Dynamical Evidence that Regge Poles Control Small-Momentum-Transfer Scattering at High Energy*

GEOFFREY F. CHEW AND VIGDOR L. TEPLITZ
Department of Physics and Lawrence Radiation Laboratory, University of California, Berkeley, California

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Assuming that Regge poles control high-energy scattering at small momentum transfers, a dynamical prediction is made for the magnitude of the total $\pi\pi$ cross section and an estimate given for the corresponding width of the forward diffraction peak, starting from a knowledge of the ρ and f^0 masses and the lifetime of the ρ. The results are in satisfactory agreement with numbers inferred from high-energy πN and NN scattering through the factorization theorem for residues.

I. INTRODUCTION

SKEPTICISM about the dominance of Regge poles in high-energy scattering at small momentum transfers, in particular for the forward elastic diffraction peak, has been expressed for three reasons: (1) The widths of the diffraction peaks observed in the 10–20-GeV range have not exhibited a consistent tendency to shrink with increasing energy.[1] (2) Mandelstam has shown that there are probably branch points to the right of the poles in the angular-momentum complex plane.[2] (3) When diffraction scattering is described phenomenologically in terms of the optical model, the unitarity condition is found to play a major role; Regge-pole parameters seem unrelated to such a constraint. It should be realized that no straightforward connection exists between these three arguments. Because the Pomeranchuk trajectory has a small slope, secondary trajectories have been shown capable at 10–20-GeV laboratory energies of suppressing or enhancing the slow shrinkage associated with the Pomeranchuk trajectory alone.[3] To see this logarithmic shrinkage, enormously higher energies will be required, where branch points in angular momentum may well be important, since their role also increases logarithmically with energy. Branch points are not needed to explain the observations at currently accessible energies. In this connection, and also in relation to point (3), it should be emphasized that the Mandelstam branch points do not arise directly from the unitarity constraint, but from singularities in the region where both energy and momentum transfer are large, singularities that are weak when evaluated in the strip approximation.[4,5] The gross features of high-energy unitarity, such as those included in the optical model, ought to be achievable without invoking branch points in angular momentum.[6]

The extremely important possibility therefore remains open that small-momentum-transfer ($\lesssim 2$ GeV/c) reactions in the 10–100-GeV range of laboratory energies are controlled to a good approximation by a modest number of high-ranking Regge poles. We present here evidence that such is in fact the case for elastic $\pi\pi$ scattering. Our main result is a dynamical prediction, having an estimated uncertainty $\lesssim 30\%$, of the total $\pi\pi$ cross section at high energies; the result agrees with experiment. A secondary result, less precise, is an estimate of the width of the $\pi\pi$ diffraction peak; again, the experimental comparison is satisfactory.

Our calculations are based on a recent version of the strip approximation that assumes both high- and low-energy phenomena at low-momentum transfers ($\lesssim 2$ GeV/c) to be controlled by the top-ranking Regge trajectories.[5] An attempt is being made to solve bootstrap equations that will generate these trajectories and the associated residues. This program is still in an early stage, but we show here that if the strip approximation succeeds in calculating the masses and widths of the ρ and f^0 mesons, it will correctly predict both the high-energy $\pi\pi$ total cross section and the width of the diffraction peak.

Since the strip approximation does not explicitly impose unitarity at high energy, how can it possibly predict the scattering there? We believe the answer to lie in the redundancy of requiring unitarity in *each* of the different reactions described by the analytic continuation of a connected part; unitarity in any one reaction is sufficient. Alternatively it may be sufficient to impose only low-energy unitarity if this be done for all the different reactions; such is the basis of the strip approach.[7] The results of this paper show that solutions of low-energy equations, when analytically continued, seem automatically to conform to high-energy unitarity limitations and to generate diffraction scat-

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[1] K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters **10**, 376 and 543 (1963).

[2] S. Mandelstam, Nuovo Cimento **30**, 1127 and 1148 (1963).

[3] B. Desai, Phys. Rev. Letters **11**, 59 (1963); A. Ahmadzadeh and I. Sakmar, *ibid.* **11**, 439 (1963); W. Rarita and V. Teplitz, *ibid.* **12**, 206 (1964).

[4] G. F. Chew and S. C. Frautschi, Phys. Rev. **123**, 1478 (1961); G. F. Chew, S. C. Frautschi, and S. Mandelstam, Phys. Rev. **129**, 2363 (1963).

[5] G. F. Chew, Phys. Rev. **129**, 2363 (1963); G. F. Chew and C. E. Jones, *ibid.* **135**, B208 (1964).

[6] We are indebted to S. Mandelstam for discussions on this point.

[7] Our equations do not correspond to a neglect of inelastic discontinuities at high energy. We simply manage to avoid in this region any explicit statement about unitarity.

tering compatible with experiment. Such a circumstance may appear miraculous, but current bootstrap dynamics unavoidably is based on apparent miracles, which will become understandable only when our viewpoint is broadened in a fundamental way.

II. THE EXPERIMENTAL SITUATION

What are the facts to be explained? At the simplest level they are the magnitudes of high-energy total cross sections[8] and the widths of the forward peaks in elastic scattering.[1] Unitarity, as expressed for example through the optical model, constrains the total cross section to be $\lesssim 2\pi R^2$, where $R^2 \propto (\Delta t)^{-1}$, Δt being the width of the forward peak. [Precisely, we define $(\Delta t)^{-1}$ as the logarithmic derivative at $t=0$ of $d\sigma/dt$, if t is the negative square of momentum transfer.] It is well known that this limit is closely approached in all the systems experimentally studied. For nucleon-nucleon scattering near 20-GeV laboratory energy, $(\Delta t)_{NN} \approx \frac{1}{10}$ GeV^2, with $\sigma_{NN}{}^{\text{tot}} \approx 40$ mb, while for pion-nucleon scattering[9] the total cross section is half as large and the peak slightly broader; $(\Delta t)_{\pi N} \approx 1/7$ GeV^2.[10] [1 GeV^{-2} $=0.4$ mb.] There are of course differences between π^+p and π^-p and between pp and $p\bar{p}$, but such differences appear to be diminishing as the energy increases and may be ignored in a first approximation. Similarly, we may temporarily ignore the small and erratic variations in energy observed for the peak widths.

It is unfortunate that $\pi\pi$ scattering cannot directly be measured, because the dynamical equations here are the simplest. Nevertheless if the Regge-pole representation is tentatively accepted, then the factorizability of the residues, as pointed out by Gell-Mann and by Gribov and Pomeranchuk,[11] allows the inference of the following high-energy $\pi\pi$ total cross section and forward peak width.

$$\sigma_{\pi\pi}{}^{\text{tot}} = (\sigma_{\pi N}{}^{\text{tot}})^2/\sigma_{NN}{}^{\text{tot}} = 10 \text{ mb},$$
$$(\Delta t)_{\pi\pi}{}^{-1} = 2(\Delta t)_{\pi N}{}^{-1} - (\Delta t)_{NN}{}^{-1} \approx 4 \text{ GeV}^{-2},$$

a combination which again is near the unitarity limit. The first task of the theory is to explain these two numbers, and it is in this connection that we have results to report.[12]

III. AN APPROXIMATE FORMULA FOR $\pi\pi$ RESIDUES

We proceed immediately to derive an approximate formula for $\pi\pi$ residues, appropriate to both the Pomeranchuk and ρ trajectories. The derivation employs the strip concept but does not neglect inelastic scattering, even in the low-energy resonance region.

Let $A_l{}^I(s)$ be the partial wave amplitude for elastic $\pi\pi$ scattering with isotopic spin I at energy squared s, normalized to $[s/(s-4m_\pi{}^2)]^{1/2}\exp(i\delta_l{}^I)\sin\delta_l{}^I$, where $\delta_l{}^I$ is the (complex) phase shift. Following Ref. 5, we write

$$\frac{A_l{}^I(s)}{q_s{}^{2l}} = \frac{N_l{}^I(s)}{D_l{}^I(s)} = B_l{}^{I(V)}(s) + \frac{1}{\pi}\int_{s_0}^{s_1} \frac{ds'}{s'-s} \frac{\operatorname{Im} A_l{}^I(s')}{q_s{}'^2}, \quad (1)$$

where $D_l{}^I(s)$ is cut only across the strip between $s_0 = 4m_\pi{}^2$ and s_1, $N_l{}^I(s)$ carrying all the remaining cuts. As explained in Ref. 5, the term $B_l{}^{I(V)}$ plays somewhat the role of a "potential" and may be calculated from the trajectories and residues of the leading Regge poles.

A Regge pole occurs at a zero of $D_l{}^I(s)$, i.e.,

$$D_{\alpha_i(s)}{}^I(s) = 0, \quad (2)$$

the residue in l of $A_l{}^I(s)$ being given by

$$\beta_i(s) = (q_s{}^2)^{\alpha_i(s)}\gamma_i(s) = (q_s{}^2)^{\alpha_i(s)} \frac{N_{\alpha_i(s)}(s)}{[\partial D_l{}^I(s)/\partial s]_{l=\alpha_i(s)}} \left(-\frac{d\alpha_i(s)}{ds}\right). \quad (3)$$

From formula (1) it is possible to show that

$$N_{\alpha_i(s)}{}^I(s) = -\frac{1}{\pi}\int_{s_0}^{s_1} \frac{ds'}{s'-s} B_{\alpha_i(s)}{}^{I(V)}(s') \operatorname{Im} D_{\alpha_i(s)}{}^I(s'), \quad (4)$$

while

$$\left[\frac{\partial}{\partial s} D_l{}^I(s)\right]_{l=\alpha_i(s)} = \frac{1}{\pi}\int_{s_0}^{s_1} \frac{ds'}{(s'-s)^2} \operatorname{Im} D_{\alpha_i(s)}{}^I(s'). \quad (5)$$

Note that inelastic scattering inside the strip has not been neglected.

Now, it turns out in all approximations studied so far that for both $I=0$ and $I=1$, when l is near 1, the functions $B_l{}^{I(V)}$ and $(-\operatorname{Im} D_l{}^I)$ are all positive across the strip. Furthermore, the "potential" $B_l{}^{I(V)}(s)$ varies

[8] S. J. Lindenbaum, W. A. Lane, J. A. Niederer, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 7, 184 (1961); G. von Dardel, D. Dekkers, R. Mermod, M. Vivargent, G. Weber, and K. Winter, *ibid.* 8, 173 (1962).

[9] In view of the success of the eightfold way, it seems safe to assume that KN scattering will be understandable if success is achieved for the πN system.

[10] This number contains a correction by Ahmadzadeh and Sakmar (Ref. 3) and by W. Rarita (private communication, Berkeley, 1964) to remove the effect of secondary trajectories. The corresponding correction in the NN case happens to be negligible.

[11] M. Gell-Mann, Phys. Rev. Letters 8, 263 (1962); V. Gribov and I. Pomeranchuk, *ibid.* 8, 343 (1962).

[12] A second task will be to explain why $\sigma_{\pi N}{}^{\text{tot}} \approx 2\sigma_{\pi\pi}{}^{\text{tot}}$ and why $(\Delta t)_{\pi N} \approx \frac{1}{2}(\Delta t)_{\pi\pi}$. Also, the tendency of the peak shape to appear exponential must be explained. If we fail on these points, our success with the $\pi\pi$ system must be counted as accidental. After such gross features are dealt with, the variations of peak widths with energy and the differences between π^+p and π^-p, as well as between pp and $p\bar{p}$, requires explanation, as does the magnitude and shape of charge-exchange scattering. All such questions involve secondary trajectories, and the phenomenological studies of Ref. 3, lead one to expect that insuperable difficulties will not arise. The greatest challenge in this area is to explain why the $N\bar{N}$ residue of at least one secondary pole changes sign; from our dynamical equations such a circumstance seems entirely possible.

slowly. Thus, we are led to the basic approximation

$$\gamma_i(s)/\alpha_i'(s)\approx(\bar{s}_i-s)B_{\alpha_i(s)}{}^{I(V)}(\bar{s}_i),\quad s\ll\bar{s}_i, \tag{6}$$

where $\bar{s}_i$ is some average energy inside the strip. Calculations with plausible choices for $B_l{}^{I(V)}$ support formula (6) and suggest that $\bar{s}_i\gtrsim 2$ GeV^2, but a precise value for $\bar{s}_i$ will not be required immediately.

IV. THE TOTAL $\pi\pi$ CROSS SECTION

Many applications of formula (6) are possible. Our first application does not require the form of $B_l{}^{I(V)}(s)$, but only the relation

$$B_l{}^{I=0(V)}(s)\approx 2B_l{}^{I=1(V)}(s),\quad s\lesssim s_1, \tag{7}$$

a result that follows from the crossing matrix if the high-energy "potential" is dominated by $I=1$ exchange. A study of $I=0$ exchange, to be published elsewhere, indicates that at most the factor 2 may be lowered to 1.5 when $l\approx 1$ and $s\approx 1$ GeV^2. It is taken for granted that $I=2$ exchange is negligible.

Assumption (7) leads to the circumstance that for the Pomeranchuk trajectory and the ρ trajectory the values of $\bar{s}_i$ are roughly the same. Noting that $\alpha_P(0)=1$ while $\alpha_\rho(m_\rho{}^2)=1$, we may combine (6) and (7) to obtain

$$\frac{\gamma_P(0)}{\alpha_P'(0)}\frac{\alpha_\rho'(m_\rho{}^2)}{\gamma_\rho(m_\rho{}^2)}\approx 2\frac{\bar{s}}{\bar{s}-m_\rho{}^2}\approx 2, \tag{8}$$

if $\bar{s}\gg m_\rho{}^2$. It is easy to verify that

$$\gamma_\rho(m_\rho{}^2)/\alpha_\rho'(m_\rho{}^2)\approx 4\Gamma_\rho/m_\rho, \tag{9}$$

where Γ_ρ is the full width of the ρ, so we find

$$\gamma_P(0)\approx 8(\Gamma_\rho/m_\rho)\alpha_P'(0). \tag{10}$$

The high-energy $\pi\pi$ total cross section, if it is in fact controlled by the Pomeranchuk-Regge pole, is given by

$$\sigma_{\pi\pi}=8\pi^2\gamma_P(0), \tag{11}$$

so we predict finally that

$$\sigma_{\pi\pi}\approx 64\pi^2(\Gamma_\rho/m_\rho)\alpha_P'(0). \tag{12}$$

The slope of the Pomeranchuk trajectory, assuming that it passes through $l=2$ at the mass of the f^0, has been estimated by Ahmadzadeh and Sakmar as $\frac{1}{3}$ GeV^{-2}.[13] Taking $\Gamma_\rho=100$ MeV and $m_\rho=750$ MeV, we then find from formula (12):

$$\sigma_{\pi\pi}\approx 11\text{ mb}.$$

V. THE WIDTH OF THE $\pi\pi$ DIFFRACTION PEAK

To estimate the width of the diffraction peak, a more specific assumption must be made about the "potential" $B_l{}^{I=0(V)}$. In general, high-energy dominance of the Pomeranchuk-Regge pole leads to

$$\begin{aligned}\mathrm{Im}A_{\pi\pi}(s,t)\underset{s\to\infty}{\longrightarrow}&\tfrac{1}{3}\pi[2\alpha_P(t)+1]\\&\times\gamma_P(t)(q_t{}^2)^{\alpha_P(t)}P_{\alpha_P(t)}\left(\frac{s}{2q_t{}^2}\right),\\\approx&\tfrac{1}{3}\pi[2\alpha_P(t)+1]\gamma_P(t)(\tfrac{1}{2}s)^{\alpha_P(t)},\end{aligned} \tag{13}$$

for $\alpha_P(t)$ near 1.[14] Using formula (6), we then have,

$$\begin{aligned}\mathrm{Im}A_{\pi\pi}(s,t)\propto&[2\alpha_P(t)+1]\alpha_P'(t)(\bar{s}-t)\\&\times B_{\alpha_P(t)}{}^{I=0(V)}(\bar{s})(\tfrac{1}{2}s)^{\alpha_P(t)}.\end{aligned} \tag{14}$$

The form most commonly used for $B_l{}^{I=0(V)}(s)$ is that based on exchange of a (fixed spin) ρ:

$$B_l{}^{I=0(V)}(s)\propto\left(1+\frac{s}{2q_\rho{}^2}\right)\frac{Q_l(1+m_\rho{}^2/2q_s{}^2)}{(q_s{}^2)^{l+1}}. \tag{15}$$

Although we expect important deviations from this behavior when the potential is carefully calculated, the form (15) may serve to indicate the l dependence of the potential, which is all we need for the shape of the diffraction peak.

For $s\sim 2$ GeV^2, most of the l dependence of (15) resides in the factor $(q_s{}^2)^l$, and the weak remaining l dependence in Q_l near $l=1$ is conveniently almost proportional to $(2l+1)^{-1}$. Thus, (14) becomes

$$\mathrm{Im}A_{\pi\pi}(s,t)\propto\alpha_P'(t)(\bar{s}-t)(s/2q_{\bar{s}}{}^2)^{\alpha_P(t)}. \tag{16}$$

At this point, evidently, an estimate of $\bar{s}$ is required as well as an estimate of the shape of the Pomeranchuk trajectory near $t=0$. From preliminary calculations of trajectories and residues with a variety of "potentials" and strip widths, when the potential and strip width are adjusted to give $\alpha_P(0)=1$, $\alpha_P'(0)\approx\frac{1}{3}$ GeV^{-2}, we find $\bar{s}\sim 2$ GeV^2. Furthermore the trajectory for $t\ll\bar{s}$, is represented roughly by the form,

$$\alpha_P(t)\approx c+\frac{1-c}{1-t/\bar{s}}, \tag{17}$$

so,

$$\alpha_P'(t)\approx\frac{\alpha_P'(0)}{(1-t/\bar{s})^2}. \tag{18}$$

Taking the logarithmic derivative of (16) at $t=0$, we then calculate

$$a_{\pi\pi}\equiv\tfrac{1}{2}(\Delta t_{\pi\pi})^{-1}\approx(\bar{s})^{-1}+\alpha_P'(0)\ln(2s/\bar{s}). \tag{19}$$

As explained in Sec. II above, we expect $(\Delta t_{\pi\pi})^{-1}$ to be ~ 4 GeV^{-2} at an s corresponding to 20-GeV laboratory energy for NN scattering. This is $s\sim 40$ GeV^2,

[13] A. Ahmadzadeh and I. Sakmar, Phys. Letters **5**, 145 (1963).

[14] The real part of the amplitude is small near $t=0$.

so taking $\bar{s}=2$ GeV^2 we have from (19)

$$a_{\pi\pi}\approx(0.5+\tfrac{1}{3}\ln 40)\ \text{GeV}^{-2} = 1.7\ \text{GeV}^{-2}, \tag{20}$$

not far from the expected 2 GeV^{-2}.

It thus appears that if the strip approximation succeeds in explaining the masses and widths of the ρ and f^0 mesons, it will correctly predict both the high-energy $\pi\pi$ total cross section and the width of the diffraction peak.[15]

An additional result not immediately subject to experimental test is the effect of the ρ trajectory on the high-energy $\pi\pi$ amplitude. Using the same approximations as above, one merely adds to formula (16) a factor

$$\begin{bmatrix}\frac{1}{3}\\ \frac{1}{3}\\ \frac{1}{3}\end{bmatrix}+\begin{bmatrix}1\\ \frac{1}{2}\\ -\frac{1}{2}\end{bmatrix}\tfrac{1}{2}(2s/\bar{s})^{\alpha_\rho(t)-\alpha_P(t)}, \tag{21}$$

where the column vectors have elements corresponding to

$$I=\begin{bmatrix}0\\1\\2\end{bmatrix}.$$

Notice that if $\alpha_P-\alpha_\rho\lesssim 0.5$ the influence of the ρ will persist to rather high energies.

[15] Note that even to get the correct sign for $\sigma_{\pi\pi}$ and $a_{\pi\pi}$ from a dynamical calculation of the Regge parameters is a nontrivial achievement.

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Search for Charge $\frac{1}{3}e$ Particles in Cosmic Rays*

A. W. SUNYAR, A. Z. SCHWARZSCHILD, AND P. I. CONNORS†
Brookhaven National Laboratory, Upton, New York
(Received 7 July 1964)

A search was conducted for particles of charge $\frac{1}{3}e$ (quarks) in cosmic rays at sea level. No such particles were found. The experiment yields an upper limit for the frequency of high-energy quarks of 1 per 2×10^5 relativistic μ mesons. This limit is related to a limit on the quark production cross section as a function of possible quark mass. A short discussion is given of macroscopic experiments which might have detected quarks in matter.

I. INTRODUCTION

RECENT speculation[1] by Gell-Mann concerning the possible existence of fractionally charged particles (quarks) has led to several experiments[2,3] designed to reveal their presence. The negative result of the experiments of Leipuner *et al.* and Bingham *et al.*, performed with high-energy proton beams, resulted in a lower limit of $\sim$2 GeV/c^2 for the mass of the charge $\frac{1}{3}e$ quark. The production of quark pairs in p-p collisions at 28 GeV is kinematically impossible for quarks with $m_Q>2.7$ GeV/c^2. Such a limitation on the mass does not exist for quarks produced by cosmic-ray proton-nucleon interactions in the atmosphere, although the cosmic proton flux is of course much smaller than is available at the Brookhaven alternating gradient synchrotron (AGS). We have performed an experiment designed to detect the possible presence of relativistic particles of charge $\frac{1}{3}e$ in the cosmic radiation at sea level. The distinctive character by which we attempt to identify particles of lower than unit charge is their small specific ionization at relativistic velocities.

II. EXPERIMENTAL EQUIPMENT AND RESULTS

The experimental apparatus consisted of the plastic scintillation counter telescope shown in Fig. 1, and associated coincidence circuitry, together with a dual-beam oscilloscope on which the linearly amplified signals from six of the counters (2, 4, 5, 6, 7, 8) were displayed and photographed.

Events satisfying coincidence requirements [1 $\bar{3}$ 4 8] (see Fig. 1) measured the number of minimum ionizing muons traversing the counter telescope. Photographs of μ-meson traces taken with these coincidence requirements served to provide pulse-height calibrations for minimum ionization for each of the detectors. In order to reduce the number of photographs to be scanned in searching for the charge $\frac{1}{3}e$ quarks, energy selection was

* Research carried out under the auspices of the U. S. Atomic Energy Commission.
† Graduate student, Pennsylvania State University, University Park, Pennsylvania.
[1] M. Gell-Mann, Phys. Letters 8, 214 (1964).
[2] L. B. Leipuner, W. T. Chu, R. C. Larsen, and R. K. Adair, Phys. Rev. Letters 12, 423 (1964); H. H. Bingham *et al.*, Phys. Letters 9, 201 (1964).
[3] D. R. O. Morrison, Phys. Letters 9, 199 (1964); and P. A. Piroué and A. J. S. Smith, Post-deadline paper, American Physical Society meeting, Washington, April 1964 (unpublished). [3a] [*Note added in proof.* After submission of this paper, two Letters have appeared on this subject. The references are: V. Hagopian *et al.*, Phys. Rev. Letters 13, 280 (1964); W. Blum *et al.*, Phys. Rev. Letters 13, 353(A) (1964).]