Joint Physics Analysis Center

Summer Workshop on the Reaction Theory
Exercise sheet 3

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June 12 – June 22 To be discussed on Wednesday of Week-I.

Classwork

πN States

(a) Consider elastic πN scattering in the center-of-momentum frame. Construct states of definite angular momentum J in the LS basis.

(b) Write the allowed J^P quantum numbers.

(c) Up through F-wave, write all the allowed states in the spectroscopic notation ${}^{2s+1}\ell_J$.

(d) Construct states of total isospin. What are the allowed isospin quantum numbers? Combining isospin states with partial waves, write all the allowed states in the spectroscopic notation ℓ_{2J2I} through F-wave.

πN Scattering

The amplitude for elastic πN scattering may be written

$$\mathcal{A}(s,t) = 8\pi\sqrt{s}\,[f(\theta)\mathbb{1} + ig(\theta)\boldsymbol{\sigma}\cdot\hat{\mathbf{n}}]\,, \tag{1}$$

where $\hat{\mathbf{n}} = \mathbf{p}\times\mathbf{p}'/|\mathbf{p}\times\mathbf{p}'|$ and $f(\theta)$ is the non-spin-flip amplitude and $g(\theta)$ is the spin-flip amplitude. We can write the differential cross section in terms of two functions $f(\theta)$ and $g(\theta)$,

$$\frac{d\sigma}{d\Omega} = |f(\theta)|^2 + |g(\theta)|^2. \tag{2}$$

The functions $f(\theta)$ and $g(\theta)$ may be expanded in partial waves:

$$f(\theta) = \frac{1}{|\mathbf{p}|}\sum_{\ell=0}^{\infty}((\ell+1)\,a_{\ell,\ell+1/2}(s) + \ell\,a_{\ell,\ell-1/2})P_\ell(\cos\theta) \tag{3}$$

and

$$g(\theta) = \frac{1}{|\mathbf{p}|}\sum_{\ell=0}^{\infty}(a_{\ell,\ell+1/2}(s) - a_{\ell,\ell-1/2})\sin\theta\,P'_\ell(\cos\theta) \tag{4}$$

Keeping only the $\ell=1$, $J=3/2$ term ($J=\ell\pm 1/2$), write the differential cross section in terms of $a_{1,3/2}$. In an experiment (see Fig. 1), at $\sqrt{s} = 1235.4$ MeV, the angular distribution was measured. Note that $|\mathbf{p}|^{-2} \approx 7.34$ mb at this energy. Assuming only the $\ell=1$, $J=3/2$ term, what is the magnitude of the partial wave amplitude $|a_{1,3/2}|$? In terms of the phase shift and inelasticity, what can you determine? The phase shift here is defined as

$$a_{\ell,J}(s) = \frac{1}{2i}(\eta_{\ell,J}(s)e^{2i\delta_{\ell,J}(s)} - 1). \tag{5}$$

πN Resonances

We consider the Δ^{++} baryon as an example of resonance phenomena. The Δ^{++} baryon can be found in the scattering of $\pi^+ p \to \pi^+ p$ (see Fig. 2). Let s be the invariant mass of the πN system.

The amplitude for such a process is approximated by the Breit-Wigner form, when s is near the resonance position.

$$a_{J=3/2}(s) \sim \frac{1}{s - M^2 + iM\Gamma(s)} \tag{6}$$

where in general $\Gamma(s)$ is a function of s.

The pole position is found from $s_p - M^2 + iM\Gamma(s_p)$=0, where $\sqrt{s_p} \equiv M_R - i\Gamma_R/2$ is the definition for the mass and width of the resonance. Find the pole mass and width in terms of M and $\Gamma(M)$ and it's derivative $\Gamma'(M)$ for s near M^2.

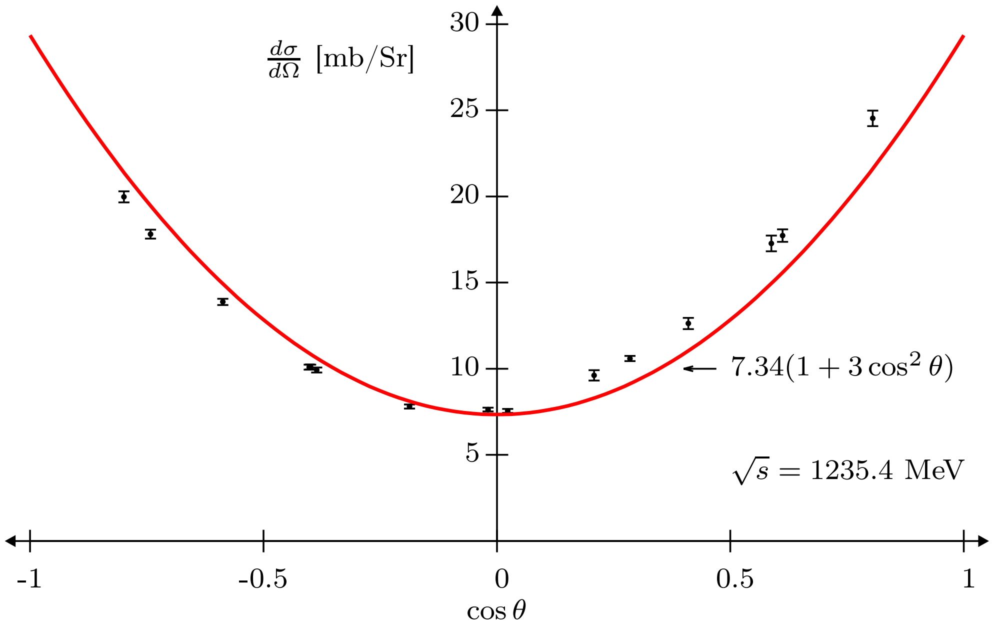

Figure 1: Experimental data for the angular distribution for elastic $\pi^+ p$ scattering at the center-of-momentum energy $\sqrt{s} = 1235.4$ MeV (Bussey 1973 [2]). The curve corresponds to the maximum of $d\sigma/d\Omega$ under the assumption that only the partial wave with $\ell = 1$, $J = 3/2$ contributes.

References

[1] V. Mathieu, I. V. Danilkin, C. Fernández-Ramírez, M. R. Pennington, D. Schott, A. P. Szczepaniak and G. Fox, Phys. Rev. D **92**, no. 7, 074004 (2015) doi:10.1103/PhysRevD.92.074004 [arXiv:1506.01764 [hep-ph]].

[2] P. J. Bussey, J. R. Carter, D. R. Dance, D. V. Bugg, A. A. Carter and A. M. Smith, Nucl. Phys. B **58**, 363 (1973). doi:10.1016/0550-3213(73)90589-0

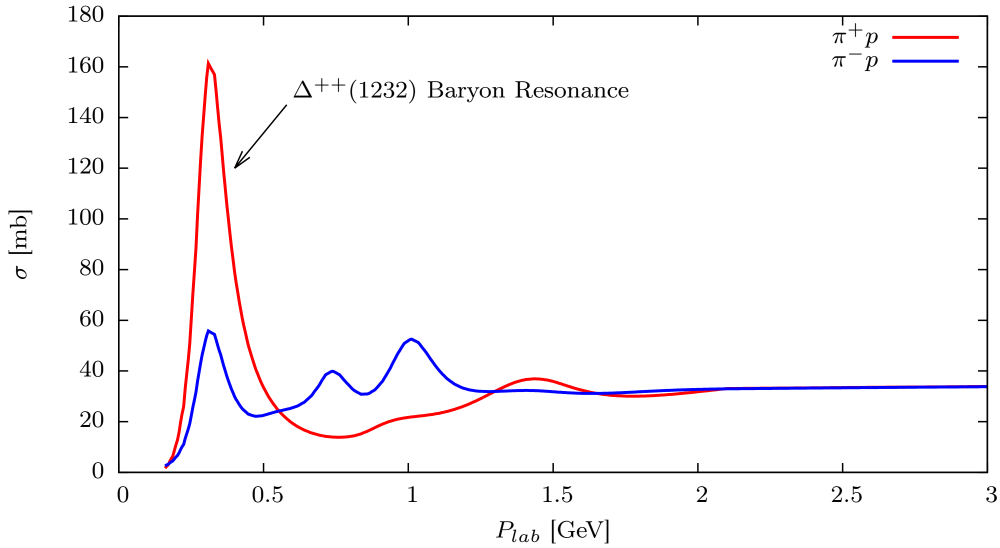

Figure 2: Cross sections for πN scattering [1].