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## POMERANCHUK TRAJECTORY AND ITS RELATION TO LOW-ENERGY SCATTERING AMPLITUDES

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While "ordinary" Regge trajectories can be bootstrapped using the resonance approximation to finite-energy sum rules, we suggest that the Pommeranchuk trajectory is mostly built from the nonresonating background at low energies. This hypothesis (a) explains the constancy of  $KN$  and  $NN$  total cross-sections (in contrast to  $\bar{K}N$  and  $\bar{N}N$ ) over a wide energy range; (b) predicts many other interesting relations which are experimentally satisfied; and (c) leads to no obvious inconsistencies.

The Pommeranchuk trajectory has always been somewhat of a mystery. There is no conclusive evidence that any known particle corresponds to it; its slope seems to be much smaller than the "almost universal" slope of most trajectories; its dynamical origin is not clear at all and the only simple intuitive picture that "explains" why such a trajectory exists is the so-called diffraction picture. In particular, it is not known whether the Pommeranchuk trajectory can be bootstrapped or whether it has to be introduced "by hand" into the description of scattering amplitudes.<sup>1</sup>

An equally strange, though probably less fundamental, puzzle is the amazing constancy of the total cross sections for  $K^+p$ ,  $K^+n$ ,  $pp$ , and  $pn$  scattering over the region 2-20 BeV, a property which is not shared by the total cross sections for  $K^-p$ ,  $K^-n$ ,  $\bar{p}p$ ,  $\bar{p}n$ , and  $\pi^\pm p$  scattering.<sup>2</sup> The usual description of small-angle, high-energy scattering amplitudes in terms of a few Regge trajectories in the  $t$  channel is consistent with these experimental facts, but it definitely does not predict them. On the contrary, from the  $t$ -channel point of view it would be extremely hard to predict that  $K^+p$  and  $K^-p$  scattering behave so differently, and that  $\sigma_t(K^+p)$  is constant while  $\sigma_t(K^-p)$  varies at high energies.<sup>3</sup> The situation becomes even more peculiar when we observe that the reactions for which  $\sigma_t$  is constant at high energies are precisely those which do not exhibit any  $s$ -channel resonances at low energies, while the processes for which  $\sigma_t$  varies considerably at high energies are precisely those which show

a rich spectrum of  $s$ -channel resonances at low energies.<sup>4</sup>

In this note we discuss the dynamical nature of the Pommeranchuk trajectory and, as a by-product, present a possible solution to our second puzzle. We utilize the recently developed technique<sup>5</sup> of computing the properties of Regge trajectories by studying the low-energy scattering region and connecting it via finite-energy sum rules (FESR)<sup>5,6</sup> to the high-energy parameters. This method has so far yielded many interesting results<sup>5-8</sup> which are mostly related to trajectories other than the Pommeranchukon. The main result of our attempt of "bootstrapping" the Pommeranchukon in this way is summarized by the following conjecture: The Pommeranchuk trajectory is mostly built by the nonresonating background in the low-energy amplitudes,<sup>9</sup> while the other ("ordinary") trajectories can be usually described in terms of the resonance approximation for the low-energy region. This conjecture leads to a simple explanation of the constant  $KN$  and  $NN$  cross sections, as well as to a large number of additional successful experimental predictions.

Our starting point is the trivial observation that some processes (such as  $K^+p$ ,  $pp$ , or  $\pi^+\pi^+$  elastic scattering) do not seem to involve any important resonances in the low-energy region, while others (e.g.,  $K^-p$ ,  $\bar{p}p$ , or  $\pi^+\pi^-$  scattering) exhibit a very rich resonance structure. On the other hand, the Pommeranchuk trajectory dominates the small- $t$ , large- $s$  scattering of all of these processes, independent of the presence or

absence of resonances. If we now use the FESR in order to relate the low-energy amplitudes to the high-energy parametrization, we must conclude that it is extremely unlikely that the Pomeranchukon is strongly correlated to the resonance structure at low energy. Explicit calculations in various specific cases actually show that if we approximate the integrals over the low-energy region by resonances only, it is essentially impossible to produce the correct properties of the  $P$  trajectory on the other side of the FESR. We mention here only two examples of such a situation<sup>10</sup>:

(1) In  $\pi\pi$  elastic scattering the resonance approximation produces correctly the properties of the  $\rho$  and  $P'$  trajectories, but it does not seem to account for the Pomeranchukon contribution.<sup>8</sup>

(2) If we consider hypothetical reactions such as  $K + \Delta(1236) \rightarrow K + \Delta$  and assume that all the non-negligible baryon resonances are in SU(3) singlets, octets, or decuplets,<sup>4</sup> we find that in  $K^+\Delta^{++}$  there are  $Y_1^*$  resonances in the  $u$  channel while in  $K^0\Delta^{++}$  there are no resonances either in the  $s$  or in the  $u$  channel. In both cases, however, the same Pomeranchukon contribution has to be obtained from the FESR. This can be easily understood if the low-energy nonresonating background amplitudes build up the contribution of the Pomeranchukon, while if we assume the usual resonance approximation for the low-energy region we are immediately led into inconsistencies.

These two examples, as well as a few other cases,<sup>10</sup> indicate (although do not prove) that the low-energy background is, in fact, largely responsible for building up the Pomeranchukon contributions. The second half of our conjecture, namely, the possibility that the "ordinary" trajectories are mostly built by the low-energy resonances, is strongly supported by the many recent successful applications<sup>5,7,8</sup> of FESR, in which the low-energy resonance approximation has provided a good description of various  $t$ -channel Regge trajectories other than Pomeranchukon.

Armed with these plausibility arguments we now proceed to assume that our conjecture is indeed correct, and to derive its various consequences. Our philosophy is the following: We believe that the usual parametrization of high-energy scattering amplitudes in terms of a few Regge poles in the  $t$  channel is valid, and we impose on it the additional "s-channel information" provided by our conjecture. Stated in a general

form, this means that if the left-hand side of the finite-energy sum rule

$$\int_0^N \nu^n \text{Im}A(\nu, t) d\nu = \sum_i \beta_i(t) \frac{N^{\alpha_i+1+n}}{\alpha_i+1+n} \quad (1)$$

is separated into a "resonant part"  $A_{\text{res}}$  and a "background part"  $A_{\text{bg}}$  then, within a good approximation,

$$\int_0^N \nu^n \text{Im}A_{\text{bg}}(\nu, t) d\nu = \beta_P(t) \frac{N^{\alpha_P+1+n}}{\alpha_P+1+n}, \quad (1a)$$

$$\int_0^N \nu^n \text{Im}A_{\text{res}}(\nu, t) d\nu = \sum_{i \neq P} \beta_i(t) \frac{N^{\alpha_i+1+n}}{\alpha_i+1+n}, \quad (1b)$$

where the summation in (1b) involves all trajectories except the Pomeranchukon. In those cases in which  $\text{Im}A_{\text{res}} \cong 0$  for  $-\infty < \nu < +\infty$  we are led, for sufficiently large  $N$  (say  $N > 2$  BeV) to the approximate relation

$$\sum_{i \neq P} \beta_i(t) \frac{N^{\alpha_i+1+n}}{\alpha_i+1+n} = 0. \quad (2)$$

Moreover, if the  $t$ -channel quantum numbers of the amplitude  $A$  do not permit the  $P$  trajectory to contribute and the low-energy integral includes no resonances, we predict that, at high energies, the amplitude  $A$  will be purely real.<sup>11</sup>

In a few cases (such as  $K^+p$  scattering), resonances are absent in the  $s$  channel while they contribute significantly in the  $u$  channel. In such cases we can use a simple generalization of Eqs. (1a) and (1b) and write

$$\int_{N_1}^{N_2} \text{Im}A d\nu \sim \int_{N_1}^{N_2} \text{Im}A_{\text{bg}} d\nu = \frac{\beta_P(t)}{\alpha_P+1} [N_2^{\alpha_P+1} - N_1^{\alpha_P+1}], \quad (3a)$$

$$0 \sim \int_{N_1}^{N_2} \text{Im}A_{\text{res}} d\nu = \sum_{i \neq P} \frac{\beta_i(t)}{\alpha_i+1} [N_2^{\alpha_i+1} - N_1^{\alpha_i+1}], \quad (3b)$$

where the interval  $(N_1, \dots, N_2)$  is chosen on the positive  $\nu$  axis ( $s$ -channel physical  $K^+p$  scattering) between, say,  $N_1 = 1$  BeV and  $N_2 = 2$  BeV (the region in which other processes are dominated by resonances while in  $K^+p$  scattering  $\text{Im}A_{\text{res}} \sim 0$ ). Since Eq. (3b) is supposed to hold for a

range of values of  $N_2, N_1$ , we are effectively led to a relation of the type of Eq. (2) for  $K^+p$  scattering. This is true in spite of the existence of  $K^-p$  resonances which at first sight might be suspected to modify this conclusion.

Some of the consequences of our conjecture are the following<sup>10</sup>:

(a) All total cross sections for reactions in which no important  $s$ -channel resonances show up should be approximately constant in energy over a very wide energy range. This successfully explains why  $\sigma_t(K^+p)$ ,  $\sigma_t(K^+n)$ ,  $\sigma_t(pp)$ , and  $\sigma_t(pn)$  are essentially constant, and why  $\sigma_t(K^+p) = \sigma_t(\bar{K}^+n)$ ,  $\sigma_t(pp) = \sigma_t(pn)$  already at relatively low energies.

(b) Total cross sections for reactions which exhibit strong resonances need not be constant and they should gradually decrease to their asymptotic value. If our description is correct, no total cross section will ever increase towards its Pomeranchuk limit. So far, this is experimentally true in all cases.

(c) In view of the absence of  $I=2$   $\pi\pi$  resonances,  $\sigma_t(\pi^+\pi^+)$  should be approximately constant in energy. If we parametrize high-energy  $\pi\pi$  scattering in terms of the  $P$ ,  $P'$ , and  $\rho$  trajectories,  $\sigma_t(\pi^+\pi^+) = \text{const}$  leads to

$$\alpha_\rho = \alpha_{P''}, \quad (4)$$

$$\gamma_{\rho^0\pi^+\pi^-} = \gamma_{P'\pi^+\pi^-}. \quad (5)$$

Equation (4) is very well satisfied. Equation (5) can be compared with the values for the factorized  $\rho$  and  $P'$  residues as obtained from the analysis of  $NN$ ,  $N\bar{N}$ , and  $\pi N$  elastic scattering. The large errors in the  $\bar{p}n$  cross sections prevent us from reaching definite conclusions, but all the published numbers<sup>12</sup> are consistent with (5).

(d) A similar analysis for  $\pi K$ ,  $KK$ , and  $KN$  scattering gives

$$\alpha_\rho = \alpha_{A_2}, \quad (6)$$

$$\alpha_\omega = \alpha_{P''}, \quad (7)$$

$$\alpha_{K^*} = \alpha_{K^{**}}, \quad (8)$$

as well as relations among the factorized residue functions  $\gamma_{A_2 KK}$ ,  $\gamma_{\rho KK}$ , etc. Equations (6)-(8) are acceptable while the residue relations cannot be tested at present.

(e) All high-energy inelastic  $KN$  and  $NN$  reactions, in which the Pomeranchukon cannot be exchanged, should have purely real amplitudes.

This is trivially correct for  $K^+n$  and  $pn$  charge exchange, since it follows from isospin and our prediction (a). For other reactions such as  $pp \rightarrow p\Delta$ ,  $Kp \rightarrow K\Delta$ ,  $Kp \rightarrow K^*p$ ,  $Kp \rightarrow K^*\Delta$ , the separation of the real and imaginary parts is experimentally very difficult. In all these reactions, however, the currently accepted high-energy descriptions are consistent with a purely real amplitude, since all the suggested parametrizations involve either an equal mixture of  $\rho$  and  $A_2$  exchange (in which case the imaginary part cancels in a similar way to the  $K^+n \rightarrow K^0p$  case) or pion exchange which, at least at small  $t$ , contributes mostly to the real amplitude.<sup>10</sup>

(f) If we assume that  $SU(3)$  is an exact symmetry of the factorized residue functions, we predict that the total meson-meson cross sections in the  $10$ ,  $10^*$ , and  $27$  representations in the  $s$  channel are constant. Since we believe that at high energies only singlets and octets contribute in the  $t$  channel, we conclude that we must have a nonet of degenerate tensor trajectories in addition to the Pomeranchukon. This is independently required by all  $SU(3)$ -invariant Regge fits to meson-baryon scattering,<sup>12</sup> if  $\alpha_P(0) = 1$ .

We conclude with a few general remarks:

(1) Our picture is perfectly consistent with the intuitive "diffraction" picture of the Pomeranchukon. It is conceivable that scattering amplitudes can be described in terms of two parts: (i) an "optical" or "geometrical" part which is represented by the Pomeranchuk pole in the  $t$  channel but is viewed as a smooth nonresonating contribution to the amplitude in the  $s$  channel; (ii) a "dynamical" part which can be approximated by a few resonances or trajectories either in the  $t$  channel or in the  $s$  channel. Since the Pomeranchukon contributes equally to all  $s$ -channel isospins it is very hard to relate it to an  $s$ -channel trajectory. On the other hand, it is reasonable that the optical or geometrical properties of the particles are independent of the isospin in the  $s$  channel.

(2) The interference model,<sup>13</sup> in which  $t$ -channel Regge trajectories are added to  $s$ -channel resonances, has been shown to be inadequate at least in a few cases.<sup>5</sup> If our description is correct, we should be allowed to have a modified version of this model in which  $s$ -channel resonances are added to the  $t$ -channel Pomeranchukon but not to any other  $t$ -channel trajectory. This should not involve any double counting. We have checked the cases in which the usual interference model is known to fail and found no con-

tradition to our modified version.

(3) The surprising success of the resonance approximation in the finite-energy sum rules for the odd  $\pi N \rightarrow \pi N$  amplitudes<sup>5</sup> as well as the success of the  $\pi\pi \rightarrow \pi\omega$ ,  $\pi\pi \rightarrow \pi A_2$  calculation<sup>7</sup> probably follows from the absence of the Pomeranchukon in these reactions. The complexity of the even  $\pi N$  amplitude<sup>5</sup> and the  $\pi\pi$  problem<sup>8</sup> can be reduced if we do use the resonance approximation but try to produce only the "ordinary" trajectories in the  $t$  channel assuming that the  $P$  trajectory is "already" taken care of by the unknown low-energy background.

(4) There is one open question which is very relevant to our discussion but does not affect any of our conclusions: Can we describe the scattering at high energies (say, at 10 BeV) in terms of many (wide, dense, and highly inelastic)  $s$ -channel resonances added to an "optical" Pomeranchukon? If this is the case we would not need the finite-energy sum rules in order to derive our results. The rapidly decreasing elasticities of the known high  $N^*$  resonances indicate, however, that a huge number of  $N^*$  trajectories is needed for such a picture to be valid.

(5) Finally we remark that all the attempts to approximate the world of strong interactions by (infinitely many) discrete states seem to be inconsistent with our picture of the Pomeranchukon, and appropriate modifications should be introduced into these programs if our model is correct.

The author wishes to thank his colleagues at the theory group at the Weizmann Institute for helpful discussions. After completing this work we have learned that many of the conclusions in this paper have been independently discovered by F. J. Gilman.

<sup>1</sup>A discussion of these and other puzzling features of the  $P$  trajectory is given by G. F. Chew, Comments Nucl. Particle Phys. 1, 121 (1967).

<sup>2</sup>The experimental total cross sections can be found, e.g., in W. Galbraith *et al.*, Phys. Rev. 138, B913 (1965).

<sup>3</sup>Throughout this paper "high energy" refers to the 2- to 30-BeV region, namely, above the usual domain of resonance dominance but below "asymptopia" where all total cross sections are presumably constant.

<sup>4</sup>The possible existence of  $S = +1$  baryons (or  $I = 2$  states, etc.) is irrelevant here, as long as their contribution to the cross sections (or to the FESR) is negligible. This seems to be the case experimentally. Similar remarks apply to the deuteron and to possible  $NN$  resonances.

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<sup>7</sup>M. Ademollo, H. R. Rubinstein, G. Veneziano, and M. A. Virasoro, Phys. Rev. Letters 19, 1402 (1967), and to be published.

<sup>8</sup>P. G. O. Freund, Phys. Rev. Letters 20, 235 (1958); C. Schmid, to be published.

<sup>9</sup>Freund (Ref. 8) has suggested that in the specific case of  $\pi\text{-}\pi$  scattering the Pomeranchukon is accounted for by the nonresonating background. We claim that this is generally the case.

<sup>10</sup>A detailed analysis of these and other cases will be published elsewhere.

<sup>11</sup>The FESR are not really used here as a tool for quantitative calculations. We refer to them only with respect to the idea of "building up"  $t$ -channel Regge trajectory in terms of  $s$ -channel resonances. This idea is essentially contained in the famous graph (see, e.g., Ref. 5) which shows how the low-energy cross-section curve oscillates around the extrapolated Regge curve until the two coincide at energies of a few BeV.

<sup>12</sup>See, e.g., V. Barger and M. Olsson, Phys. Rev. 146, 1080 (1966); V. Barger, M. Olsson, and K. V. L. Sarma, Phys. Rev. 147, 1115 (1966).

<sup>13</sup>V. Barger and M. Olsson, Phys. Rev. 151, 1123 (1966).