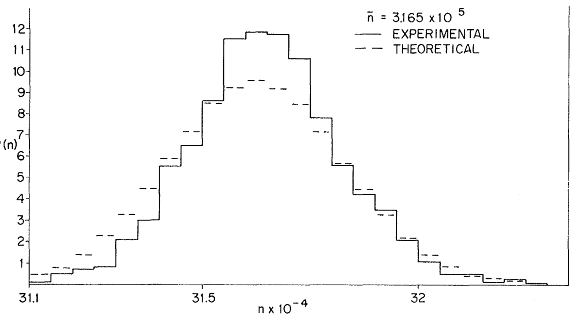

FIG. 3. Probability distribution $[P(n_T)]$ versus photoelectron count (n_T) for $T=10^{-1}$ sec.

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REGGE RECURRENCES AND π^-p ELASTIC SCATTERING AT 180°*

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(Received 14 April 1966)

In a recent experiment at the zero-gradient synchrotron, the differential cross section for π^-p elastic scattering was measured at a fixed angle of 180° over the laboratory momentum range 1.6-5.3 BeV/c.[1] The results of this experiment show considerable structure in the 180° differential cross section as a function of momentum.

In this Letter we present the essential results of a theoretical calculation which is in good quantitative agreement with the experimental data on 180° π^-p elastic scattering.[1,2] Our calculation is based on the following hypotheses:

(i) The known $Y=+1$ fermion resonances are recurrences of three Regge trajectories, namely $\Delta_\delta(I=\frac{3}{2}, P=+, \tau=-)$, $N_\alpha(I=\frac{1}{2}, P=+, \tau=+)$, $N_\gamma(I=\frac{1}{2}, P=-, \tau=-)$.[3]

(ii) The π^-p scattering amplitude at 180° is given by the sum of amplitudes arising from (a) the presence of the $\Delta_\delta, N_\alpha, N_\gamma$ Regge recurrences in the direct s-channel $[\pi^-+p\to\pi^-+p]$ and (b) the exchange of the Δ_δ Regge-pole trajectory in the crossed u channel $[\pi^++p\to\pi^++p]$. (The value of the Δ_δ trajectory intercept for 180° scattering is estimated by extrapolation in a Chew-Frautschi plot.[4])

These hypotheses lead to a consistent description of the detailed structure of 180° π^-p scattering based on the spin-parity assignments expected from the Regge-recurrence concept. The success of this approach supports in general the theoretical treatment of fermion exchange by Reggeization of the amplitude.[5]

Chew-Frautschi plot.[3,4]—In Table I the masses of the established resonance states of the $Y=+1$ fermion system are listed along with their quantum numbers that have been experimentally determined.[6] Figure 1 shows a possible theoretical assignment of these baryon states according to the Regge-recurrence concept, assuming approximate straight-line trajectories. On the basis of this Chew-Frautschi plot, the spin-parity assignments of these $Y=+1$ fermion states are predicted as listed in Table I. In this classification scheme there is at present no candidate for a third member of the N_α trajectory. As is shown below, a further recurrence on the N_α trajectory is not required by the present 180° π^-p scattering data.[1,2]

Resonance amplitude $(\Delta_\delta, N_\alpha, N_\gamma)$.—The amplitude resulting from a Regge recurrence in the s channel reduces to a Breit-Wigner resonance form for s in the vicinity of the physical resonance. We represent the total amplitude due to the successive recurrences as a sum of relativistic resonance amplitudes:

$$f^{\mathrm{Res}}(\sqrt{s}, \theta=\pi)=\frac{1}{k}\left[\frac{1}{3}\sum_{\Delta_\delta}\frac{x_{3/2}(-1)^l(J+\frac{1}{2})}{\epsilon-i}+\frac{2}{3}\sum_{N_\alpha,N_\gamma}\frac{x_{1/2}(-1)^l(J+\frac{1}{2})}{\epsilon-i}\right]. \quad (1)$$

The sums in Eq. (1) refer to the resonant states

Table I. Experimental parameters for $Y=+1$ fermion resonance states. The predicted values of spin and parity (J^P) are based on the Regge-recurrence model discussed in the text. Most of the quoted experimental elasticity parameters x_I are based on the proposed J assignments and are calculated from the height of the resonance bumps in total cross-section data. The final column indicates the elasticity parameters x_I used in the calculation of the 180° π^-p differential cross section.

Resonance[a] (mass in MeV)	Mass² (BeV²)	Experimental J^P	Predicted J^P	Experimental width Γ (BeV)	Experimental elasticity (x_I)	Value of x_I used
$\Delta_\delta(1236)$	1.53	$3/2^+$	$3/2^+$	0.125	1[b]	1
$\Delta_\delta(1924)$	3.70[b]	$7/2^+$ [c]	$7/2^+$	0.17	0.33-0.41[c,d]	0.35
$\Delta_\delta(2420)$	5.86[e,f]	?	$11/2^+$	0.31	~0.11[f]	0.15
$\Delta_\delta(2840)$	8.07[f]	?	$15/2^+$	0.40	≈0.03[f]	0.06
$\Delta_\delta(3220)$	10.37[f]	?	$19/2^+$	0.44	≈0.006[f]	0.01
$N_\alpha(938)$	0.88	$1/2^+$	$1/2^+$	...	...	...
$N_\alpha(1688)$	2.85[b]	$5/2^+$ [c]	$5/2^+$	0.10	0.80[c]	0.80
$N_\gamma(1518)$	2.30[b]	$3/2^-$ [b]	$3/2^-$	0.125	0.76[b]	0.76
$N_\gamma(2190)$	4.80[e]	?[g]	$7/2^-$	0.20	0.15-0.25[h]	0.15
$N_\gamma(2640)$	6.97[f]	?	$11/2^-$	0.36	≈0.07[f]	0.07
$N_\gamma(3020)$	9.12[f]	?	$15/2^-$	0.40	≈0.007[f]	0.01

[a]Using the quantum numbers Y for hypercharge, I for isospin, P for parity, and τ for signature, the symbol Δ denotes $Y=1$, $I=\frac{3}{2}$; N denotes $Y=1$, $I=\frac{1}{2}$; the subscript α denotes $P=+$, $\tau=+$; γ denotes $P=-$, $\tau=-$; and δ denotes $P=+$, $\tau=-$.

[b]A. H. Rosenfeld, Rev. Mod. Phys. 37, 633 (1965).

[c]P. J. Duke et al., Phys. Rev. Letters 15, 468 (1965).

[d]See Layson, Ref. 7.

[e]A. N. Diddens et al., Phys. Rev. Letters 10, 262 (1963).

[f]W. Galbraith, Proc. Roy. Soc. (London) 289, 521 (1966); A. Citron et al., to be published.

[g]See Carroll et al., Ref. 11.

[h]Elasticity estimated from Fig. 2 of Ref. c.

listed in Table I. x_I represents the elasticity parameter of the resonance; J, l are the total and orbital angular momenta, and $\epsilon=(M^2-s)/M\Gamma$,[7,8] where M and Γ are the mass and full width of the resonance, respectively. Since the resonances are, for the most part, highly inelastic, i.e., $x_I \ll 1$, we use constant widths as is done in the experimental analysis of resonances from total cross-section data. It should be noted that very inelastic resonances make only small contributions to Eq. (1).[9]

Exchange amplitude (Δ_δ).—The contribution to the π^-p scattering amplitude at 180° [where $u \equiv u_B = (M^2-\mu^2)^2/s$] from the u-channel exchange of the Δ_δ Regge trajectory is given by[10,11]

$$f^{\mathrm{Regge}}(\sqrt{s}, u_B)$$

$$=\left(\frac{s+M^2-\mu^2}{2s}\right)[g(\sqrt{u}_B, s)+g(-\sqrt{u}_B, s)]$$

$$+\left(\frac{M^2-\mu^2}{s}\right)\frac{M}{\sqrt{u}_B}[g(\sqrt{u}_B, s)-g(-\sqrt{u}_B, s)], \quad (2)$$

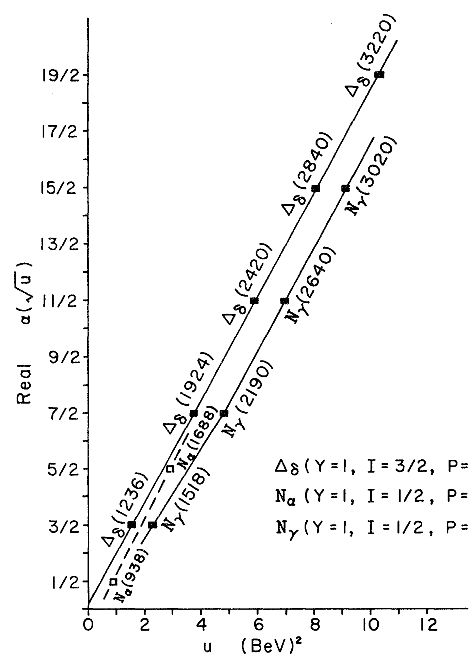

FIG. 1. Chew-Frautschi plot[3,4] of the $Y=+1$ fermion Regge recurrences.

where

$$g(\sqrt{u_B}, s) = \frac{\gamma(\sqrt{u_B})}{\sqrt{s_0}} \frac{1 - i\exp[-i\pi\alpha(\sqrt{u_B})]}{\cos\pi\alpha(\sqrt{u_B})} \times \left(\frac{s - M^2 - \mu^2}{s_0}\right)^{\alpha(\sqrt{u_B}) - \frac{1}{2}}. \quad (3)$$

The arbitrary scaling factor s_0 in Eq. (3) is chosen to be $(1\ \mathrm{BeV})^2$. $\alpha(\sqrt{u})$ is the trajectory and $\gamma(\sqrt{u})$ is the dimensionless residue of the Δ_δ Regge pole. At $\sqrt{u} = 1236$ MeV these Regge parameters are related to the spin and width of the (3, 3) resonance, thus $\alpha(1236) = \frac{3}{2}$ and $\gamma(1236) > 0$. We assume that $\gamma(\sqrt{u_B})$ has the same sign as $\gamma(1238)$. $\alpha(\sqrt{u})$ satisfies a dispersion relation as a function of $\sqrt{u}$ with cuts starting at thresholds $\pm(M + \mu)$. Although there exists no theoretical basis for a straight-line trajectory of the form $\mathrm{Re}\alpha(\sqrt{u}) = a + c(\sqrt{u})^2$, the empirical classification of Δ_δ in Fig. 1 gives such a behavior for $\sqrt{u} \geqslant 1236$ MeV.[12] We make the simplest possible assumption that this form for $\mathrm{Re}\alpha(\sqrt{u})$ is valid down to $\sqrt{u} = \sqrt{u_B}$ for the purpose of estimating $\alpha(\sqrt{u_B})$.[13] From a least-squares fit to the masses of the Δ_δ resonances, we find

$$\mathrm{Re}\alpha(\sqrt{u}) = 0.15 + 0.9(\sqrt{u})^2. \quad (4)$$

We use this form for $\alpha(\sqrt{u_B})$ in the calculation of the amplitude in Eq. (2); however, the variation with u_B is unimportant to the quality of our fit inasmuch as u_B varies only from $u_B = 0.19\ (\mathrm{BeV})^2$ at 1.6 BeV/c to $u_B = 0.05\ (\mathrm{BeV})^2$ at 8.0 BeV/c. We also take $\gamma(\sqrt{u_B})$ to be constant. Our approximations include neglecting the second term in Eq. (2).

Interference of amplitudes.—The idea that the interplay of a direct-channel resonance with a crossed-channel fermion-pole background would produce characteristic features in the angular distribution was suggested by Heinz and Ross.[14] The energy dependence of the 180° scattering can also yield information on the parity of the direct-channel resonance as pointed out by Kormanyos *et al.*[1] For example, these authors suggest that the $I = \frac{1}{2}$, 2190-MeV resonance should have $J^P = \frac{7}{2}^-$ in agreement with the assignment in Table I.

The basic premise of our calculation is that the resultant 180° π^-p scattering amplitude can be written as the sum of the amplitudes of Eqs. (1) and (2). Thus,

$$(d\sigma/d\Omega)_{\theta=\pi} = |f^{\mathrm{Res}} + f^{\mathrm{Regge}}|^2. \quad (5)$$

We are aware that unitarity is not necessarily insured by the sum of amplitudes in Eq. (5). However, since the π^-p angular distribution is sharply peaked near 180°,[15] f^{Regge} is expected to contribute predominantly to the high partial waves. Furthermore, the s-channel resonances contribute only a fraction of their unitarity limits due to their small elasticities. Hence, violations of unitarity are not expected to occur in Eq. (5).

We have calculated on a CDC-3600 the 180° π^-p differential cross section from Eqs. (1), (2), and (5), using an $\alpha(\sqrt{u_B})$ from Eq. (4) and the resonance parameters (Γ, M, x_I) from Table I. The residue γ is essentially the only free parameter in our calculation. No appreciable variation of the x_I from their experimental values was allowed. The theoretical 180° π^-p differential cross section for $\gamma = 0.15$ is shown in Fig. 2(a) along with the 180° experimental data. The model is apparently in good agreement with the experimental data considering the minimal number of free parameters. It should be emphasized that the theoretical curve in Fig. 2(a) represents the absolute differential cross section and does not involve an arbitrary normalization. In order to elucidate the interference between f^{Regge} and f^{Res}, the real and imaginary parts of the amplitudes as a function of laboratory momentum are presented in Fig. 2(b). The amplitude due to the direct-channel resonances cannot by itself accommodate the principal features of the experimental data. In view of the fact that the relative size (and also sign) of the real and imaginary parts of f^{Regge} is tied to its energy dependence through the value of $\alpha(\sqrt{u_B})$ [cf. Eq. (3)], it is rather remarkable that this amplitude allows just the proper interference to yield the results in Fig. 2(a). In addition f^{Regge} has the proper magnitude to explain the 8-BeV data point where resonance contributions are negligible (cf. Fig. 2).

The $I = \frac{1}{2}$, 2190-MeV state is the key resonance in our model. *A priori* this resonance could be assigned (cf. Fig. 1) as the first recurrence of $N_\gamma(\frac{7}{2}^-)$ or the second recurrence of $N_\alpha(\frac{9}{2}^+)$. With our model it is possible to reproduce the two-decade valley in $d\sigma/d\Omega$ near $p_{\mathrm{lab}} = 2.1$ BeV/c *only* with a negative parity assignment for the $I = \frac{1}{2}$, 2190 resonance and a positive parity assignment for the $I = \frac{3}{2}$, 2420 resonance. The fact that the $I = \frac{3}{2}$, 2420 resonance lies on a straight-line extrapolation of the Δ_δ trajectory lends additional support to a positive parity

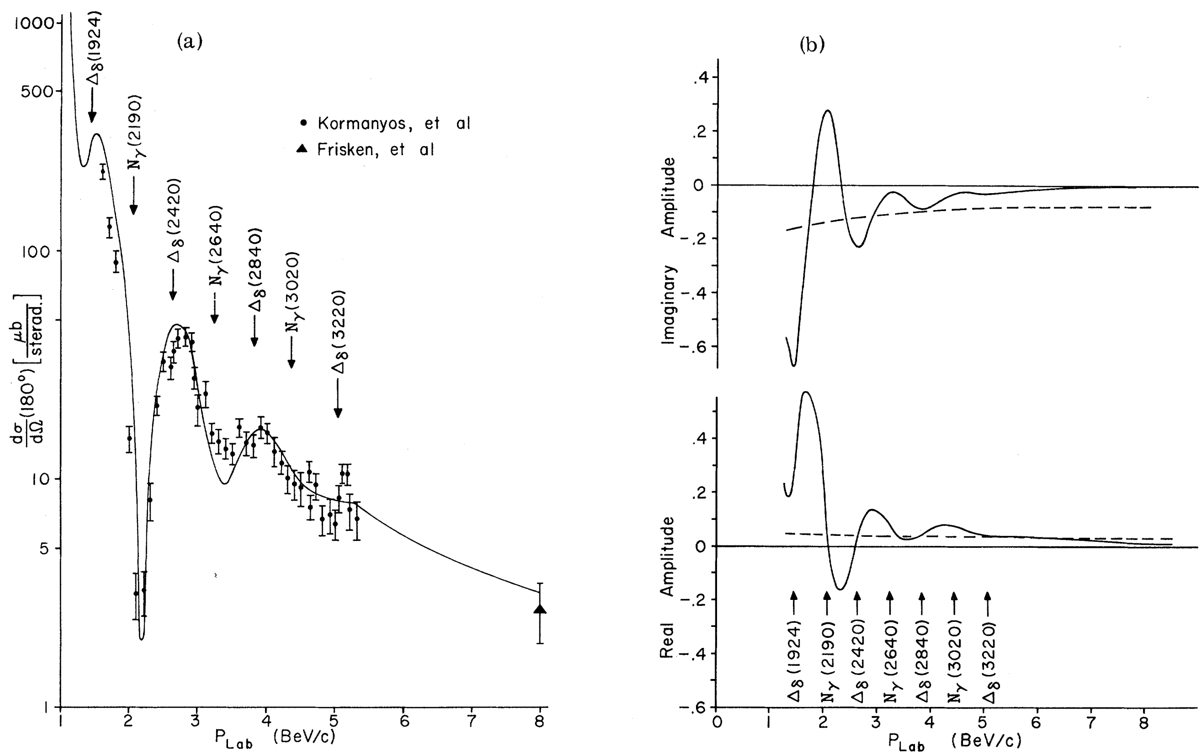

FIG. 2. (a) Theoretical curve for the 180° π^-p elastic scattering differential cross section as a function of laboratory momentum (the theoretical curve involves one free parameter, the residue of the Δ_δ Regge pole). The arrows indicate the positions of the Δ_δ, N_α, and N_γ Regge recurrences. Expermental data are taken from Refs. 1 and 2. (b) Real and imaginary parts of f^{Regge} and f^{Res}. The Δ_δ, N_α, N_γ resonance amplitude (f^{Res}) is represented by the solid curves and the Δ_δ exchange amplitude (f^{Regge}) is represented by the dashed curves. Relative units are used on the ordinate.

assignment. We have also investigated opposite parity assignments to that given in Table I for the higher mass resonances and find qualitative disagreement with the experimental trend of $d\sigma/d\Omega$ in Fig. 2(a). We conclude that there is strong evidence for the spin and parity assignments given in Table I according to the Regge-recurrence hypothesis.

In conclusion, we have constructed an internally consistent model for both the $Y=1$ fermion Regge recurrences and the experimental data on the 180° π^-p differential cross section. Although the present analysis has been restricted to 180° scattering in order to avoid spin-flip complications, this method can probably be extended to the study of angular distributions as well. The interference technique discussed here may eventually provide more accurate determinations of the resonance elasticity parameters x_I than those obtained from analysis of total cross-section data.

We thank Professor M. Ross and Professor G. Kane for a discussion and Dr. M. Olsson for his interest in this work.

Note added in proof.—We have explored the possibility that the N_α trajectory recurs at 2200 MeV with $J^P = \frac{9}{2}^+$ as would be suggested by extrapolation in the Chew-Frautschi plot of Fig. 1. We find that the same quantitative agreement with the data of Fig. 2(a) is possible for elasticity parameters of $x = 0.22$ for the $\frac{7}{2}^-$, 2190-MeV resonance and $x = 0.07$ for a $\frac{9}{2}^+$, 2200-MeV resonance. These elasticity parameters are consistent with the total cross-section data. Our results require that the negative-parity state in this region dominates. This conclusion agrees with a recent phase-shift analysis of Yokosawa et al.[17] Further study of parity doubling due to N_α, N_γ recurrences at similar mass values is in progress.

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*Work supported in part by the U. S. Atomic Energy Commission under Contract No. AT(11-1)-881. COO-881-62.

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[3]Notation from A. H. Rosenfeld, in Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962, edited by J. Prentki (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 325: I (isospin), P (parity), $\tau=(-)^{J-1/2}$ (signature). s and u are the squares of the c.m. energy and four-momentum transfer; k is the c.m. momentum.

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[6]The few other possible $Y=+1$ resonance states suggested by recent phase-shift analysis [for a critical evaluation, see K. Draxler and R. Hüper, Phys. Letters 20, 199 (1966)] lie to the right of the trajectories in Fig. 1. Arguments based on the expected trajectory intercepts at $u\approx 0$ and on the elasticity and J values of these resonances indicate that these trajectories will not contribute appreciably to our analysis.

[7]W. Layson, Nuovo Cimento 27, 724 (1963); J. D. Jackson, Nuovo Cimento 34, 1644 (1964).

[8]S. R. Deans and W. G. Holladay, to be published.

[9]There have been a few other resonance states suggested by recent phase-shift analyses. [For references, see J. Cence, Phys. Letters 20, 306 (1965).] Most of these resonances appear to be quite inelastic, have a small value of $J+\frac{1}{2}$, and occur in the vicinity of highly elastic resonances. Consequently, these very inelastic resonances are not expected to contribute significantly to the resonance contribution to elastic scattering. In our analysis we take into account only the resonances that show up strongly in the total cross-section data.

[10]The standard technique of performing a Sommerfeld-Watson transformation is used in obtaining Eq. (2) [cf. W. Singh, Phys. Rev. 129, 1889 (1963)]. In addition we tacitly assume that Regge behavior $[P_{\alpha+1/2}'(x_u)\sim(s-M^2-\mu^2)^{\alpha-1/2}]$ continues to hold at $\theta=\pi$ despite the kinematic limitation which occurs due to unequal masses, $\mu\neq M$ [cf. D. A. Atkinson and V. Barger, Nuovo Cimento 38, 634 (1965)]. This conjecture is also employed in the paper by Stack, Ref. 5.

[11]There are indications from charge-exchange reactions and total cross-section data that Regge-like behavior survives down to low energies ~2-3 BeV. [A. S. Carroll et al., Phys. Rev. Letters 16, 288 (1966); G. Höhler et al., Phys. Letters 20, 79 (1966); I. Butterworth et al., Phys. Rev. Letters 15, 734 (1965); V. Barger and M. Olsson, to be published.] Consequently, we shall use the Regge amplitude of Eq. (2) for the entire laboratory momentum range 1.6-8 BeV/c.

[12]A similar empirical observation is true for the ρ trajectory obtained from analysis of $\pi^-+p\rightarrow\pi^0+n$ charge-exchange data. (See Höhler et al., Ref. 11.) The real part of the ρ trajectory determined for $t<0$ extrapolates by a straight line to $\alpha\approx 1$ at $t=m_\rho^2$ (in this case the threshold occurs at $t=4\mu^2$).

[13]This form is only meant as a crude approximation to $\mathrm{Re}\,\alpha(\sqrt{u})$, adequate to fit present data. The actual power-series expansion of $\mathrm{Re}\,\alpha(\sqrt{u})$ around $\sqrt{u}=0$ has, in general, odd terms in $\sqrt{u}$ and does not converge beyond the branch points at $\sqrt{u}=\pm(M+\mu)$.

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[15]H. Brody et al., Phys. Rev. Letters 16, 828 (1966). Other experimental references to the π^-p angular distribution near 180° are listed in this paper and in Ref. 1.

[16]Carroll et al., Ref. 11, also present evidence that the $I=\frac{1}{2}$, 2190-MeV is a $\frac{7}{2}^-$ resonance.

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