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Moving Branch Points and Two-Body Inelastic Reactions (*) (**).

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Summary. — An approach for calculating Regge-cut contributions in certain two-body inelastic reactions is formulated and it is shown that it leads to amplitudes with all the properties established in dynamical models of cuts in complex angular momentum. With this, a model based on a nonconspiring pion accounts well for near-forward $\gamma p \to \pi^+ n$, with one free parameter which is approximately obtained from independent information. The same model (based on ω Regge exchange) describes well $\gamma p \to \pi^0 p$ for momentum transfers $(-t)^{\frac{1}{2}} \leqslant 1\,\text{GeV}$; the t-dependence of $d\sigma/dt$ in a wide range of energies and the ratio of cross-sections with polarized photons is obtained with no free parameters. Finally, the model accounts well for the absolute magnitude and variation of $\pi^+ p \to \rho^0 \Delta^{++}$ at small $|t|$.

1. – Introduction.

It is well known that a Regge-pole description of $np \to pn$ and of $\gamma p \to \pi^+ n$ necessitates a pion conspiring with a trajectory of opposite parity ([1,2]). However, if a conspiring pion controls $\pi\mathcal{N} \to \rho\Delta$, the differential cross-section of this reaction should exhibit a forward dip ([3]); in constrast, a forward peak is experimentally observed ([4]). Regge-pole fits of $\pi\mathcal{N} \to \rho\Delta$ with a conspiring pion are possible; but they correspond to a complicated physical picture and

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([4]) M. ADERHOLZ, J. BARTSCH, M. DEUTSCHMANN, G. KRAUS, R. SPETH, C. GROTE, contain a large number of free parameters [5]. Because of these undesirable features and the absence of physical particles lying on conspirator trajectories, other, conspiracy-free, descriptions of the experimental situation are of importance.

Such a possibility exists if, apart from moving poles, branch points in complex angular momentum are also considered. The theoretical importance of Regge cuts has been known for several years [6,7]. From the phenomenological point of view there are several models which can be interpreted in terms of Regge poles and cuts and successfully describe elastic scattering [8-14]. Also, a similar interpretation holds for models combining Regge poles with absorption.

Our purpose is to provide a simple and unique description of certain two-body inelastic processes with models combining nonconspiring Regge poles plus cuts and containing a small number of free parameters. In their construction we shall be guided by certain general features common to all models of ref. [8-12]. In our approach the dips observed in certain two-body reactions are related to nonsense factors; for this and other well-known reasons these reactions are treated more easily if decomposed in crossed-(t-)channel helicity amplitudes. Then the kinematical constraints known to hold in certain cases will be used to further reduce the number of free parameters.

In Sect. **2** we discuss certain features in common to several [8-12] Regge-cut models successfully describing elastic scattering. In Sect. **3** and **4** we formulate a prescription for the magnitude of the Regge cuts in inelastic reactions; and we show that in most cases of interest this leads to asymptotic expansions with all the properties established in dynamical models of cuts. In Sect. **5** our approach is applied to near-forward photoproduction of charged pions; the

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model contains only one free parameter which determines the cut contributions; in order of magnitude even this parameter can be inferred from independent information. Section **6** studies $\gamma p \to \pi^0 p$ for momentum transfers $0 < -t \leqslant 1.2\ (\text{GeV})^2$ with a similar model; it turns out that the t-dependence of the cross-sections in a wide range of energies and the magnitude and variation of the data with polarized photons can be obtained essentially with no free parameters. Finally, Sect. **7** shows that our model of charged-pion photoproduction successfully accounts for the magnitude and variation of $\pi\mathcal{N} \to \rho\Delta$ as well with essentially no free parameters.

Regge-cut models based on field theory or unitary iterations are known to lead to infinite series of moving branch points [6,7]; this is well accounted for in our general formulation (Sect. **3** and **4**). However, several phenomenological analyses, in particular for $-t \leqslant 1\ (\text{GeV})^2$ can well proceed with only the first cut, which then represents the complete series in an average sense.

I. – Genetal Formulation.

2. – Henkel transforms and moving branch points in elastic scattering.

As in most of ref. [9-12] we begin with the impact parameter expansion of the amplitude $f(s, t)$ for the elastic scattering of two spinless particles. As the square of the total c.m. energy $s \to \infty$ this is given by the Hankel transform:

$$f(s, t) = -is\int_0^\infty E(b, s) J_0(bq)\, b\, \mathrm{d}b \tag{2.1}$$

with $q = (-t)^{\frac{1}{2}} =$ momentum transfer. Assume further that $F(b, s)$ depends on the variable b as follows:

$$F(b, s) = \frac{1}{2\lambda}\big(\varphi(x) - \varphi(0)\big) \tag{2.2a}$$

with

$$x \equiv \frac{\beta}{\varrho}\cdot \exp\left[-\frac{b^2}{4\lambda\varrho}\right], \tag{2.2b}$$

where β and λ are real constants (to be identified below) and

$$\varrho \equiv \ln s - \frac{i\pi}{2}. \tag{2.2c}$$

$\varphi(x)$ is assumed to be a real function of x expandable in a Taylor series around $x=0$, so that

$$F(b,s)=\frac{1}{2\lambda}\sum_{n=1}^{\infty}\frac{\varphi^{(n)}(0)}{n!}\left(\frac{\beta}{\varrho}\right)^{n}\cdot\exp\left[-\frac{nb^2}{4\lambda\varrho}\right], \tag{2.3}$$

where $\varphi^{(n)}(0)=(\mathrm{d}^n\varphi(x)/\mathrm{d}x^n)_{x=0}$. Replacing in (2.1) and interchanging summation and integration we obtain after integrating in b

$$f(s,t)=\beta\sum_{n=1}^{\infty}\frac{1}{n!n}\varphi^{(n)}(0)\cdot\left(\frac{\beta}{\varrho}\right)^{n-1}\cdot\left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_n(t)}, \tag{2.4}$$

where

$$\alpha_n(t)=1+(\lambda/n)t\,. \tag{2.5}$$

Suppose now that for $n=1$ (2.5) represents the Pomeranchuk Regge trajectory (slope λ). Then the term $n=1$ in (2.4) can be identified with the contribution $f^{(P)}(s,t)$ of the Pomeranchuk pole to the scattering amplitude:

$$f^{(P)}(s,t)=\beta\varphi^{(1)}(0)\cdot(\exp[-i\pi/2]\cdot s)^{1+\lambda t}\,. \tag{2.6}$$

For a given function $\varphi(x)$ this has a residue determined essentially by the real constant β; also, it has the correct phase determined by the signature factor, here reduced to the form $\exp[-i(\pi/2)\alpha_1(t)]=\exp[-i(\pi/2)(1+i\lambda t)]$. The argument x of (2.2b), which is the basis for the construction of the whole series (2.4), is easily seen to be related to the inverse Hankel transform of $f^{(P)}(s,t)$:

$$\int_0^{\infty}f^{(P)}(s,t)J_0(bq)\,q\,\mathrm{d}q=\frac{\varphi^{(1)}(0)}{2\lambda}\cdot\frac{\beta}{\varrho}\cdot\exp\left[-\frac{b^2}{4\lambda\varrho}\right]. \tag{2.7}$$

The terms of (2.4) with $n\geqslant 2$ can also be associated with singularities in complex angular momentum J; for:

i) It is well known [6,7] that exchange of $(n+1)$ Pomeranchukons with trajectories $\alpha_1(t)$ (in proper nonplanar diagrams) leads to a moving branch point $\alpha_n(t)$ which at small $|t|$ varies as

$$\alpha_n(t)=n\alpha_1\left(\frac{t}{n^2}\right)-n+1\,; \tag{2.8}$$

with $\alpha_1(t)=1+\lambda t$ we immediately obtain (2.5).

ii) The phase of the leading contribution of the corresponding cut is $\exp[-i(\pi/2)\alpha_n(t)]$, in accord with general theorems on crossing-symmetric asymptotic expansions of the scattering amplitude [14].

iii) In the crossing symmetric form [14], as the number of the exchanged Reggeons increases, the asymptotic cut contributions contain decreasing powers of $\ln s - i\pi/2$.

Thus it is reasonable to associate the $n \geqslant 2$ terms of (2.4) with Regge cuts and write

$$f(s, t) = f^{(P)}(s, t) + f^{\text{cuts}}(s, t)\,, \tag{2.9}$$

$$f^{\text{cuts}}(s, t) = \beta \sum_{n=2}^{\infty} \frac{1}{n!\,n} \varphi^{(n)}(0) \left(\frac{\beta}{\varrho}\right)^{n-1} \cdot \left(\exp\left[-\frac{i\pi}{2}\right] \cdot s\right)^{\alpha_n(t)}. \tag{2.10}$$

The specific form of the function $\varphi(x)$ represents an ansatz about the cut discontinuity near the branch point; on this very little is known theoretically. In ref. [8,9] the form

$$\varphi(x) = \tfrac{1}{2}[1 - (1 - 4x)^{\frac{1}{2}}]$$

is used; its derivation is based on the mechanism that produces, via elastic unitarity, the Amati-Fubini-Stanghellini cuts. Reference [10] uses

$$\varphi(x) = \frac{x}{1 - x}\,,$$

which can be derived by a similar mechanism applied on a somewhat different model. Finally ref. [11,12] use the form

$$\varphi(x) = e^{x} \qquad (x < 0)\,, \tag{2.11}$$

for which some analogy with Glauber's formula for multiple scattering on nuclei might be invoked.

3. – Henkel transforms and moving branch points in inelastic reactions.

Let $f_{\lambda_3\lambda_4,\lambda_1\lambda_2}$ be the helicity amplitudes of the t-channel reaction $1+\bar{2} \to \bar{3}+4$. Standard procedures allow us to define parity-conserving ones by

$$\begin{aligned}\bar{f}^{\sigma=\pm}_{\lambda\mu} = \bar{f}^{\pm}_{\lambda_3\lambda_4,\lambda_1\lambda_2} = \left(\sqrt{2}\cos\frac{\theta_t}{2}\right)^{-|\lambda+\mu|} \left(\sqrt{2}\sin\frac{\theta_t}{2}\right)^{-|\lambda-\mu|} f_{\lambda_3\lambda_4,\lambda_1\lambda_2} \pm \\ \pm (-)^{\lambda+\lambda_m}\sigma_3\sigma_4 \left(\sqrt{2}\cos\frac{\theta_t}{2}\right)^{-|\lambda-\mu|} \left(\sqrt{2}\sin\frac{\theta_t}{2}\right)^{-|\lambda+\mu|} f_{-\lambda_3-\lambda_4;\lambda_1\lambda_2}\,,\end{aligned} \tag{3.1}$$

where $\lambda = \lambda_1 - \lambda_2$, $\mu = \lambda_3 - \lambda_4$, $\lambda_m = \max(|\lambda|, |\mu|)$, θ_t the c.m. scattering angle in the t-channel and $\sigma = +\,(-)$ denotes natural (unnatural) parity. The kine-

Fig. 5. – Percentage πp and $\pi\pi$ effective-mass distributions for the reaction $\pi^+ p \to p 2\pi^+\pi^-\pi^0$ at 8 GeV. Curves as for Fig. 4.

is displayed clearly. It was pointed out by CLA that the charged pions of the final state must be ordered on the multi-Regge graph. For example, consider the reaction $K^- p \to \Lambda\pi^+\pi^-$ where the only allowed graphs are those of Fig. 6.

The great advantage is that now we can distinguish the pions immediately. The consequences of this ordering of the pions, as far as single-particle distributions are concerned, were pursued by CLA. We can now study the same effects in two-body distributions. According to our parametrization for including resonances, only those particles which are adjacent to each other on

$$\tilde{f}^{+}_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s, 0) = \pm \tilde{f}^{-}_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s, 0) \tag{3.4}$$

provided that $|\lambda| + |\mu| \neq 0$.

Suppose that the amplitude $\tilde{f}^{\sigma}_{\lambda\mu}(s, t)$ receives a contribution $\tilde{f}^{\sigma(\mathrm{R})}_{\lambda\mu}(s, t)$ from a Regge trajectory, say, $\alpha_{\mathrm{R}}(t)$ of signature ξ_{R}; this can be written

$$\tilde{f}^{\sigma(\mathrm{R})}_{\lambda\mu}(s, t) = b^{\sigma}_{\lambda\mu}(t) \cdot \frac{\exp[-(i\pi/4)(2\alpha_{\mathrm{R}} + \xi_{\mathrm{R}} - 1)]}{\sin(\pi/4)(2\alpha_{\mathrm{R}} - \xi_{\mathrm{R}} + 1)} \cdot \left(\frac{s}{s_0}\right)^{\alpha_{\mathrm{R}} - \lambda_m} . \tag{3.5}$$

We shall be interested in two-body inelastic reactions for $0 \leqslant -t \leqslant 1$ (GeV)^2; then $\sin(\pi/4)(2\alpha_{\mathrm{R}}(t) - \xi_{\mathrm{R}} + 1)$ either is a nonzero function—in general slowly varying—or it has a zero cancelled by a proper ghost-killing factor of $b^{\sigma}_{\lambda\mu}(t)$. In these cases one can write

$$\tilde{f}^{\sigma(\mathrm{R})}_{\lambda\mu}(s, t) \simeq \beta^{\sigma}_{\lambda\mu}(t) \cdot \exp\left[-\frac{i\pi}{4}\left(2\alpha_{\mathrm{R}}(t) + \xi_{\mathrm{R}} - 1\right)\right]\left(\frac{s}{s_0}\right)^{\alpha_{\mathrm{R}}(t) - \lambda_m}, \tag{3.6}$$

where $\beta^{\sigma}_{\lambda\mu}(t)$ may contain only factors due to nonsense transitions and factors vanishing at $t = 0$ due to analyticity-factorization requirements ([16]). There is one case of interest in this paper when $\sin(\pi/4)(2\alpha_{\mathrm{R}} - \xi_{\mathrm{R}} + 1)$ is rapidly varying in the low-$|t|$ physical region; this will be treated in detail in Sect. **7**.

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In our approach it is assumed that $\tilde{f}^{\sigma}_{\lambda\mu}(s, t)$ receives also contributions from Regge cuts due to exchange of the trajectory R plus n Pomeranchuk trajectories $(n \geqslant 1)$. Thus

$$\tilde{f}^{\sigma}_{\lambda\mu}(s, t) = \tilde{f}^{\sigma(\mathrm{R})}_{\lambda\mu}(s, t) + \tilde{f}^{\sigma(\mathrm{cuts})}_{\lambda\mu}(s, t) . \tag{3.7}$$

To construct $\tilde{f}^{\sigma(\mathrm{cuts})}_{\lambda\mu}(s, t)$ we shall proceed in close analogy with the expansions of Sect. 2. For purposes of comparison with recent analysis of elastic scattering we shall use the form (2.11). Then, in our model the cut contributions are assumed to be defined by the following Hankel transforms:

$$\tilde{f}^{\sigma(\mathrm{cuts})}_{\lambda\mu}(s, t) = \int F^{\sigma(\mathrm{R})}_{\lambda\mu}(b, s)(e^{x} - 1) J_0(bq) b \,\mathrm{d}b \tag{3.8}$$

with x given by (2.2b) and

$$F^{\sigma(\mathrm{R})}_{\lambda\mu}(b, s) = \int \tilde{f}^{\sigma(\mathrm{R})}_{\lambda\mu}(s, t) J_0(bq) q \,\mathrm{d}q . \tag{3.9}$$

$\tilde{f}^{\sigma(\mathrm{R})}_{\lambda\mu}$ is given by (3.6). It is now our purpose to show that in all cases of physical interest (3.8) and (3.9) generate a series of cuts with the properties discussed in Sect. 2. Note that in (3.8) and (3.9) b is not an impact parameter but merely a variable defining the inverse function $F^{\sigma(\mathrm{R})}_{\lambda\mu}(b, s)$.

We start with the case where $\beta^{\sigma}_{\lambda\mu}$ of (3.6) is finite at $t = 0$, does not contain nonsense factors and stays approximately constant in $0 \leqslant -t \leqslant 1$ (GeV)^2. Taking a linear trajectory for R,

$$\alpha_{\mathrm{R}}(t) = \alpha_{\mathrm{R}}(0) + \lambda_{\mathrm{R}} t , \tag{3.10}$$

and fixing $s_0 = 1$ (GeV)^2 we get

$$F^{\sigma(\mathrm{R})}_{\lambda\mu}(b, s) = \beta^{\sigma}_{\lambda\mu} s^{\alpha_{\mathrm{R}}(0) - \lambda_m} \cdot \exp\left[-\frac{i\pi}{4}(2\alpha_{\mathrm{R}}(0) + \xi_{\mathrm{R}} - 1)\right] \cdot \cdot \frac{1}{2\varrho\lambda_{\mathrm{R}}} \cdot \exp[-b^2/4\varrho\lambda_{\mathrm{R}}] , \tag{3.11}$$

$\varrho = \ln s - i\pi/2$. Expanding $e^{x} - 1 = \sum_{n=1}^{\infty} 1/n! x^n$ and integrating over b in (3.8) we get

$$\tilde{f}^{\sigma(\mathrm{cuts})}_{\lambda\mu}(s, t) = \beta^{\sigma}_{\lambda\mu} s^{-\lambda_m} \cdot \exp\left[\frac{i\pi}{4}(1 - \xi_{\mathrm{R}})\right] \sum_{n=1}^{\infty} \frac{1}{n!} \left(\frac{\beta}{\varrho}\right)^n \cdot \frac{\lambda_n}{\lambda_{\mathrm{R}}} \cdot \left(\exp\left[-\frac{i\pi}{2}\right] \cdot s\right)^{\alpha_n(t)} , \tag{3.12}$$

where

$$\lambda_n^{-1} = \lambda_{\mathrm{R}}^{-1} + n\lambda^{-1} , \qquad \alpha_n(t) = \alpha_{\mathrm{R}}(0) + \lambda_n t . \tag{3.13}$$

Next we turn to the case $\beta^{\sigma(\mathrm{R})}_{\lambda\mu}(t) \sim t$ as $t \to 0$ so that an acceptable parametrization is

$$\beta^{\sigma(\mathrm{R})}_{\lambda\mu}(t) = t\gamma^{\sigma}_{\lambda\mu}\,, \qquad \gamma^{\sigma}_{\lambda\mu} = \text{const}\,. \tag{3.14}$$

With $\alpha_{\mathrm{R}}(t)$ as in (3.10), eq. (3.11) gives

$$F^{\sigma(\mathrm{R})}_{\lambda\mu}(s, b) = -\gamma^{\sigma}_{\lambda\mu} s^{\alpha_{\mathrm{R}}(0)-\lambda_m} \exp\left[-\frac{i\pi}{4}\left(2\alpha_{\mathrm{R}}(0) + \xi_{\mathrm{R}} - 1\right)\right] \cdot \\ \cdot \frac{1}{2(\lambda_{\mathrm{R}}\varrho)^2} \cdot \exp[-b^2/4\lambda_{\mathrm{R}}\varrho]\, L_1\left(\frac{b^2}{4\lambda_{\mathrm{R}}\varrho}\right), \tag{3.15}$$

where $L_n(z)$ is the Laguerre function of order n ([19]). The Hankel transform of $L_n(z)$ exists in closed form:

$$\int_0^\infty b\,\mathrm{d}b\, J_0(bq) \exp[-\beta b^2] L_n(\alpha b^2) = \frac{(\beta-\alpha)^n}{2\beta^{n+1}} \cdot \exp\left[-\frac{q^2}{4\beta}\right] L_n\left(\frac{\alpha q^2}{4\beta(\alpha-\beta)}\right). \tag{3.16}$$

Thus replacing (3.15) in (3.8) we get as before

$$\tilde{f}^{\sigma(\mathrm{cuts})}_{\lambda\mu}(s, t) = -\gamma^{\sigma}_{\lambda\mu} s^{-\lambda_m} \cdot \exp\left[\frac{i\pi}{4}(1-\xi_{\mathrm{R}})\right] \cdot \\ \cdot (\varrho\lambda)^{-1} \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \left(\frac{\beta}{\varrho}\right)^n \left(\frac{\lambda_n}{\lambda_{\mathrm{P}}}\right)^2 \left(\exp\left[-\frac{i\pi}{2}\right] \cdot s\right)^{\alpha_n(t)} \cdot L_1(z) \tag{3.17}$$

with λ_n and $\alpha_n(t)$ as in (3.13) and

$$z = t\varrho \frac{\lambda_n^2}{\lambda_n - \lambda_{\mathrm{R}}}\,. \tag{3.18}$$

For sufficiently small values of $|t|$ so that $|z| \ll 1$ we can approximate

$$L_n(z) \simeq 1\,. \tag{3.19}$$

The analogy between the expansions (3.12), (3.17) and (2.10) is easily seen. The n-th term in the sum of (3.12) or (3.17) can be identified with the asymptotic contribution to $\tilde{f}^{\sigma}_{\lambda\mu}$ from a moving branch point in complex J due to exchange of one R-trajectory plus n Pomeranchuk trajectories. Again: i) the exponent of s, ii) the phase of each term and iii) the increasing power of

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$1/(\ln s - i\pi/2)^n$ is precisely what results from cuts in multi-Reggeon exchange models. Moreover, the forms (3.12) and (3.17) considered as asymptotic expansions of $\tilde{f}^{\sigma}_{\lambda\mu}(s, t)$ for $s \to \infty$ and $t = \text{fixed} \leqslant 0$ are in full accord with general theorems on the s-dependence and the phase of crossing-symmetric asymptotic expansions of t-channel helicity amplitudes [14].

With the slope λ of the Pomeranchukon taken from elastic scattering the expansions (3.12) and (3.17) introduce only one free parameter: the real constant β. In these expansions as well as in (2.4) the power n of β is just the number of Pomeranchukons forming the cut; this suggests that β can be loosely interpreted as the overall « coupling » of the Pomeranchukon to the scattered initial and final particles. A further assumption throughout this work is that β is independent of the helicity indices λ, μ and thus essentially the same as in elastic scattering of *spinless* particles; this allows a comparison of the values we obtain in Sect. **5-7** to independent analysis of elastic scattering [11,12].

Suppose now that $\tilde{f}^{\sigma(\mathrm{R})}_{\lambda\mu}(s, t)$ has a residue function behaving as in (3.14) and at the same time $\tilde{f}^{\sigma}_{\lambda\mu}(s, t)$ satisfies at $t = 0$ a kinematical constraint of the type (3.3a) or (3.4) (these cover all cases of interest in the present work). In terms of kinematically singularity-free amplitudes these constraints take the form

$$\tilde{f}^{\sigma}_{\lambda\mu}(s, 0) = h(\lambda, \mu; m_i)\tilde{f}^{-\sigma}_{\lambda'\mu}(s, 0)\,, \tag{3.20}$$

where $h(\lambda, \mu; m_i)$ is a known nonvanishing factor depending on the helicities and masses of the scattered particles. To order $s^{\alpha_{\mathrm{R}}-\lambda_m}$ the constraint is satisfied by evasion, but to orders $s^{\alpha_{\mathrm{R}}-\lambda_m}/(\ln s)^k$ it requires a conspiracy between cut contributions. Now, it is well known that each Regge cut contributes to helicity amplitudes with both $\sigma = +$ and $\sigma = -$; in view of (3.17) $\tilde{f}^{-\sigma}_{\lambda'\mu}(s, t)$ is expected to have an asymptotic expansion that can be written in the form

$$\tilde{f}^{-\sigma}_{\lambda'\mu}(s, t) = -\gamma_{\lambda\mu} s^{-\lambda_m} \exp\left[\frac{i\pi}{4}(1-\xi_{\mathrm{R}})\right] \cdot \\ \cdot (\varrho\lambda)^{-1} \sum_{n=1}^{\infty} \frac{1}{(n-1)!} \frac{\beta'_n}{\varrho^n} \left(\frac{\lambda_n}{\lambda_p}\right)^2 \left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_n(t)} + O(t)\,,$$

where the β'_n are unknown constants. However, the constraint (3.20), which must be satisfied to all orders in s, demands

$$\beta'_n = \beta^n\,, \qquad \forall n\,. \tag{3.21}$$

Hence, in the present model

$$\tilde{f}^{-\sigma}_{\lambda'\mu}(s, t) = h^{-1}(\lambda, \mu; m_i)\tilde{f}^{\sigma(\mathrm{cuts})}_{\lambda\mu}(s, t) + O(t)\,. \tag{3.22}$$

$O(t)$ stands for terms which as $t \to 0$ vanish like t^N, $N \geqslant 1$; moreover the analyticity of $\tilde{f}^{-\sigma}_{\lambda'\mu}(s, t)$ demands that these terms vary smoothly with t, so that for sufficiently small $|t|$ they can be neglected; then $\tilde{f}^{-\sigma}_{\lambda'\mu}(s, t)$ is determined without any additional parameter.

4. – Residues with nonsense factors.

In our approach, the dips observed in certain two-body inelastic reactions (like $\pi^- p \to \pi^0 n$ and $\gamma p \to \pi^0 p$ at photon laboratory energy $\leqslant 10$ GeV) are due to nonsense factors of the residue functions. Thus, it is necessary to consider Hankel transforms and the resulting cuts for more complicated forms of $\beta^\sigma_{\lambda\mu}(t)$ as well.

We start with the form

$$\beta^\sigma_{\lambda\mu}(t) = \gamma^\sigma_{\lambda\mu}\alpha_R(t)\big(\alpha_R(t) + 1\big) \qquad \gamma^\sigma_{\lambda\mu} = \text{const}\,, \tag{4.1}$$

which will be used in our treatment of $\gamma p \to \pi^0 p$. Using [19]

$$\int_0^\infty y^{2n} J_0(xy) \exp[-\tfrac{1}{4}y^2] y\,\mathrm{d}y = n!\,2^{2n+1}\cdot\exp[-x^2]\cdot L_n(x^2) \tag{4.2}$$

and the linear trajectory (3.10),

$$\begin{aligned} F^{\sigma(R)}_{\lambda\mu}(b, s) = {} & \gamma^\sigma_{\lambda\mu} s^{\alpha_R(0)-\lambda_m} \exp\left[-\frac{i\pi}{4}\big(2\alpha_R(0)+\xi_R-1\big)\right]\cdot(2\varrho\lambda_R)^{-1}\exp[-b^2/4\varrho\lambda_R]\,\cdot \\ & \cdot\left\{\alpha_R(0)\big(\alpha_R(0)+1\big) - \big(1+2\alpha_R(0)\big)\varrho^{-1}L_1(b^2/4\varrho\lambda_R) + (2/\varrho^2)L_2(b^2/4\varrho\lambda)\right\}. \end{aligned} \tag{4.3}$$

Introducing this in (3.8), expanding $e^x - 1 = \sum_{n=1}^{\infty}(x^n/n!)$ and using (3.16),

$$\begin{aligned} \tilde{f}^{\sigma(\text{cuts})}_{\lambda\mu}(s, t) = {} & \gamma^\sigma_{\lambda\mu} s^{-\lambda_m} \exp\left[\frac{i\pi}{4}(1-\xi_R)\right]\sum_{n=1}^{\infty}\frac{1}{n!}\left(\frac{\beta}{\varrho}\right)^n \frac{\lambda_n}{\lambda_R}\big(1+\alpha_R(0)\big)\,\cdot \\ & \cdot\left\{\alpha_R(0) - G_n^{(1)} + G_n^{(2)}\right\}\left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_n(t)}, \end{aligned} \tag{4.4}$$

where

$$G_n^{(1)} \equiv \frac{1+2\alpha_R(0)}{1+\alpha_R(0)}\cdot\left(\frac{n\lambda_n}{\varrho\lambda}\right)L_1(z)\,, \qquad G_n^{(2)} \equiv \frac{2}{1+\alpha_R(0)}\cdot\left(\frac{n\lambda_n}{\varrho\lambda}\right)^2 L_2(z)\,, \tag{4.5}$$

with z as in (3.18).

The last expansion can be interpreted similarly to (3.12) and (3.17). With $L_n(z) \simeq 1$ which is a good approximation in all cases of present interest, (4.4) represents a superposition of 3 series of Regge cuts differing in the type of discontinuity near the branch point. Again each term has the correct exponent of s and the correct asymptotic phase.

Next, consider the form

$$\beta^{\sigma}_{\lambda\mu}(t) = t\gamma^{\sigma}_{\lambda\mu}\alpha_{\mathrm{R}}(t)\big(\alpha_{\mathrm{R}}(t)+1\big) \tag{4.6}$$

again of interest in $\gamma p \to \pi^0 p$. The same approach gives

$$\begin{aligned}\tilde{f}^{\sigma(\text{cuts})}_{\lambda\mu}(s,t) = -\gamma^{\sigma}_{\lambda\mu}s^{-\lambda_m}\exp\left[\frac{i\pi}{4}(1-\xi_{\mathrm{R}})\right]\sum_{n=1}^{\infty}\frac{1}{n!}\left(\frac{\beta}{\varrho}\right)^{n}\frac{\lambda_n}{\lambda_{\mathrm{R}}}\cdot \\ \cdot\left\{\alpha_{\mathrm{R}}(0)\big(\alpha_{\mathrm{R}}(0)+1\big)-2G^{(1)}_n+3G^{(2)}_n\right\}\left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_n(t)},\end{aligned} \tag{4.7}$$

for which the interpretation is similar.

Of special interest is the exchange of the pion Regge trajectory

$$\alpha_{\pi}(t) = \alpha_{\pi}(0) + \lambda_{\pi}t\,. \tag{4.8}$$

In $\gamma p \to \pi^+ n$ and $\gamma n \to \pi^- p$ this contributes only to $\tilde{f}^-_{01}(s,t)$; and gauge-invariance requirements are known to lead to [20]

$$\tilde{f}^{-(\pi)}_{01}(s,t) = \frac{1}{t-\mu^2}\cdot\beta_{\pi}(t)\alpha_{\pi}(t)\cdot\frac{1+\exp[-i\pi\alpha_{\pi}(t)]}{\sin\pi\alpha_{\pi}(t)}\cdot s^{\alpha_{\pi}(t)-1}\,.$$

$\alpha_{\pi}(t)$ is a factor due to the sense-nonsense transition at $\alpha_{\pi}(t)=0$ and $\mu=$ pion mass. We shall adopt a nonconspiring pion so that, as $t\to 0$, $\beta_{\pi}(t)\sim t$. At least for small $|t|$ we may then approximate

$$\tilde{f}^{-(\pi)}_{01}(s,t) \simeq \frac{t}{t-\mu^2}\cdot\sqrt{8}\,eg\cdot\exp\left[-\frac{i\pi}{2}\alpha_{\pi}(t)\right]\cdot s^{\alpha_{\pi}(t)-1}, \tag{4.9}$$

where e and g are respectively the electromagnetic and the pion-nucleon coupling constants. Replacing in (3.9),

$$F^{-(\pi)}_{01}(b,s) = \sqrt{8}\,egs^{-1}\left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_{\pi}(0)}\int_{q_{\min}}^{\infty}\frac{q^2}{q^2+\mu^2}\cdot\exp[-\varrho\lambda_{\pi}q^2]J_0(bq)\,q\,\mathrm{d}q\,.$$

([20]) J. S. Ball: *Phys. Rev.*, **124**, 2014 (1961); N. Dombey: *Nuovo Cimento*, **31**, 1025 (1964); G. Zweig: *Nuovo Cimento*, **32**, 689 (1964).

With increasing q the function $q^2/(q^2+\mu^2)$ tends rapidly to 1; however, the integrand is important up to, roughly,

$$q^2 \sim 2(\lambda_\pi|\varrho|)^{-1} \sim 35\mu^2$$

for the energies of interest ($s\sim 20$ (GeV)^2); hence we can take

$$F_{01}^{-(\pi)}(b,s) \simeq \sqrt{8}\, egs^{-1}\left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_\pi(0)}\cdot r\int\limits_0^\infty \exp[-\varrho\lambda_\pi q^2]J_0(bq)q\,\mathrm{d}q\,,$$

where the factor r depends weakly (logarithmically) on s only and, for the energies of interest, is of order 1 ([21]). With (4.2) we finally have

$$F_{01}^{-(\pi)}(b,s) \simeq \sqrt{8}\, egs^{-1}\left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_\pi(0)}\frac{r(\ln s)}{2\varrho\lambda_\pi}\cdot\exp[-b^2/4\varrho\lambda_\pi]\,. \tag{4.10}$$

This is essentially of the form (3.11) and leads to a series of Regge cuts such as (3.12).

II. – Applications.

5. – Near-forward photoproduction of charged pions.

In our model photoproduction of charged pions near the forward direction is assumed to be due to a nonconspiring pion Regge pole plus the cuts due to exchange of one pion with one or more Pomeranchukons. As said, pion exchange contributes only to $\tilde{f}_{01}^{-}(s,t)$, which has the complete form

$$\tilde{f}_{01}^{-}(s,t) = \tilde{f}_{01}^{-(\pi)}(s,t) + \tilde{f}_{01}^{-(\text{cuts})}(s,t)\,. \tag{5.1}$$

For $\tilde{f}_{01}^{-(\pi)}$ we shall use throughout

$$\tilde{f}_{01}^{-(\pi)}(s,t) = \sqrt{2}\,\pi eg\frac{t}{t-\mu^2}\cdot\alpha_\pi\frac{1+\exp[-i\pi\alpha_\pi]}{\sin\pi\alpha_\pi}\cdot s^{\alpha_\pi-1} \tag{5.2}$$

([21]) An estimate of r can be obtained through the integral (7.6); for the energies of interest it follows that $r\leqslant 0.1$.

with pion trajectory

$$\alpha_\pi = \alpha_\pi(t) = -\mu^2 + t .$$

$\tilde{f}_{01}^{-(\text{cuts})}$ will be determined on the basis of (4.10).

In $\gamma\mathcal{N} \to \pi\mathcal{N}$ there is one kinematical constraint (eq. (3.3a)), which in terms of kinematic-singularity-free helicity amplitudes reads

$$\tilde{f}_{11}^{+}(s, 0) = 2M \cdot \tilde{f}_{01}^{-}(s, 0) \tag{5.3}$$

(M = nuclear mass). The corresponding Hara-Wang kinematical factors are

$$K_{11}^{+}(t) = \tfrac{1}{4} t^{-\frac{1}{2}}(t - \mu^2) , \qquad K_{01}^{-}(t) = \tfrac{1}{4} t^{-\frac{1}{2}}(t - \mu^2)(t - 4M^2) . \tag{5.4}$$

With a nonconspiring pion, $\tilde{f}_{01}^{-(\pi)}(s, 0) = 0$; thus the discussion at the end of Sect. **3** implies

$$\tilde{f}_{11}^{+}(s, t) = 2M \cdot \tilde{f}_{01}^{-}(s, t) . \tag{5.5}$$

This is expected to hold in a range of $|t|$ of a few μ^2, at least.

We shall study two different models for the cut contributions. In the first, we keep only the term $n = 1$ of (3.12), *i.e.*

$$\tilde{f}_{01}^{-(\text{cut})}(s, t) = \sqrt{8}\, eg \frac{\lambda_1}{\lambda_\pi} s^{-1} \left(\frac{r\beta}{\varrho}\right)\left(\exp\left[-\frac{i\pi}{2}\right] \cdot s\right)^{\alpha_1(t)} . \tag{5.6}$$

This is expected to reflect the average behaviour of the series of cuts. We fix the Pomeranchukon slope at $\lambda = 0.3\ (\text{GeV})^{-2}$ as in several Regge-pole analyses of elastic scattering; and for simplicity take $\varrho = \ln s$ (asymptotic form of one-cut contribution); we also fix $r = 1$. Using

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{1}{2\pi s q_s^2} \sum_{\lambda_\gamma \lambda_{\mathcal{N}} \lambda_{\bar{\mathcal{N}}}} |f_{\lambda_\gamma 0, \lambda_{\mathcal{N}} \lambda_{\bar{\mathcal{N}}}}|^2 \tag{5.7}$$

(q_s = c.m. initial momentum in s-channel), we calculate $\mathrm{d}\sigma/\mathrm{d}t$ for $E_\gamma = 8.11$ and 16 GeV. Our results (Fig. 1, continuous lines) correspond to

$$\beta = -8.1 . \tag{5.8a}$$

The second model keeps the complete series of cuts (3.12) with exact factors $\varrho = \ln s - i\pi/2$. To facilitate comparison with a similar model applied to elastic scattering use $\lambda = 0.8\ (\text{GeV})^{-2}$ ([12]) and again fix $r = 1$. The remaining free parameter is taken to be

$$\beta = -10.5 \tag{5.8b}$$

and the results are presented in Fig. 1 (broken lines).

The essential feature of charged-pion photoproduction is that the differential cross-section shows a forward peak of width $\sim \mu^2$. One can see how our models operate to produce this situation. With increasing $|t|$ the factor $t(t-\mu^2)^{-1}$ in (5.2) increases very rapidly (rate $\sim \mu^{-2}$); however, at a fixed s, the cuts provide $\tilde{f}_{01}^-$ and $\tilde{f}_{11}^+$ with a slowly varying contribution. If the phases of $\tilde{f}_{01}^{-(\pi)}$ and $\tilde{f}_{01}^{-(\text{cuts})}$ differ by $\sim \pi$, due to destructive interference the amplitude may develop a narrow forward peak [22]. It is remarkable that β, the only free parameter of the model, can be chosen so that both the absolute magnitude of the forward $\mathrm{d}\sigma/\mathrm{d}t$ as well as the width of the peak are correctly reproduced.

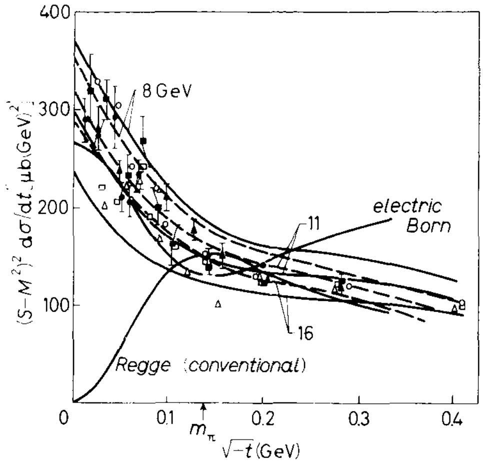

Fig. 1. – Near-forward $\gamma p \to \pi^+ n$. Continuous lines: one-cut model. Broken lines: model with series of cuts eq. (3.12). Data as in ref. [23]. ○, ●, 8 GeV; □, ■, 11 GeV; △, ▲, 16 GeV. Open symbols, SLAC 1967; full symbols, SLAC 1968 (new data).

More remarkable, the needed value of β (5.8a, b) is fairly close to that describing elastic scattering in similar models (in ref. [12], $\beta = -10$ for $\bar{p}p \to \bar{p}p$, -7 for $pp \to pp$, -4 for $\pi p \to \pi p$). We argue that this is understandable. Accept, for definiteness, vector dominance; then β represents an average «coupling» of the Pomeranchukon to the nucleon, charged pion and neutral ρ. However, a similar interpretation holds for elastic scattering, so that β should be, in order of magnitude, the same. Granted this interpretation, we conclude that even β is, to some extent, fixed.

Both our models give a forward $\mathrm{d}\sigma/\mathrm{d}t$ varying slightly, with s. Note that in the second model the variation is significantly reduced; this arises because of cancellations between successive cuts and is in accord with the most recent data on $\gamma p \to \pi^+ n$ [23].

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(22) A physically similar description of forward $\gamma p \to \pi^+ n$ has been advanced by F. HENYEY *et al.* (University of Michigan preprint) who generate Regge cuts by absorption corrections to Regge-pole exchanges.

(23) B. RICHTER: in *Proceedings of the XIV International Conference on H.E. Physics* (Vienna, 1968), p. 3.

Finally, note that a crucial point in the fit of $\gamma p \to \pi^+ n$ by a conspiring parity doublet of Regge poles is the use of a pion residue function

$$\beta_\pi(t) \sim 1 + \lambda(t-\mu^2)\mu^{-2} \tag{5.9}$$

with $\lambda = 0.4$ [2]; thus $\beta_\pi(t)$ must vary rapidly and even have a zero at small $-t$. To justify this behaviour ref. [2] invokes the work of Mandelstam on relations between PCAC and conspiracy theory [24]. In view of the theoretical difficulties of this subject, the form (5.9) must be considered as a purely phenomenological ansatz. In our models (and in ref. [22]) a zero in $\tilde{f}_{01}^-(s, t)$ at small $-t$ naturally arises due to pole-cut interference.

6. – Photoproduction of neutral pions.

We shall describe $\gamma p \to \pi^0 p$ by a mechanism very similar to forward $\gamma p \to \pi^+ n$ except that now it is the ω Regge pole (instead of π) that provides the driving force. The ω pole will be accompanied by a series of moving branch points due to simultaneous exchange of one ω plus one or more Pomeranchukons. Contributions from either pole (ρ or B) are possible, but turn out unnecessary.

The ω pole contributes to the helicity amplitudes $\tilde{f}_{11}^+$ and $\tilde{f}_{01}^+$ (corresponding Hara-Wang factor $K_{01}^+ = \frac{1}{4}(t-\mu^2)$). In accord with finite-energy sum rules [25] we assume that its trajectory chooses nonsense at $\alpha_\omega(t) = 0$. Also, it is generally accepted that ω does not conspire at $t = 0$, so that combined requirements of analyticity and factorization (for the ω pole alone) imply that the residue of ω in $\tilde{f}_{11}^{+(\omega)}$ must be $\sim t$ at small $|t|$ [16]. Thus we proceed with the following pole contributions:

$$\tilde{f}_{\lambda\mu}^{+(\omega)}(s, t) = b_{\lambda\mu}^+ \alpha_\omega(\alpha_\omega + 1)\frac{1-\exp[-i\pi\alpha_\omega]}{\sin \pi\alpha_\omega}\cdot s^{\alpha_\omega - 1}, \tag{6.1}$$

where

$$b_{01}^+ = \gamma_{01}^+, \qquad b_{11}^+ = \gamma_{11}^+ t \tag{6.2}$$

($\gamma_{\lambda\mu}^+ =$ constants) and

$$\alpha_\omega = \alpha_\omega(0) + \lambda_\omega t\,. \tag{6.3}$$

The series of cuts arising from (6.1) has been given in Sect. **4**. Here we con-

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([24]) S. MANDELSTAM: *Phys. Rev.*, **168**, 1884 (1968).
([25]) P. DI VECCHIA, F. DRAGO and M. PACIELLO: *Nuovo Cimento*, **55** A, 809 (1968).

sider the equivalent to the first model of Sect. 5 (one-cut), when

$$\tilde{f}^{+(\text{cut})}_{01}(s,t)=\gamma^{+}_{01}\,\frac{\lambda_1\beta}{\lambda_\omega}\cdot\frac{1+\alpha_\omega(0)}{\ln s}\left\{\alpha_\omega(0)-G^{(1)}+G^{(2)}\right\}\cdot\left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_1(t)-1}, \tag{6.4}$$

$$\tilde{f}^{+(\text{cut})}_{11}(s,t)=-\gamma^{+}_{11}\left(\frac{\lambda_1}{\lambda_\omega}\right)^2\cdot\frac{\beta}{\lambda}\cdot\frac{1+\alpha_\omega(0)}{(\ln s)^2}\cdot \\ \cdot\left\{\alpha_\omega(0)-2G^{(1)}+3G^{(2)}\right\}\left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_1(t)-1}, \tag{6.5}$$

where

$$G^{(1)}=\frac{1+2\alpha_\omega(0)}{1+\alpha_\omega(0)}\cdot\frac{\lambda_1}{\lambda\ln s}\,,\qquad G^{(2)}=\frac{2}{1+\alpha_\omega(0)}\cdot\left(\frac{\lambda_1}{\lambda\ln s}\right)^2 \tag{6.6}$$

and the trajectory of the moving branch point

$$\alpha_1(t)=\alpha_\omega(0)+\lambda_1 t\,,\qquad \lambda_1=\frac{\lambda\lambda_\omega}{\lambda+\lambda_\omega}\,. \tag{6.7}$$

As we are interested in a description of $\gamma p\to\pi^0 p$ in $0\leqslant|t|\leqslant 1\ (\text{GeV})^2$ we can well approximate $L_m(x)\simeq 1$ in eqs. (4.4) and (4.7). Furthermore, the constraint (5.5) and the discussion following eq. (3.22), extended now to $|t|\approx 1\ (\text{GeV})^2$, implies that the unnatural-parity amplitude $\tilde{f}^{-}_{01}(s,t)$ will also receive a contribution fully determined from (6.5).

In the following calculation we take $\lambda_\omega=0.86\ (\text{GeV})^{-2}(\alpha_\omega=0$ at $t\simeq-0.55)$ and, as in the one-cut model of Sect. 5, a Pomeranchukon slope $\lambda=0.3\,(\text{GeV})^{-2}$. The residue constants γ^{+}_{01} and γ^{+}_{11} can be treated as free parameters; however, an estimate of the ratio $\gamma^{+}_{11}/\gamma^{+}_{01}$ can be inferred from Regge-pole analysis of $pp\to pp$ and $\bar{p}p\to\bar{p}p$, where ω exchange is also very important. It can be seen ([26]) that, roughly, $1\leqslant\gamma^{+}_{11}/\gamma^{+}_{01}\leqslant 3$; and we shall take $\gamma^{+}_{11}/\gamma^{+}_{01}=2$.

Thus, apart from the magnitude of, say, γ^{+}_{01} ([27]) there remains only β as free parameter. However, with the interpretation of β as an average Pomeranchukon « coupling » and with vector dominance as a guide, we can claim that at least in order of magnitude β must be the same, as in the one-cut model of Sect. 5; in both cases (π^+ exchange in $\gamma p\to\pi^+ n$ and ω exchange in $\gamma p\to\pi^0 p$) the Pomeranchukon couples to nucleon, pion and ρ-meson. Thus we fix β as in (5.8a).

([26]) W. RARITA, R. RIDDELL, C. CHIU and R. J. N. PHILLIPS: *Phys. Rev.*, **165**, 1615 (1968).

([27]) Even γ^{+}_{01} can be inferred through vector dominance from the ω-exchange contribution to $\pi\mathcal{N}\to\rho\mathcal{N}$; see A. DAR, V. WEISSKOPF, C. LEVINSON and H. LIPKIN: *Phys. Rev. Lett.*, **20**, 1261 (1968).

Our final assumption concerns the relative sign of the pole and cut contribution, which for inelastic reactions is, in principle, completely undetermined. Note that in our examples the cut contributions vary slowly with t, but, in general, the pole contributions vary rapidly (due to nonsense factors and factors $\sim t$). It is obvious that a clear prescription can be formulated only near $t = 0$ when pole and cut have the same phase. In $\gamma p \to \pi^+ n$ the form of $\tilde{f}_{01}^{-(\pi)}$ (eq. (5.2)) and the requirement of forward peak imply that, as $t \to 0+$, the pole and the first cut contribution to the t-channel helicity amplitude $\tilde{f}_{01}^{-}$ have the same relative sign. In $\gamma p \to \to \pi^0 p$ we obtain best agreement with the data if we extend this convention to both $\tilde{f}_{01}^{+}$ and $\tilde{f}_{11}^{+}$. We shall keep it throughout all our work ([28]).

With the model thus formulated we present $d\sigma/dt$ in Fig. 2a), and the ratio

$$R = \frac{d\sigma_\perp/dt - d\sigma_\parallel/dt}{d\sigma/dt}$$

($\sigma_\perp(\sigma_\parallel)$ is the cross-section with photons polarized perpendicular (parallel) to the production plane) in Fig. 2b). As we are interested in producing basic features rather than to fit data, no effort was made to vary β. We note the following:

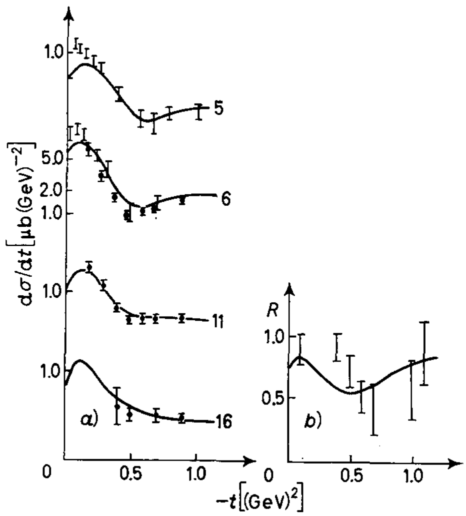

Fig. 2. – a) Differential cross-sections for $\gamma p \to \pi^0 p$. Data: • ANDERSON *et al.*: *Phys. Rev. Lett.*, **21**, 384 (1968); I BRAUNSCHWEIG *et al.*: *Phys. Lett.*, **26** B, 405 (1968). b) The asymmetry ratio R calculated at $E_\gamma = 3$ GeV. Data: BELLENGER *et al.*: MIT preprint (also ref. ([23])).

i) The forward dip of $d\sigma/dt$ is explained by the vanishing of $\tilde{f}_{11}^{+(\omega)}(s, t)$ at $t = 0$ (eq. (6.2)) and the fact that $\tilde{f}_{01}^{+}(s, t)$ enters in $d\sigma/dt$ with a kinematical factor vanishing at $t \simeq 0$.

ii) The dip at $t \simeq -0.6$, which disappears with increasing s, is due to the vanishing of $\alpha_\omega(t)$ in the pole terms, which at relatively low energy control $d\sigma/dt$ (in particular $f_{01}^{+(\omega)}$). However, at $t \simeq -0.8$ (GeV)^2, the $f_{\lambda\mu}^{+(\omega)}$ decrease rather fast with s. On the contrary, our cut contributions (6.4) and (6.5) decrease very slowly; this holds in particular for $\tilde{f}_{11}^{+(\text{cut})}$ which is (relatively) very important for $-t \geqslant 0.5$ (GeV)^2 (note that between $E_\gamma = 3$ and 16 (GeV)^2 the quan-

([28]) For $\gamma p \to \pi^+ n$, $\gamma n \to \pi^- p$ in $0 < -t \leqslant 1.2$ (GeV)^2 and for $\pi\mathcal{N} \to \omega\mathcal{N}$ a similar analysis (in progress) with the same sign convention leads to good results, as well.

tity $\alpha_\omega(0) - 2G^{(1)} + 3G^{(2)}$ of (6.5) increases slowly). Thus, for $E_\gamma \gtrsim 10\ (\text{GeV})^2$ the cuts, which are smooth in t, dominate and the dip is washed out.

iii) As is well known, $\mathrm{d}\sigma_\perp/\mathrm{d}t$ contains only natural-parity exchanges ($\tilde{f}^+_{01}$ and $\tilde{f}^+_{11}$) and $\mathrm{d}\sigma_\parallel/\mathrm{d}t$ only unnatural ($\tilde{f}^-_{01}$). At $t \simeq -0.2$ and $t \simeq -1$ the ω-pole contributions enhance $\mathrm{d}\sigma_\perp/\mathrm{d}t$ so that $R \approx 0.8$. At $t \simeq -0.55$ the $\tilde{f}^{+(\omega)}_{\lambda\mu}$ vanish and R shows a dip; however, $\tilde{f}^{+(\text{cut})}_{01}(s, t \simeq -0.55)$ is significant and this gives $R \approx 0.5$ in the position of the dip.

A somewhat similar model of $\gamma p \to \pi^0 p$ has been advanced in ref. [29]; however, it contains 5 completely free parameters and is not concerned with possible relations between pole and cut contributions or correlation of cuts in $\gamma p \to \pi^0 p$ and $\gamma p \to \pi^+ n$. On the other hand [30], ω Regge exchange supplemented only by cuts calculated with the conventional absorption prescription fails completely to explain the disappearance of the dip with increasing s and gives $R \geqslant 0.92$ for all t; good agreement is obtained only by adding ρ and B exchanges with a B trajectory slope $\lambda_B = 0.4\ (\text{GeV})^{-2}$.

7. – The process $\pi^+ p \to \rho^0 \Delta^{++}$ at small $|t|$.

Recent experiments [4] on $\pi^+ p \to \rho^0 \Delta^{++}$ at 8 GeV establish a forward peak of width $\sim \mu^2$; moreover, for $|t_{\min}| \leqslant |t| \leqslant 0.04\ (\text{GeV})^2$ the density matrix element ϱ_{00} is very close to 1. These features intuitively suggest that the forward amplitude is controlled by pion exchange.

As in $\gamma p \to \pi^+ n$, our model describes $\pi^+ p \to \rho^0 \Delta^{++}$ at small $|t|$ by exchange of a nonconspiring Regge pion plus the cuts due to exchange of the pion with one or several Pomeranchukons. Here, however, (unlike $\gamma p \to \pi^+ n$) a nonconspiring pion has, at $t = 0$, a finite residue function. Due to the proximity of the pion pole it turns out that simple pion exchange gives indeed most of the forward cross-section; the cuts calculated according to our prescription contribute to order 10% only.

Neglecting the helicity double-flip amplitude we have [5]

$$\varrho_{00} \frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{1}{64\pi s p_s^2} \cdot \frac{\mu^2 - (m_\Delta - M)^2}{t - (m_\Delta - M)^2} \cdot \frac{[\mu^2 - (m_\Delta + M)^2]^2}{[t - (m_\Delta + M)^2]^2} \cdot \left\{ |\tilde{f}_{00}(s, t)|^2 + 4\Phi(stu) |\tilde{f}_{01}(s, t)|^2 \right\}, \tag{7.1}$$

where $\Phi(stu) = 0$ is the boundary of the physical region and the t-channel

([29]) A. Capella and J. Tran Thanh Van: Orsay preprint LPTHE 68/41.

([30]) M. Blackmon, G. Kramer and K. Schilling: Argonne preprint.

helicity amplitudes

$$\tilde{f}_{|\lambda_{\mathcal{N}}-\lambda_{\bar{\mathcal{N}}}|,0} \equiv [t-(m_\Delta+M)^2][t-(m_\Delta-M)^2]^{\frac{1}{2}}\cdot\Phi(stu)^{|\lambda|/2}\cdot f_{00,\lambda_\Delta\lambda_{\bar{\mathcal{N}}}}\,. \tag{7.2}$$

For $\pi\mathcal{N}\to\rho\Delta$, at the pseudothreshold $t_p=(m_\Delta-M)^2$ there is the constraint [31]

$$\tilde{f}_{01}(s,t_p)\simeq g^{-1}(s)\tilde{f}_{00}(s,t_p)\,, \tag{7.3}$$

as $s\to\infty$, $g(s)\to 2s\sqrt{t_p}$. The pion Regge-pole contribution is

$$\tilde{f}^{(\pi)}_{00}=\gamma^{(\pi)}_{00}\cdot\left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_\pi(t)}\cdot\left(\sin\frac{\pi}{2}\alpha_\pi(t)\right)^{-1}\underset{t\to\mu^2}{\approx}\gamma^{(\pi)}_{00}\cdot\frac{2}{\pi\lambda_\pi}\cdot(t-\mu^2)^{-1}, \tag{7.4a}$$

$$s\tilde{f}^{(\pi)}_{10}=\gamma^{(\pi)}_{10}\cdot\left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_\pi(t)}\cdot\left(\sin\frac{\pi}{2}\alpha_\pi(t)\right)^{-1}\underset{t\to\mu^2}{\approx}\gamma^{(\pi)}_{01}\cdot\frac{2}{\pi}\,. \tag{7.4b}$$

In accord with our general procedure, we shall take a constant pion residue, equal to its perturbation theory value at $t=\mu^2$ [32]:

$$\gamma^{(\pi)}_{10}=G\lambda_\pi\mu^2\,,$$

where

$$G=4\pi^2\,\frac{m_\rho m_\Delta}{\mu^2}\left(\frac{\sigma\Gamma_\rho\Gamma_\Delta}{p_\rho p_\Delta}\right)^{\frac{1}{2}} \tag{7.5}$$

with p_ρ, p_Δ the momenta of ρ, Δ in the c.m. system of the t-channel and the corresponding widths.

In the near-forward region ($-t\leqslant 2.5\mu^2$ at 8 GeV) the helicity-flip contribution is suppressed by the factor $\Phi(stu)$; thus we consider the nonflip amplitude. Our prescription for the cut contribution (eq. (3.9)) gives

$$F^{(\pi)}_{00}(b,s)=\frac{2}{\pi\lambda_\pi}\cdot\gamma^{(\pi)}_{00}\cdot\left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_\pi(0)}\mathscr{F}(b,s)$$

with

$$\mathscr{F}(b,s)\equiv\int\limits_{q_{\min}}^{\infty}\frac{q\,\mathrm{d}q}{q^2+\mu^2}\exp[-\lambda_\pi\varrho q^2]J_0(bq)\,, \tag{7.6}$$

([31]) J. D. Jackson and G. Hite: *Phys. Rev.*, **169**, 1248 (1968).
([32]) J. D. Jackson and H. Pilkuhn: *Nuovo Cimento*, **33**, 916 (1964).

the pion pole enhances the low-q region of (7.6), for $b \ll |\varrho\lambda_\pi|^{\frac{1}{2}}$ a good approximation is

$$\text{(7.7)} \qquad \begin{cases} \mathscr{F}(b, s) = (1 + b^2\mu^2/4)E_1(z) - (b^2/4\varrho\lambda_\pi)e^{-z}\,, \\ z = \varrho\lambda_\pi(q^2_{\min} + \mu^2)\,, \end{cases}$$

with $E_1(z)$ the exponential integral [33]; note that for energies $\leqslant 30$ GeV we can further approximate

$$E_1(z) \simeq z - \ln z - \gamma$$

($\gamma = 0.577 =$ Euler's constant).

Here we shall be contented in an estimate of the cut contribution and will proceed with the one-cut model (and again $\varrho = \ln s$). Thus we find

$$\text{(7.8)} \qquad F^{(\pi)}_{00}(b, s) \approx \frac{1}{\pi\lambda_\pi}\gamma^{(\pi)}_{00}\left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_\pi(0)}\delta_1(s)\exp[-b^2/4\lambda_\pi \ln s \cdot \delta_2(s)]\,,$$

where

$$\delta_1(s) = \exp[\lambda_\pi\mu^2\ln s]\,E_1(z) \qquad (\simeq 1.65 \text{ at } 8 \text{ GeV})$$

$$\delta_2(s) = \exp[-z]E_1^{-1}(z) - \lambda_\pi\mu^2\ln s \qquad (\simeq 0.55 \text{ at } 8 \text{ GeV})\,.$$

Then, in the one-cut approximation (3.8) gives

$$\text{(7.9)} \qquad \tilde{f}^{(\text{cut})}_{00}(s, t) \approx \frac{2}{\pi\lambda_\pi}\gamma^{(\pi)}_{00}\lambda_1\beta\,\delta_1(s)\left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_1(t)}$$

with

$$\alpha_1(t) = \alpha_\pi(0) + \lambda_1 t\,, \qquad \lambda_1 = 0.257\ (\text{GeV})^{-2}\,.$$

Using the asymptotic form of $\Phi(stu)$ for $s \to \infty$ and $t \neq 0$ [31] we reduce (7.1) to the form

$$\text{(7.10)} \qquad \varrho_{00}\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \left(\frac{G}{4\pi^2}\right)^2\frac{\pi\mu^4}{sq_s^2}\cdot\frac{t_p - \mu^2}{t_p - t}\cdot\left(|\varphi_0|^2 - \frac{t}{t_p}|\varphi_1|^2\right)\,,$$

where, in view of (7.9),

$$\varphi_0(s, t) = (t - \mu^2)^{-1}\left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_\pi(t)} + \lambda_1\beta\,\delta_1(s)\left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_1(t)}.$$

([33]) M. Abramowitz and I. A. Stegun: *Handbook of Mathematical Functions*, (Washington, D.C., 1964).

Assuming that $\varphi_1(s, t)$ is a smooth function of t, the simplest way to construct it is by means of the pseudothreshold constraint (7.3); thus we take

$$\varphi_1(s, t) = (t_p - \mu^2)^{-1} \left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_\pi(t)} + \lambda_1 \beta\, \delta_1(s) \left(\exp\left[-\frac{i\pi}{2}\right]\cdot s\right)^{\alpha_1(t)},$$

so that $\varrho_{00}\,\mathrm{d}\sigma/\mathrm{d}t$ has no pole at $t = t_p$ ([31]); also, it has no pole at $t = \mu^2$.

In our numerical calculation we take $G = 2200$ (corresponding to $\Gamma_\rho \simeq 150\,\mathrm{MeV}$). The value (5.8$a$) for β $(= -8.1)$ gives fair agreement with experiment (Fig. 3). However, in $\pi^+\mathrm{p} \to \rho^0\Delta^{++}$, β can well differ from its value in $\gamma\mathrm{p} \to \pi^+\mathrm{n}$; with $\beta = -16$ the agreement is good. Figure 3 shows clearly that, with β of the established order of magnitude, the cut contribution affects forward $\pi\mathcal{N} \to \rho\Delta$ very little.

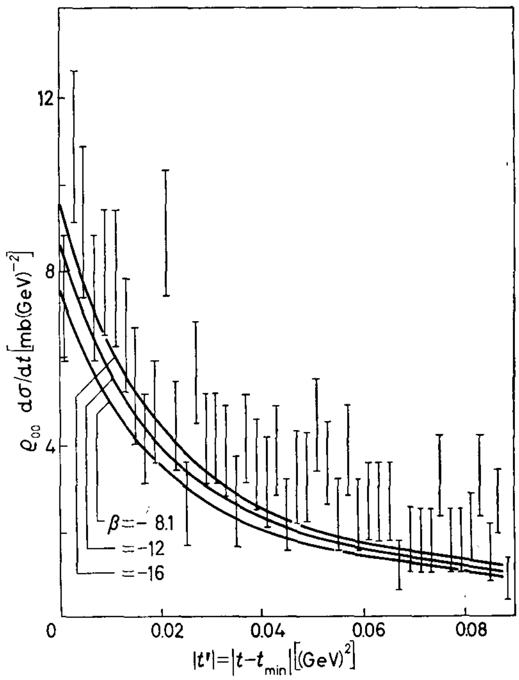

Fig. 3. – Calculations of $\varrho_{00}\,(\mathrm{d}\sigma/\mathrm{d}t)$ for $\pi^+\mathrm{p} \to \rho^0\Delta^{++}$ at 8 GeV. Data: ref. ([4]).

8. – Conclusions.

Given a Regge pole and the Pomeranchukon exchange, we have formulated a prescription for a parameter-free estimate of Regge-cut contributions in two-body reactions. We have shown in detail that this prescription leads to amplitudes in full agreement with the forms established by dynamical models of Regge cuts and by general theorems on crossing-symmetric asymptotic expansions. Applied to near-forward $\gamma\mathrm{p} \to \pi^+\mathrm{n}$, to $\gamma\mathrm{p} \to \pi^0\mathrm{p}$ in the range $0 < -t \leqslant 1.2\,(\mathrm{GeV})^2$ and to $\pi^+\mathrm{p} \to \rho^0\Delta^{++}$ at small $|t|$ our approach leads to nearly parameter-free, simple and unique descriptions of the experimental facts. The same approach applied to $\gamma\mathrm{p} \to \pi^+\mathrm{n}$, $\gamma\mathrm{n} \to \pi^-\mathrm{p}$ and $\pi\mathcal{N} \to \omega\mathcal{N}$ in $0 < -t \leqslant 1.2\,(\mathrm{GeV})^2$ leads to very encouraging results ([28]); and in other charge exchange reactions to simple, at least qualitative, understanding.

A different approach to estimate Regge-cut contributions proceeds through absorption-type corrections to Reggeized particle exchange. From the S-matrix point of view, this also is a prescription on an equal footing to ours. However, for $\gamma p \to \pi^0 p$ the absorption prescription fails to give a simple picture and does not substantially differ from multiparameter pure Regge-pole fits.

A shortcoming of our approach is that its details have not, so far, been established in any explicit dynamical model of inelastic reactions. Such a possibility is an interesting open question.

* * *

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RIASSUNTO (*)

Si formula un'approssimazione per calcolare i contributi dei tagli di Regge in alcune reazioni anelastiche a due corpi e si dimostra che ciò conduce ad ampiezze con tutte le proprietà stabilite nei modelli dinamici dei tagli nel momento angolare complesso. Con ciò, un modello basato su un pione non cospirante spiega bene lo scattering $\gamma p \to \pi^+ n$ quasi in avanti, con un parametro libero che si ottiene approssimativamente da informazioni indipendenti. Con lo stesso modello (basato sullo scambio di Regge di ω) si descrive bene $\gamma p \to \pi^0 p$ per impulsi trasferiti $(-t)^{\frac{1}{2}} \leqslant 1$ GeV; si ottiene senza parametri liberi la dipendenza da t di $d\sigma/dt$ in un ampio intervallo di energie ed il rapporto delle sezioni d'urto con fotoni polarizzati. Infine, col modello si spiega la grandezza assoluta e la variazione di $\pi^+ p \to \rho^0 \Delta^{++}$ per $|t|$ piccolo.

(*) *Traduzione a cura della Redazione.*

Движущиеся точки ветвления и двух-частичные неупругие реакции.

Резюме (*). — Формулируется подход для вычисления вкладов разрезов Редже в некоторые двух-частичные неупругие реакции, и показывается, что этот подход приводит к амплитудам со всеми свойствами, установленными в динамических моделях разрезов в плоскости комплексного момента. При этом, модель, основанная на неконспиративном пионе, хорошо объясняет $\gamma p \to \pi^+ n$ вблизи направления вперед, с помощью одного свободного параметра, который приблизительно получается из независимой информации. Такая же модель (основанная на Редже-обмене ω) хорошо описывает $\gamma p \to \pi^0 p$ для передаваемых импульсов $(-t)^{\frac{1}{2}} \leqslant 1$ ГэВ; t-зависимость $d\sigma/dt$ в широкой области энергий, и получается отношение поперечных сечений поляризованных фотонов без свободных параметров. Наконец, эта модель хорошо объясняет абсолютную величину и изменение $\pi^+ p \to \rho^0 \Delta^{++}$ при малых $|t|$.

(*) *Переведено редакцией.*