Joint Physics Analysis Center

Summer Workshop on the Reaction Theory
Exercise sheet 2

Team 2: Andrew Jackura and Marc Vanderhaeghen

Contact: http://www.indiana.edu/~ssrt/index.html

June 12 – June 22 To be discussed on Wednesday of Week-I.

Classwork

Wigner D-Matrices

The Wigner matrices of the rotation group are given as

$$\mathcal{D}^{(j)}_{\sigma'\sigma}(\alpha,\beta,\gamma) = e^{-i\alpha\sigma' - i\gamma\sigma} d^{(j)}_{\sigma'\sigma}(\beta) \tag{1}$$

where α, β, and γ are the Euler angles (see Fig. 1) and the little d-matrices are defined by

$$d^{(j)}_{\sigma'\sigma}(\beta) \equiv \langle j,\, \sigma' |\, e^{-i\beta J_y}\, | j,\, \sigma \rangle\,. \tag{2}$$

Using the properties of the rotation generators:

$$\mathbf{J}^2 = J_x^2 + J_y^2 + J_z^2, \tag{3}$$
$$J_\pm = J_x \pm i J_y, \tag{4}$$
$$\mathbf{J}^2\, |j,\, \sigma\rangle = j(j+1)\, |j,\, \sigma\rangle\,, \tag{5}$$
$$J_z\, |j,\, \sigma\rangle = \sigma\, |j,\, \sigma\rangle\,, \tag{6}$$
$$J_\pm\, |j,\, \sigma\rangle = \sqrt{j(j+1) - \sigma(\sigma \pm 1)}\, |j,\, \sigma \pm 1\rangle\,, \tag{7}$$

derive the little d-matrices for the spin-1/2 representation and the spin-1 representation. If we define the half-angle factor $\xi^{\sigma'\sigma}(z) = \sqrt{(1+z)/2}^{|\sigma'+\sigma|}\sqrt{(1-z)/2}^{|\sigma'-\sigma|}$, write the d-matrices in the form $d^{(j)}_{\sigma'\sigma}(z) = \xi^{\sigma'\sigma}(z) P^{(j)}_{\sigma'\sigma}(z)$, where $z = \cos\beta$, and $P^{(j)}_{\sigma'\sigma}(z)$ is a regular polynomial in z.

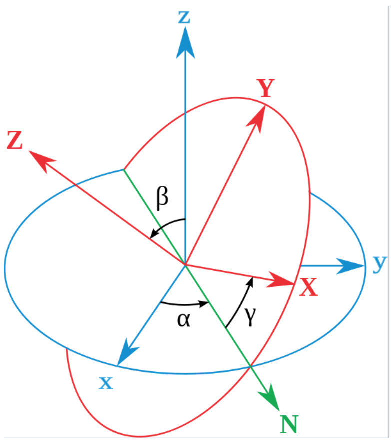

Figure 1: Euler Angles.