Joint Physics Analysis Center

Summer Workshop on the Reaction Theory
Exercise sheet 2

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Classwork

2.1 Elementary complex algebra

- Find solution $z^8 = 1$.
- Simplify $\frac{1+i}{2+i}$, $\sqrt{1+\sqrt{i}}$.
- Show that maximum absolute value of z^2+1 in a unit disk $|z|<1$ is 2.
- Show that
$$1+\cos\phi+\cos 2\phi+\cdots+\cos n\phi = \frac{1}{2}+\frac{\sin(n+\frac{1}{2})\phi}{2\sin\frac{\phi}{2}}$$
- solve the equation
$$\frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2}+\omega^2x^2(t)=0.$$

2.2 Complex functions

- Show that $\cos z = \frac{1}{2}$ has only real solutions.
- Find all values of i^i.
- Show that $\sin(z_1+z_2)=\sin z_1\cos z_2+\sin z_2\cos z_1$ using that $e^{iz}=\cos z+i\sin z$.
- Show that under the map $z\to\sin z$ lines parallel to the real axis are mapped to ellipses and that lines parallel the imaginary axis are mapped to hyperbolas.

2.3 Complex integrals

We define the paths in the complex plane: γ is a right hand unit circle $|z|=1$, γ' is a unit square passed in counter-clockwise direction.

- $\int_\gamma \mathrm{d}z$ and $\int_{\gamma'}\mathrm{d}z$,
- $\int_\gamma \frac{\mathrm{d}z}{z}$ and $\int_{\gamma'}\frac{\mathrm{d}z}{z}$,
- $\int_\gamma \frac{\mathrm{d}z}{z^2}$.

2.4 More integrals

Calculate real integrals using the Cauchy theorem in the complex plane:

$$\int_{-1}^{1} \frac{1}{\sqrt{1-x^2}}, \qquad \int_{1}^{\infty} \mathrm{d}x \frac{1}{x\sqrt{x^2-1}}.$$