Joint Physics Analysis Center

Summer Workshop on the Reaction Theory
Exercise sheet 1

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June 12 – June 22 To be discussed on Monday of Week-I.

Classwork

1.1 $2 \to 2$ scattering

Consider the scattering of $12 \to 34$, with $m_1 \neq m_2 \neq m_3 \neq m_4$. In the center of mass frame, derive the relations between incoming momentum p_i, the outgoing momentum p_f, the energies of the particles, the scattering angle, and the Mandelstam variables $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$, $u = (p_1 - p_4)^2$.

1.2 $n - body$ phase space

(a) Calculate the two-body phase space, $\frac{d^3p_1}{(2\pi)^3 2E_1} \frac{d^3p_2}{(2\pi)^3 2E_2} (2\pi)^4 \delta^4(P - p_1 - p_2)$.

(b) Use the previous result to calculate the three-body phase space, by inserting $d^4q\, \delta^4(p_2 + p_3 - q)$.

(c) What if identical particles are present?

1.3 Wigner rotations

Consider a state with helicity μ moving in the x direction, $|\vec{p}, \mu\rangle$. We want to boost the state in the z direction with β.

(a) Calculate the new momentum of the state $|\vec{p}'|$, and the angle θ' with respect to the z-axis. Check that the non-relativistic limit is reasonable.

(b) Remember the definition of helicity state. We are calculating $L_z(b)\, R(0, \pi/2, 0)\, L_z(p) \left|\vec{0}, \mu\right\rangle$. If the state were equal to $\left|\vec{p}', \mu\right\rangle = R(0, \theta', 0)\, L_z(p') \left|\vec{0}, \mu\right\rangle$, the two combinations of boost and rotation would coincide. Check that this is not the case.

(c) The right answer is realized by adding another rotation $R(0, \theta', 0)\, L_z(p')\, R(0, \omega, 0) \left|\vec{0}, \mu\right\rangle$. Calculate the ω you need for the two combinations to match.

(d) What's the relation between the boosted $L_z(b)\, |\vec{p}, \mu\rangle$ and $\left|\vec{p}', \mu\right\rangle$?