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Solutions to Approximate Integral Equations for Regge Pole Parameters*

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A method described in a previous paper is tested in the light of potential theory. The method is based on dispersion relations for Regge pole parameters. The approximations consist in coupling the β's to the α's by applying unitarity at $l=\alpha$ and considering only a few poles. When the generalized potential is replaced by a nonrelativistic potential, the coupling equations and solutions to the integral equations can be compared to exact results. Various representations are tested, and it is found that the "modified Khuri" representation for $A(l,s)$ gives good results for $\alpha_1(s)$ in the one-trajectory approximation for Yukawa potentials strong enough to cause bound S states. The results for $\beta_1(s)$ are less satisfactory. The effect of coupling in the second trajectory is considered.

I. INTRODUCTION

In a previous paper[1] a method was suggested for approximately bootstrapping Regge trajectories. The method is based on dispersion relations for Regge trajectories and on unitarity applied at $l=\alpha$. In a self-consistent calculation the potential is to be described in terms of scattering in crossed channels. Numerical calculations in terms of generalized potentials[2] are being formulated. The present paper presents results based on the potential theory relations of I. Comparison with the exact results of Ahmadzadeh[3] *et al.*[4] gives a feeling for the accuracy of uncoupled trajectory approximations and for the rate of convergence of the method to the full potential-theory answer in terms of the number of trajectories coupled.

The results depend greatly on the partial-wave representation used in the coupling equation. The representations are fully described in I and are called here "universal," "Khuri," and "modified Khuri," respectively.

II. EQUATIONS

The results of I are summarized in this section for clarity. The integral equations are based on the analytic properties of "normal" trajectories. By considering the functions $\alpha_n(s)$ and $b_n(s)=\beta_n(s)s^{-\alpha_n(s)}$, we obtain the following exact equations

$$\beta_n(s)=(g^2/2s)\prod_{i=1}^{n-1}\left(\frac{s-s_i}{s}\right)\exp\left[(1/\pi)\int_0^\infty\frac{ds'}{s'-s-i\epsilon}\times\left[\mathrm{Im}\alpha_n(s')\ln(s/s')+\tan^{-1}\left(\frac{\mathrm{Im}\beta_n(s')}{\mathrm{Re}\beta_n(s')}\right)\right]\right];$$

$$\alpha_n(s)=-n+\frac{1}{\pi}\int_0^\infty\frac{ds'}{s'-s-i\epsilon}\,\mathrm{Im}\alpha_n(s')\quad(1)$$

for the nth trajectory. The s_i are the $n-1$ indeterminacy points [zeros of $\beta_n(s)$] of the nth trajectory.

The $\beta_n(s)$ are coupled to the $\alpha_m(s)$ by unitarity,

$$[A(s,l)-A(s,l^*)^*]/2i=s^{1/2}A(s,l)A(s,l^*)^*,\quad(2)$$

which is applied at $l=\alpha(s)$ to give

$$1/2i=s^{1/2}A(s,\alpha^*(s))^*.\quad(3)$$

The potential is given by[5]

$$V(r)=-g^2e^{-mr}/r.\quad(4)$$

The representations used are in turn:

Universal:

$$A(s,l)=\sum_{n<L}\frac{\beta_n(s)}{l-\alpha_n(s)}+B_L(s,l),\quad(5a)$$

where $B_L(s,l)$ is the contribution of the background integral.

Khuri:

$$A(s,l)=\sum_n\frac{\beta_n(s)}{l-\alpha_n(s)}\exp[(\alpha_n(s)-l)\xi(s)],\quad(5b)$$

where

$$\xi=\cosh^{-1}(1+m^2/2s).$$

Modified Khuri[3]:

$$A(s,l)=\sum_{n<L}\frac{\beta_n(s)}{l-\alpha_n(s)}\exp[(\alpha_n(s)-l)\xi(s)]+(g^2/2s)Q_l(1+m^2/2s)-(g^2/2s)\times\sum_{m=1}^{\infty}\frac{P_{m-1}(1+m^2/2s)}{l+n}\exp[-(l+n)\xi(s)],\quad(5c)$$

where

$$\xi=\cosh^{-1}(1+4m^2/2s).$$

When (3) is applied to these representations, we obtain the following coupling equations:

$$\left(\frac{1}{2i}\right)=s^{1/2}\sum_{n<L}\frac{\beta_n(s)}{\alpha_n(s)-\alpha_m^*(s)}-s^{1/2}B_L(s,\alpha_m^*(s)),\quad m<L,\quad(6a)$$

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[1] S. C. Frautschi, P. Kaus, and F. Zachariasen, Phys. Rev. **133**, B1607 (1964), hereinafter referred to as I.

[2] G. F. Chew and S. C. Frautschi, Phys. Rev. **124**, 264 (1961); G. F. Chew, *ibid.* **129**, 2363 (1963).

[3] A. Ahmadzadeh, Lawrence Radiation Laboratory Reports UCRL-10929 and UCRL-11096 (unpublished).

[4] A. Ahmadzadeh, P. Burke, and C. Tate, Phys. Rev. **131**, 1315 (1963).

[5] The particle mass is chosen equal to $\frac{1}{2}$. The range of the potential $(1/m)$ is chosen equal to 1 in all numerical examples.

$$\left(\frac{1}{2i}\right)=s^{1/2}\sum_{n=1}^{\infty}\frac{\beta_n(s)}{\alpha_n(s)-\alpha_m^*(s)}\exp[(\alpha_n(s)-\alpha_m^*(s))\xi(s)]\,,\tag{6b}$$

$$\left(\frac{1}{2i}\right)=s^{1/2}\sum_{n=1}^{\infty}\frac{\beta_n(s)}{\alpha_n(s)-\alpha_m^*(s)}\exp[(\alpha_n(s)-\alpha_m^*(s))\bar{\xi}(s)]$$
$$-\left(\frac{g^2}{s}\right)s^{-1/2}\Bigg(Q_{\alpha_m^*(s)}(1+m^2/2s)$$
$$-\sum_{n=1}^{\infty}\frac{P_{n-1}(1+m^2/2s)}{\alpha_m^*(s)+n}\exp[-(\alpha_m^*(s)+n)\bar{\xi}(s)]\Bigg).\tag{6c}$$

Equations (6) are an infinite set of relations connecting the α's and β's, leading to an infinite set of coupled integral equations.[1] The set is made finite by discarding all but the first N poles and residues. This leads to N complex equations for the N residues β in terms of the N trajectories α. Equations (6) reduce to (7) in the case of $N=1$.

One-trajectory approximation: $s>0$

$$s^{1/2}\beta(s)=\mathrm{Im}\alpha(s)\quad\text{(Background neglected)}\,,\tag{7a}$$

$$s^{1/2}\beta(s)=\mathrm{Im}\alpha(s)\exp[-2i\,\mathrm{Im}\alpha(s)\xi(s)]\,,\tag{7b}$$

$$s^{1/2}\beta(s)=\mathrm{Im}\alpha(s)R(s)\exp\{i[\theta(s)-2\,\mathrm{Im}\alpha(s)\bar{\xi}(s)]\}\,,\tag{7c}$$

where

$$R(s)\exp i\theta(s)=1+ig^2s^{-1/2}(Q_{\alpha^*(s)}(1+m^2/2s)$$
$$-(\alpha^*(s)+1)^{-1}\exp\{-[\alpha^*(s)+1]\bar{\xi}(s)\})\,.$$

Substitution of (7) into (1) then yields the integral equations:

Universal:

$$\mathrm{Im}\alpha(s)=\frac{g^2}{2s^{1/2}}\exp\left((P/\pi)\int_0^{\infty}\frac{ds'}{s'-s}\mathrm{Im}\alpha(s')\ln(s/s')\right).\tag{8a}$$

Khuri:

$$\mathrm{Im}\alpha(s)=\frac{g^2}{2s^{1/2}}\exp\Bigg((P/\pi)\int_0^{\infty}\frac{ds'}{s'-s}\mathrm{Im}\alpha(s')$$
$$\times[\ln(s/s')-2\xi(s')]\Bigg).\tag{8b}$$

Modified Khuri:

$$\mathrm{Im}\alpha(s)=\frac{g^2}{2s^{1/2}R(s)}\exp\Bigg((P/\pi)\int_0^{\infty}\frac{ds'}{s'-s}\{\mathrm{Im}\alpha(s')$$
$$\times[\ln(s/s')-2\bar{\xi}(s')]+\theta(s')\}\Bigg).\tag{8c}$$

III. NUMERICAL RESULTS AND CONCLUSIONS

The Coupling Equations

For a simple attractive Yukawa potential, the top trajectory is always normal. Equations (1) are therefore exact, and the only approximation involved in the integral equations (8) comes from the one-trajectory approximation in the coupling equations (7). When this is not good enough, more trajectories must be coupled. To test the coupling relations, (7) was evaluated for the Yukawa potential (4) with $g^2=0.05$ and $g^2=1.8$, $m=1$. For the input $\mathrm{Im}\alpha(s)$, the exact results of Ahmadzadeh[3] *et al.*[4] were used. The resulting $\beta_1(s)$ for the top trajectory are shown in Figs. 1 through 3, compared with the exact $\beta_1(s)$ of Ahmadzadeh[3] obtained by solving the Schrödinger equation. Since (7a) makes $s^{1/2}\beta_1(s)$ real, it does not appear in these figures. It is seen that even for weak coupling (7b) is not a good approximation. But the modified Khuri (7c) is much better for the weak coupling and good at large energies for the stronger coupling $g^2=1.8$.

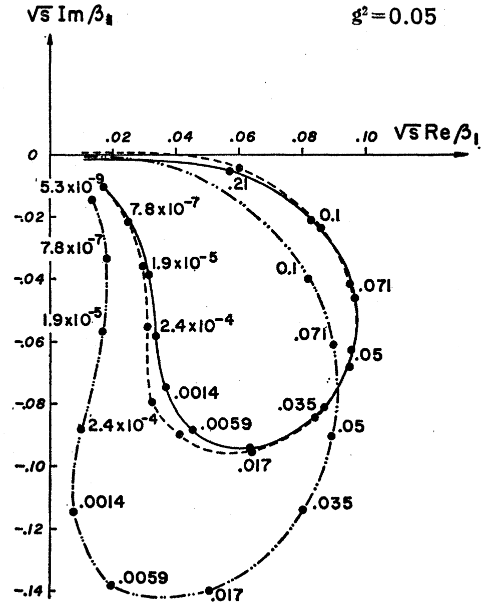

FIG. 1. $s^{1/2}\,\mathrm{Im}\beta_1$ versus $s^{1/2}\,\mathrm{Re}\beta_1$; $m=1$, $g^2=0.05$. One-trajectory approximation ($N=1$). Exact (Ref. 3) $\mathrm{Im}\alpha_1(s)$ is used as input. The three curves are: Exact (Ref. 3) $\beta_1(s)$ (———), Khuri (7b) (— · · —), and modified Khuri (7c) (— — —). The energy s is shown as a parameter.

Figure 2 also shows the results for $\beta_1(s)$ when the second trajectory is coupled into Eqs. (6c). Unitarity is applied at $l=\alpha_1(s)$ through (3) and the top $\beta_1(s)$ is calculated using as input $\mathrm{Im}\alpha_1(s)$ and $\mathrm{Im}\alpha_2(s)$ from the exact results. The modified Khuri representation now is converging well at all but the intermediate energies. At $g^2=1.8$ the top trajectory causes a shallow bound S-state and the second trajectory has become normal.[6]

The Integral Equations

The solutions to the integral equations (8) for coupling constants g^2 ranging from 0.05 to 3 are shown

[6] However, $\alpha_2(0)<-\frac{1}{2}$. This causes some trouble as $s\to 0$ because of the infinitely many trajectories streaming into the accumulation point $\alpha(0)=-\frac{1}{2}$. In this connection see the Appendix.

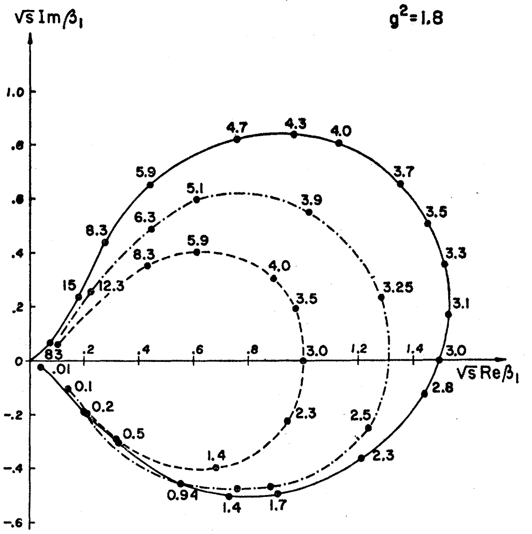

FIG. 2. $s^{1/2}\,\mathrm{Im}\beta_1$ versus $s^{1/2}\,\mathrm{Re}\beta_1$; $m=1$, $g^2=1.8$. One- and two-trajectory approximations. Exact (Ref. 3) $\beta_1(s)$ (———); modified Khuri (7c) (— — —) with exact $\mathrm{Im}\alpha_1(s)$ input; modified Khuri (6c), $N=2$) (— · — ·) with exact $\mathrm{Im}\alpha_1(s)$ and $\mathrm{Im}\alpha_2(s)$ input.

in Figs. 4 through 9. These are all one-trajectory approximations and in view of the failure of the coupling equations (6) for $N=1$ for strong couplings, it was considered pointless to go beyond $g^2=3$ with any of the one-trajectory equations (8). Coupling the trajectories is very laborious and will be reserved for the fully relativistic problem. However, since Eq. (1) is exact,

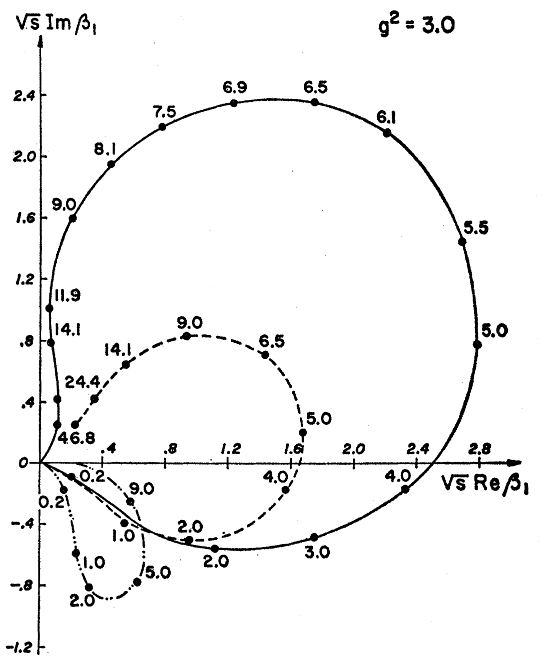

FIG. 3. Same as Fig. 1, with $g^2=3.0$.

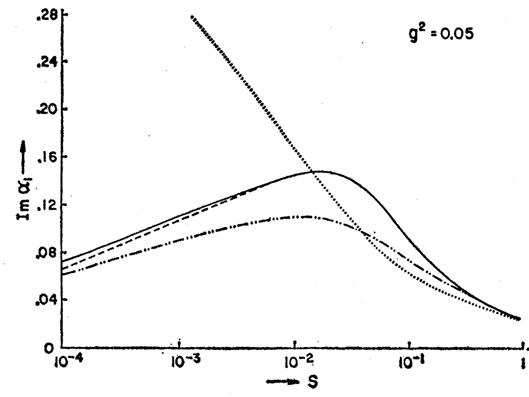

FIG. 4. $\mathrm{Im}\alpha_1$ versus s; $m=1$, $g^2=0.05$. Exact (Ref. 3) $\mathrm{Im}\alpha_1(s)$ (———). Solutions to integral equations (8): Universal (8a) (· · ·), Khuri (8b) (— · · —), and modified Khuri (8c) (— — —).

the trajectories will converge to the exact value if the coupling equations (6) converge. Some indication of this convergence was given by Fig. 2. It is interesting to note that the $\alpha_1(s)$ from the modified Khuri (7c) is

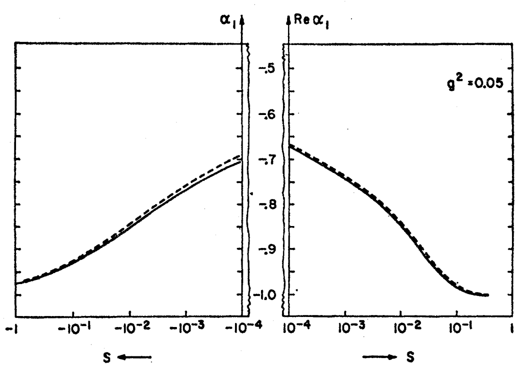

FIG. 5. $\mathrm{Re}\alpha_1$ versus s; $m=1$, $g^2=0.05$. Exact (Ref. 3) $\mathrm{Re}\alpha_1(s)$ (———). The other curve (— — —) is obtained by applying the dispersion relation (1) for $\alpha_1(s)$ to the results in Fig. 4. Modified Khuri (8c) only.

considerably better than the test of the coupling equation (6c) at $g^2=1.8$, Fig. 2, would let one expect.

When $\alpha_1(s)$ obtained from (8c) is used as input for the $N=1$ coupling equation (7c) in order to get the residue

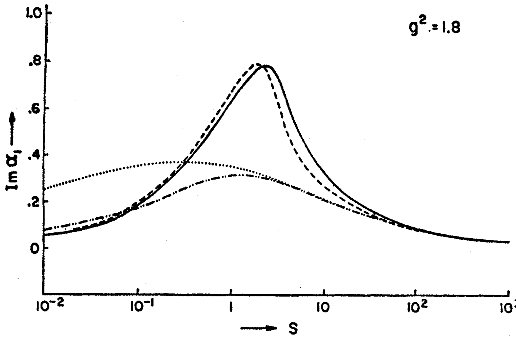

FIG. 6. Same as Fig. 4 with $g^2=1.8$.

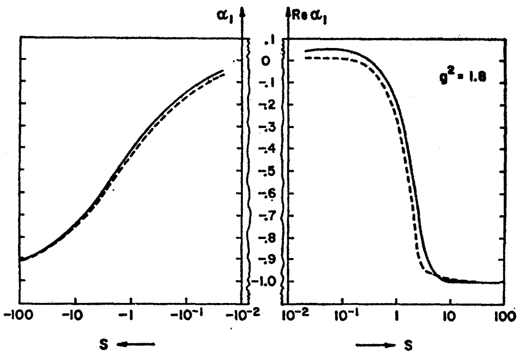

FIG. 7. Same as Fig. 5 with $g^2=1.8$.

function $\beta_1(s)$, the results are similar to the result from the exact $\mathrm{Im}\alpha_1(s)$ input. This is shown in Figs. 10 through 12. Thus, within reasonable limits, one sees that the β's suffer more from the approximation (7) than the α's.

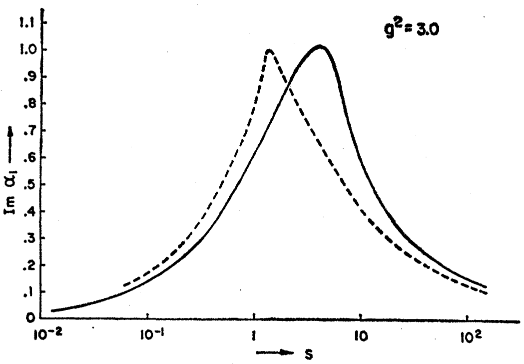

FIG. 8. Same as Fig. 4 with $g^2=3.0$. (Universal and Khuri omitted.)

Finally, Fig. 13 gives the $\mathrm{Re}S(l,s)$ and $\mathrm{Im}S(l,s)$ for $g^2=1.8$ and $l=0$ as they are obtained from (5c) using as input the output of the integral equation (8c), with $\beta_1(s)$ calculated from (7c).

In conclusion, the solutions indicate that the modified

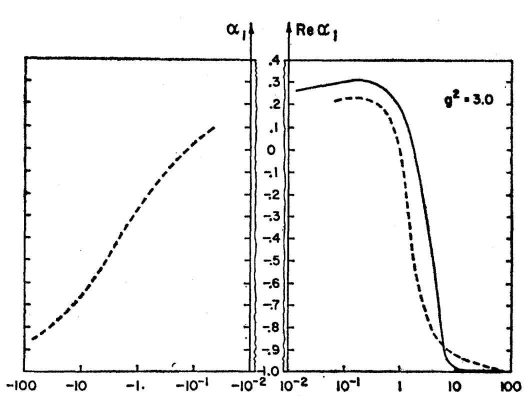

FIG. 9. Same as Fig. 5 with $g^2=3.0$.

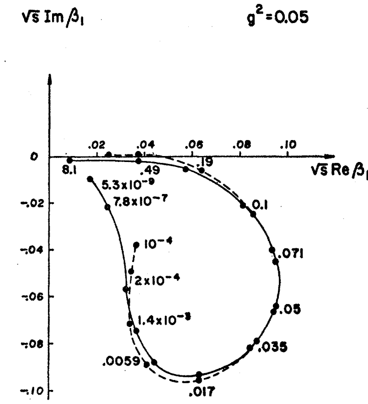

FIG. 10. $(s)^{1/2}\,\mathrm{Im}\beta_1$ versus $(s)^{1/2}\,\mathrm{Re}\beta_1$; $g^2=0.05$. Exact (Ref. 3) (———). The modified Khuri $\beta_1(s)$ (— — —) is computed using the result for $\mathrm{Im}\alpha_1(s)$ from the integral (8c) in the coupling equation (7c).

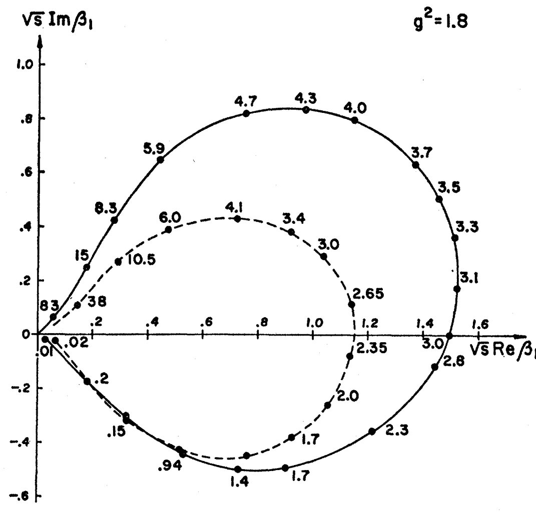

FIG. 11. Same as Fig. 10 with $g^2=1.8$.

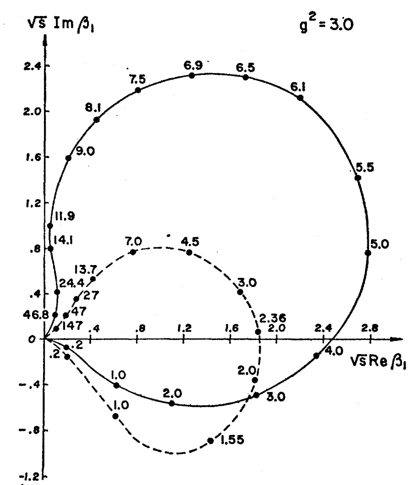

FIG. 12. Same as Fig. 10 with $g^2=3.0$.

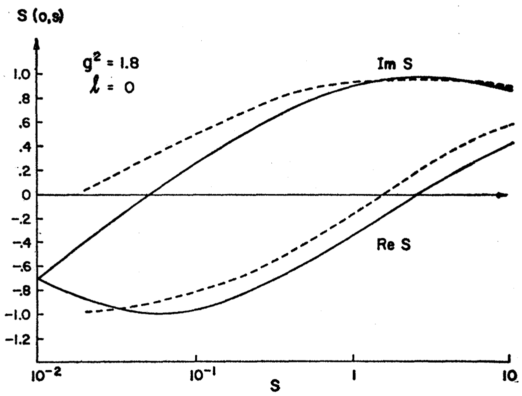

FIG. 13. Real and imaginary parts of the S matrix, $S(l,s)$; $l=0$, $g^2=1.8$. Exact (Ref. 3) (———). The modified Khuri $S(0,s)$ is computed from (5c) using the results for $\mathrm{Im}\alpha_1(s)$ from (8c) and the corresponding $\beta_1(s)$ from (7c).

Khuri representation (5c) is a much better basis for the coupling of residues to trajectories than the other representations tested. It is also indicated that coupling in higher trajectories is not necessary until these trajectories become normal, i.e., $g^2\approx 1.8$ in our example. For $g^2=3.0$, clearly the second trajectory should be coupled in, but it is now normal [i.e., $\alpha_2(s)$ is analytic with only a right-hand cut] and should present no difficulty.

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APPENDIX: ACCUMULATION POINT TRAJECTORIES AND THRESHOLD BEHAVIOR

When $\alpha_2(0)<-\frac{1}{2}$, the phase of $\beta_2(s)$ near threshold is given by[3]:

$$s^{1/2}\beta_2(s) \underset{s\to 0}{\longrightarrow} \mathrm{Im}\alpha_2(s)e^{2\pi i[\alpha_2(0)+1/2]} \quad . \tag{A1}$$

The coupling equations (6) cannot account for the phase of $\beta_2(s)$ when $\alpha_2(0)<-\frac{1}{2}$, whether $\alpha_1(s)$ is coupled in or not.

In this connection it is important to note that when $\alpha_2(0)<-\frac{1}{2}$, it ceases to be the "second" trajectory somewhere near threshold, because there are infinitely many trajectories streaming into the accumulation point at $\alpha(0)=-\frac{1}{2}$. We wish to show that these must be dealt with collectively if at all, because only collectively do these accumulation point trajectories reproduce a reasonable partial-wave threshold behavior.

The accumulation point trajectories were analyzed by Newton and Desai[7] using the following form for $S(l,k)$:

$$S(l,s)=\frac{1-s^{l+\frac{1}{2}}e^{i\pi(l+\frac{1}{2})}C(l)}{1-s^{l+\frac{1}{2}}e^{-i\pi(l+\frac{1}{2})}C(l)}, \quad C(-\tfrac{1}{2})=1. \tag{A2}$$

The position of the poles of the S matrix are then given near $l=-\frac{1}{2}$ by

$$\alpha_n(s)=-\tfrac{1}{2}-\frac{2\pi in}{\ln s}+\frac{2n\pi^2}{(\ln s)^2},$$
$$n=\pm1, \pm2, \cdots, \quad |n|\ll|\ln s|/2\pi. \tag{A3}$$

When the residue of the pole is examined, one finds

$$\beta_n(s)=\frac{2\pi in}{(\ln s)^2}. \tag{A4}$$

Combining the two trajectories with the same absolute n in (5b), for example, we find the following contribution:

$$A_n(s,l) \underset{s\to 0+}{\longrightarrow} \frac{(4\pi)^2}{(2l+1)}[s^{l+\frac{1}{2}}/(\ln s)^3]n^2, \tag{A5}$$

which is a vanishing contribution compared to the normal $s^{l+\frac{1}{2}}$ threshold contribution of normal trajectories. But as $s\to 0$ the number of trajectories in this regime goes up as $|\ln s|$. One therefore finds that the total threshold contribution of the accumulation point trajectories is given by

$$A(s,l) \underset{s\to 0+}{\longrightarrow} \sum_{n=1}^{|\ln s|} A_n(s,l) \propto s^{l+\frac{1}{2}}. \tag{A6}$$

It is thus plausible to say that when $\alpha_2(0)<-\frac{1}{2}$, it is the collective bunch of accumulation point trajectories which play the role of the "second" trajectory.

The coupling equations (6) fail near threshold for trajectories for which $\alpha(0)<-\frac{1}{2}$, even though $\mathrm{Im}\alpha(s)$ vanishes at threshold. The reason is the threshold behavior of the S matrix for $\mathrm{Re}l<-\frac{1}{2}$.[3,8] Except in the immediate vicinity of a Regge pole, the threshold behavior is given by

$$\begin{aligned} S(l,s) &\underset{s\to 0}{\longrightarrow} 1, && \mathrm{Re}l>-\tfrac{1}{2} \\ S(l,s) &\longrightarrow e^{2\pi i(l+\frac{1}{2})}, && \mathrm{Re}l<-\tfrac{1}{2}. \end{aligned} \tag{A7}$$

[7] B. R. Desai and R. G. Newton, Phys. Rev. **130**, 2109 (1963).
[8] A. O. Barut and D. Zwanziger, Phys. Rev. **127**, 974 (1962).

It follows that the one-trajectory coupling equation, which is obtained from

$$A(s,l)=\frac{\beta(s)}{l-\alpha(s)} \quad \text{as} \quad s\to 0^+, \quad \alpha(0)>-\tfrac{1}{2} \quad \text{(A8)}$$

and (3)

$$(1/2i)=s^{1/2}A(s,\alpha^*(s))^*,$$

which leads to

$$s^{-1/2}\beta(s)=\mathrm{Im}\alpha(s), \quad s\to 0^+, \quad \alpha(0)>-\tfrac{1}{2},$$

should be replaced by

$$A(s,l)=\frac{\beta(s)}{l-\alpha(s)}+(1/2i)[e^{2\pi i(l+\frac{1}{2})}-1], \quad s\to 0 \quad \alpha(0)<-\tfrac{1}{2}. \quad \text{(A9)}$$

This implies that, except in the neighborhood of a pole, $S(l,s)$ is given by (A7). When the unitarity condition (3) is applied to (A9) we obtain

$$s^{1/2}\beta(s)\underset{s\to 0+}{\longrightarrow}\mathrm{Im}\alpha(s)e^{2\pi i[\alpha(0)+1/2]}, \quad \alpha(0)<-\tfrac{1}{2}. \quad \text{(A10)}$$

This is the correct residue near threshold for $\alpha(0)<-\frac{1}{2}$. The "width" of the threshold region depends on the strength of the interaction. There is therefore no obvious rule as to how and at what s one should change from (A9) to (5) in the coupling equations. If the expansions (5b) or (5c) converge in the left-hand l plane, (A10) should obtain if all the trajectories are coupled in, but as the discussion of the accumulation point indicates, it is not likely that any finite set will lead to the correct threshold behavior when $\alpha_n(0)<-\frac{1}{2}$.

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New Resonances and the Vector-Meson System*

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Under the assumption of exact SU_3 symmetry we investigate the force between two degenerate vector-meson octets due to the exchange of a vector, pseudoscalar, and scalar octet. It is found that many of the recently discovered particles may fit into this scheme as bound states. However, the model does not reproduce the well-known pseudoscalar and vector-meson octets which are its input, but suggests a second octet of each kind at higher mass. It also gives a 0^+ and 2^+ singlet and octet as well as a 1^+ and a 2^- octet.

I. INTRODUCTION

Recently, several new resonances have been reported in the $\pi\rho$ (A),[1] $\pi\omega$ (B),[2] πK^*,[3] $\bar{K}K^*$,[4] and $\eta\pi\pi$ (X^0)[5] system. This indicates that resonances may cluster to form new particles. In this paper we investigate the force between two octets of vector mesons. Under the assumption of exact SU_3 symmetry we calculate the input for an N/D calculation of the vector-meson–vector-meson scattering amplitude. From the sign and the magnitude of the Born-amplitude in the various channels, we conclude what particles may emerge from the vector-meson system and where their masses may range. We do not try, however, to determine the masses and coupling constants by solving the equations as, for instance, in the models for the π-ω resonance,[6] since it involves some parameters. A determination of these parameters by a self-consistency condition as in the bootstrap calculations[7] seems impossible in our case since the attraction in the vector-meson channel is, as we shall see, weaker than the one for other particles like 1^+ and 2^+ which have not been observed at masses below or about the vector-meson mass. Vector meson scattering as a qualitative model for SU_3 symmetry has previously been studied by Cutkosky *et al.*[8] These authors do not, however, study the dynamical details.

* This work supported by the U. S. Atomic Energy Commission.
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[1] G. Goldhaber, J. Brown, S. Goldhaber, J. Kadyk, B. Shen, and G. Trilling, Phys. Rev. Letters **12**, 336 (1964); S. U. Chung, O. Dahl, L. Hardy, R. Hess, G. Kalbfleish, *et al.*, *ibid.* **12**, 621 (1964); M. Aderholz *et al.*, Phys. Letters **10**, 226 (1964).

[2] M. Abolins, R. Lander, W. Melhop, N. Xuong, and P. Yeager, Phys. Rev. Letters **11**, 381 (1963); D. Duane Carmony *et al.*, *ibid.* **12**, 254 (1964).

[3] T. P. Wangler, A. R. Erwin, W. D. Walker Phys. Letters **9**, 71 (1964), R. Armenteros, D. N. Edwards, T. Jacobsen, L. Montanet, A. Shapira, *ibid.* **9**, 207 (1964).

[4] Proceedings of the 1964 Conference on High Energy at Dubna (to be published).

[5] G. R. Kalbfleisch, L. Alvarez, A. Barbaro-Galtieri, O. Dahl, P. Eberhard *et al.*, Phys. Rev. Letters **12**, 527 (1964); M. Goldberg, M. Gundzik, S. Lichtman, I. Leitner, M. Primer *et al.*, *ibid*, **12**, 546 (1964); M. Goldberg, M. Gundzik, I. Leitner, M. Primer, P. Connolly *et al.*, *ibid.* **13**, 249 (1964).

[6] R. F. Peierls, Phys. Rev. Letters **12**, 50 (1964); T. K. Kuo, *ibid.* **12**, 465 (1964); C. Goebel, Phys. Letters **9**, 67 (1964); E. Abers, Phys. Rev. Letters **12**, 55 (1964).

[7] F. Zachariasen and C. Zemach, Phys. Rev. **128**, 849 (1962); R. H. Capps, *ibid.* **134**, B460 (1964), see also references to previous papers. Chan Hong-Mo, P. C. De Celles, J. E. Paton, Nuovo Cimento **33**, 70 (1964), A. Pignotti Phys. Rev. **134**, B630 (1964), M. L. Mehta, *ibid.* **134**, B1377 (1964).

[8] R. E. Cutkosky, Phys. Rev. **131**, 1888 (1963); R. E. Cutkosky and P. Tarjanne, *ibid.* **132**, 1354 (1963).