Joined Physics Analysis Center

Summer Workshop on the Reaction Theory
Exercise sheet 7

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Classwork

1. Using $\int_0^\infty e^{-ax}dx = 1/a$, compute the series (and make the factor s^α appear)

$$F(s,\alpha) = \sum_{J=0}^{\infty} \frac{s^J}{J-\alpha}. \tag{1}$$

2. Consider $\pi\pi \to \pi\pi$ with m being the pion mass. The reduce amplitude φ_ℓ is defined by removing the barrier factor $B_\ell = (t-4m^2)^\ell$ from the (elastic) partial amplitude $t_\ell(s) = B_\ell(s)\varphi_\ell$. The phase space factor is $\rho(s) = (1/16\pi)\sqrt{1-4m^2/s}$. Use the unitarity equation $\operatorname{Im} t_\ell(s) = \rho(s)|t_\ell|^2$ to deduce the unitarity equation for the reduce amplitude.

3. Consider $\pi\pi \to \pi\pi$ and $\pi\pi \to K\bar{K}$ with m_1 being the pion mass and m_2 being the kaon mass. Let us denote by 1 (2) the $\pi\pi$ ($K\bar{K}$) channel so that $t_\ell^{ij}(s)$ is the partial wave for the scattering $i \to j$. The reduce amplitude φ_ℓ^{ij} is defined by removing the barrier factors $B_\ell^i = (t-4m_i^2)^\ell$ from the (elastic) partial amplitude $t_\ell^{ij}(s) = \sqrt{B_\ell^i(s)B_\ell^j(s)}\varphi_\ell^{ij}(s)$. Note that $t_\ell^{ji}(s) = t_\ell^{ij}(s)$. The phase space factors are $\rho_i(s) = (1/16\pi)\sqrt{1-4m_i^2/s}$. Use the unitarity equation $\operatorname{Im} t_\ell^{ij}(s) = \sum_{k=1,2}\rho_k(s)t_\ell^{ik*}(s)t_\ell^{kj}(s)$ or equivalently

$$\operatorname{Im} t_\ell^{11}(s) = \rho_1(s)|t_\ell^{11}(s)|^2 + \rho_2(s)|t_\ell^{12}(s)|^2, \tag{2a}$$
$$\operatorname{Im} t_\ell^{12}(s) = \rho_1(s)t_\ell^{11*}(s)t_\ell^{12}(s) + \rho_2(s)t_\ell^{12*}(s)t_\ell^{22}(s), \tag{2b}$$
$$\operatorname{Im} t_\ell^{22}(s) = \rho_1(s)|t_\ell^{12}(s)|^2 + \rho_2(s)|t_\ell^{22}(s)|^2, \tag{2c}$$

to derive the unitarity equations for the reduce amplitudes φ_ℓ^{ij}.

4. In the single channel case $\pi\pi \to \pi\pi$, assume the following form for the reduce amplitude $\varphi_\ell(s) = \beta(s)/(\ell-\alpha(s))$ and derive the unitarity equation for the Regge trajectory $\alpha(s)$ using the unitarity equation for the reduce amplitude, assuming the residue $\beta(s)$ is real.

5. In the coupled channel case $\pi\pi \to \pi\pi, K\bar{K}$, assume the following form for the reduce amplitude $\varphi_\ell^{ij}(s) = \beta_{ij}(s)/(\ell-\alpha(s))$. Derive the unitarity equation for the Regge trajectory $\alpha(s)$ using the three unitarity equations for the reduce amplitudes $\varphi^{11}, \varphi^{12}$ and φ^{22}, assuming the residues $\beta_{ij}(s)$ are real (and $\beta_{12} = \beta_{21}$). Since these unitarity equations are equal, what are the resulting constraints on the residues $\beta_{ij}(s)$?