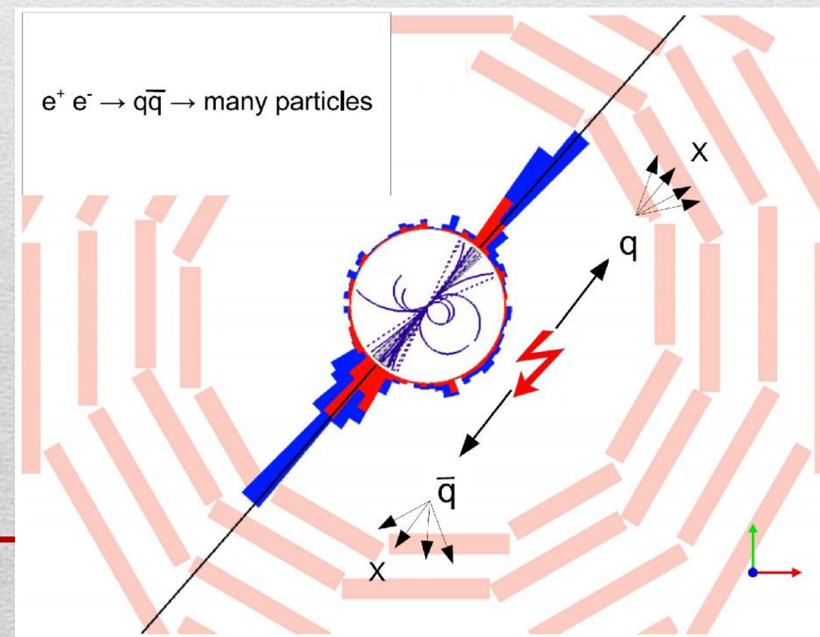
We have the final answer: QCD!...

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\psi}_f \left(i\not{D} - m_f \right) \psi_f - \frac{1}{2g_s^2} \mathrm{Tr}\, G_{\mu\nu} G^{\mu\nu}$$



...with the residual problem that we have no idea* of how to solve it!

- At high energies, the coupling $\alpha_s = \frac{g_s^2}{4\pi} \ll 1$ (asymptotic freedom), perturbation theory works



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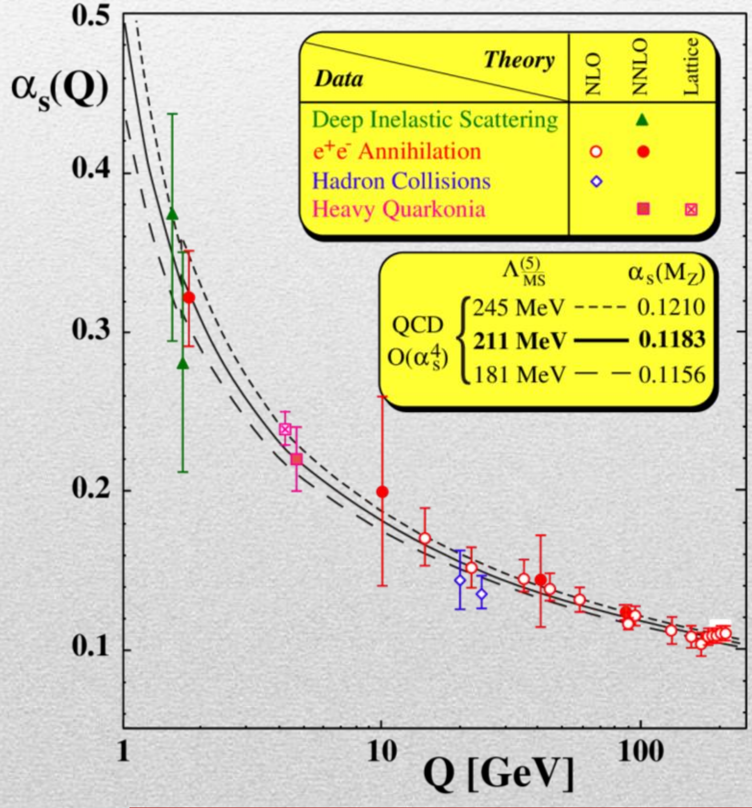

...with the residual problem that we have no idea* of how to solve it!

- At low energies, the coupling $\alpha_s \gg 1$ thinking in terms of quarks and gluons make no sense anymore;
- They «arrange» themselves in a incalculable way into colourless hadrons (confinement)

Weak interactions do not confine
because the Higgs mechanism stops the running

* we have Lattice QCD, with many warnings and caveats

What can we say then?

When you are desperate, don't panic and look for symmetries:

- Symmetries are beautiful

- Symmetries constrain your results no matter how complicated your theory is

Luckily, strong interactions are the ones with more symmetry:

- Under Parity (someone wonders why)
- Under Charge Conjugation
- Under Time reversal
- Conserve Flavor (isospin, strangeness...)
- Conserve electric charge and baryonic number

Moreover, there are some generic properties that any interaction has to satisfy

The *S*-Matrix principles

- Future cannot change the past
- 100%, something will happen
- The anti-particle is an anti-particle and not just a different particle

Jacques de Lapalisse, QFT

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Jacques de Lapalisse, QFT

Even though these look so obvious, there is no amplitude which is known to satisfy all these principles at the same time

In the '60s, people tried to guess how the real solution looks like, just by implementing these principles. It did not work. Now we have QCD, but it doesn't work either

Imposing those in a clever way allow us to constrain as much as possible the arbitrariness of choosing a model to extract physics from experiments

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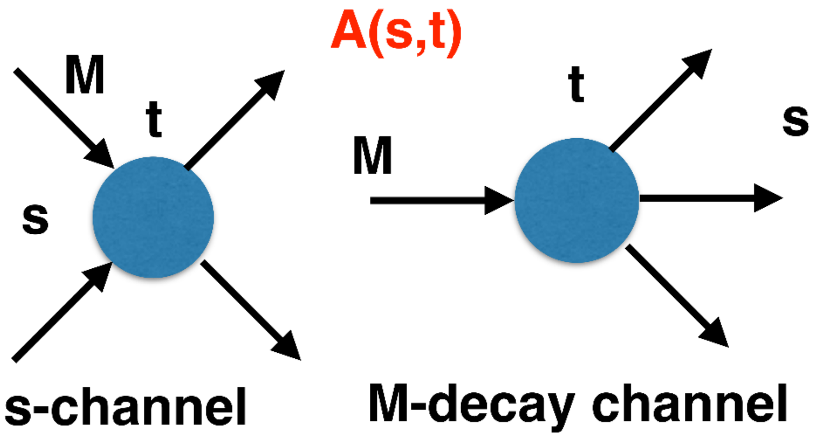
Parametrize your ignorance. Build a reasonable model. Fit data. Have fun.

S-Matrix principles



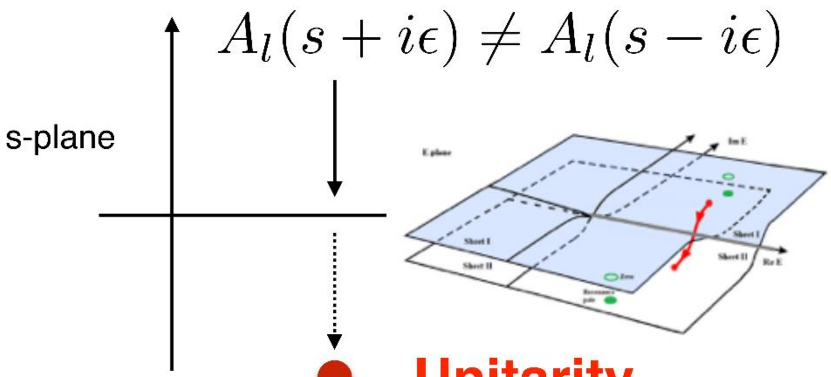
Crossing

$$A(s,t) = \sum_l A_l(s) P_l(z_s)$$

Analyticity

$$A_l(s) = \lim_{\epsilon \to 0} A_l(s + i\epsilon)$$

$$A_l(s + i\epsilon) \neq A_l(s - i\epsilon)$$



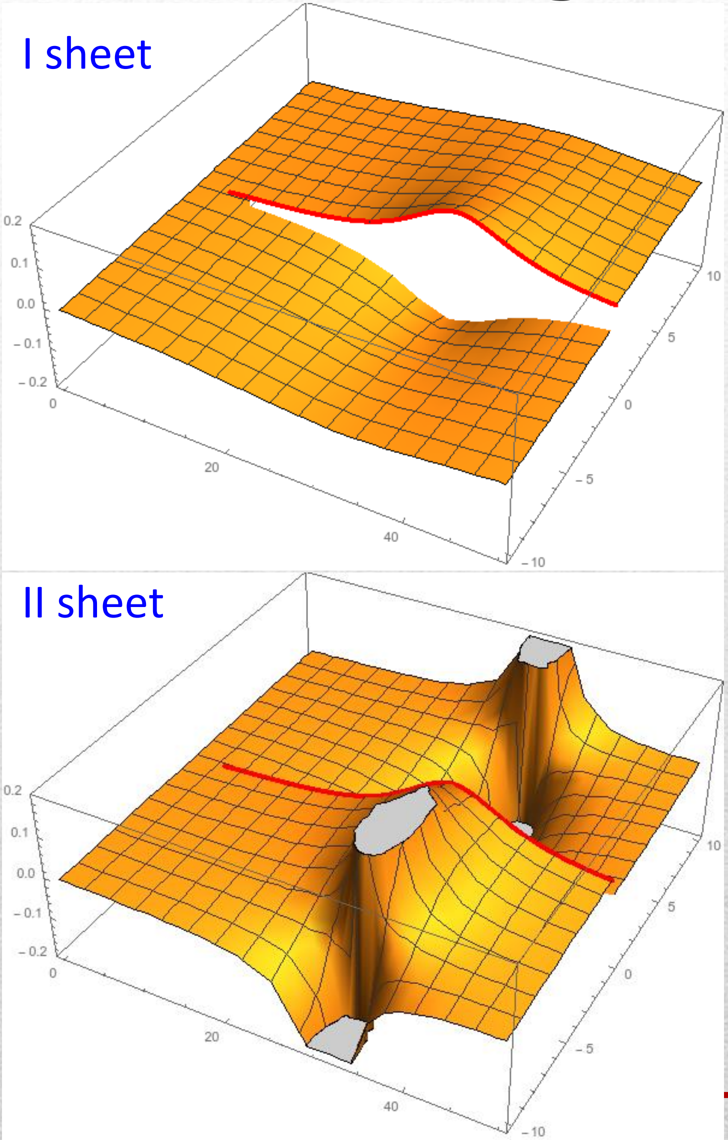
Unitarity

These are constraints the amplitudes have to satisfy, but do not fix the dynamics

Resonances are poles in the unphysical Riemann sheets

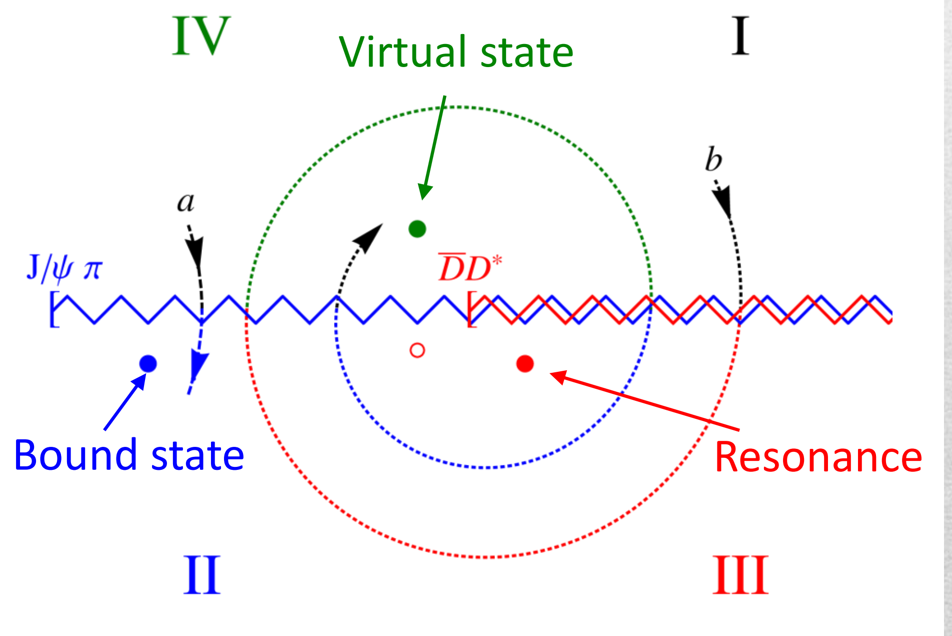
Need for complex analysis

Pole hunting



Extracting physics information means to hunt for poles in the complex plane

Pole position → Mass and width
Residues → Couplings



Why strong interactions are strong

We don't experience strong interactions in everyday life*.
They happen on much shorter scales

- Gravity $V(r) = G\frac{M_1 M_2}{r}, G \sim 10^{-39} m_p^{-2}$
- Electromagnetism, $V(r) = \alpha\frac{1}{r}, \alpha \sim \frac{1}{137}$
- NN interaction, $V(r) \sim \frac{f_{\pi NN}^2}{4\pi}\frac{1}{r}\exp\left(-\frac{r}{r_0}\right)$,
  $\frac{f_{\pi NN}^2}{4\pi} \sim 0.075, r_0 \sim 1\text{ fm} \sim m_\pi^{-1}$ (Rutherford)
- πN interaction, $\frac{g_{\pi N}^2}{4\pi} \sim 14$

*At least, out of office/class/lab hours

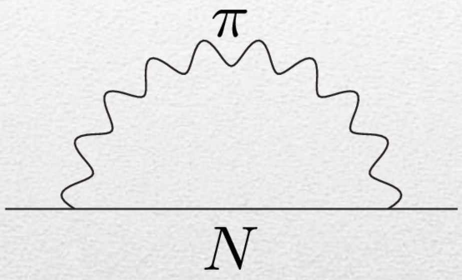
Why strong interactions are strong

In nonrelativistic quantum mechanics I can define an interaction radius

$$f(k,\theta) = \sum_l (2l+1) f_l(k) P_l(\cos\theta)$$

$$f_l(x) \sim \begin{cases} 1, & l \sim kr_0 \\ 0, & l \gg kr_0 \end{cases}$$

$$r_0 \sim 1\ \mathrm{fm} \sim m_\pi^{-1}$$



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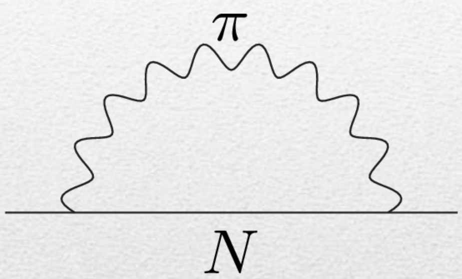


- $\sigma(\nu p)$ is ~ $10\ \text{fb} \sim 10^{-8}\ \text{fm}^2$;

$\sigma(pp)$ is ~ $50\ \text{mb} \sim 5\ \text{fm}^2$;

Symmetries of strong interactions

Discrete symmetries:
- Parity
- Charge conjugation
- Time reversal

First two give rise to multiplicative quantum numbers which strong interaction conserve

They reduce the number of independent amplitudes we need

Continuous symmetries:

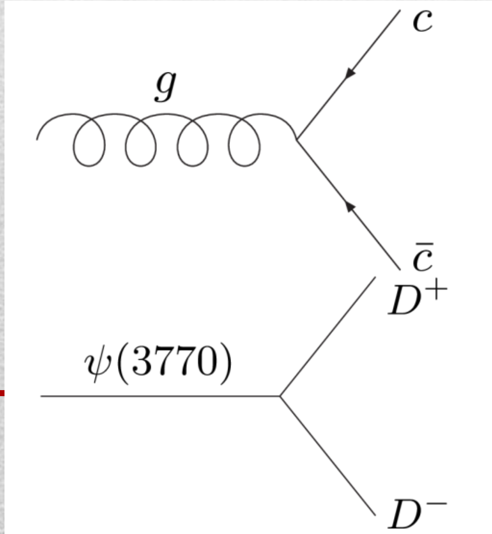
Common to any interaction:
- Poincaré transformations (translation, rotations, boosts)
- Baryon number and Electric Charge

- Flavor conservation
- Isospin (or more), approximate

Internal $U(1)$ symmetries give rise to additive quantum numbers

Flavor conservation is a $U(1)^6$ symmetry,
Separate conservation of flavor quantum numbers

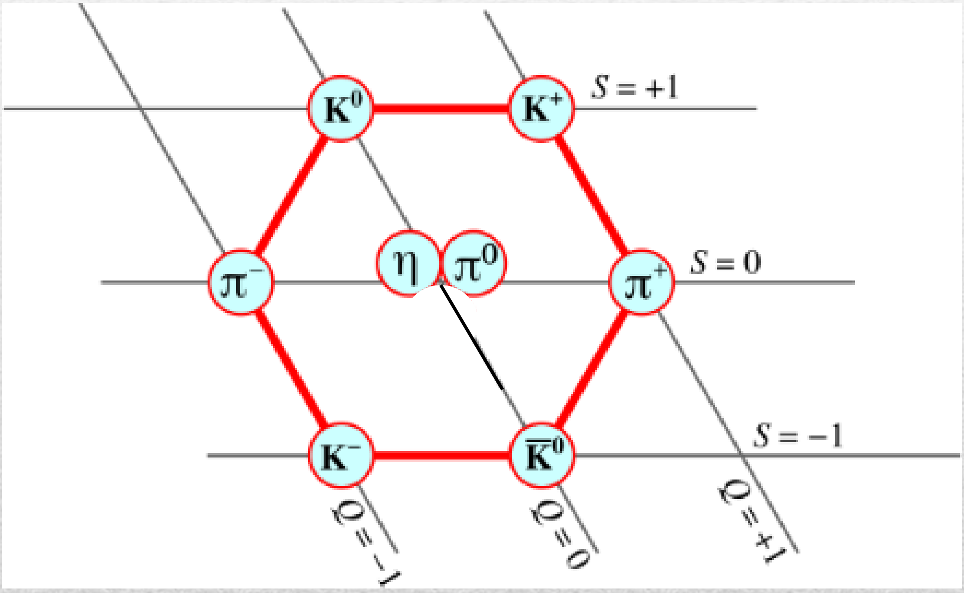
Consequence: particles with open flavor are created in pairs

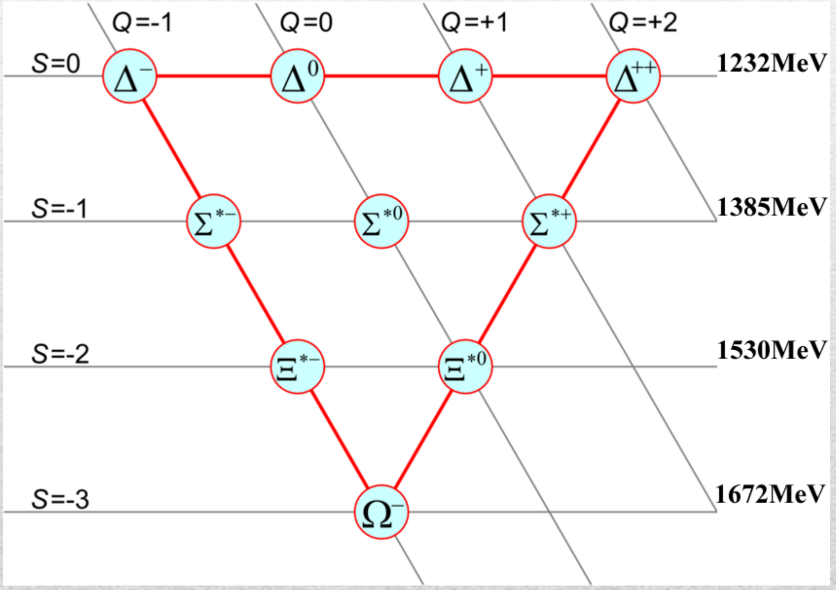


Isospin and Flavor

If I consider p, n (or u, d quark) as degenerate, can embed $U(1)_u \otimes U(1)_d$ into $SU(2)_I$
If we also add $U(1)_s$ we have $SU(3)_F$

Particles belongs to irreducible representations of $SU(3)_F$





Amplitudes of particles in the same multiplet are related by Clebsh-Gordan coefficients (Wigner-Eckart theorem)

Charge conjugation and G-parity

Totally neutral particles are eigenstate of charge conjugation

$$C|\pi^0\rangle = +|\pi^0\rangle \qquad C|\rho^0\rangle = -|\rho^0\rangle$$
$$C|\pi^+\rangle = +|\pi^-\rangle \qquad C|\rho^+\rangle = -|\rho^-\rangle$$
$$C|\pi^-\rangle = +|\pi^+\rangle \qquad C|\rho^-\rangle = -|\rho^+\rangle$$

I can add a rotation of π in isospin space

$$e^{-i\pi I_y}C|\pi^0\rangle = +e^{-i\pi I_y}|\pi^z\rangle = -|\pi^z\rangle = -|\pi^0\rangle$$
$$e^{-i\pi I_y}C|\pi^+\rangle = +e^{-i\pi I_y}|\pi^-\rangle = +e^{-i\pi I_y}(|\pi^x\rangle - i|\pi^y\rangle) = +e^{-i\pi I_y}(-|\pi^x\rangle - i|\pi^y\rangle) = -|\pi^+\rangle$$
$$e^{-i\pi I_y}C|\pi^-\rangle = +e^{-i\pi I_y}|\pi^+\rangle = +e^{-i\pi I_y}(|\pi^x\rangle + i|\pi^y\rangle) = +e^{-i\pi I_y}(-|\pi^x\rangle + i|\pi^y\rangle) = -|\pi^-\rangle$$

Unflavored mesons are eigenstates of G parity

$$\rho^0\ (I^G = 1^+) \rightarrow \pi^+\pi^-$$
$$\omega^0\ (I^G = 0^-) \nrightarrow \pi^+\pi^-$$

Isospin breaking

Isospin violation is due to

a) electromagnetic interactions, $Q(u) = \frac{2}{3}, Q(d) = -1/3$,

b) unequal quark masses, $m_u \neq m_d$

$m_{\pi^+} - m_{\pi^0} \simeq 4 \text{ MeV}$

Mass corrections cancel out at lowest order,
pure electromagnetic effect

$\eta \to \pi^+\pi^-\pi^0$

EM corrections cancel out at lowest order,
pure mass difference effect

$m_p - m_n \simeq -1.3 \text{ MeV}$

Both are present and give different sign contributions,
mass difference roughly 2 times EM effect
pure mass difference effect

(if you forget this sign,
we all die)

Ingredients we need

- First we need to define the states, and their transformation properties
- We define the scattering problem and introduce the S-matrix
- We relate the S-matrix to observables