EXAMPLE OF A REGGE DAUGHTER TRAJECTORY FOR $I=1$, $Y=0$ BOSON RESONANCES

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A number of known $I=1, Y=0$ boson resonances are assigned to two Regge trajectories by combining the information from both a Chew-Frautschi plot and an imaginary spin versus real spin (Regge trajectory) plot. One trajectory is conjectured to be the daughter of the other and several predictions are made about the quantum numbers of known resonances. In addition, the masses and widths of some new resonances are predicted.

It has recently been proved theoretically by Freedman and Wang [1,2] that certain Regge trajectories imply the existence of associated daughter trajectories. The first daughter will have particle resonances at approximately the same energies as its parent but with the corresponding spins of each precisely one unit lower. For non-strange bosons the G parity of the parent and daughters would be the same.

Freedman [3] has conjectured that the δ resonance [4,5] might be a 0^{++} (J^{PG}) daughter of the $1^{-+}\rho$ meson. In this paper we wish to demonstrate that the assignment of the δ to the ρ in a filial relationship can in fact be interpreted as only one of several connections between the $I=1$, $Y=0$ resonances lying on two particular Regge trajectories. In doing this we will also demonstrate how additional information may be extracted from a Regge plot that is not available from the Chew-Frautschi (CF) plot of spin versus mass squared. The CF plot alone does not provide sufficient or very convincing evidence for a daughter trajectory, but the plot of the imaginery part of the particle spin (α) versus the real part (J) provides an important constraint on the Regge classification.

Obviously α cannot be measured directly, but it is a standard result of (potential) Regge theory, proved in Newton [6], that the full width at half maximum of a resonance caused by a Regge pole passing near a physical J is given by

$$\Gamma = \frac{2\,\alpha\, \mathrm{d}J/\mathrm{d}E}{(\mathrm{d}J/\mathrm{d}E)^2 + (\mathrm{d}\alpha/\mathrm{d}E)^2}\,, \tag{1}$$

where E is the energy and the derivatives are evaluated at the energy for which J is equal to the resonance spin. In a relativistic generalization of (1) we must replace E by s, the Mandelstam invariant which is the energy squared in the center of mass system. We have

$$\Gamma = \frac{2\alpha J'}{J'^2 + \alpha'^2} \tag{2}$$

where the prime now indicates differentiation with respect to s. If, as we shall see, we have

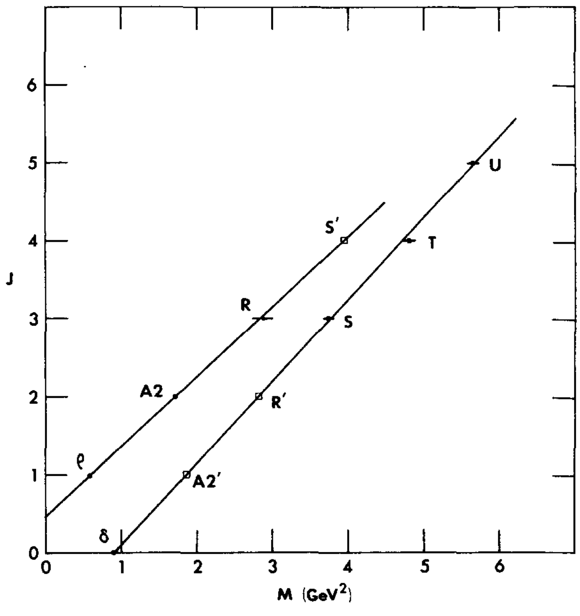

Fig. 1. The Chew-Frautschi plot of a parent and a daughter trajectory. Exchange degeneracy is assumed and the relevant quantum numbers are discussed in the text. Notice that the ρ-A2 line is consistent with the scattering predictions of $J \approx 0.4$ at zero energy.

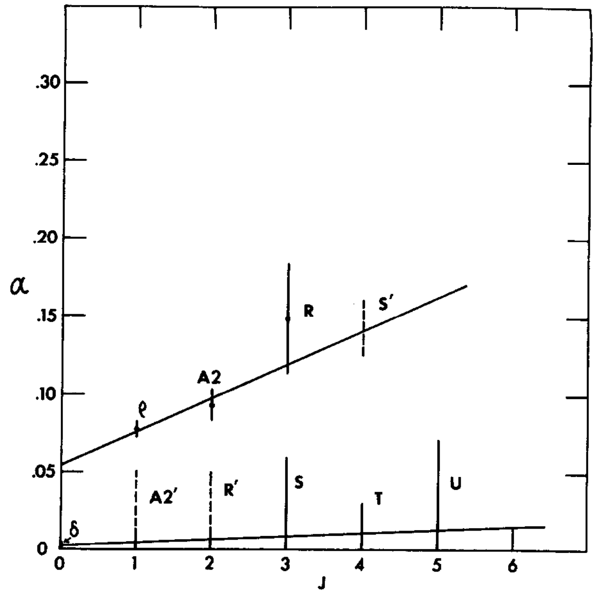

Fig. 2. A Regge pole trajectory plot of the parent and daughter. The size of the A2', R' and S' dashed lines are only suggestive.

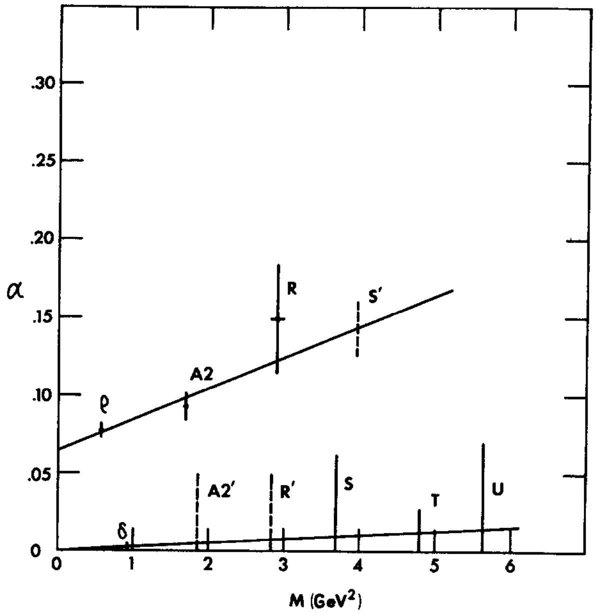

Fig. 3. A Regge pole using mass squared instead of J. Such a plot is easier to view if several particles of the same spin are involved as would be the case if we added more meson trajectories.

$J' \gg \alpha'$, (2) gives us *

$$\Gamma = \tfrac{1}{2}\Gamma J' \tag{3}$$

and a Regge plot may be obtained from the CF slope and the experimental widths.

In fig. 1 we exhibit two straight lines whose slopes are close to 1 GeV^{-2}. This is the situation found in the baryon CF plots [7]. The ρ, A2, δ, S, T and U are thought to be well established and the quantum numbers of the ρ and A2 are considered well established as [5] 1^{-+} and 2^{+-}. In making fig. 1 we have used the hypothesis of exchange degeneracy as suggested by Arnold [8]. This states that for natural parity mesons, for example, the $0^+, 2^+, 4^+$, etc. and $1^-, 3^-, 5^-$, etc. trajectories will be degenerate and have opposite G parity. This results from a virtual absence of exchange forces under some conditions, as discussed by Arnold.

The situation just below 3 GeV^2 is very confused since a large number of experiments have detected several resonances of varying mass and width. A comprehensive summary of the confusion can be found in Goldhaber's talk in ref. 5. Consistent with his comments we assume two resonances in this region: one (the R) is taken to be a 3^{-+} resonance at about 1700 MeV while another (the R', its daughter) is a 2^{++} at about 1630 MeV. We shall see below that R and R' are also characterized by having widths that differ by a factor of perhaps 5 with the R' width smaller.

The A2 and A2' are also taken as two resonances as was found by the Magliĉ group [4]. Here the A2' at about 1370 MeV is a 1^{--} and again will have a distinctly smaller width than the parent A2.

Finally we indicate an S' which should be a broad 3^{--} resonance just under 2000 MeV which is the parent of the narrow S. A T' and U' could also be assumed but we do not indicate them. The S' has not yet been observed.

Using the values of J' from fig. 1 we may make a Regge plot with the aid of eq. (3). We also show in fig. 3 an α versus M^2 plot which is easier to read for some purposes. We notice in fig. 2 that the two Regge trajectories are both quite distinct and have $\alpha \approx 0$. The narrow width meson resonances could not be on the same CF plot as the broad ones without terrible twisting of the Regge trajectories ‡.

* There is also a self-consistent solution to the CF plot and the Regge plot with $\alpha' \ll J'$ but this would cause a large energy shift of the resonance peak and invalidate eq. (1); this is discussed in ref. 6.

‡ Nothing forbids this, but both Yukawa potential theory and our experience with baryons makes this a very undesirable and unlikely occurance.

This dichotomy into two distinct classes of width is more useful than it has appeared so far. It excludes the higher Maglić resonances S, T and U from lying on the same CF curves with the B, A1, π_V(1003) or the π(1640) all of which are broad resonances, as well as the ρ, A1 and R. Such classifications by Cline [3,9] and by Dalitz [5] on the basis of CF plots alone would thus seem very suspect.

As another example of the usefulness of a Regge plot we see that on the one hand, the assumption that the δ is 1^{++} and lies with the π and S, T, U would cause a bad bump in the CF plot, while on the other hand any other third known resonance on the π-δ CF plot would cause a sharp curvature of the Regge trajectory in fig. 2 because the δ is so narrow.

Unlike the case of the baryons where at present no useful information can be extracted from the widths, the α versus J plot for mesons is important to consider when making a CF diagram. For example, in a CF plot a nearly straight line can be made to connect the π, B(1^{++}), and the π(1640) (2^{--}), but the π(1640) width clearly excludes such as assignment when viewed on a Regge plot unless sharply curved trajectories (with $d^2\alpha/dJ^2 < 0$!) are allowed. We summarize here the quantum number assignments of the particles in fig. 1 as determined from the Regge lines:

ρ	1^{-+}	δ	0^{++}
A2	2^{+-}	A2'	1^{--}
R	3^{-+}	R'	2^{++}
S'	4^{+-}	S	3^{--}
		T	4^{++}
		U	5^{--}

Experimentally three tasks stand out in meson resonance work. There is the verification or rejection of the quantum assignments above, the search for a broad S' resonance, and the clarification of the R and A2 multi-peaks.

By sharpening the experimental errors on all meson resonances it might become possible to draw other Regge trajectories by utilizing this new information as we have done above.

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