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DISPERSION SUM RULES AND HIGH ENERGY SCATTERING

A. A. LOGUNOV, L. D. SOLOVIEV and A. N. TAVKHELIDZE
Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research, Dubna

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Dispersion sum rules which connect the high-energy scattering parameters with the integrals of low-energy cross sections are obtained.

It was recently shown that the dispersion sum rules [1] which follow from the ordinary one-dimensional dispersion relations under certain assumptions about the high-energy behaviour of the scattering amplitudes allow one to obtain a great number of relations between the coupling constants of particles and resonances [2-9]. These sum rules are of the form

$$\int_{\nu_0}^{\infty} \operatorname{Im} f(\nu)\, d\nu = 0 \qquad (1)$$

where $f(\nu)$ is crossing-odd invariant amplitude of the reaction $a+b \rightarrow c+d$ which satisfies the dispersion relations in the invariant energy variable ν at fixed momentum transfer. The dispersion sum rules (1) holds only if the function $f(\nu)$ decreases with increasing energy. Therefore, for each process only those amplitudes were considered up to now which, for kinematical reasons, must decrease quicker than the remaining ones.

The purpose of this note is to show that the sum rule (1) can be generalized to non-decreasing amplitudes if we known from experiment the high-energy behaviour of these amplitudes. At present it is known [11] that the total and differential (for small momentum transfers) cross sections in the range 5-30 GeV can be described (with the experimental accuracy) by a sum over Regge poles $\sum_i b_i \nu^{\alpha_i}$ with the parameters b_i and α_i determined from experiment. We will show that the dispersion sum rules permit one to connect these parameters with the integrals of the cross sections over the region of lower energies. This makes it possible, on the one hand, to use in a simple manner the experimental information on the low energy cross sections for the analysis of high energy scattering, and, on the other, to connect the low-energy parameters (coupling constants, resonance widths) with the high-energy ones.

So, we suppose that at high energies $\nu > A$ and fixed momentum transfer the amplitude $f(\nu)$ can be represented as

$$f(\nu) = \sum_i c_i \nu^{\alpha_i} + \epsilon(\nu), \qquad (2)$$

where the function $\epsilon(\nu)$ quickly decreases as $\nu \rightarrow \infty$ and is negligibly small at $\nu > A$.

Applying the Cauchy theorem to the function $\epsilon(\nu)$, using the crossing-symmetry, and neglecting the integral

$$\int_A^{\infty} \operatorname{Im} \epsilon(\nu)\, d\nu, \qquad (3)$$

we get the following sum rule

$$\int_{\nu_0}^{A} \operatorname{Im} f(\nu)\, d\nu - \sum_i \frac{\operatorname{Im} c_i}{\alpha_i + 1} A^{\alpha_i + 1} = 0 \qquad (4)$$

which generalizes the sum rule (1) to the case of non-decreasing amplitudes.

We note that the analogous sum rule can be easily written down if the sum over Regge poles in eq. (2) is replaced by any known analytic function. For these case of an arbitrary behaviour of $f(\nu)$ at infinity one may consider the sum rule for the function $f(\nu)\,\psi(\nu)$, where $\psi(\nu)$ is a crossing-even analytic function holomorphic at $|\nu| < A$ and decreasing quickly enough as $\nu \to \infty$.

As an example, we consider the sum rules (4) for the forward pion-nucleon scattering. Taking as $f(\nu)$ the combinations of amplitudes $A^{(-)} + \nu B^{(-)}$ and $\nu(A^{(+)} + \nu B^{(+)})$ (the amplitudes $A^{(\pm)}$ and $B^{(\pm)}$ are defined, e.g. in ref. 10) we get

$$-2\pi^2 g^2 \left(\frac{\mu}{m}\right)^2 + \int_\mu^A k(\sigma^{(-)} - \sigma^{(+)})\,\mathrm{d}\nu = \sum_i \frac{b_i^{(-)} - b_i^{(+)}}{\alpha_i + 1} \left(\frac{A}{\nu_1}\right)^{\alpha_i - 1} A^2, \quad (5)$$

$$\pi^2 g^2 \left(\frac{\mu}{m}\right)^4 + \int_\mu^A \frac{k\nu}{m} (\sigma^{(-)} + \sigma^{(+)})\,\mathrm{d}\nu = \sum_i \frac{b_i^{(-)} + b_i^{(+)}}{\alpha_i + 2} \left(\frac{A}{\nu_1}\right)^{\alpha_i - 1} \frac{A^3}{m}, \quad (6)$$

where $g^2 = 14.6$, μ is the pion mass, m is the nucleon mass, k and ν stand for the lab. momentum and energy of the incident pion and $\sigma^{(\pm)}$ is the total $\pi^{\pm}p$ scattering cross section.

The coefficients $b_i^{(\pm)}$ are normalized by the equality (at $\nu > A$)

$$\sigma^{(\pm)} = \sum_i b_i^{(\pm)} (\nu/\nu_1)^{\alpha_i - 1}. \quad (7)$$

Doing as in the current high-energy analysis [11] we take A = 5-7 GeV and restrict ourselves to the two vacuum and ρ-meson terms in the right-hand sides of eqs. (5) and (6).

Then only the ρ-meson term contributes to eq. (5). The numerical calculations of the integrals of the cross sections yields the following results. For $\alpha_\rho = 0.54$ the coefficient b_ρ is found to be $b_\rho = 2.48$ (in mb) at A = 5 GeV and $b_\rho = 2.60$ at A = 7 GeV, while directly from the analysis of the cross sections at $\nu > A$ [11], it follows that $b_\rho = 2.75 \pm 0.25$. The vacuum terms give the contribution to the sum rule (6). Using for the parameters of the vacuum terms the values found from the high energy region [11] we obtain at A = 5 and 7 GeV the right-hand side of eq. (6) to be larger than the left-hand one merely by 2% what is within the experimental errors. A detailed analysis of the sum rules for different processes will be published elsewhere.

We see that the obtained sum rules can be easily used in the analysis of the high-energy scattering. On the other hand, they generalize the low energy sum rules treated earlier [1-9] and allow on e to obtain a great number of relations between the decay widths of the resonances as well as their connection with the high-energy scattering parameters.

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