We again find that the resonant contributions saturate Eq. (16) if we neglect the continuum contribution to the kaon propagator. We may combine Eqs. (4), (11), and (16) to obtain a sum rule involving σ_T:

$$8\pi^3\alpha^2 f_\pi^{\ 2}\left[1+\frac{1}{3}\left(\cos^2\theta_Y+\frac{m_\omega^{\ 2}}{m_\Phi^{\ 2}}\sin^2\theta_Y\right)\left(\frac{\cos\theta_N}{\cos\theta_Y\cos(\theta_Y-\theta_N)}\right)\right]+\text{const.}=\int_{2m_\pi}^{\infty}W^3\sigma_T(W)dW, \tag{17}$$

where W is the center-of-mass energy of the lepton pair, and $f_\pi = f_K$ according to the Cabibbo hypothesis. If we assume that the pion and kaon continuum contributions are small, which seems plausible because of the a^{-2} factor in the spectral integrals, then Eq. (17) provides a test of our assumptions about currents and fields that could be verified experimentally by colliding-beam experiments.

I wish to thank T. D. Lee for a helpful conversation.

---

*Grumman Research Department Staff, Summer 1967.

[1]R. Gatto, Theoretical Physics (International Atomic Energy Agency, Vienna, Austria, 1963), p. 569.

[2]B. Richter, Stanford Linear Accelerator Center Report No. SLAC-PUB-240, 1966 (unpublished).

[3]J. D. Bjorken, Phys. Rev. 148, 1467 (1966).

[4]V. N. Gribov, B. L. Ioffé, and I. Ya. Pomeranchuk, Phys. Letters 24B, 554 (1967).

[5]N. M. Kroll, T. D. Lee, and B. Zumino, Phys. Rev. 157, 1376 (1967).

[6]M. Gell-Mann, Phys. Rev. 125, 1067 (1962); Physics 1, 63 (1964).

[7]T. D. Lee, S. Weinberg, and B. Zumino, Phys. Rev. Letters 18, 1029 (1967).

[8]J. Dooher, to be published.

[9]K. Kawarabayashi and M. Suzuki, Phys. Rev. Letters 16, 255 (1966).

SU(3), MESON-BARYON SCATTERING, AND ASYMPTOTIC LIMITS*

S. Meshkov
National Bureau of Standards, Washington, D. C.

and

G. B. Yodh
University of Arizona, Tucson, Arizona and University of Maryland, College Park, Maryland

(Received 5 July 1967)

The meson-baryon total cross sections and elastic-scattering data in the forward direction are fitted using SU(3)-invariant t-channel amplitudes. The analysis implies that (1) $0 \leq \sigma_{\text{tot}}(s\to\infty) \leq 15.5$ mb, and (2) $-0.11 \leq \text{Re}T(s,t=0)/\text{Im}T(s,t=0) \leq 0$ as $s\to\infty$.

During the past few years, a number of different, yet in some ways similar, approaches[1-3] have been used to describe high-energy reaction and scattering data successfully. All of them invoke SU(3) invariance to some extent, with the consequence that it is not clear how much of their success is due to SU(3) invariance or to the detailed features of the models.

In the present work, we analyze high-energy meson-baryon scattering data in as model independent a manner as possible by assuming that these processes are described by SU(3)-invariant, octet and singlet, t-channel S-matrix elements. Using, as input, (i) the experimentally observed energy dependence of meson-baryon total cross sections[4-6] and (ii) the ratio α of real-to-imaginary part of forward scattering amplitudes, in the momentum range 6-22 GeV/c, we obtain the following results:

(1) A good fit to all meson-baryon total cross sections and to $\alpha(\pi^\pm p)$ and $\alpha(K^\pm p)$ is obtained, using only ten parameters.

(2) The unitary singlet amplitude, which is just the sum $\frac{1}{6}(\pi^+p+\pi^-p+K^+p+K^-p+K^+n+K^-n)$, is dominant and clearly decreasing with increasing s in the observed energy range.

(3) If we assume that the energy dependence of the t-channel amplitudes continue to be the same at higher energies and that SU(3) invariance continues to hold, the asymptotic limit for all meson-baryon total cross sections must be equal and less than 15.5 mb unless the ra-

Table I. t-chanel SU(3) invariant S-matrix elements. The $\underline{10}$ and $\underline{10}^*$ amplitudes are equal.

Process \ Amplitude	27	10	$\overline{10}$	$a=(\frac{1}{10})8_{ss}$	$b=(\frac{1}{2\sqrt{5}})8_{sa}$	$c=(\frac{1}{2\sqrt{5}})8_{as}$	$d=(\frac{1}{6})8_{aa}$	$e=(\frac{1}{8})1$
$(K^+p\vert K^+p)$	7/40	-1/12	-1/12	2	0	0	-2	1
$(K^-p\vert K^-p)$	7/40	1/12	1/12	2	0	0	2	1
$(\pi^+p\vert \pi^+p)$	-1/40	1/12	1/12	-1	1	-1	-1	1
$(\pi^-p\vert \pi^-p)$	-1/40	-1/12	-1/12	-1	1	1	1	1
$(K^+n\vert K^+n)$	-1/40	1/12	1/12	-1	-1	1	-1	1
$(K^-n\vert K^-n)$	-1/40	-1/12	-1/12	-1	-1	-1	1	1
$(\pi^-p\vert \pi^0n)$	0	$\frac{1}{6\sqrt{2}}$	$\frac{1}{6\sqrt{2}}$	0	0	$-\sqrt{2}$	$-\sqrt{2}$	1
$(\pi^-p\vert \eta n)$	$-\frac{3}{5\sqrt{6}}$	$-\frac{1}{2\sqrt{6}}$	$\frac{1}{2\sqrt{6}}$	$\sqrt{6}$	$\sqrt{6}/3$	0	0	
$(K^-p\vert \bar{K}^0n)$	-1/5	-1/6	-1/6	-3	-1	-1	-1	
$(K^+n\vert K^0p)$	1/5	-1/6	-1/6	3	1	-1	-1	

tio α were to become large. In particular, there exists a simple fit to the singlet amplitude which predicts that all $\sigma_T \to 0$ at a rate $P_L^{-0.074}$.

(4) We predict the forward differential cross sections for charge exchange processess.[7] They are in reasonable agreement with experiment.

The t-channel, SU(3)-invariant S-matrix elements are listed in Table I. There are seven amplitudes in the reaction system: meson+ +baryon→meson+baryon; $\underline{27}$, $\underline{10}$, $\underline{8}_{ss}$, $\underline{8}_{sa}$, $\underline{8}_{as}$, $\underline{8}_{aa}$, and $\underline{1}$.[8] (Due to time-reversal invariance, the amplitudes $\underline{10}$ and $\underline{10}^*$ are equal in the t-channel representation.) The octet subscripts refer to the symmetry or antisymmetry of the meson-meson and baryon-antibaryon states, respectively, reading from left to right.

The Barger-Rubin sum rule[9] (also called the weak Johnson-Treiman relation) holds over a very wide energy range, indicating even down to quite low energies that the 10 amplitude has no effect. Therefore we set the $\underline{10}$ amplitude $\equiv 0$. In addition we set the $\underline{27}$ amplitude $\equiv 0$. Keeping the $\underline{27}$ finite in our analysis would require knowledge of the ηp total cross section, or would require, as input, the forward cross sections of inelastic channels, which are proportional to the squares of matrix elements. As a result of neglecting the 27 amplitude, the unitary singlet amplitude e is the sum of all six meson-baryon elastic amplitudes. The amplitudes a, b, and e are even under charge conjugation ($C=+$), whereas c and d are odd ($C=-$).

The imaginary parts of the invariant amplitude T in the forward direction are determined from $\sigma_{\rm tot}$ (MB) by the optical theorem, ${\rm Im}T(s, t=0)=0.127 m_t p_L \sigma_{\rm tot}$, where m_t is the target mass (GeV) and p_L is the laboratory momentum (GeV/c) of the incident meson. We define $\Sigma(MB)={\rm Im}T(\bar{M}B)+{\rm Im}T(MB)$ and $\Delta(MB)={\rm Im}T(\bar{M}B)-{\rm Im}T(MB)$. Five linear combinations X_j are used to determine the imaginary parts of a_I through e_I. They are the following:

$$X_1=e_I=\tfrac{1}{6}[\Sigma(Kp)+\Sigma(Kn)+\Sigma(\pi p)],\quad X_2=4d_I=\Delta(Kp),$$

$$X_3=4b_I=[\Sigma(\pi p)-\Sigma(Kn)],\quad X_4=2c_I+2d_I=\Delta(\pi p),$$

and

$$X_5=2b_I-6a_I=[\Sigma(\pi p)-\Sigma(Kp)].$$

Figure 1(a) shows the momentum variation of these combinations on a plot of $\log X_j$ vs

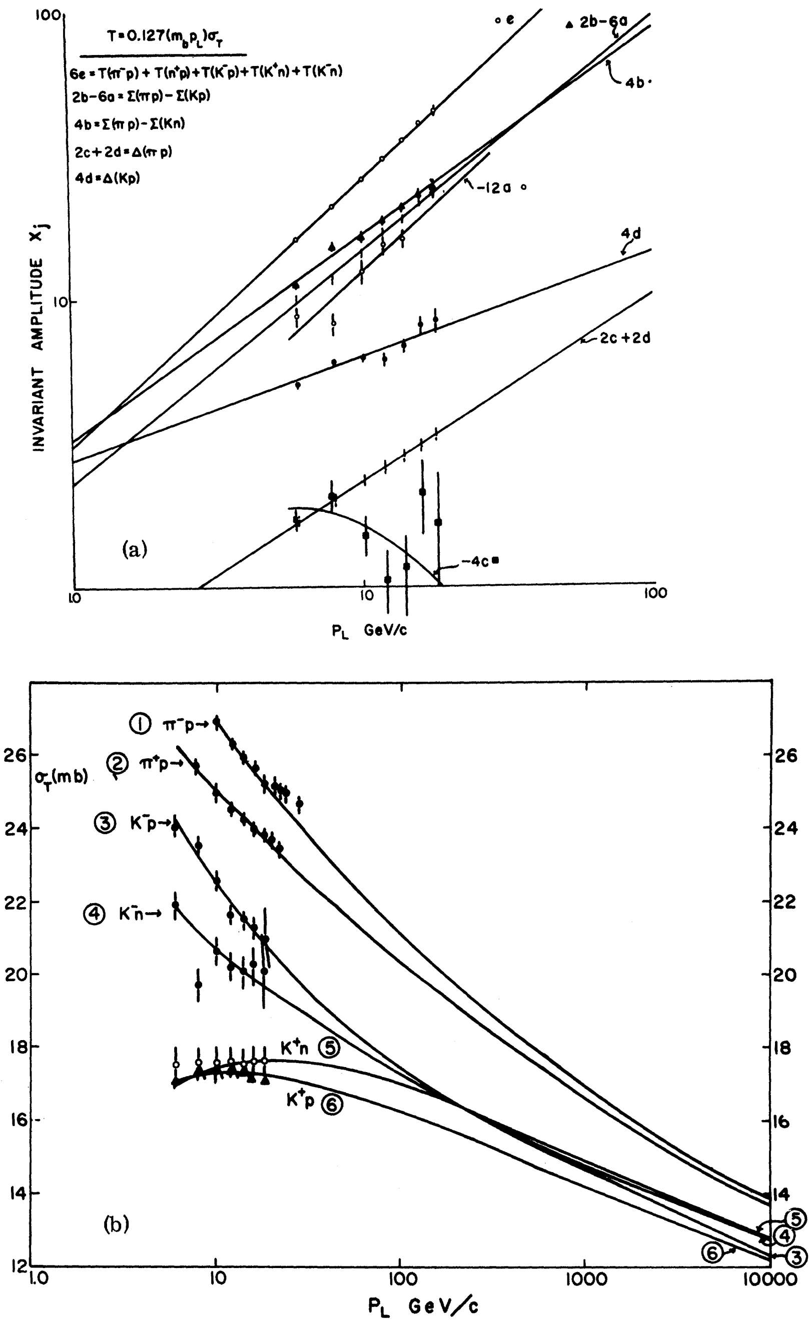

FIG. 1. (a) Invariant amplitudes (b) obtained from the total cross sections. (a) Display of the invariant amplitudes xj, which are obtained form the input total cross sections. The fits and cross-section predictions based on their use are shown in (b) (solid curves). The errors of the fit are ±0.5 mb.

$\log p_L$. Guided by these plots we parametrize the momentum dependence of X_j by $X_j = A_j p^{u_j}$. We obtain a good fit to the 44 data points with these ten parameters.

Because of the unavailability of KN data above 18 GeV/c, we could not use the recent precise $\pi^{\pm}p$ data[6] above 18 GeV/c in obtaining X_1, X_2, X_3. In determining X_4 we were able to use the new $\pi^{\pm}p$ data up to 22 GeV/c. In the present work p_{lab} plots are used rather than Q plots[10] because at these energies, where t channel dominates, it is hoped that threshold effects will be unimportant. The best fits to X_j, obtained by minimizing χ^2, are found to be the following: $X_1 = (3.07 \pm 0.11)p^{0.926 \pm 0.015}$, $X_2 = 2.76 \pm 0.16)p^{0.353 \pm 0.025}$, $X_3 = (3.29 \pm 0.39)p^{0.701 \pm 0.051}$, $X_4 = (0.533 \pm 0.061)p^{0.634 \pm 0.048}$ and $X_5 = (2.29 \pm 0.16)p^{0.802 \pm 0.031}$. The units are $(\mathrm{mb})^{1/2}$ GeV.

The curves of Fig. 1(b) display the fit to the individual meson-baryon total cross sections between 6 and 18 GeV/c. Within the errors of the fit, which are ±0.5 mb (primarily due to K^-n and K^-p), there is reasonable agreement with $\sigma(\pi^\pm p)$ above 18 GeV/c. These points were not used in obtaining X_1, X_3, and X_5. The near equality and approximate constancy of $\sigma(K^+p)$ and $\sigma(K^+n)$ from 6 to 18 GeV/c is due to a delicate balancing of all five SU(3) amplitudes (see Table I).

In order to deduce asymptotic limits on total cross sections we examine the relative magnitudes and energy variation of the SU(3) amplitudes determined above. The most striking feature of the amplitude analysis is the dominance of the unitary singlet amplitude e_I. Furthermore, the cross section corresponding to the e_I amplitude (computed via the optical theorem) is decreasing with energy less rapidly than the analogous cross sections for the octet amplitudes. Assuming that SU(3) invariance holds and that the energy dependence of the SU(3) amplitudes continues unchanged at higher energies, asymptotic limits of total cross sections will be determined primarily by the behavior of the unitary singlet term. The fit obtained above corresponds to $\sigma_e = 25.6p^{-0.074 \pm 0.015}$, resulting in vanishing cross sections at infinite energy.

A natural question to ask is why we should restrict our data fitting procedure to a two-parameter fit of the form Ap_L^u. In fact, it is possible to fit e_I by a three parameter form

$$e_I = C_1 p_L + C_2 p_L^{u_e} \quad u_e \leq 1 \tag{1}$$

which corresponds to

$$\sigma_e = C_1' + C_2' p_L^{u_e - 1} \tag{2}$$

which results in a constant cross section C_1' as $p_L \to \infty$.

In order to make a quantitative estimate of the range of allowed values of C_1', experimental information on α, the ratio of real-to-imaginary parts of elastic-forward-scattering amplitudes, must be used. The evaluation of α is simplified by the $p_L^{u_j}$ behavior of the imaginary parts X_j. It has been shown[11] that the real part of an amplitude which varies as p_L^u is related to its imaginary part by (in the asymptotic region)[12]

$$\alpha_j = -\cot \tfrac{1}{2}\pi u_j \tag{3}$$

for $C = +$ amplitudes and

$$\alpha_j = \tan \tfrac{1}{2}\pi u_j \tag{4}$$

for $C = -$amplitudes. The singlet amplitude e has $C = +1$; consequently, using Eqs. (3) and (1) we find that

$$\alpha_e = -\cot \tfrac{1}{2}\pi u_e \left\{ \frac{1}{1 + (C_1/C_2) p_L^{i - u_e}} \right\}. \tag{5a}$$

The allowed range of values for u_e can now be drastically limited by relating α_e to experiment as follows:

$$\alpha_e = \frac{\mathrm{Re}(e)}{\mathrm{Im}(e)} = \frac{\sum_i \alpha_i \sigma_i}{\sum_i \sigma_i} = \sum_i \alpha_i \left(\frac{\sigma_i}{\sigma_t}\right), \tag{5b}$$

where σ_i are the six meson-baryon total cross sections, σ_t is their sum, and $\alpha_i = \mathrm{Re}[T_i(s,t=0)]/\mathrm{Im}[T_i(s,t=0)]$. The evaluation of α_e is performed at the low-momentum point, $p_L = 6$ GeV/c. At this momentum, $|\alpha(\pi^-p)| = 0.15$ and $|\alpha(\pi^+p)| = 0.22$[6]; $|\alpha(K^-p)|$ and $|\alpha(K^-n)|$ are consistent with zero above 4 GeV/c[13]; $|\alpha(K^+p)|$[4] is taken as 0.3 and $|\alpha(K^+n)|$ as 0.1. (The K^+n value is estimated from the calculation described in the succeeding paragraphs.) The values of σ_i/σ_t at 6 GeV/c are taken from experiment and are 0.21, 0.19, 0.13, and 0.13 for π^-p, π^+p, K^+p, and K^+n, respectively. Consequently, $\alpha_e = 0.126$.

A best fit to the e_I amplitude of Eq. (1), together with Eq. (5a), shows that the value $\alpha_e = 0.126$ requires that $u_e \geq 0.75$. The cross section σ_e, which corresponds to $u_e = 0.75$, is

$$\sigma_e = 15.5 + 11 p_L^{-0.25} \text{ mb}. \tag{6}$$

In the range, $0.926 < u_e < 1$, all solutions are excluded because $C_1 < 0$. For $u_e = 0.926$, $C_1 = 0$ and we have the two-parameter fit. The solution for $u_e = 1$ corresponding to constant σ_e is not acceptable because the confidence level for fitting the present data is less than 3×10^{-4}. Our conclusion is that all solutions with $0.75 \leq u_e \leq 0.926$ are acceptable, implying that $\sigma_e(s \to \infty) \leq 15.5$ mb.

It is also interesting to estimate the varia-

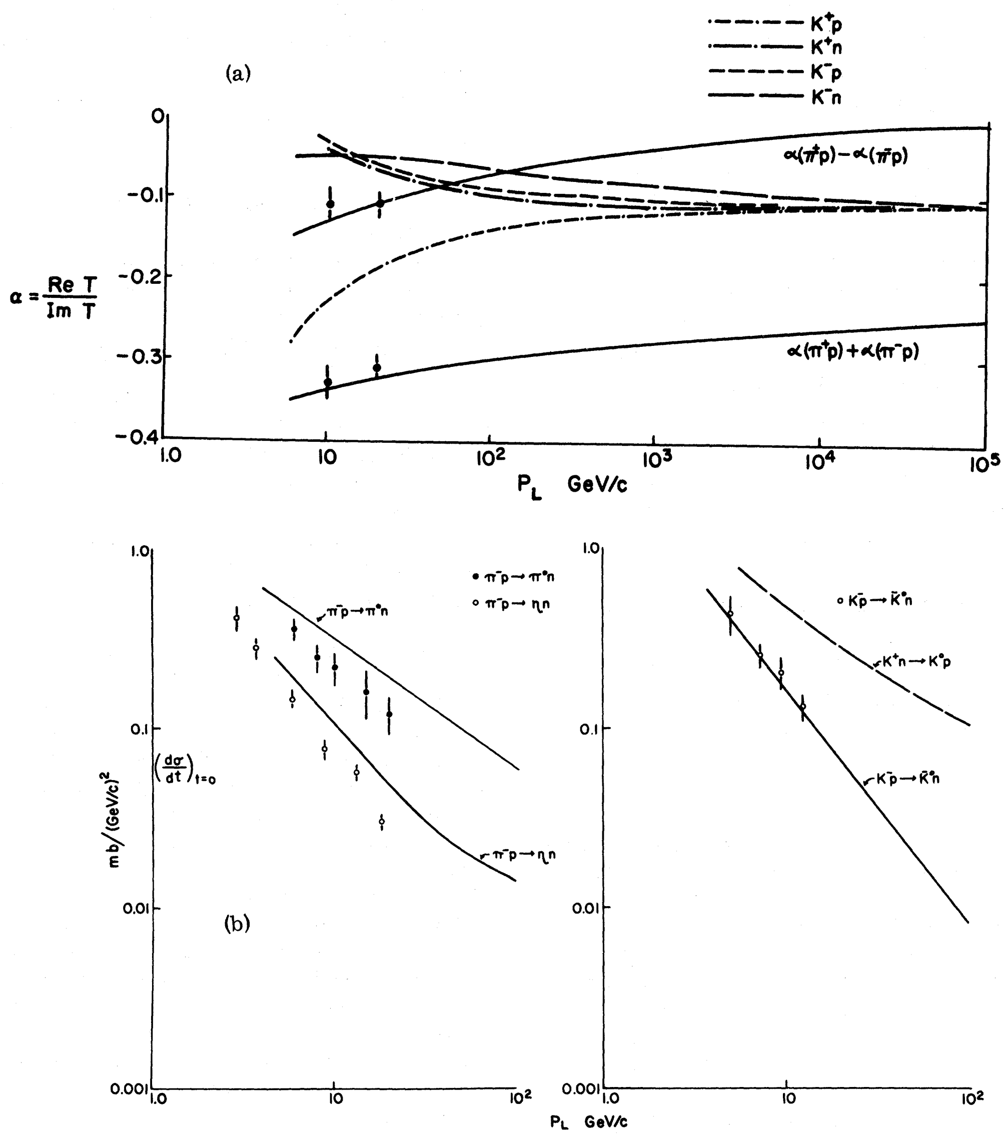

FIG. 2. Experimental data and predictions for (a) $\alpha = \mathrm{Re}T/\mathrm{Im}T$, and (b) forward charge-exchange proceses. The predicted values (curves) are based on the use of the amplitudes shown in Fig. 1(a). The uncertainty in the predicted values of (b) are 25% for $\pi^-p\to\pi^0n$, 40% for $\pi^-p\to\eta n$, and 20% for $K^-p\to\bar{K}^0n$.

tion of the individual cross sections at higher energy on the basis of two-parameter fits to X_j. The predictions are plotted in Fig. 1(b). The salient features are the following: (1) All σ_T decrease eventually because of the dominance of the e amplitude; (2) π^-p approaches π^+p, K^+p approaches K^-p, and K^+n approaches K^-n; (3) the π^-p cross section is bigger than π^+p at all energies and K^+p is the smallest at all energies. The precise details of the approach and crossings are affected by the relatively poorly determined K^-p and K^-n input.

By applying Eqs. (3) and (4) to the octet amplitudes (a_I, b_I) and (c_I, d_I), respectively, we can calculate the real part of a, b, c, d at $t=0$. In Fig. 2(a) we compare our predictions for $\alpha(\pi^+p)+\alpha(\pi^-p)$ and $\alpha(\pi^-p)-\alpha(\pi^+p)$ with recent measurements.[6] The agreement is good. Also given in Fig. 2 are predicted values of $\alpha(K^\pm p)$ and $\alpha(K^\pm n)$. The value obtained for $\alpha(K^-p)$ at 6 GeV/c is small and that for $\alpha(K^+p)$ is large, in agreement with present data.

Using the amplitudes a through e, we calculate $(d\sigma/dt)_{t=0}$ for the following charge-exchange processes: $K^-p\to\bar{K}^0n$, $K^+n\to K^0p$, $\pi^-p\to\pi^0n$, and $\pi^-p\to\eta n$. Figure 2(b) shows the comparison of our predictions with experiment. The uncertainty in the predicted values is 25% for $\pi^-p\to\pi^0n$, 40% for $\pi^-p\to\eta n$, and 20% $K^-p\to\bar{K}^0n$. The agreement with experiment is good for $K^-p\to\bar{K}^0n$ and reasonable for $\pi^-p\to\pi^0n$ and $\pi^-p\to\eta n$.

We conclude that it is possible to describe the existing high-energy meson-baryon total cross sections, and forward elastic scattering and charge exchange data, in terms of t-channel SU(3) invariant amplitudes. Using the ratio α, as a powerful tool for restricting the energy dependence of the SU(3) singlet amplitude, we make quantitative predictions about asymptotic limits. Not only should all asymptotic meson-baryon total cross sections become equal, but their limit must be less than 15.5 mb.

We are indebted to S. J. Lindenbaum for many informative discussions. We wish to thank V. Barger, T. Bowen, J. Coyne, E. Jenkins, H. Harari, G. Kane, M. Kugler, W. A. Love, H. J. Lipkin, and M. Olsson for their comments, suggestions and criticism.

---

*Work supported in part by U. S. Atomic Energy Commission under Contract No. AEC-ORO-2504-92, National Science Foundation Contract No. GP-6827, and Office of Naval Research Contract No. NAonr-6-67.

[1]R. J. N. Phillips and W. Rarita, Phys. Rev. 139, B1336 (1965); Phys. Rev. Letters 15, 807 (1965); Phys. Letters 19, 598 (1965). V. Barger and M. Olsson, Phys. Rev. Letters 15, 930 (1965); Phys. Rev. 146, 1080 (1966). A. Ahmadzadeh and C. H. Chan, Phys. Letters 22, 692 (1966).

[2]E. M. Levin and L. L. Frankfurt, Zh. Eksperim. i Teor. Fiz.—Pis'ma Redakt. 2, 105 (1965) [translation: JETP Letters 2, 65 (1965)]; H. J. Lipkin and F. Scheck, Phys. Rev. Letters 16, 71 (1966); H. J. Lipkin, Phys. Rev. Letters 16, 1015 (1966); J. J. J. Kokkedee and L. Van Hove, Nuovo Cimento 42A, 711 (1966).

[3]N. Cabibbo, L. Horwitz, and Y. Ne'eman, Phys. Letters 22, 336 (1966); N. Cabibbo, J. J. J. Kokkedee, L. Horwitz, and Y. Ne'eman, Nuovo Cimento 45A, 275 (1966).

[4]K. J. Foley et al., Phys. Rev. Letters 14, 862 (1965); 15, 45 (1965).

[5]W. Galbraith et al. Phys. Rev. 138, B913 (1965).

[6]S. J. Lindenbaum, in Proceedings of the Fourth Coral Gables Conference on Symmetry Principles at High Energy, University of Miami, January 1967, edited by A. Perlmutter and B. Kurşunoglu (W. H. Freeman & Company, San Francisco, California, 1967); also K. J. Foley et al., Phys. Rev. Letters 19, 397 (1967).

[7]G. Hohler, J. Baacke, and R. Strauss, Phys. Letters 21, 233 (1966); O. Guisan et al., Phys. Letters 18, 200 (1965).

[8]For convenience, the octet and singlet amplitudes are listed in Table I as multiples of $a=(1/10)\underline{8}_{ss}$, $b=(1/2\sqrt{5})\underline{8}_{sa}$, $c=(1/2\sqrt{5})\underline{8}_{as}$, $d=(1/6)\underline{8}_{aa}$, and $e=(1/8)\underline{1}$.

[9]V. Barger and M. Rubin, Phys. Rev. 140, B1365 (1965); K. Johnson and S. B. Treiman, Phys. Rev. Letters 14, 189 (1965). The Barger-Rubin sum rule says that $K^+p-K^-p=\pi^+p-\pi^-p+K^+n-K^-n$, independent of the value of the $\underline{27}$ amplitude. (See Table I.) We denote a total cross section for a given process by the symbols for the particles involved.

[10]S. Meshkov, G. A. Snow, and G. B. Yodh, Phys. Rev. Letters 12, 87 (1964).

[11]L. van Hove, Rev. Mod. Phys. 36, 655 (1964).

[12]The validity of the use of these equations down to 6 GeV/c may be questioned. Nevertheless, one may hope that the asymptotic forms may give the dominant behavior because the contribution of the pole terms in meson-baryon forward dispersion relations is no more than 30% even at 1 GeV/c. We thank Professor G. Kane and Professor M. Olsson for valuable discussions on this subject.

[13]M. Aderholz et al., Phys. Letters 24B, 434 (1967); J. Mott et al., Phys. Letters 23, 171 (1966).