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MESON-NUCLEON ELASTIC SCATTERING AND PHOTO-MESON PRODUCTION VIA THE u-CHANNEL†

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ABSTRACT

An explanation of the available pion-nucleon and photo-meson production data in the backward direction is proposed using Baryon trajectories. Special emphasis is given to the constraints, the unequal mass kinematics and their implications.

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Backward scattering is of great interest to the Regge Pole model for several reasons. From the theoretical point of view it involves several constraints imposed by (a) the McDowell Symmetry[1], (b) Conspiracy, (c) Gribov's Theorem[2], and, in addition, the unequal mass complications[3]. It is important to note that contrary to forward scattering, all the constraints occur in regions accessible to experiment. From the experimental point of view there are, or there will soon be available, data on several processes[4].

We discuss here the importance of the N_α and the Δ trajectory in explaining the elastic pion-proton data. We propose an explanation for the dominance of the nucleon trajectory in π^+p elastic scattering and we point out some observable consequences of the unequal mass kinematics. Finally, we make some general observations concerning the photoproduction processes.

For fixed u and large s, even within the backward cone, the Regge Representation of the helicity amplitudes is given by[5]:

$$F_{\substack{++\\+-}}(u, z_u) = \left\{ \sum_{\substack{i,\ \text{over}\\ \text{trajectories}}} \frac{1}{2} c_i \beta_{\alpha_i}(w) \frac{1 + \eta_i e^{-i\pi(\alpha_i - 1/2)}}{\sin \pi(\alpha_i - 1/2)} \frac{2\alpha_i + 1}{\Gamma(\alpha_i + 3/2)} s^{\alpha_i - 1/2} \right. \tag{1}$$

$$\left. \mp \begin{bmatrix} \text{Same expression with } \alpha_i(w) \text{ and } \beta_{\alpha_i}(w) \\ \text{replaced by the complex conjugates} \end{bmatrix} \right\} \begin{pmatrix} \cos \frac{\theta u}{2} \\ \sin \frac{\theta u}{2} \end{pmatrix}$$

where η_i is the signature factor and c_i is the isospin factor. Eq. (1) is obtained by Reggeizing the helicity amplitudes, which are free of kinematic singularities. The unequal mass complications are treated in the manner described in the next paragraph. The normalization is as follows:

$$\frac{d\sigma}{du} = \pi^2 \left\{ \left| F_{++}(u, z_u) \right|^2 + \left| F_{+-}(u, z_u) \right|^2 \right\} \tag{2}$$

Several authors[3] have shown that if we neglect the background term and represent those amplitudes, which are free of kinematic singularities, by a sum over finite many Regge poles in the form[6]:

$$F_{++}(u, z_u)\left(\cos\frac{\theta u}{2}\right)^{-1} = \sum_{\alpha} \beta_\alpha(w)\, P^{(0,1)}_{\alpha-1/2}(z_u) \tag{3}$$

then we can find models which are compatible with Regge asymptotic behavior and Mandelstam analyticity. It must be emphasized that although none of these constitutes a rigorous proof, they all arrive at the conclusion that asymptotically the above mentioned amplitudes behave as if all the masses were equal. We conclude that for very large s and $u \approx 0$:

$$F_{++}(u, z_u)\left(\cos\frac{\theta u}{2}\right)^{-1} \sim \beta_{++}(w)\; s^{\alpha-1/2} \tag{4}$$

$$F_{+-}(u, z_u)\left(\sin\frac{\theta u}{2}\right)^{-1} \sim \beta_{+-}(w)\; s^{\alpha-1/2} \tag{5}$$

and therefore when we calculate the differential cross section we expect to observe a new feature[7] coming from the facts that: (1) $\cos\frac{\theta u}{2}$ and $\sin\frac{\theta u}{2}$ remain finite and small within the backward cone, $(M^2 - \mu^2)^2/s \geq u \geq 0$, even when $s \to \infty$; (2) $\beta_{+-}(w) \to 1/w$ when $w \to 0$. Unfortunately, all previous analyses[8] of the data did not emphasize the significance of the half-angles. We discuss later some evidence in favor of their presence.

The residues are obtained from the appropriate partial waves of the Born diagrams evaluated at the poles and corrected for threshold factors.

We evaluated the residues of the three leading trajectories at the poles and we obtained the ratios:

$$\left(\frac{\beta(w)}{d\alpha/dw}\right)_{\alpha=N_\alpha} : \left(\frac{\beta(w)}{d\alpha/dw}\right)_{\alpha=N_\gamma} : \left(\frac{\beta(w)}{d\alpha/dw}\right)_{\alpha=\Delta} = 11.9 : 1.5 : 1.0 \tag{6}$$

The residue of the N_γ is considerably smaller than the nucleon residue (provided the two trajectories have comparable slopes); in addition, we expect its trajectory to lie below the nucleon trajectory. For these reasons we included in the calculation only the N_α and the Δ trajectory. The N_γ could be important at the position of the dip in the π^+p cross section, but at the moment it is impossible to disentangle its contribution. The trajectories:

$$\alpha_\Delta = -.075 + .75\,w + .70\,w^2 \tag{7a}$$

$$\alpha_{N_\alpha} = -.39 + .725\,w^2 \tag{7b}$$

can account for the angular dependence. This suggests that the residues are slowly varying functions of w. We can estimate the contribution of each trajectory to the differential cross sections at $u \approx 0$ and $s = 19.5$ using (1), (6) and (7). We find the following ratios:

$$\frac{\text{Nucleon to } \pi^+p}{\text{Delta to } \pi^-p} \approx .11 \text{ and } \frac{\text{Nucleon to } \pi^+p}{\text{Delta to } \pi^+p} \approx 1.0$$

The exact results of the calculation are shown in Figure 1 and 2. The constant residues which we used to fit the data are:

$$\beta_\Delta = .35 \frac{\mu b^{1/2}}{\text{BeV}} \tag{8a}$$

$$\beta_N = 13.7 \frac{\mu b^{1/2}}{\text{BeV}} \tag{8b}$$

The Δ-residue is 38% of its value at the pole; while the nucleon residue is 59% larger than its value at the pole. The π^+p process is dominated by the N_α-trajectory. The contribution of the Δ employed in the calculation is just what is required by isospin invariance. The interference between the two trajectories is destructive for $u < -.2\ (\text{BeV/c})^2$ and constructive in the remaining region. The presence in Eq. (7a) of a large term linear in w gives two helicity amplitudes of comparable magnitude at $u < -.3\ (\text{BeV/c})^2$, $(|F_{++}|^2 \approx \frac{1}{2}|F_{+-}|^2)$. This forces the cross section to level off at large values of u and it eliminates a dip at $u \approx -1.8\ (\text{BeV/c})^2$, which is present in π^-p when the trajectory is linear in u. We conclude that the trajectories given in (7) together with residues which are slowly varying functions of w can give the correct ratio of cross sections with the nucleon trajectory dominating in π^+p.

The half-angles cause the following effects:

(1) The high energy dependence of the helicity amplitudes at $u = 0$ is as follows:

$$F_{++}(u, z) \sim s^{\alpha-1/2} \tag{9a}$$

$$F_{+-}(u, z) \sim s^{\alpha} \tag{9b}$$

(2) Whenever the diffraction peak is not very steep, the half angles make some of the amplitudes level off at $u \approx 0$. For pion-nucleon scattering, for example, the helicity non-flip amplitude goes to zero at $u = \frac{(M^2-\mu^2)^2}{s}$. This creates antishrinkage and possibly shallow minima. In photoproduction, where three of the helicity amplitudes go to zero at 180° the effects should be much more prominent.

Photoproduction. Presently there are data for two photoproduction processes in the backward direction:[4] $\gamma p \to \pi^+ n$ and $\gamma p \to \pi^0 p$. Both of these processes deserve a detailed analysis, which includes all the spin dependence and the corresponding constraints. We discuss here some very distinct features, which appear in the data and comment on their implications.

(a) Although the angular distribution of $\gamma p \to \pi^+ n$ can, in general, come from an interference of the Δ and the N_α trajectory, the absence of a dip in the data of R. Anderson et al.,[4b] eliminates the possibility of dominance by the nucleon trajectory. Figure 3 shows the data and our calculations using only the Δ trajectory and Regge representations for the helicity amplitudes similar to Eq. (1). The main difference is that now three of the helicity amplitudes go to zero at 180^o; the sum of their contributions to the cross section at $u \approx -.4(\mathrm{BeV/c})^2$ and $s = 19.26$ is three times the contribution of the non-vanishing amplitude. As a result they have a significant contribution even at small values of u and make the cross section decrease at large energies like:

$$\frac{d\sigma}{du} \sim s^{2\alpha_\Delta - 3} \quad .$$

Our curves give the observed angular distribution and energy dependence. The overall normalization was adjusted at the highest energy. We observe a small dip at $u \approx 0$, but no shrinkage.

(b) In the vector meson dominance model the isovector part of the process $\gamma p \to \pi^+ n$ is related to $\rho^o p \to \pi^+ n$, which in turn is related by isospin invariance and time reversal to $\pi^- p \to \rho^o n$ through the relation[9]

$$\frac{d\sigma}{du}\,(\gamma p \to \pi^+ n) = \left(\frac{e}{f_\rho}\right)^2 \frac{d\sigma}{du}\,(\pi^- p \to \rho^o n)\, \rho_{11}^{hel}\,(u) + \begin{matrix}\text{Isoscalar}\\ \text{contribution}\end{matrix} \tag{10}$$

where $\rho_{11}^{hel}(u)$ is the density matrix element. From Figure 3 and (10) we can predict $\frac{d\sigma}{du}(\pi^- p \to \rho^o n)\,\rho_{11}^{hel}(u)$ for several incident energies and scattering angles. Preliminary results for all existing data of $\pi^- p \to \rho^o n$ at 4 BeV are in agreement with the vector dominance model.[10] The above discussion does not necessarily imply that $\pi^- p \to \rho^o n$ is dominated by the Δ-trajectory. On the contrary, the density matrix element[10] $\rho_{11}^{hel} \approx .25$ allows considerable contribution from the nucleon trajectory. In fact, if the ρp interaction is similar to the πp interaction, we expect the nucleon trajectory to be very important in the charge exchange modes. The obvious implications will be more structure in the angular distribution and a faster fall of the differential cross section with increasing energy.

(c) The absence of a sharp dip at $u \approx -.2\ (\mathrm{BeV}/c)^2$ in $\gamma p \to \pi^o p$ eliminates the possibility that it is purely nucleon exchange. Could it then be mostly Δ exchange? A pure Δ exchange involves only the isovector part of the photon and it implies that:

$$\left(\frac{d\sigma}{du}\right)_{\gamma p \to p\pi^o} = 2\left(\frac{d\sigma}{du}\right)_{\gamma p \to \pi^+ n} \tag{11}$$

Recent experimental results show that[4c]

$$\left(\frac{d\sigma}{du}\right)_{\gamma p \to p\pi^o} \approx \left(\frac{d\sigma}{du}\right)_{\gamma p \to \pi^+ n} \tag{12}$$

for $-.3\ (\mathrm{BeV}/c)^2 \leq u \leq 0.0\ (\mathrm{BeV}/c)^2$. Therefore both the Δ and the nucleon contribute. Equation (12) implies that the contribution of the $I = 1/2$ trajectories at the position of the dip cannot be equal to zero, but that there is a remainder which interferes destructively with the Δ. Such a remainder can come from either an imaginary part in the nucleon trajectory, or from some other $I = \frac{1}{2}$ trajectory.

We conclude that a simple Regge pole model can account for the pion-proton and $\gamma p \rightarrow \pi^{+} n$ data. There are two features in the model which should be present in all analyses: (a) Estimates of the residues can be obtained from the Born diagrams, and (b) the half-angles should be present, eliminating shrinkage at small u but creating some other noticeable effects. Both aspects could and should be checked in other processes. On the experimental side, the photoproduction data should be extended to larger ranges of u and s . These processes allow studies of interferences between several trajectories. Production of isoscalar mesons allows the study of $I = \frac{1}{2}$ trajectories alone. With new data accumulating, this should become an exciting region of research.

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   (b) Photoproduction of π^+ mesons on hydrogen was performed at SLAC. R. Anderson et al., to be published in Phys. Rev. Letters.

   (c) Photoproduction of π^o mesons on hydrogen was performed at SLAC. B. Gittelman, private communication.
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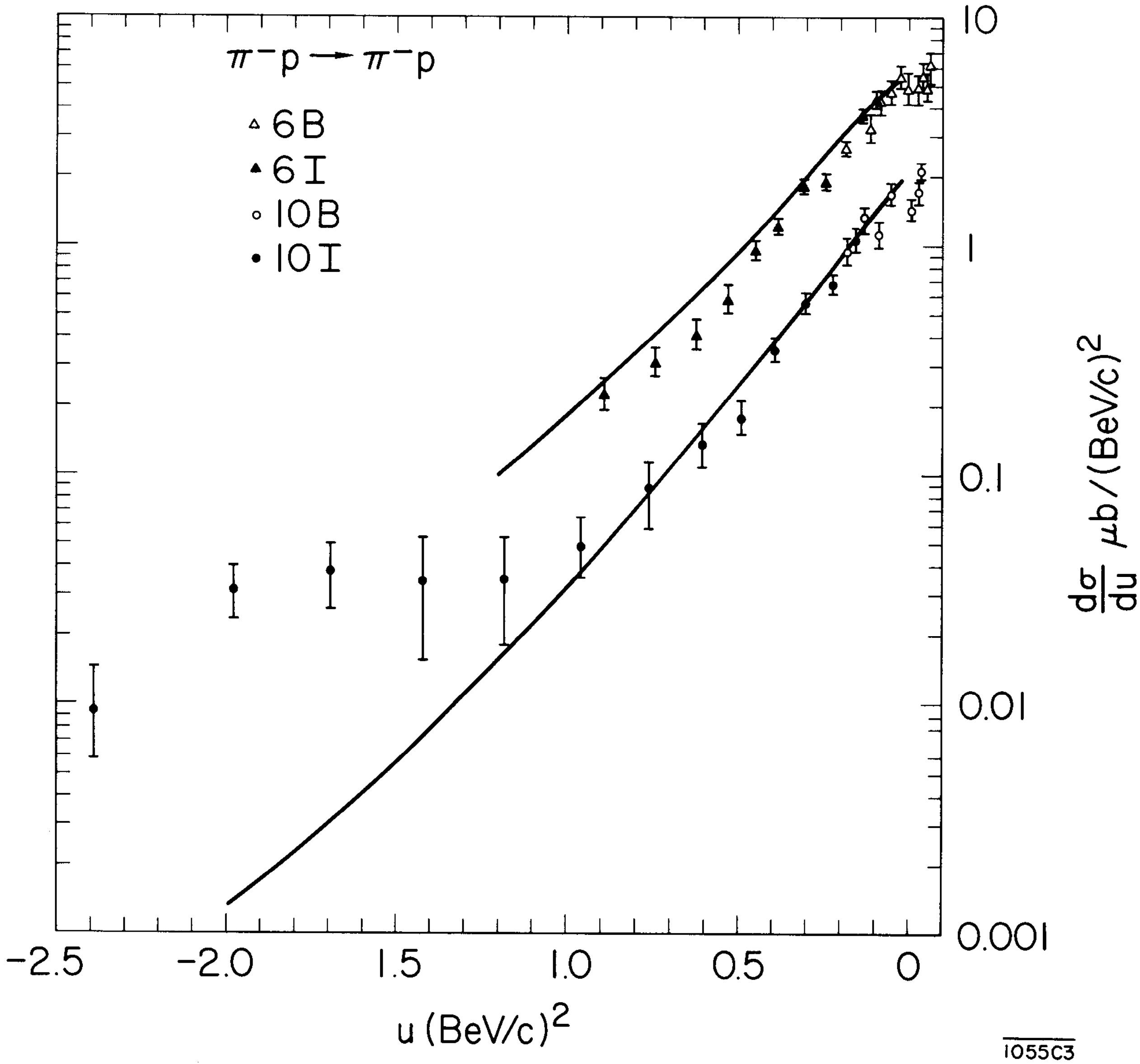
FIGURE CAPTIONS

1. Angular distribution and energy dependence for backward $\pi^- p$ elastic scattering at 6 and 10 BeV. The data are taken from Orear <u>et al.</u> The open triangles and circles correspond to the backward geometry and the rest to the intermediate region. The curves represent the fits described in the text. The intercept of the trajectory can also account for the data at 13.7 and 16.3 BeV. The discrepancy at large values of u can be accounted easily by adding to the trajectory a small term quadratic in u.
2. Angular distribution and energy dependence for backward $\pi^+ p$ elastic scattering. The notation and source of the data are the same as in Fig. 1. Our curves can be extended to larger values of u and s in good agreement with the data.
3. Photoproduction of π^+ mesons at 4.3, 6.7 and 9.8 BeV. The curves correspond to fits using only the Δ trajectory. The data were taken at SLAC by R. Anderson <u>et al.</u>



$\pi^- p \rightarrow \pi^- p$
6B
6I
10B
10I
10
1
0.1
0.01
0.001
$\frac{d\sigma}{du}$ μb/(BeV/c)^2
−2.5
−2.0
1.5
1.0
0.5
0
u (BeV/c)^2
1055C3

Fig. 1

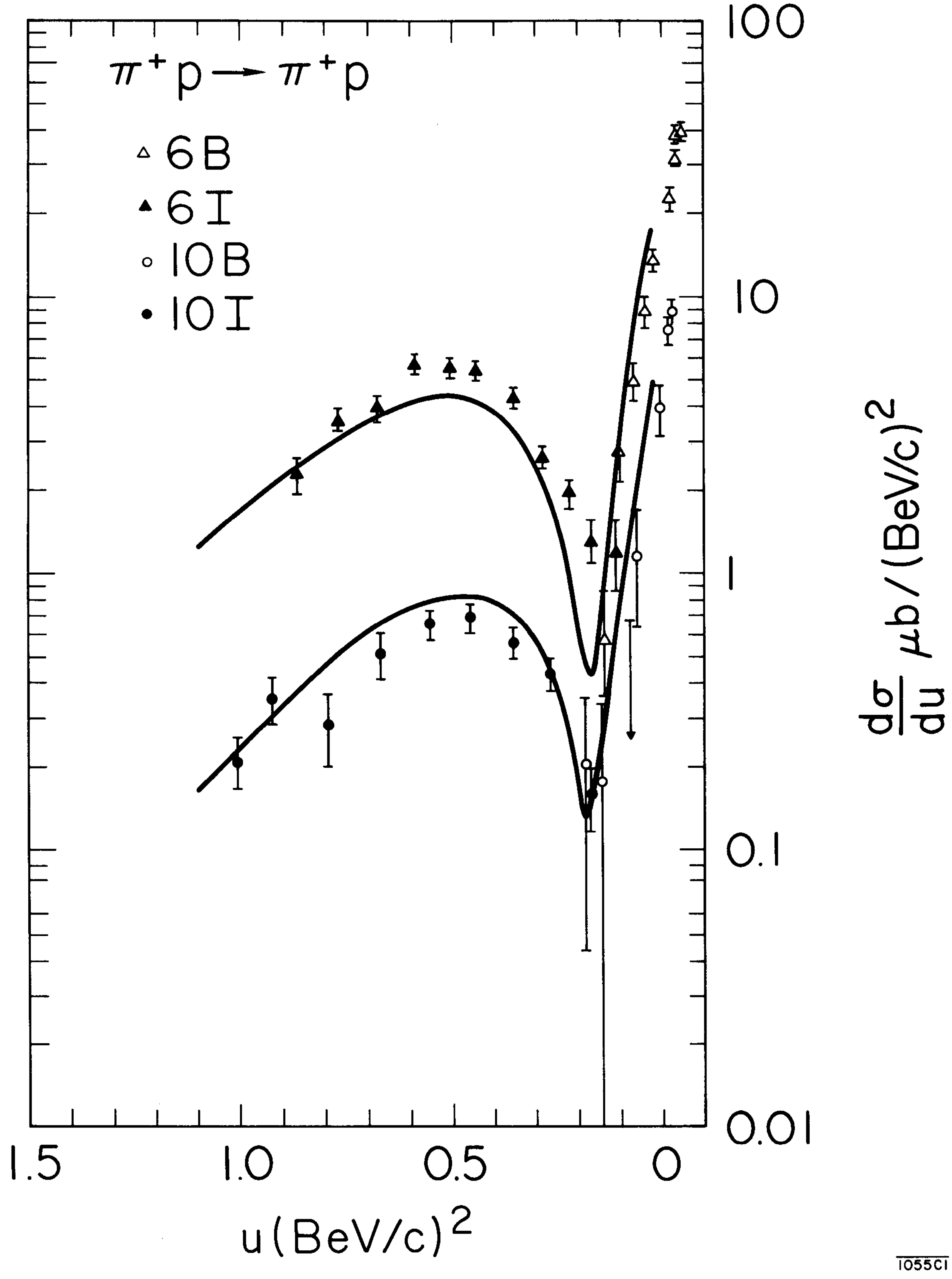
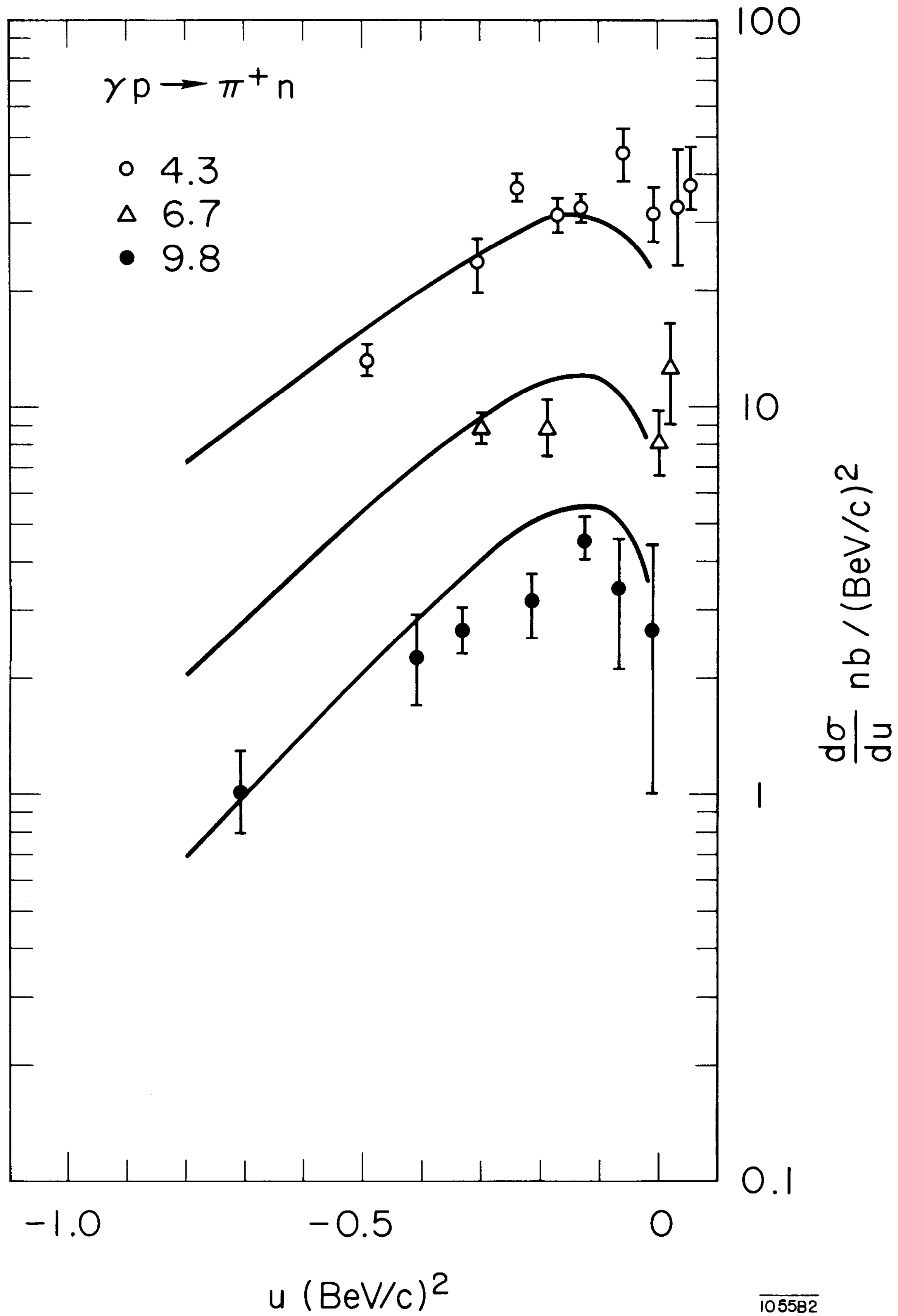

$\pi^+ p \rightarrow \pi^+ p$
6B
6I
10B
10I
100
10
1
0.1
0.01
$\frac{d\sigma}{du}$ $\mu b/(BeV/c)^2$
1.5
1.0
0.5
0
$u (BeV/c)^2$
1055C1

Fig. 2



$\gamma p \rightarrow \pi^+ n$
○ 4.3
△ 6.7
● 9.8
100
10
1
0.1
$\frac{d\sigma}{du}$ nb / (BeV/c)^2
−1.0
−0.5
0
u (BeV/c)^2

1055B2

Fig. 3