Joint Physics Analysis Center

Summer Workshop on the Reaction Theory
Exercise sheet 6

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June 12 – June 22 To be discussed on Thursday of Week-II.

Classwork

6.1 Analytic functions

A Real (Hermitian) Analytic function is an analytic function which satisfies Schwarz reflection principle.

- Check whether the following functions are real (Hermitian) analytic functions, investigate their analytic structure:
  - $\sqrt{s}$ with the standard definition of the square root function (as in any programming language: $\sqrt{\pm i} = (1 \pm i)/\sqrt{2}$).
  - the two-body phase space $\Phi(s) = \frac{1}{8\pi}\sqrt{1 - \frac{4m^2}{s}}$,
  - $\log(x)$ with the standard definition of the argument function (as in any programming language $\log(\pm i) = \pm\pi/2$).
- What are the asymptotic limits $s \to \infty$ for the functions above. Write down the integral representations using subtraction if needed. Prove the relation explicitly for the function $\log(s)$.
- Reconstruct function $\Sigma(s)$ which is real analytic everywhere except the unitarity cut $s > 4m^2$. The imaginary part of $\Sigma(s)$ just above the cut is equal to $\theta(s-4m^2)\,\rho(s) = \theta(s-4m^2)\,\Phi(s)/2$. Compare the analytic structures of $\Sigma(s)$ and $i\rho(s)$.

6.2 Omnès problem for the Breit-Wigner amplitude

The Omnès function $\Omega(s)$ is a solution of the equation

$$\Delta\Omega(s) = 2i\rho(s)t^*(s)\Omega(s),$$

where $\Delta\Omega(s) = \Omega(s+i\epsilon) - \Omega(s-i\epsilon)$. The right hand part of the equation is calculated in the physical s-channel, therefore, the real value of s is approached from above. The phase space factor $\rho(s)$ is introduced above. The amplitude $t(s)$ satisfies elastic unitarity.

- Derive the expression for the omnès function

$$\Omega(s) = \Omega_0 \mathrm{exp}\left(\frac{s}{\pi}\int_{4m^2}^{\infty}\frac{\delta(s')}{s'(s'-s)}\mathrm{d}s'\right)$$

- Calculate the phase shift and the Omnès function explicitly for a Breit-Wigner amplitude in the limit of zero width.

- The Breit-Wigner amplitude with the finite energy-dependent width follows.

$$F_{\mathrm{BW}}(s) = \frac{g^2}{m^2 - s - im\Gamma(s)}, \quad \Gamma(s) = \frac{g^2}{2m}\Phi(s) \tag{1}$$

  What is the value of the phase at threshold? What is the limit of the phase when $s \to \infty$? Sketch the phase a function of s.

- How does the phase change when the width gets smaller? Find a function to approximate the phase in the limit $\Gamma \to 0$.

- Calculate the integral for the phase explicitly.

• Consider the most fancy modification of the Breit-Wigner amplitude where the left-hand cut in the phase space is removed by replacing the phase space by the dispersive integral.

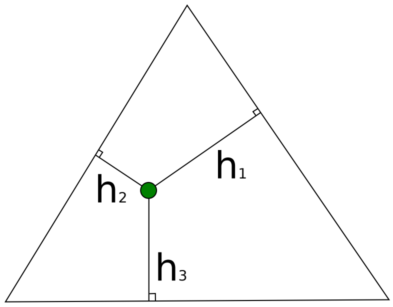
$$F_{\mathrm{BW}}(s) = \frac{g^2}{m^2 - s - im\tilde{\Gamma}(s)}, \quad \tilde{\Gamma}(s) = \frac{g^2}{2m}\frac{s}{\pi i}\int_{4m^2}^{\infty}\frac{\Phi(s')}{s'(s'-s)}\mathrm{d}s' \tag{2}$$

- Show that the imaginary part of the denominator in Eq. 2 is the same as the one in Eq. 1

- Guess the Omnès function which can be calculated from Eq. 2 without evaluating it.

6.3 Dalitz plot for $\eta \to \pi^+\pi^-\pi^0$ decay

The Dalitz plot of $\eta \to 3\pi$ is usually drawn in the symmetric triangle coordinates (see sheet 4): the kinetic energies of the pions $h_1 = T_+$, $h_2 = T_-$ and $h_3 = T_0$ are given by the distances to the sides of the equilateral triangle.



- Find out the side length of the triangle for the decay $\eta \to \pi^+\pi^-\pi^0$.
- What are the relations between the kinetic energies and the Cartesian coordinates, say T_x, T_y.

Usually the axes are linearly rescalled to fit the Dalitz plot to the range $X \in [-1, 1]$, $Y \in [-1, 1]$. $X = p_1 T_x + q_1$, $Y = p_2 T_x + q_2$.

- What is the minimal and maximal values for T_+, T_-, T_0? What are ranges for T_x and T_y?
- Find out the transformation $T_x, T_y \to X, Y$, namely p_1, q_1, p_2, q_2.
- Calculate $\mathrm{d}^2\Gamma/(\mathrm{d}X\mathrm{d}Y)$.

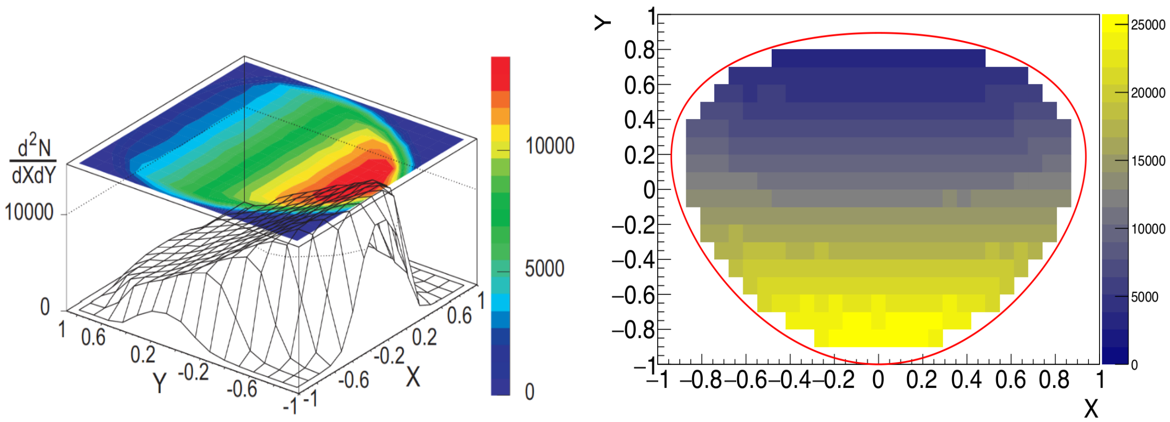


Figure 1: The Dalitz plots for the decay $\eta \to \pi^+\pi^-\pi^0$ measured at KLOE [JHEP 0805 (2008) 006]

The density of the dalitz plot is proportional to square of the matrix element.

- What is reason of the symmetry with respect to $X \to -X$ reflection?
- Why does the intensity change towards Y axis?
- How would the Dalitz plot differ to the one for $\eta \to 3\pi$?

6.4 $\eta \to 3\pi$ decay

- Consider the decay channels of the η-meson:
  - $\eta \xrightarrow{?} \pi^+\pi^-/\pi^0\pi^0$: find possible J^P quantum numbers of the system of two pions.

- $\eta \stackrel{?}{\to} \gamma\gamma$: find possible J^P quantum numbers of the system of two gamma quanta.
- Which quantum number is violated in the decay $\eta \to 3\pi$.

6.5 Black disc model

In 1935, Hideki Yukawa postulated that the short-range nuclear force may be mediated by massive particles.

- What are those particles?
- What is the range of the interaction?
- How long is the mean free path length of these particles in iron $\rho_{\mathsf{Fe}} = 7.8\,\mathrm{g/cm^3}$ given that the absorption cross section is 500 mbarn?
- How many partial waves are expected to be significant when those particles with energy 30 GeV are scattered off iron.
- Calculate the scattering cross section under the assumption that the iron nucleus can be regarded as a total absorbing disk.

Hints:

- the mass of the iron atom is $m_0 = 55.85$ u,
- the radius of the iron atom is 140 pm.
- The nucleus radius is approximately $r = 1.25 A^{1/3}$ fm.