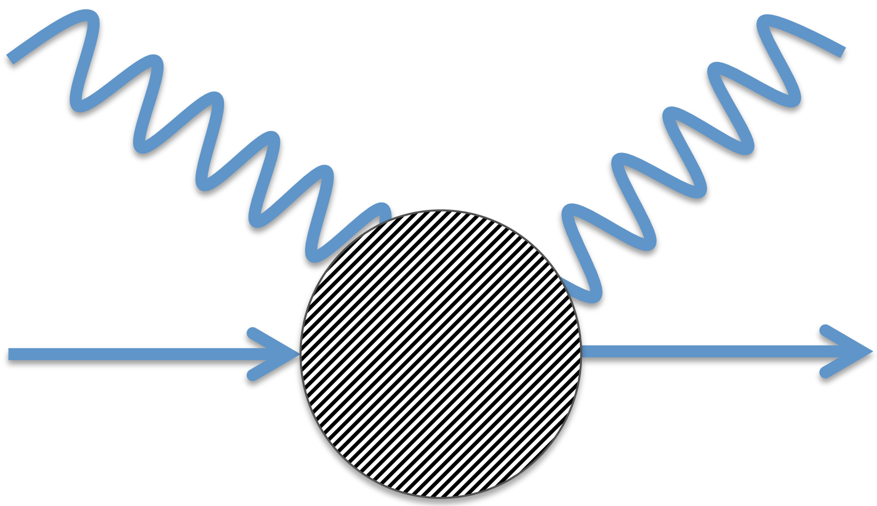
Stefano Sconfietti

Nucleon Compton Scattering: DIPOLE DYNAMIC POLARIZABILITIES from experimental data.

UNIVERSITÀ DI PAVIA



INFN

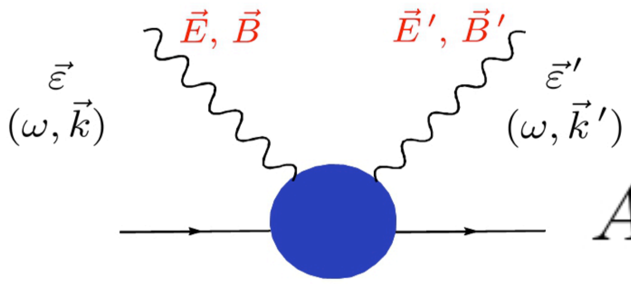
2017 Reaction Theory Summer Workshop – Indiana University

Stefano Sconfietti

Beer: which Country is the best producer?

2017 Reaction Theory Summer Workshop – Indiana University

RCS amplitudes and **Dispersion Relations**

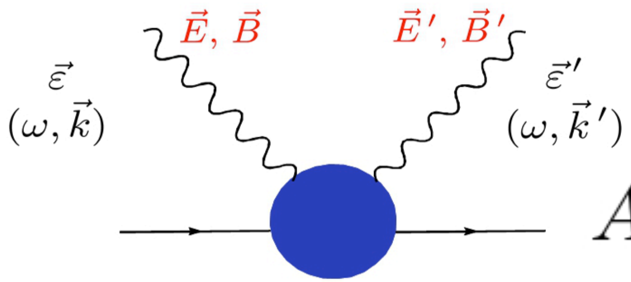


$$A_i(\nu, t) = A_i^s(\nu, 0) + A_i^t(0, t) + A_i(0, 0)$$

Subtracted Dispersion Relations (*s*-channel)

$$A_i^s(\nu, 0) \;=\; \frac{2}{\pi}\,\nu^2\,\mathrm{P}\int_{\nu_{thr}}^{\infty} \mathrm{Im_s}\;A_i(\nu', t)\,\frac{d\nu'}{\nu'(\nu'^2 - \nu^2)}$$

RCS amplitudes and Dispersion Relations



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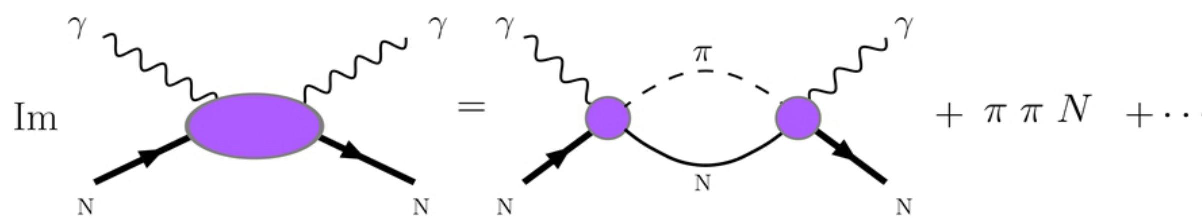
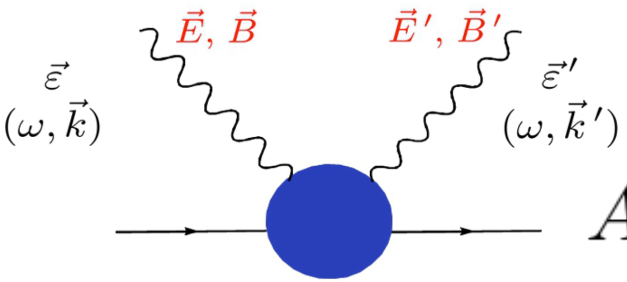
S - CHANNEL



RCS amplitudes and Dispersion Relations



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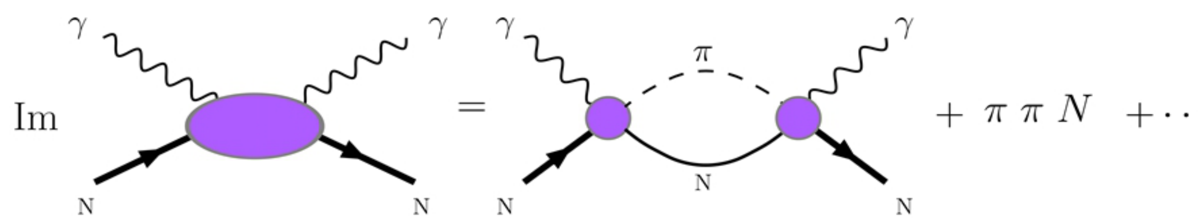
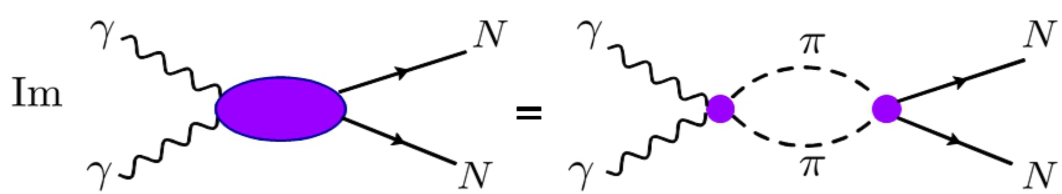
S - CHANNEL



T - CHANNEL



Low Energy Expansion

$$
\begin{aligned}
A_i(\nu,t) &= A_i(\nu,t)|_{(0,0)} + \left.\frac{\partial A_i(\nu,t)}{\partial \nu^2}\right|_{(0,0)} \nu^2 + \left.\frac{\partial A_i(\nu,t)}{\partial t}\right|_{(0,0)} t \\
&+ \frac{1}{2}\left(\left.\frac{\partial^2 A_i(\nu,t)}{\partial \nu^4}\right|_{(0,0)} \nu^4 + \left.\frac{\partial^2 A_i(\nu,t)}{\partial t}^2\right|_{(0,0)} t^2 + 2\left.\frac{\partial^2 A_i(\nu,t)}{\partial \nu^2 \partial t}\right|_{(0,0)} \nu^2 t \right)
\end{aligned}
$$

Low Energy Expansion

$$A_i(\nu,t) = A_i(\nu,t)|_{(0,0)} + \left.\frac{\partial A_i(\nu,t)}{\partial \nu^2}\right|_{(0,0)} \nu^2 + \left.\frac{\partial A_i(\nu,t)}{\partial t}\right|_{(0,0)} t$$
$$+ \frac{1}{2}\left(\left.\frac{\partial^2 A_i(\nu,t)}{\partial \nu^4}\right|_{(0,0)} \nu^4 + \left.\frac{\partial^2 A_i(\nu,t)}{\partial t}^2\right|_{(0,0)} t^2 + 2 \left.\frac{\partial^2 A_i(\nu,t)}{\partial \nu^2 \partial t}\right|_{(0,0)} \nu^2 t \right)$$

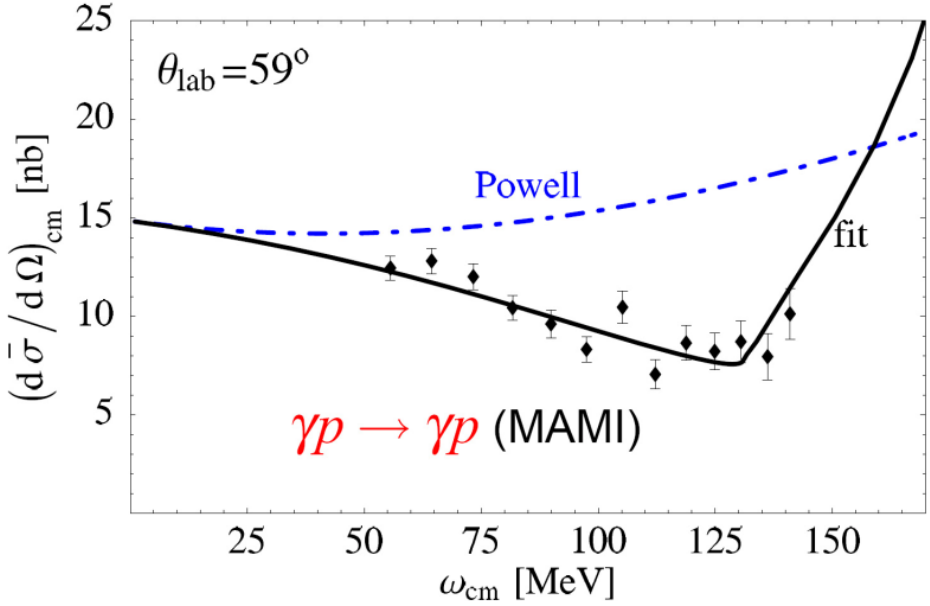
ν and ***t*** as **INDEPENDENT VARIABLES** (*fixed-t*)

manifestly invariant structure

$R_i = R_i(A_i)$

choiche of a reference system: *cm*

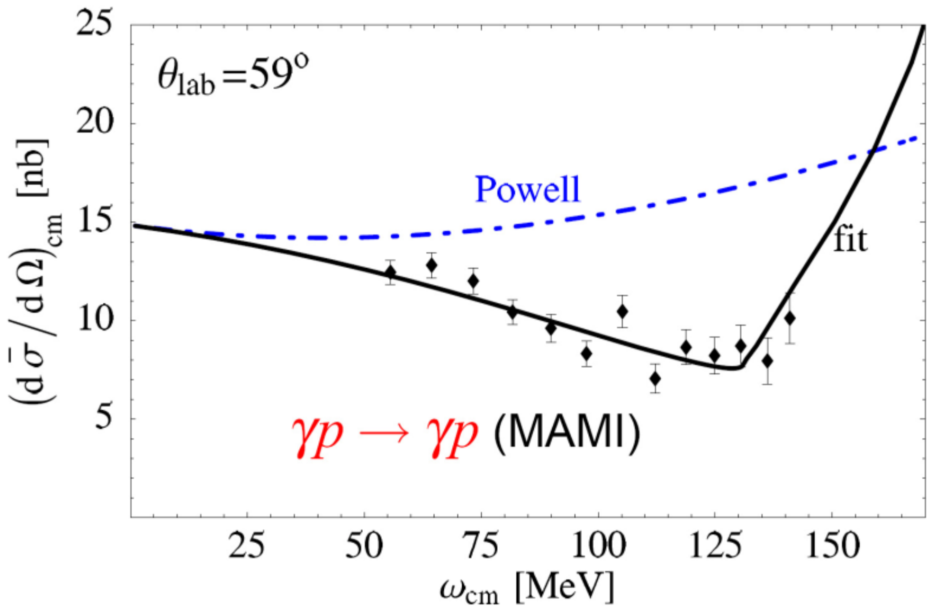
Effective Hamiltonian and Static Polarizabilities



Powell cross section: pointlike nucleon with anomalous magnetic moment

Static polarizabilities: response of the internal nucleon degrees of freedom to a **static** electric and magnetic field

Effective Hamiltonian and Static Polarizabilities



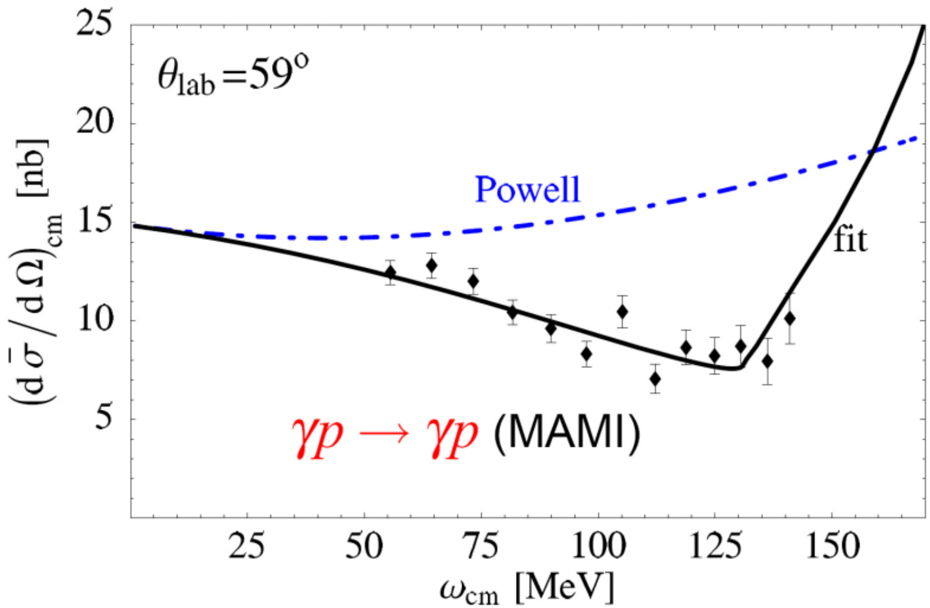
Powell cross section: pointlike nucleon with anomalous magnetic moment

Static polarizabilities: response of the internal nucleon degrees of freedom to a **static** electric and magnetic field

$$H_{\rm eff}^{\rm pol} = -2\pi\left\{\omega^2\left[\alpha_{E1}\,\vec{E}^2+\beta_{M1}\,\vec{B}^2\right]\right.$$

spin-independent dipole

Effective Hamiltonian and Static Polarizabilities



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Static polarizabilities: response of the internal nucleon degrees of freedom to a **static** electric and magnetic field

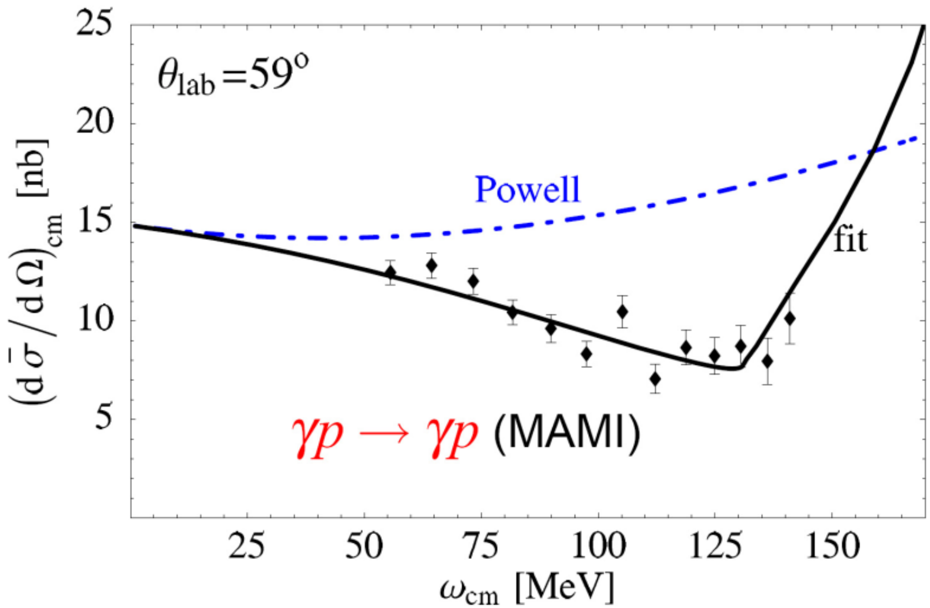
$$H_{\rm eff}^{\rm pol} = -2\pi\Big\{\omega^2\left[\alpha_{E1}\,\vec{E}^2 + \beta_{M1}\,\vec{B}^2\right]$$

spin-independent dipole

$$+\,\omega^3\left[\gamma_{E1E1}\,\vec{\sigma}\cdot(\vec{E}\times\dot{\vec{E}}) + \gamma_{M1M1}\,\vec{\sigma}\cdot(\vec{B}\times\dot{\vec{B}})\right.$$

spin-dependent dipole

Effective Hamiltonian and Static Polarizabilities



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Static polarizabilities: response of the internal nucleon degrees of freedom to a **static** electric and magnetic field

$$H_{\rm eff}^{\rm pol} = -2\pi\Big\{\omega^2\left[\alpha_{E1}\,\vec{E}^2+\beta_{M1}\,\vec{B}^2\right]$$ **spin-independent dipole**

$$+\,\omega^3\Big[\gamma_{E1E1}\,\vec{\sigma}\cdot(\vec{E}\times\dot{\vec{E}})+\gamma_{M1M1}\,\vec{\sigma}\cdot(\vec{B}\times\dot{\vec{B}})$$ **spin-dependent dipole**

$$-2\,\gamma_{M1E2}\,\sigma_i B_j E_{ij}+2\,\gamma_{E1M2}\,\sigma_i E_j B_{ij}\Big] + \mathcal{O}(\omega^3)\Big\}$$ **spin-dependent dipole-quadrupole**

Multipole Expansion and Dipole Dynamical Polarizabilities (DDP)

$$R_1 = \sum_{l\geq 1}\{[(l+1)f_{EE}^{l+} + lf_{EE}^{l-}](lP_l' + P_{l-1}'') - [(l+1)f_{MM}^{l+} + lf_{MM}^{l-}]P_l''\}$$

$$R_2 = \sum_{l\geq 1}\left\{[(l+1)f_{MM}^{l+} + lf_{MM}^{l-}](lP_l' + P_{l-1}'') - [(l+1)f_{EE}^{l+} + lf_{EE}^{l-}]P_l''\right\}$$

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$$R_1 = \sum_{l\geq 1} \{[(l+1)f_{EE}^{l+} + lf_{EE}^{l-}](lP_l' + P_{l-1}'') - [(l+1)f_{MM}^{l+} + lf_{MM}^{l-}]P_l''\}$$

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$$\alpha_{El} = a(l)\frac{(l+1)f_{EE}^{l+} + lf_{EE}^{l-}}{\omega^{2l}}$$

$$\beta_{Ml} = a(l)\frac{(l+1)f_{MM}^{l+} + lf_{MM}^{l-}}{\omega^{2l}}$$

Multipole Expansion and Dipole Dynamical Polarizabilities (DDP)

$$R_1 = \sum_{l\geq 1}\{[(l+1)f_{EE}^{l+} + l f_{EE}^{l-}](lP_l' + P_{l-1}'') - [(l+1)f_{MM}^{l+} + l f_{MM}^{l-}]P_l''\}$$

$$R_2 = \sum_{l\geq 1}\left\{[(l+1)f_{MM}^{l+} + l f_{MM}^{l-}](lP_l' + P_{l-1}'') - [(l+1)f_{EE}^{l+} + l f_{EE}^{l-}]P_l''\right\}$$

DIPOLE DYNAMICAL POLARIZABILITIES

$$\alpha_{El} = a(l)\frac{(l+1)f_{EE}^{l+} + l f_{EE}^{l-}}{\omega^{2l}}$$

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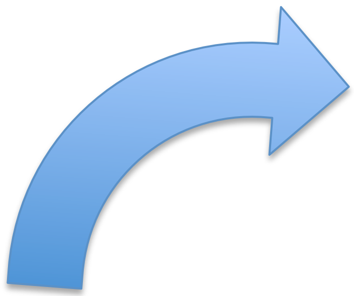
DDP: ready for the **fit** (nearly)



multipoles

DDP

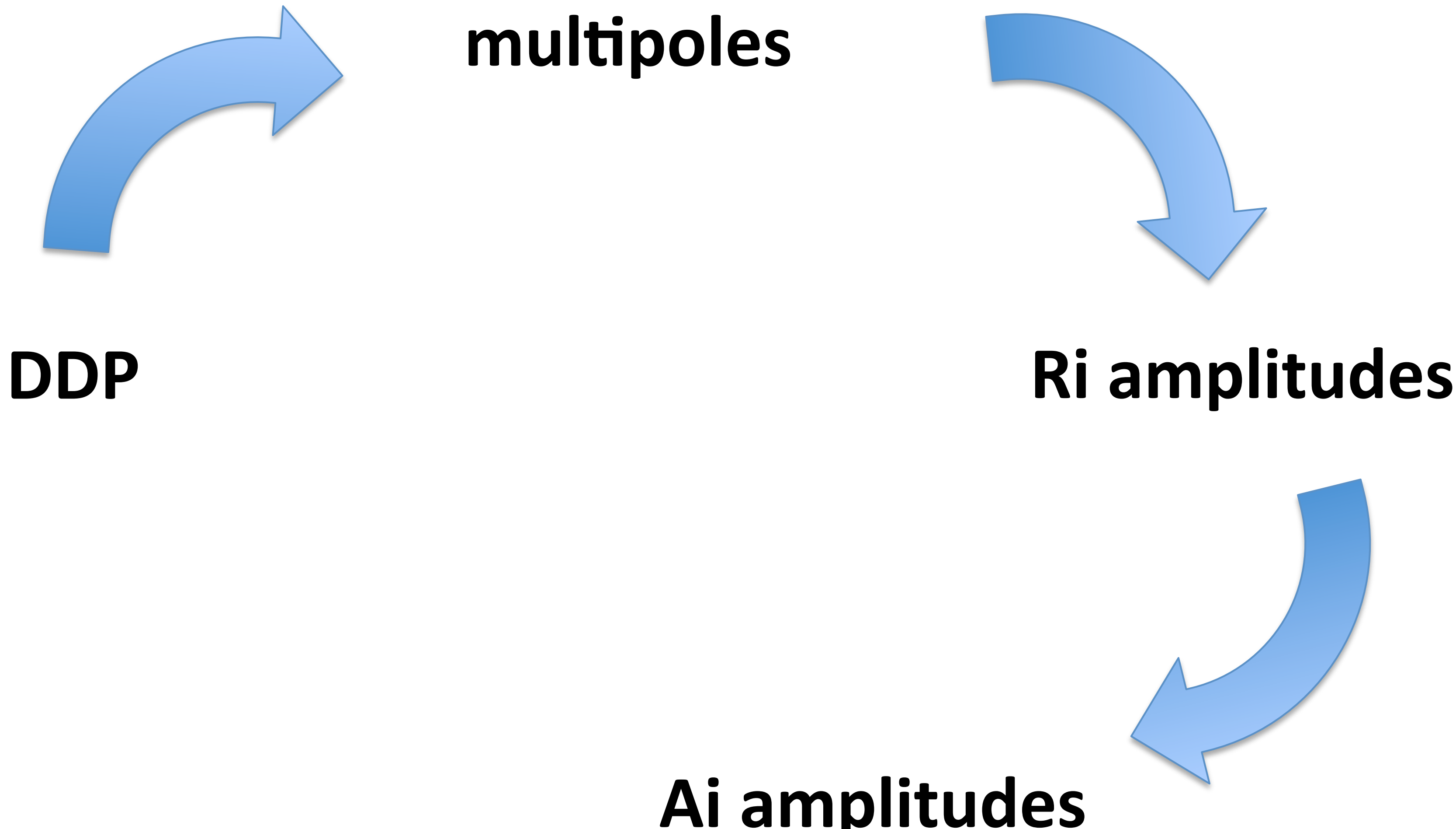
DDP: ready for the **fit** (nearly)

multipoles

DDP

Ri amplitudes

DDP: ready for the **fit** (nearly)



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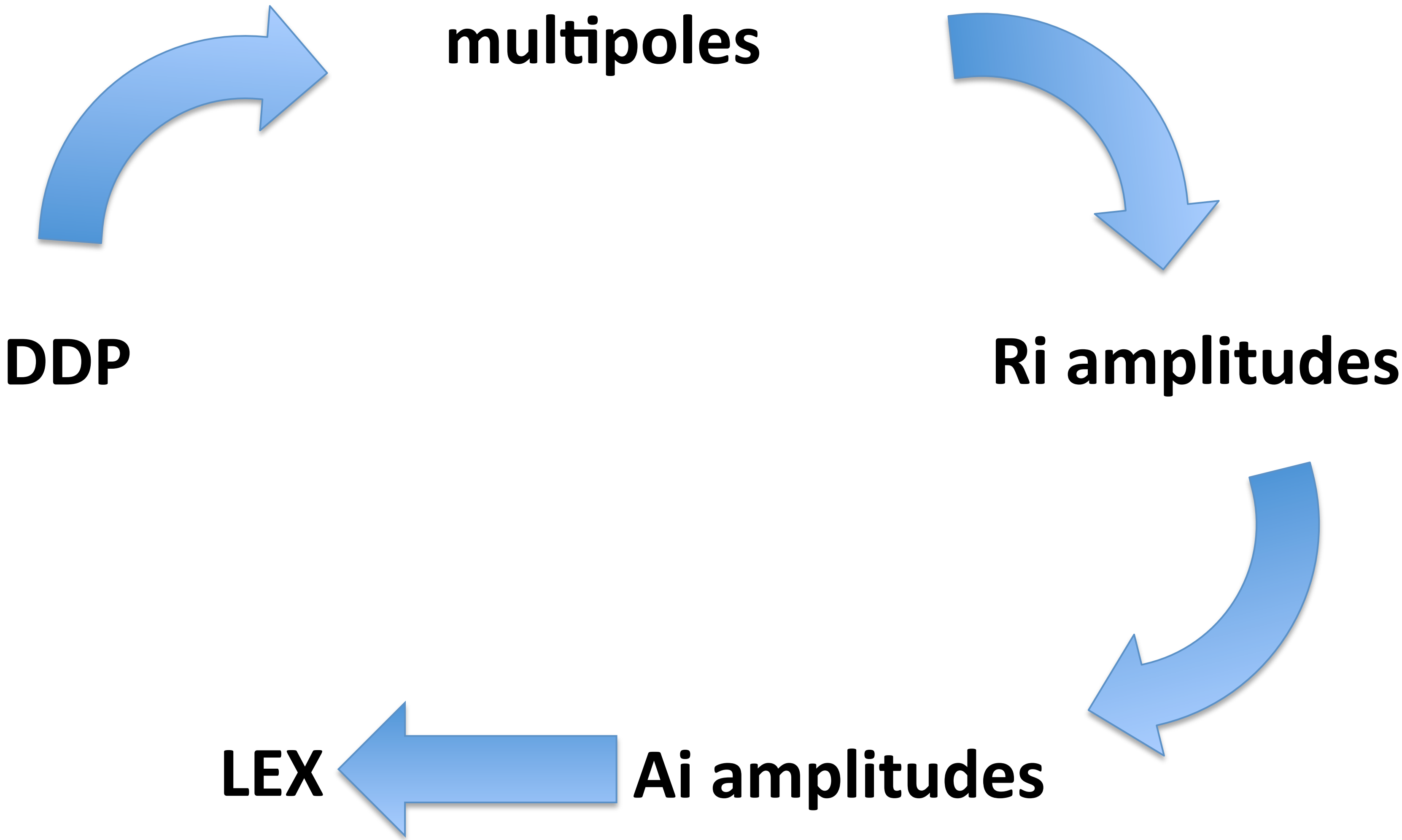

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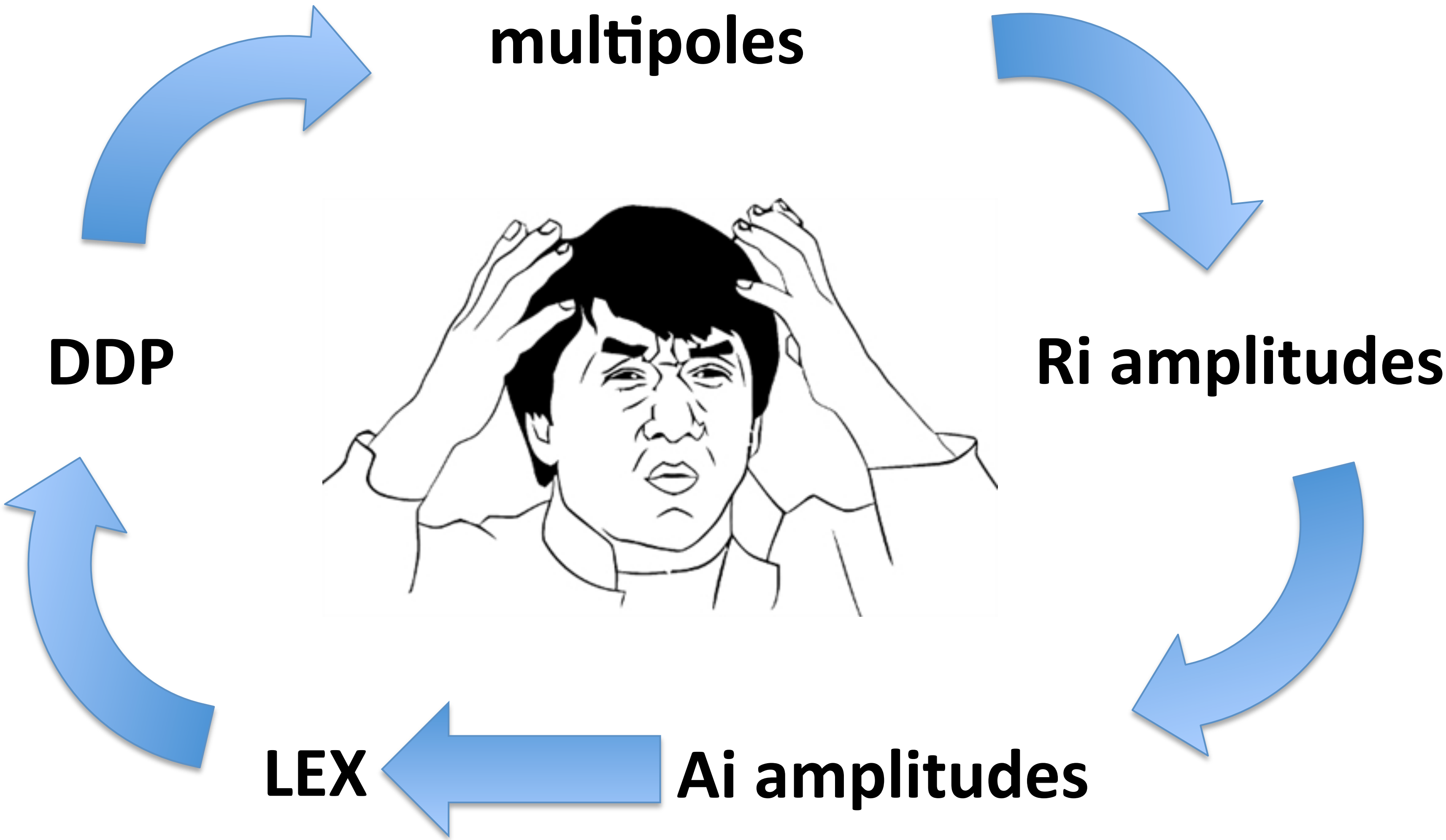

DDP: ready for the **fit** (really)

$$\alpha(\omega) = \alpha E10 + \alpha E11\ \omega + \alpha E12\ \omega 2 + \alpha E13\ \omega 3 + \alpha E14\ \omega 4 + \alpha E15\ \omega 5$$

$$\beta(\omega) = \beta M10 + \beta M11\ \omega + \beta M12\ \omega 2 + \beta M13\ \omega 3 + \beta M14\ \omega 4 + \beta M15\ \omega 5$$

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$\alpha(\omega) = \alpha E10 + \alpha E11\ \omega + \alpha E12\ \omega 2 + \alpha E13\ \omega 3 + \alpha E14\ \omega 4 + \alpha E15\ \omega 5$

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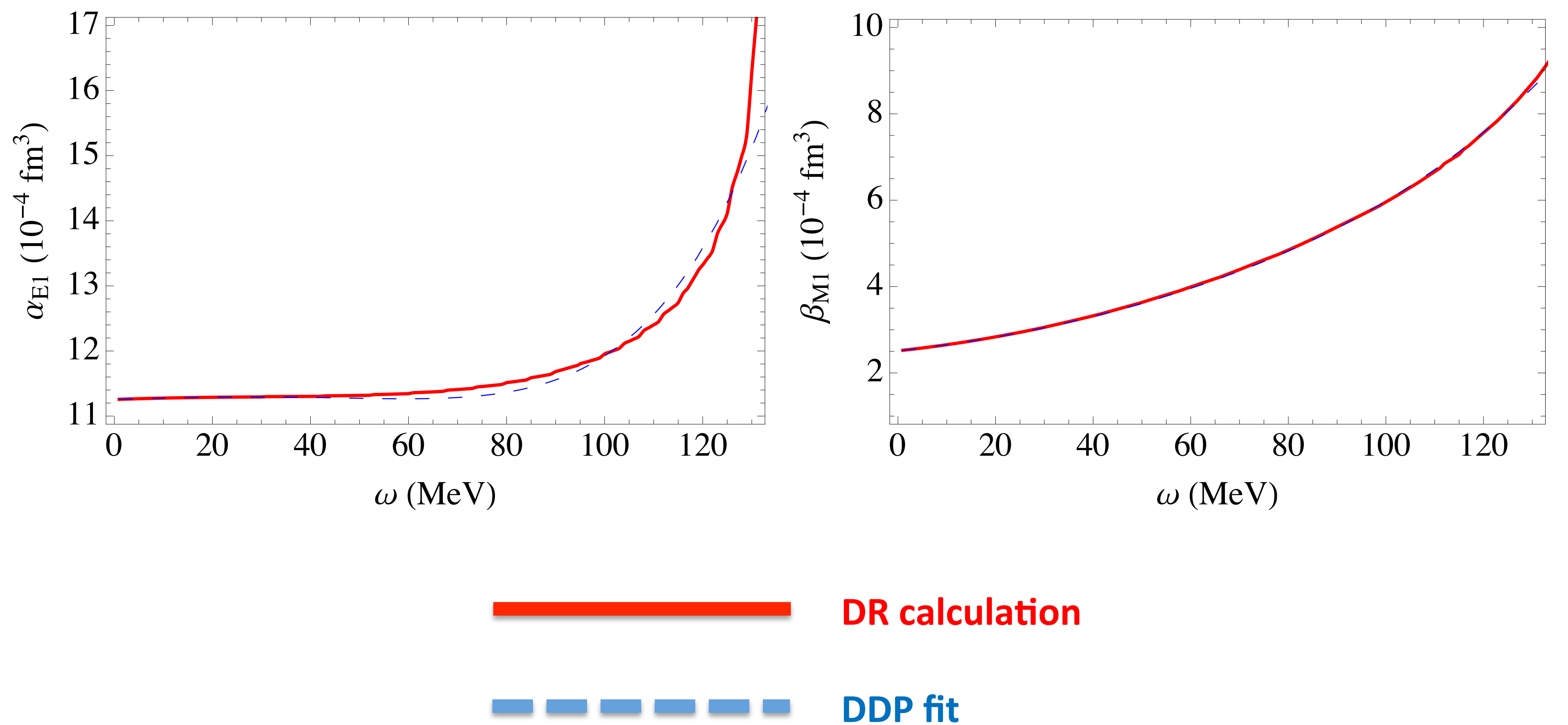
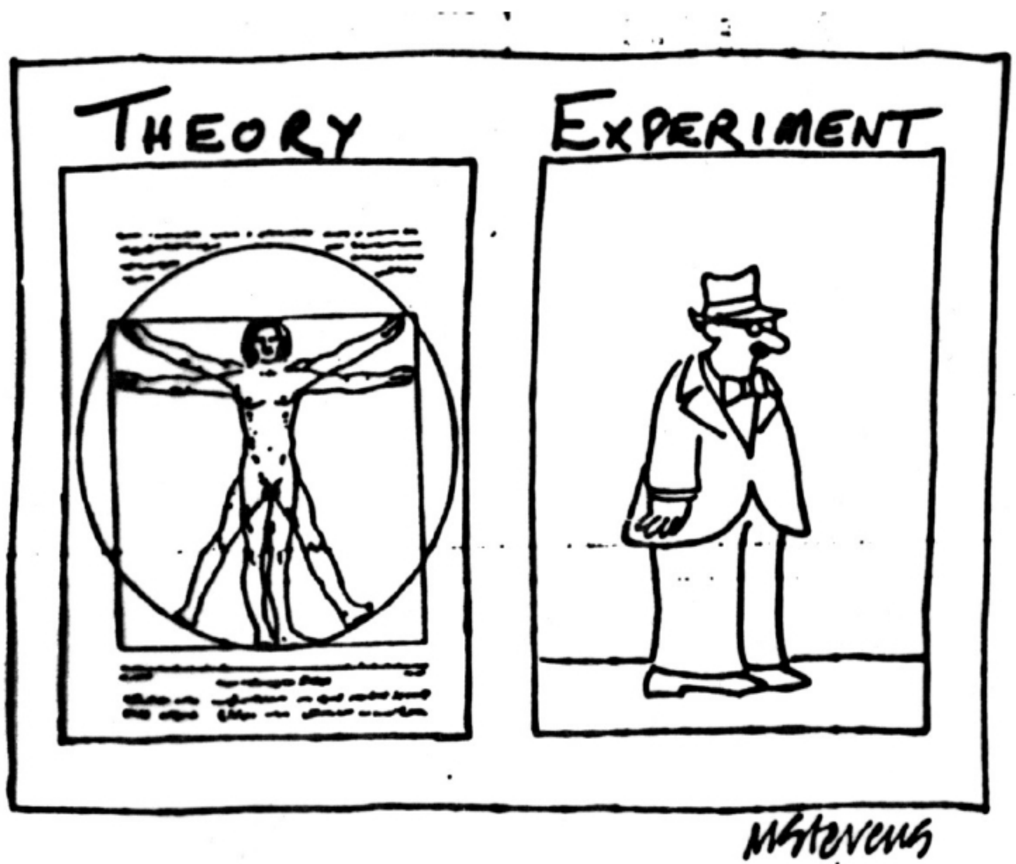


Data analysis: new method



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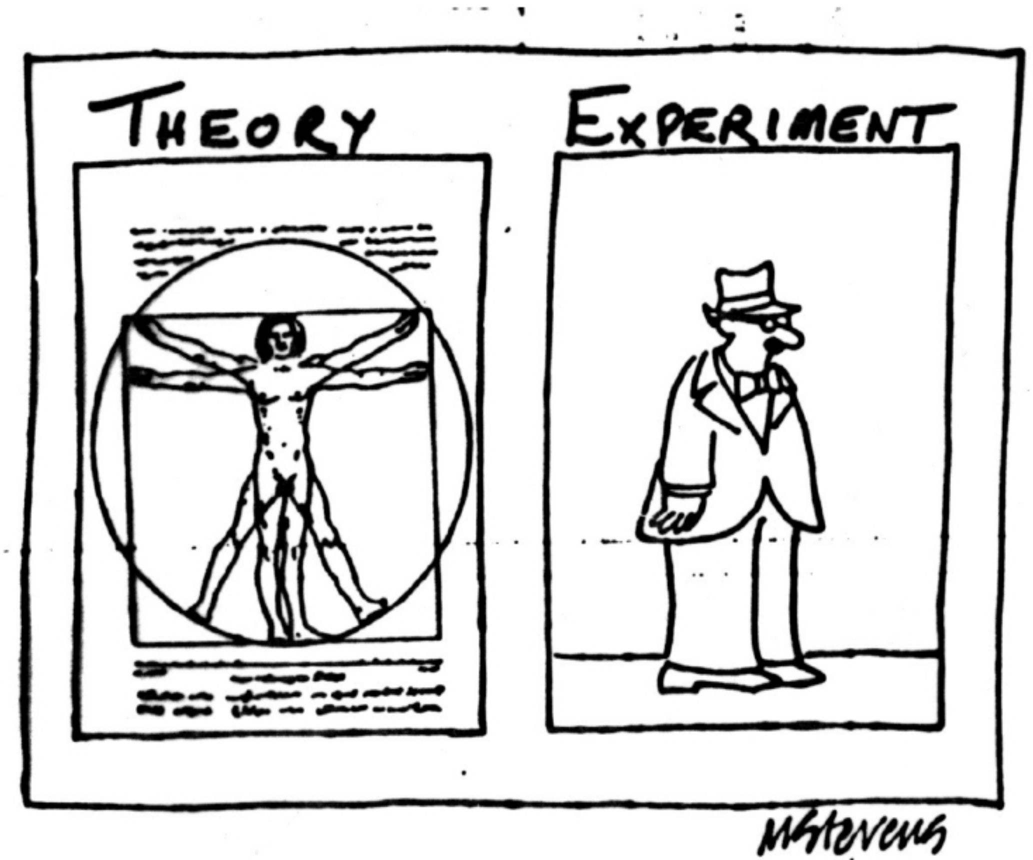
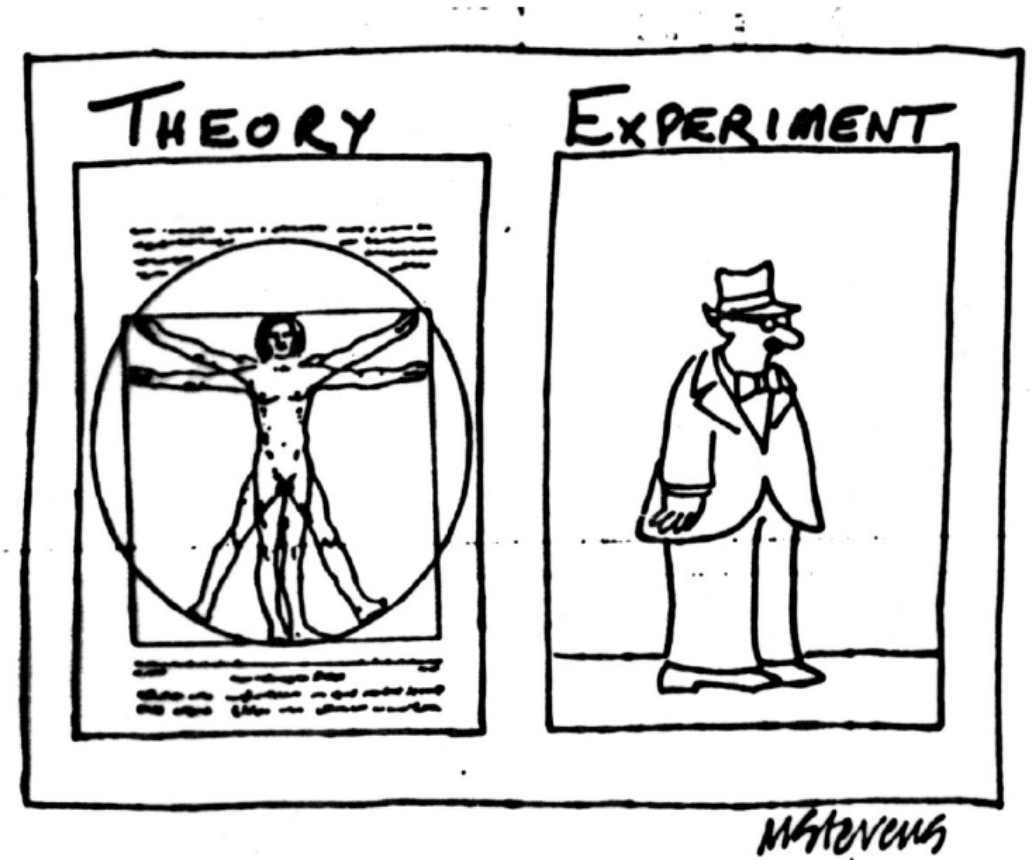


LOW SENSITIVITY

GRADIENT METHOD NOT TO BE USED

Data analysis: new method



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MONTE CARLO TECHNIQUE

NO ERROR ESTIMATION

NO ASSUMPTIONS ON PARAMETERS DISTRIBUTIONS

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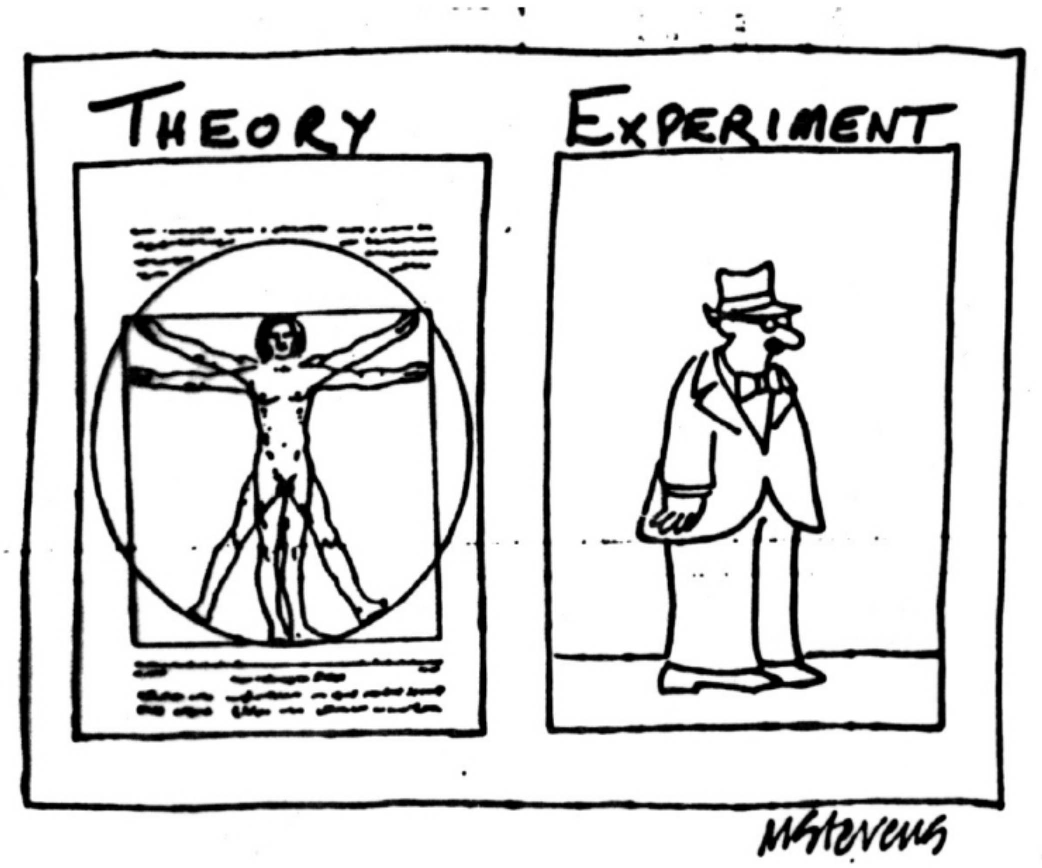

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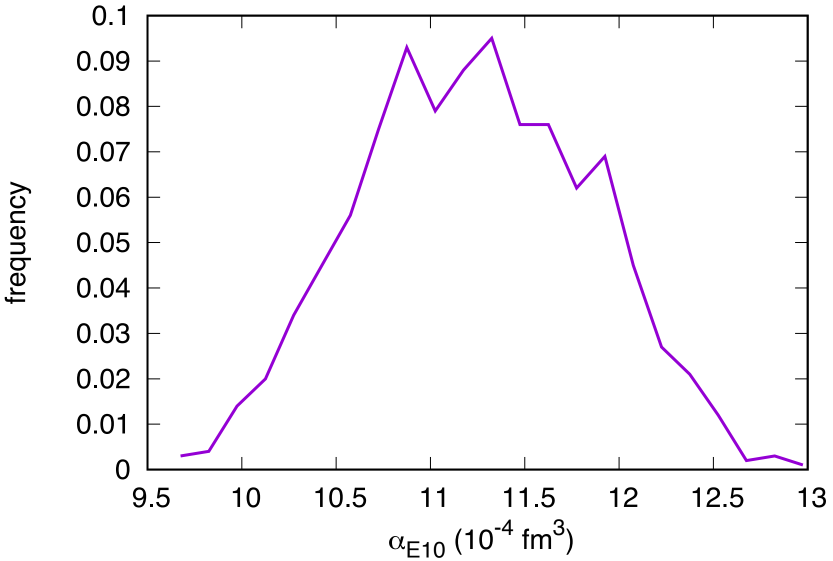
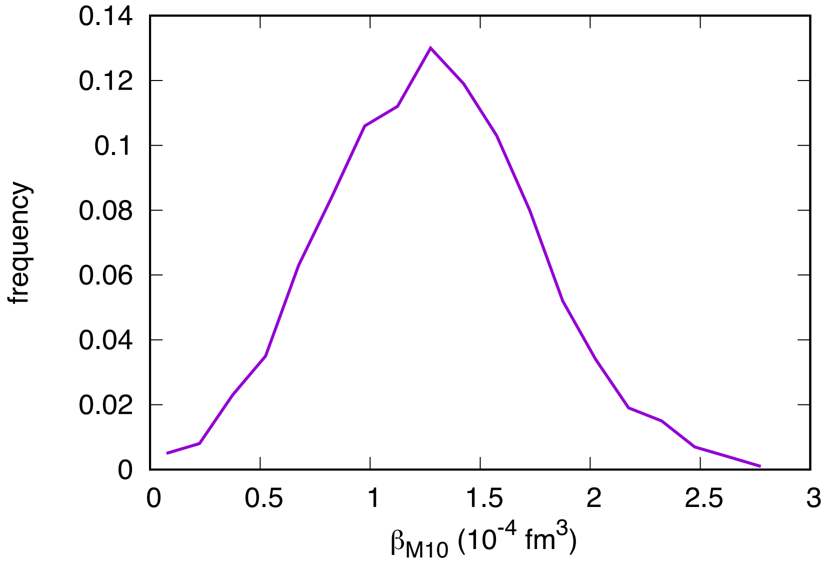
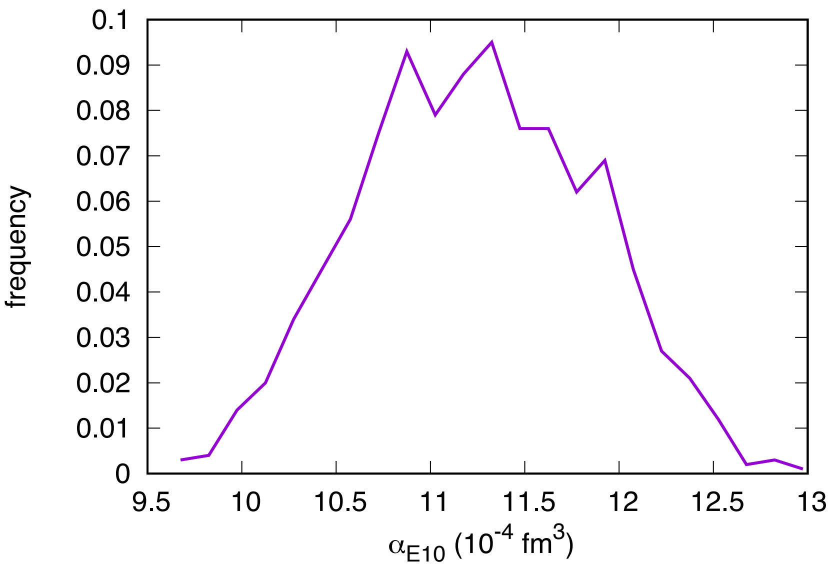
BOOTSTRAP

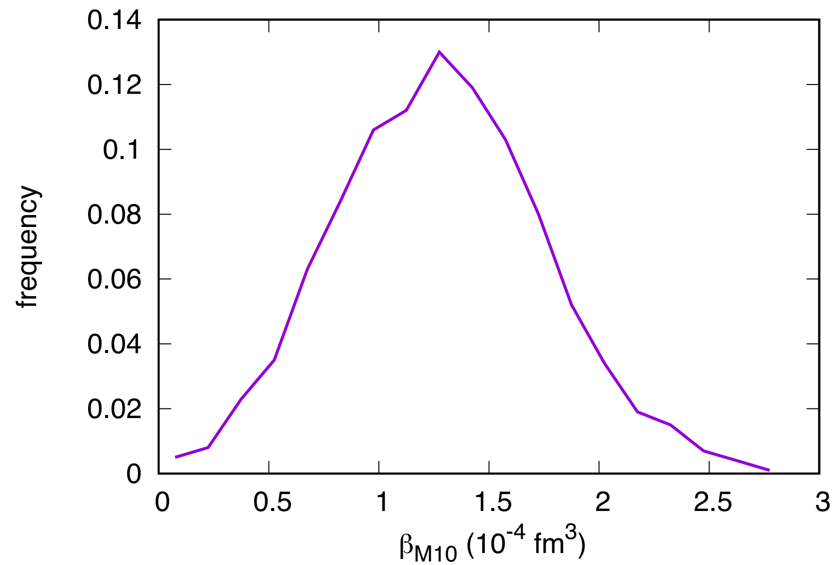
Gradient VS Bootstrap

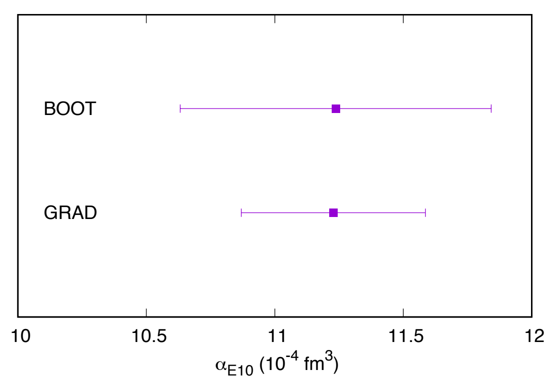


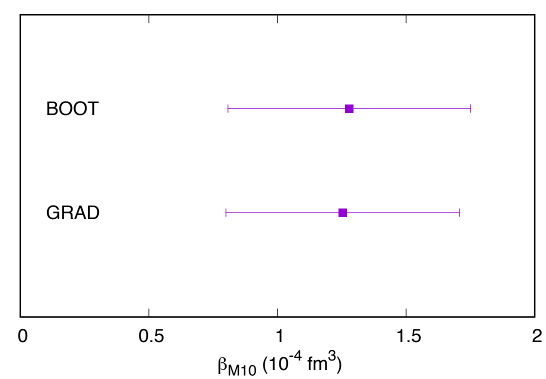


Gradient VS Bootstrap





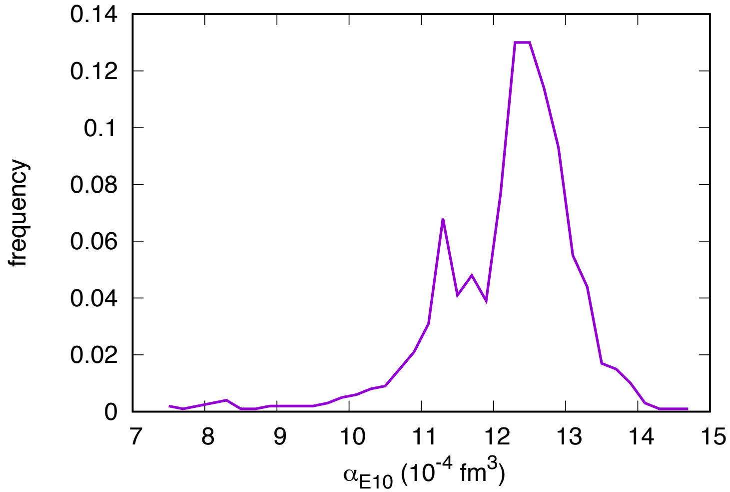


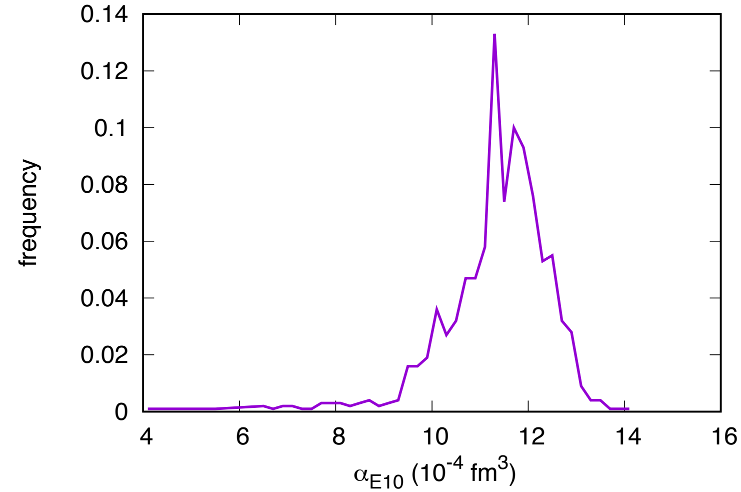


RESULTS (1)

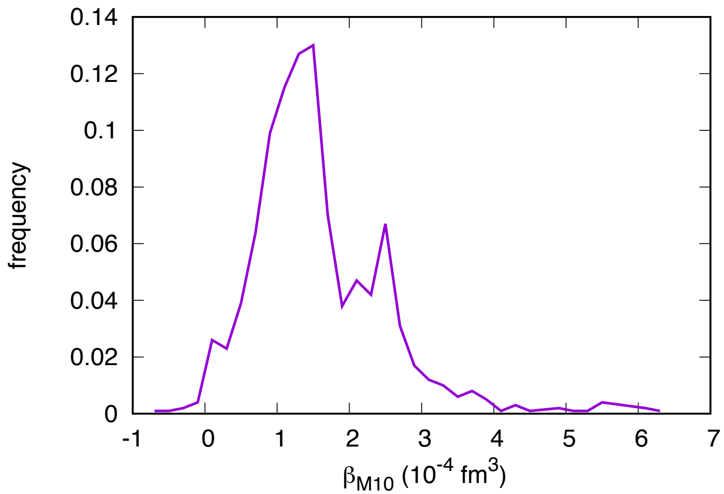
FULL DATA SET | **TAPS DATA SET (MAINZ)**

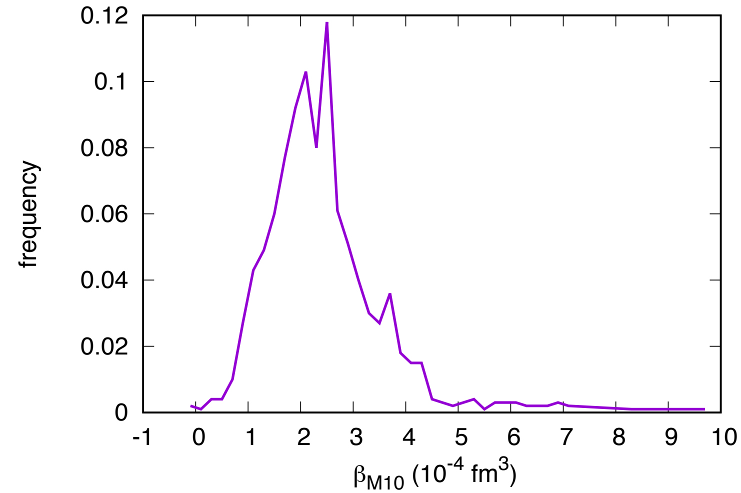
α_{E10}



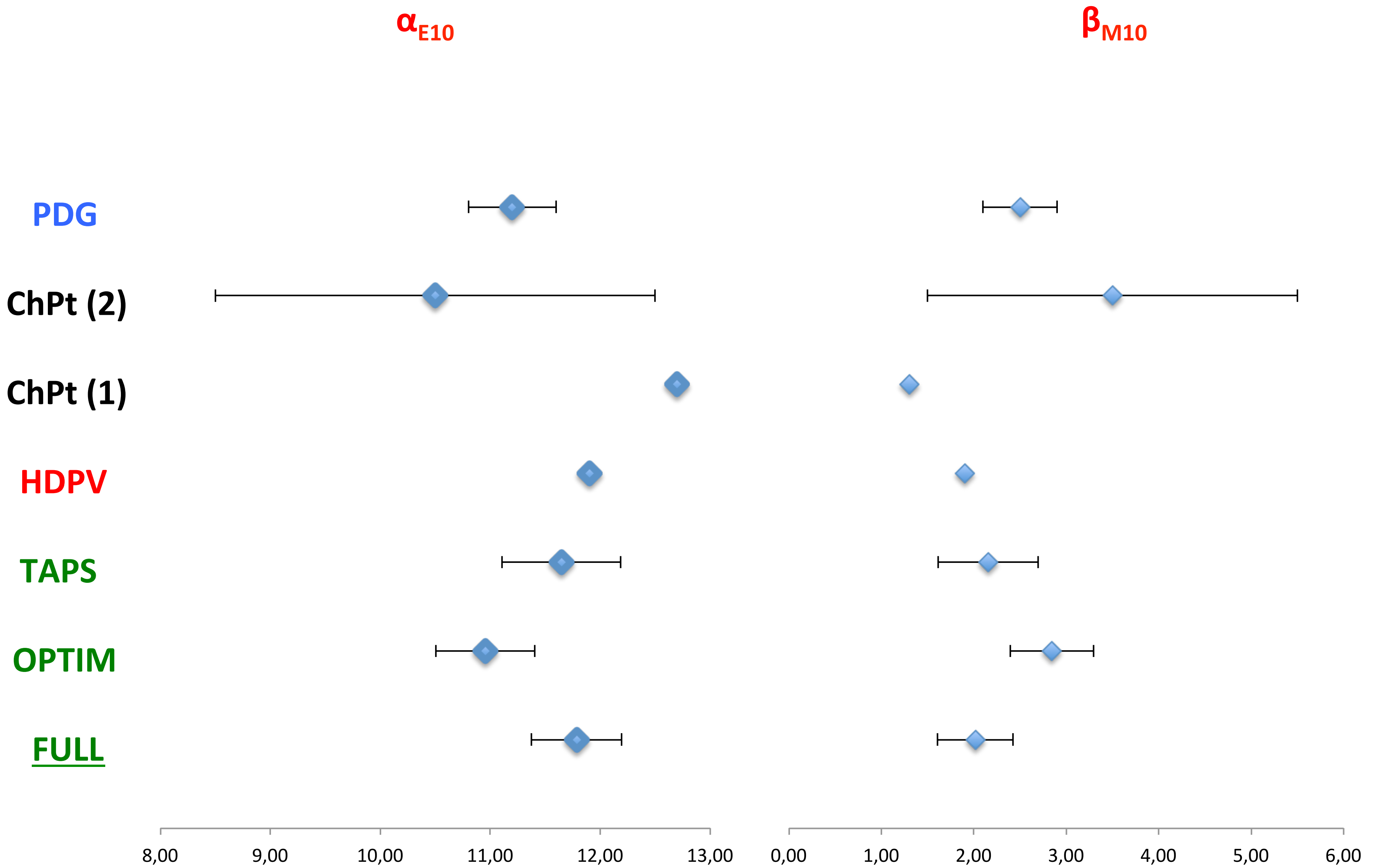


β_{M10}

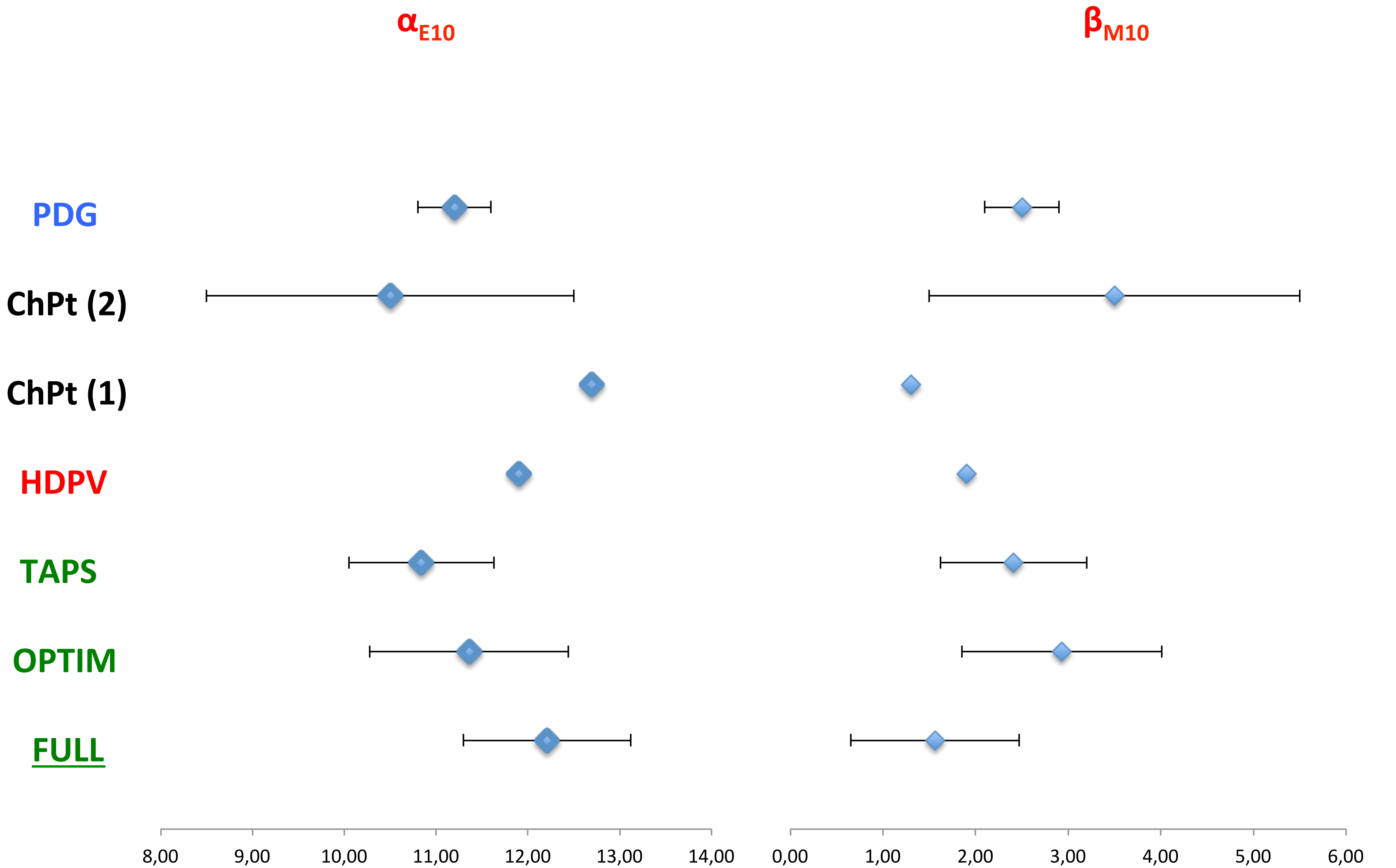




RESULTS (2): static situation

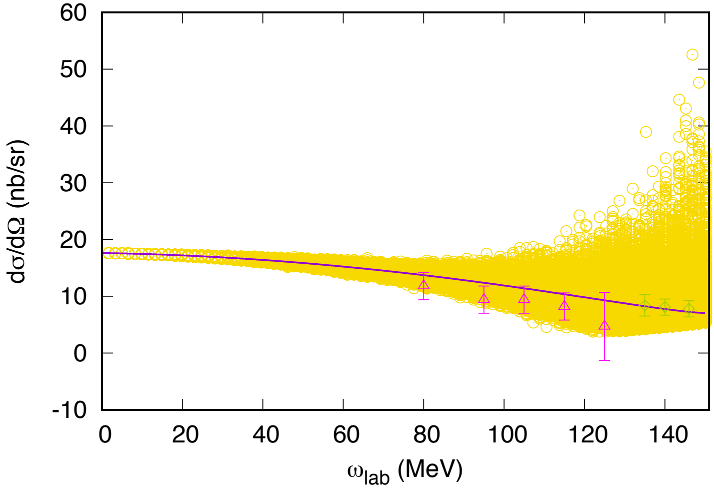


RESULTS (3): dynamical situation

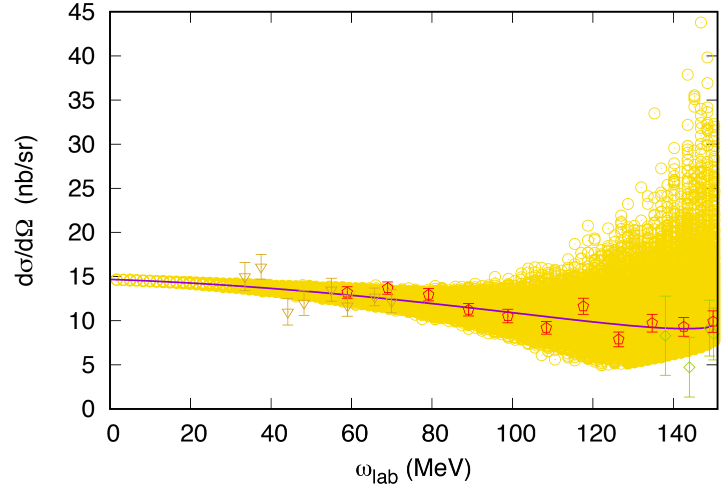


Differential cross section and DDP

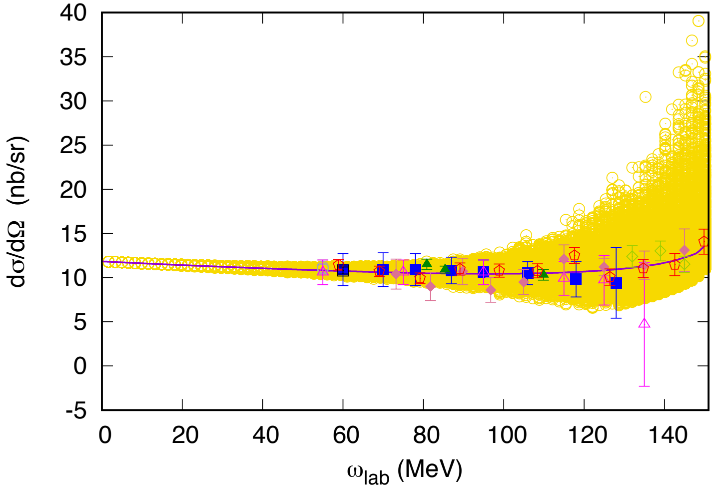
45°



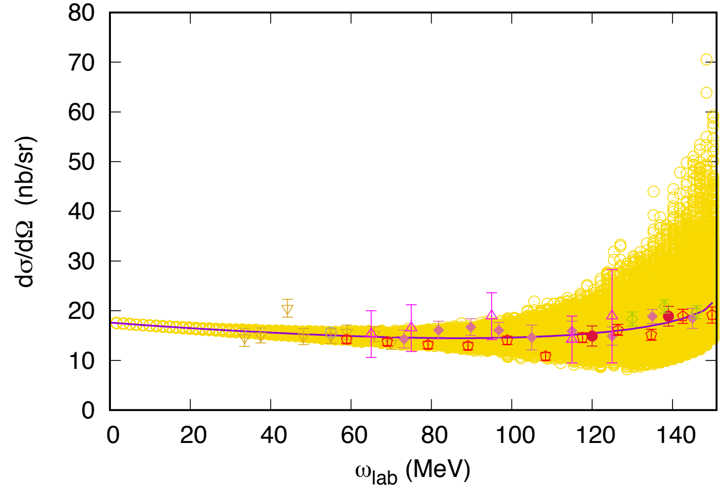
60°



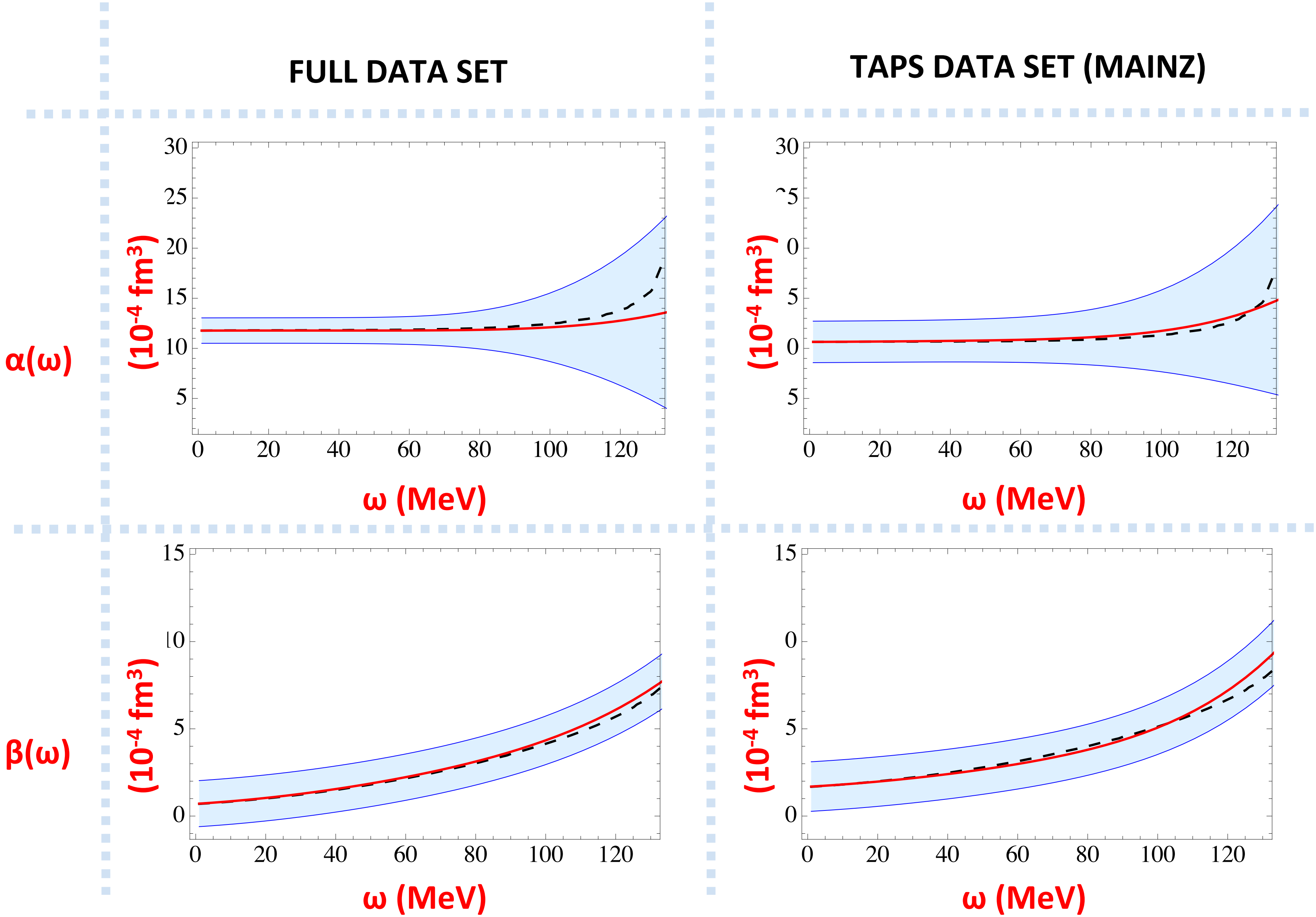
85°



135°



DDP: prediction with error



BACKUP

Systematical error: modifyed χ^2

$$\chi^2_{mod} = \sum_{i=1}^{N_{tot}} \left[\frac{\mathcal{N}\mathcal{S}_{i,exp} - \mathcal{S}_{i,theory}}{\mathcal{N}\sigma_{i,exp}} \right]^2 + \left(\frac{\mathcal{N} - 1}{\sigma_{i,sys}} \right)^2$$

$$S_{i,exp}^{boot} = \xi \left[S_{i,exp} \pm \gamma \, \sigma_{i,exp} \right]$$

From multipoles to DDP

$$\alpha_{E1}(\omega) = -\frac{-10R_1(w,0) - 2R_1^{(2)}(w,0) - 5R_2^{(1)}(w,0)}{10w^2}$$

$$\beta_{M1}(\omega) = -\frac{-5R_1^{(1)}(w,0) - 10R_2(w,0) - 2R_2^{(2)}(w,0)}{10w^2}$$

DDP: α(ω) expansion

$$
\begin{aligned}
\alpha_{E1}(\omega) &= \alpha_{E10} + \frac{\beta_{M10}}{m}\omega + \left(\alpha_{E1,\nu} + \frac{5\alpha_{E10} - 2\beta_{M10}}{8m^2}\right)\omega^2 \\
&+ \left(\frac{8\alpha_{E1,\nu} + \alpha_{E20} + 12\beta_{M1,\nu}}{8m} + \frac{\gamma_{M1E2} - \gamma_{M1M1}}{8m^2} + \frac{\beta_{M10} - 2\alpha_{E10}}{8m^3}\right)\omega^3 \\
&+ [\alpha_4^h + \frac{1}{480m^4}(-72\alpha_{E10} - 57\beta_{M10} + 6m(25\gamma_{E1E1} - 25\gamma_{E1M2} + 39(\gamma_{M1E2} - \gamma_{M1M1})) \\
&+ m^2(1248\alpha_{E1,\nu} + 95\alpha_{E20} + 540\beta_{M1,\nu} + 26\beta_{M20}) \\
&- 12m^3(15\gamma_{E1E1,\nu} - 15\gamma_{E1M2,\nu} - 69\gamma_{E2E2} + 12\gamma_{E2M3} + 25\gamma_{M1E2,\nu} \\
&- 25\gamma_{M1M1,\nu} - 12\gamma_{M2E3} + 51\gamma_{M2M2}))]\omega^4 \\
&+ [\alpha_5^h + \frac{1}{2400m^5}(15\alpha_{E10} + 5m^2(612\alpha_{E1,\nu} + 38\alpha_{E20} + 1008\beta_{M1,\nu} + 89\beta_{M20}) \\
&- 210\beta_{M10} + 15m(-46\gamma_{E1E1} + 46\gamma_{E1M2} + 33(\gamma_{M1M1} - \gamma_{M1E2})) \\
&+ 12m^3(55\gamma_{E1E1,\nu} - 55\gamma_{E1M2,\nu} - 6(35\gamma_{E2E2} - 22\gamma_{E2M3} + 5\gamma_{M1E2,\nu} \\
&- 5\gamma_{M1M1,\nu} + 38\gamma_{M2E3}) + 555\gamma_{M2M2}))\omega^5.
\end{aligned}
$$

DDP: β(ω) expansion

$$
\begin{aligned}
\beta_{M1}(\omega) &= \beta_{M10} + \frac{\alpha_{E10}}{m}\omega + \left(\frac{5\beta_{M10} - 2\alpha_{E10}}{8m^2} + \beta_{M1,\nu}\right)\omega^2 \\
&+ \left(\frac{\alpha_{E10} - 2\beta_{M10}}{8m^3} + \frac{8\beta_{M1,\nu} + \beta_{M20} + 12\alpha_{E1,\nu}}{8m} + \frac{\gamma_{E1M2} - \gamma_{E1E1}}{8m^2}\right)\omega^3 \\
&+ [\beta_4^h + \frac{1}{480m^4}(-72\beta_{M10} + m^2(1248\beta_{M1,\nu} + 95\beta_{M20} + 540\alpha_{E1,\nu} + 26\alpha_{E20}) \\
&- 57\alpha_{E10} + 6m(25\gamma_{M1M1} - 25\gamma_{M1E2} + 39(\gamma_{E1M2} - \gamma_{E1E1})) \\
&- 12m^3(15\gamma_{M1M1,\nu} - 15\gamma_{M1E2,\nu} - 69\gamma_{M2M2} + 12\gamma_{M2E3} + 25\gamma_{E1M2n} \\
&- 25\gamma_{E1E1,\nu} - 12\gamma_{E2M3} + 51\gamma_{E2E2}))]\omega^4 \\
&+ [\beta_5^h + \frac{1}{2400m^5}(15\beta_{M10} + 5m^2(612\beta_{M1,\nu} + 38\beta_{M20} + 1008\alpha_{E1,\nu} + 89\alpha_{E20}) \\
&- 210\alpha_{E10} + 15m(-46\gamma_{M1M1} + 46\gamma_{M1E2} + 33(\gamma_{E1E1} - \gamma_{E1M2})) \\
&+ 12m^3(55\gamma_{M1M1,\nu} - 55\gamma_{M1E2,\nu} - 6(35\gamma_{M2M2} - 22\gamma_{M2E3} + 5\gamma_{E1M2,\nu} \\
&- 5\gamma_{E1E1,\nu} + 38\gamma_{E2M3}) + 555\gamma_{E2E2}))\omega^5.
\end{aligned}
$$

χ^2: that's NOT all folks!