is known. F/D ratios which differ greatly from the statistical values are also interesting, because in the present context they provide rather strong statements about the dynamics; i.e., about the relative strength of the invariant amplitudes.

As a case in point, consider the $N^*(1570)$ resonance with $J^P=\frac{1}{2}^-$. There is evidence that this state has a coupling to $N\eta$ at least comparable to its coupling to $N\pi$.[10] This means that r is unlikely to lie between 0 and 1. Tables III and IV reveal that the $8_B{}^2$ and $8_C{}^2$ of **1134**, and the $\mathbf{8}^2$ of either **56** are the only likely assignments, using the statistical values. The former possibilities fit in well with the work of Ref. 7. On the other hand, this state has previously been mentioned as a candidate for the **70**$^-$ by Gyuk and Tuan.[11] If their hypothesis is accepted, then the observed coupling gives a fairly strong constraint on the dynamics; for example, **70** dominance is obviously favored.

[10] F. Uchiyama-Campbell and R. K. Logan, Phys. Rev. **149**, 1220 (1966).

[11] I. P. Gyuk and S. F. Tuan, Phys. Rev. Letters **14**, 121 (1965).

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Some Consequences from Superconvergence for πN Scattering

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Sum rules of the superconvergent type are obtained for the πN helicity-flip and helicity-nonflip amplitudes. The sum rules for the helicity-nonflip amplitudes are shown to be consistent with the Regge-pole-dominance model. Investigation of the sum rule for $B^{(-)}(\nu)$ leads us to speculate as to the existence of resonances on N_β, Δ_γ, and N_δ baryon trajectories.

I. INTRODUCTION

RECENTLY, an exact sum rule for the π^-p helicity-nonflip forward-scattering amplitude with charge exchange has been proposed in order to investigate singularities in the complex J plane.[1,2] Assuming that there are no other singularities in the complex J plane except the ρ Regge pole above $J=-1$ at $t=0$, we separated the helicity-nonflip amplitude $f^{(-)}(\nu)$ into the ρ-pole term $f_\rho(\nu)$ and the remaining term $f^{(-)\prime}(\nu)$, which vanishes faster than ν^{-1} at infinity. Since the $f^{(-)\prime}(\nu)$ is odd under crossing symmetry, satisfies an unsubtracted dispersion relation, and vanishes faster than ν^{-1} for $\nu\to\infty$, we were immediately led to the following sum rule of the superconvergent type:

$$4\pi f^2-\frac{1}{2\pi}\int_\mu^\infty d\nu\{(\nu^2-\mu^2)^{1/2}[\sigma_{\pi^-p}(\nu)-\sigma_{\pi^+p}(\nu)] -4\pi\beta_\rho P_{\alpha_\rho}(\nu/\mu)\}=0\,, \quad (1)$$

with[3]

$$f^2=\frac{g_r{}^2}{4\pi}\left(\frac{\mu}{2m}\right)^2=0.081\pm0.002\,. \quad (2)$$

An experimental check of the above sum rule has suggested to us that Eq. (1) holds[4] within the present accuracy of the total cross-section measurements. Therefore, we have concluded that the experiments support the ρ Regge-pole-dominance model at high energy, even though we cannot rule out the possibility of the existence of other singularities (including a ρ' pole or a cut) if the pole residue or discontinuities are reasonably small.

This ρ-pole-dominance model has also been strongly favored[5] by the remarkable diffraction shrinkage at high energy for the reaction $\pi^-+p\to\pi^0+n$, and the dip phenomena[6] observed in the above and other reactions. The single-ρ-exchange model, however, predicted no polarization for the above reaction $\pi^-+p\to\pi^0+n$, which was not consistent with the observed nonzero polarization.

Recently, a possible model to overcome this difficulty was proposed by Desai, Gregorich, and Ramachandran.[7] They pointed out that if baryon trajectories continue to rise for quite large energies, then, as a consequence of assuming the total amplitude to be given by the single ρ Regge-pole term and the direct-channel contribution from baryon trajectories, it is possible to explain the

[1] K. Igi and S. Matsuda, Phys. Rev. Letters **18**, 625 (1967), hereafter referred to as I.

[2] A. A. Logunov, L. D. Soloviev, and A. N. Tavkhelidze, Phys. Letters **24B**, 181 (1967); D. Horn and C. Schmid, California Institute of Technology Report, CALT-68-127 (unpublished).

[3] W. S. Woolcock, in *Proceedings of the Aix-en-Provence Conference on Elementary Particles, 1961* (Centre d'Etudes Nucléaires, Saclay, France, 1961), Vol. I, p. 459.

[4] See Table I and Fig. 1 of I.

[5] R. Logan, Phys. Rev. Letters **14**, 414 (1965).

[6] F. Arbab and C. Chiu, Phys. Rev. **147**, 1045 (1966); S. Frautschi, Phys. Rev. Letters **17**, 722 (1966).

[7] B. R. Desai, D. T. Gregorich, and R. Ramachandran, Phys. Rev. Letters **18**, 565 (1967).

magnitude and the energy dependence of the polarization as well as the differential cross section.

In the present paper we take the ρ-dominance model as our starting point for both the helicity-flip and helicity-nonflip amplitudes, and investigate the superconvergence relations derived from the above assumption. In Sec. II, we obtain a sum rule[8] for the helicity-flip amplitude with charge exchange, which connects the integral over direct-channel resonances with ρ Regge parameters. We then show that the sum rule does not seem to hold for the currently known baryon resonances. Therefore, we speculate as to the possible existence[9] of N_β, Δ_γ, and N_δ resonances connected by the MacDowell principle[10] to the N_α, Δ_δ, and N_γ resonances, since these possible new resonances will contribute in such a way that the sum rule may hold.

In Sec. III a sum rule for $B^{(+)}(\nu)$ which connects the low-energy resonance parameters with the P and P' Regge residues for the helicity-flip amplitude is discussed. Since the convergence of the integral is rather rapid in this case, we use the above relation as an additional constraint for the P and P' parameters determined from experimental data, which will be shown to be useful in the investigation of πN polarization.

In Sec. IV, two kinds of superconvergence relations are investigated from two different assumptions in order to probe the J singularities with the vacuum quantum numbers. One of them will be proved to be equivalent to the sum rule for the s-wave scattering length.[11]

II. A SUM RULE FOR THE HELICITY-FLIP AMPLITUDE $B^{(-)}(\nu)$

Let us begin with summarizing briefly the sum rule derived in II for the helicity-flip amplitude $B^{(-)}(\nu)$. We assume that $B^{(-)}(\nu)$ can be separated as

$$B^{(-)}(\nu)=B_\rho(\nu)+B^{(-)\prime}(\nu)\,, \tag{3}$$

with

$$\operatorname{Im}B_\rho(\nu)=\alpha_\rho(\alpha_\rho+1)\frac{\tilde{C}_\rho}{\mu^2}\left(\frac{\nu}{\mu}\right)^{\alpha_\rho-1} \tag{4}$$

in an asymptotic form at high energies. The factor $(\alpha_\rho+1)$ is included to emphasize the zero at $\alpha_\rho=-1$. The quantity $\tilde{C}_\rho$ is defined to be dimensionless, and is regular except for additional zeroes at $\alpha_\rho=-2$, -3, -4, $\cdots$. Then, the function $\nu B^{(-)\prime}(\nu)$, which is odd under crossing symmetry, satisfies an unsubtracted dispersion relation and the following asymptotic behavior: $\nu B^{(-)\prime}(\nu)<\nu^{-1}$. Therefore, we obtain the following superconvergence sum rule:

$$\int_0^\infty d\nu\,\nu\,\operatorname{Im}B^{(-)\prime}(\nu)=0\,. \tag{5}$$

Assuming the Regge asymptotic behavior to be already established at high energies, $\nu>\nu_A$ (we take $\nu_A=5.46$ GeV as in I), we obtain

$$-4\pi f^2+\frac{1}{\pi}\int_\mu^{\nu_A} d\nu\,\nu\,\operatorname{Im}B^{(-)}(\nu)=\frac{\tilde{C}_\rho\alpha_\rho}{\pi}\left(\frac{\nu_A}{\mu}\right)^{\alpha_\rho+1}. \tag{6}$$

Experimental Test

Unfortunately, $\operatorname{Im}B^{(-)}(\nu)$ is not simply related to cross sections. Therefore, we make the following approximations[12] in the evaluation of the integral

$$\int_\mu^{\nu_A} d\nu\,\nu\,\operatorname{Im}B^{(-)}(\nu):$$

For $\nu>\nu_A$, we may put

$$\operatorname{Im}B^{(-)}(\nu)\cong\operatorname{Im}B_\rho(\nu)\,, \tag{7}$$

since we have already assumed that the ρ Regge asymptotic behavior is established at these high energies. Chew and Jones[13] have argued that the resonance and Regge regions may overlap in some intermediate-energy regions. Practically, Barger *et al.*[14] have shown that a considerable number of experimental data down to 1.4 GeV can be explained by a superposition of direct-channel resonances and the Regge background term. Therefore, we will assume

$$\operatorname{Im}B^{(-)}(\nu)\cong B_{\mathrm{Res}}{}^{(-)}(\nu)+\operatorname{Im}B_\rho(\nu) \tag{8}$$

for $\nu_A>\nu>\nu_M$. (ν_M is supposed to be the lowest energy at which Barger's analysis is applicable; consequently $\nu_M=1.4$ GeV was chosen.[14]) Here, the subscript Res stands for the contribution of all the direct-channel resonances. Thus, it would be reasonable to use the values of the direct-channel resonance parameters cited in Ref. 14 (especially the total width and elasticity) for $\nu_A>\nu>\nu_M$.

For $\nu<\nu_M$, we may assume

$$\operatorname{Im}B^{(-)}(\nu)\cong\operatorname{Im}B_{\mathrm{Res}}{}^{(-)}(\nu)\,. \tag{9}$$

In this energy region, as direct-channel resonance parameters, those tabulated in the Rosenfeld table[15] have been used.

[8] K. Igi and S. Matsuda, University of Tokyo Report, March, 1967 (unpublished), hereafter referred to as II.
[9] C. B. Chiu and J. D. Stack, Phys. Rev. **153**, 1575 (1967).
[10] S. W. MacDowell, Phys. Rev. **116**, 774 (1959).
[11] K. Igi, Phys. Rev. Letters **9**, 76 (1962); Phys. Rev. **130**, 820 (1963).
[12] One of the authors (K.I.) wishes to thank Professor G. F. Chew for helpful comments in the approximation for evaluating the integral in II.
[13] G. F. Chew and E. Jones, Phys. Rev. **135**, B208 (1964).
[14] V. Barger and M. Olsson, Phys. Rev. **151**, 1123 (1966); V. Barger and D. Cline, *ibid.* **155**, 1792 (1967). In the present paper we have calculated the low-energy integrals in the narrow-width approximation, using the interference model by the above authors. However, if the phase shifts of πN scattering become available up to reasonably high energies in the near future, we can argue more precisely about the superconvergence relations.
[15] A. H. Rosenfeld *et al.*, Rev. Mod. Phys. **39**, 1 (1967).

TABLE I. Contributions to

$$I_1 \equiv \frac{1}{\pi}\int_\mu^{\nu_A} d\nu\, \nu\, \mathrm{Im}B_{\mathrm{Res}}^{(-)}(\nu) \quad \text{and} \quad I_2 \equiv \frac{m}{2\pi^2}\int_\mu^{\nu_A} d\nu\, \mathrm{Im}B_{\mathrm{Res}}^{(+)}(\nu)$$

($\nu_A = 5.46$ GeV) from each resonance in the narrow-width approximation.

Resonance (mass in MeV)	Spin parity J^P	Total width $\Gamma_{l\pm}$ (BeV)	Elasticity $\eta_{l\pm}$	Contribution to I_1	Contribution to I_2
$\Delta_\delta(1236)$	$\frac{3}{2}^+$	0.12	1.0	16.4	−14.6
$\Delta_\delta(1920)$	$\frac{7}{2}^+$	0.17	0.33 —0.50	9.8—14.8	(−2.0)—(−3.0)
$\Delta_\delta(2420)$	$\frac{11}{2}^+$	0.28	0.11 —0.12	10.4—11.4	(−1.2)—(−1.3)
$\Delta_\delta(2850)$	$\frac{15}{2}^+$	0.40	0.03 —0.05	6.9—11.4	(−0.5)—(−0.9)
$\Delta_\delta(3230)$	$19/2^+$	0.44	0.003—0.02	1.1—7. 6	(−0.1)—(−0.5)
$N_\gamma(1525)$	$\frac{3}{2}^-$	0.11	0.65	10.6	2.1
$N_\gamma(2190)$	$\frac{7}{2}^-$	0.24	0.15 —0.25	10.5—17.5	0.8—1.3
$N_\gamma(2650)$	$\frac{11}{2}^-$	0.40	0.05 —0.08	10.7—17.1	0.5—0.8
$N_\gamma(3030)$	$\frac{15}{2}^-$	0.40	0.007—0.015	2.3—5.0	0.1—0.2
$N_\gamma(3350)$	$19/2^-$	0.10	0.003—0.01	0.3— 1.2	0.0—0.1
$N_\alpha(\ 938)$	$\frac{1}{2}^+$	...	...	−1.1	14.8
$N_\alpha(1688)$	$\frac{5}{2}^+$	0.10	0.65	14.7—25.5	2.1—3.7
$N_\alpha(2220)$	$\frac{9}{2}^+$	0.20	0.05	4.3	0.3
$N_\alpha(2610)$	$\frac{13}{2}^+$	0.30	0.025	5.4	0.3
$N_\alpha(2970)$	$17/2^+$	...	...	...	...
$N_\alpha'(1400)$	$\frac{1}{2}^+$	0.20	0.70	9.7	2.6
$N_\beta(1570)$	$\frac{1}{2}^-$	0.13	0.30	0.1	0.0
$N_\beta(1670)$	$\frac{5}{2}^-$	0.14	0.40	−6.1	−0.9
$N_\beta'(1700)$	$\frac{1}{2}^-$	0.24	1.0	0.8	0.1
$\Delta_\beta(1670)$	$\frac{1}{2}^-$	0.18	0.40	−0.2	0.1
Total				106.6—151.4	2.6—7.1

By making use of approximations (7), (8), and (9), the sum rule (6) would be modified as

$$-4\pi f^2 + \frac{1}{\pi}\int_\mu^{\nu_A} d\nu\, \nu\, \mathrm{Im}B_{\mathrm{Res}}^{(-)}(\nu) = \frac{\tilde{C}_\rho \alpha_\rho}{\pi}\left(\frac{\nu_M}{\mu}\right)^{\alpha_\rho+1}. \quad (10)$$

In the narrow-width approximation, we can obtain

$$\frac{1}{\pi}\int_\mu^{\nu_A} d\nu\, \nu\, \mathrm{Im}B_{\mathrm{Res}}^{(-)}(\nu)$$

$$= \sum_{l\pm, I} \pm C_I \frac{\pi}{2} \frac{\Gamma_{l\pm}\eta_{l\pm}(M_{l\pm}^2 - m^2 - \mu^2)}{m^2 q_{l\pm}^3}$$

$$\times\left[\{(M_{l\pm}-m)^2-\mu^2\}\binom{l+1}{-l} - 2M_{l\pm}ml(l+1)\right]. \quad (11)$$

The summation extends over resonances with $2m\nu_A > M_{l\pm}^2 - m^2 - \mu^2$. Here C_I takes the values $\frac{1}{3}$ and $-\frac{1}{3}$ for $I=\frac{1}{2}$ and $I=\frac{3}{2}$, respectively. $\Gamma_{l\pm}$, $\eta_{l\pm}$, and $M_{l\pm}$ are the total width, elasticity, and mass of the resonance of total angular momentum $J=l\pm\frac{1}{2}$; m is the nucleon mass; and $q_{l\pm}$ is the center-of-mass momentum given by

$$q_{l\pm} = \frac{[(M_{l\pm}+m)^2-\mu^2]^{1/2}[(M_{l\pm}-m)^2-\mu^2]^{1/2}}{2M_{l\pm}}. \quad (12)$$

For each resonance, we tabulate the contribution to

$$\frac{1}{\pi}\int_\mu^{\nu_A} d\nu\, \nu\, \mathrm{Im}B_{\mathrm{Res}}^{(-)}(\nu)$$

in Table I. We then obtain the following numerical values: The left-hand side of Eq. (10) is given by[16] 106.6–151.4. The right-hand side of Eq. (10) is given by 23.2 for the solution (a) by Chiu, Phillips, and Rarita[17] ($\alpha_\rho = 0.576$, $\tilde{C}_\rho = 3.36$); or 23.1 for the solution (b)[17] ($\alpha_\rho = 0.576$, $\tilde{C}_\rho = 3.35$). Therefore, the sum rule (10) does not seem to hold as long as we use only the currently known baryon resonances in the above approximations.

Speculations

We wish to make some theoretical speculations as to the reason why there is such a discrepancy.

1. *Other Possible Baryon Trajectories*

Some time ago the following speculation was made[9]: The N_β (Δ_γ,N_δ) resonances could possibly exist, too, as a consequence of the MacDowell principle,[10] if the N_α (Δ_δ,N_γ) resonances were to lie on the Chew-Frautschi plot.[18] As is evident from Eq. (11) and Table I, the contributions to the integral

$$\frac{1}{\pi}\int_\mu^{\nu_A} d\nu\, \nu\, \mathrm{Im}B_{\mathrm{Res}}^{(-)}(\nu)$$

from resonances on the N_α, Δ_δ and N_γ trajectories are all positive except for the nucleon. However, if reso-

[16] The range of values is due to the uncertainty in the elasticity determination of Barger *et al.* (see Ref. 14).

[17] C. B. Chiu, R. J. N. Phillips, and W. Rarita, Phys. Rev. **153**, 1485 (1967).

[18] G. F. Chew and S. Frautschi, Phys. Rev. Letters **7**, 394 (1961).

nances on the N_β, Δ_γ, N_δ trajectories exist at all,[19] their contributions all become negative. Therefore, the sum rule (10) has a tendency to hold. Further investigations on baryon resonances are thus quite important.

2. *Possible Ambiguities in the Approximations*

First, the widths and elasticities of higher baryon resonances have not been definitely determined. Secondly, the contribution of such a resonance, like the Roper resonance (which is not found in the total cross-section measurements), in our analysis, has been estimated using the Breit-Wigner formula. This might also be an ambiguous point. Thirdly, we assumed the Regge asymptotic behavior to hold above ν_A ($=5.46$ GeV). If, however, baryon trajectories which might rise to infinite energies could give non-negligible contributions even at high energies for $\nu>\nu_A$, these effects would also have to be taken into account. Experimentally, the higher the energy, the more extraordinarily small the elasticities become. Therefore, they may be expected to be small.

If the sum rule (10) still does not hold even after all the above possibilities have been checked, then the assumption of no singularities for $\alpha_\rho \geq J \geq -1$ must be abandoned.

III. A SUM RULE FOR THE AMPLITUDE $B^{(+)}(\nu)$ AND πN POLARIZATION

Similarly, we can derive the following sum rule[20] for the helicity-flip amplitude $B^{(+)}(\nu)$ which is odd under crossing symmetry:

$$\int_0^\infty d\nu \, \mathrm{Im} B^{(+)\prime}(\nu)=0 . \tag{13}$$

We have separated $B^{(+)}(\nu)$ as

$$B^{(+)}(\nu)=\sum_{P(i)=P,P'} B_{\alpha P(i)}(\nu)+B^{(+)\prime}(\nu) , \tag{14}$$

with

$$\mathrm{Im} B_{\alpha P(i)}(\nu)=\alpha_{P(i)}\frac{\tilde{C}_{P(i)}}{\mu^2}\left(\frac{\nu}{\mu}\right)^{\alpha P(i)-1} . \tag{15}$$

We assumed here that no J singularities extend above $J=0$ at $t=0$, except for the P and P' poles.

If the Regge behavior is assumed to hold at high energies $\nu>\nu_A$ as before, one can obtain

$$\frac{g_r^2}{4\pi}+\frac{m}{2\pi^2}\int_\mu^{\nu_A} d\nu \, \mathrm{Im} B^{(+)}(\nu)=\frac{1}{2\pi^2}\frac{m}{\mu} \times \sum_{P(i)=P,P'} \tilde{C}_{P(i)}\left(\frac{\nu_A}{\mu}\right)^{\alpha P(i)} . \tag{16}$$

Following the same procedure as in the previous section, Eq. (16) becomes

$$\frac{g_r^2}{4\pi}+\frac{m}{2\pi^2}\int_\mu^{\nu_A} d\nu \, \mathrm{Im} B_{\mathrm{Res}}{}^{(+)}(\nu)=\frac{1}{2\pi^2}\frac{m}{\mu} \times \sum_{P(i)=P,P'} \tilde{C}_{P(i)}\left(\frac{\nu_M}{\mu}\right)^{\alpha P(i)} . \tag{17}$$

The speculations 1 and 2 in Sec. II should also apply in evaluating $\mathrm{Im}B_{\mathrm{Res}}{}^{(+)}(\nu)$. However, since the convergence of the above integral is very rapid, the main contribution comes from low-energy resonances like N and $\Delta(1236)$ (see Table I). Thus, it is probable that the other possible higher baryon resonances and ambiguities in their parameters will not affect the sum rule so much.

Therefore, assuming Eq. (19) to be practically useful, we can obtain an additional constraint to be imposed on the helicity-flip Regge parameters for the P and P' determined from experimental data. This constraint would become useful in the investigation of πN polarizations. Chiu, Phillips, and Rarita[17] obtained two possible solutions, (a) and (b), using total cross sections, differential cross sections for πN elastic and charge-exchange scattering, and $\pi^- p$ elastic polarization.

Let us impose the above constraint on the solutions. Numerically, the left-hand side of Eq. (16) is calculated to be 2.6–7.1.[16] On the other hand, solution (a) ($\tilde{C}_P=-2.74$, $\tilde{C}_{P'}=-8.93$) predicts the right-hand side of Eq. (16) to be -25.3. Solution (b) ($\tilde{C}_P=-0.355$, $\tilde{C}_{P'}=-0.951$) predicts the right-hand side to be -2.7. Both values should be compared with the magnitude of the nucleon term $g_r^2/4\pi$ ($\cong 15$). Therefore, the solution (b) would be preferred.

IV. SUM RULES FOR THE $f^{(+)}(\nu)$ AMPLITUDE AND SINGULARITIES IN THE COMPLEX J PLANE

Let us define the amplitude[21] $f^{(+)\prime}(\nu)$ by

$$f^{(+)\prime}(\nu)\equiv f^{(+)}(\nu)-\sum_{P(i)=P,P'} f_{P(i)}(\nu) ,$$

with

$$\mathrm{Im} f_{P(i)}(\nu)=\frac{1}{4\pi}\alpha_{P(i)}\frac{C_{P(i)}}{\mu}\left(\frac{\nu}{\mu}\right)^{\alpha P(i)} . \tag{18}$$

Since $f^{(+)\prime}(\nu)/\nu$ and $\nu f^{(+)\prime}(\nu)$ are odd in crossing symmetry, we shall derive sum rules for each of the two amplitudes and investigate the consequences.

(A) The dispersion relation for $f^{(+)\prime}(\nu)/\nu$ can be written

$$\mathrm{Re}\frac{f^{(+)\prime}(\nu)}{\nu}=\frac{f^{(+)\prime}(0)}{\nu}+\frac{g_r^2}{4\pi}\frac{1}{2m}\left(\frac{1}{\nu_B-\nu}-\frac{1}{\nu_B+\nu}\right) +\frac{1}{\pi}\int_\mu^\infty d\nu'\left(\frac{1}{\nu'-\nu}-\frac{1}{\nu'+\nu}\right)\mathrm{Im}\frac{f^{(+)\prime}(\nu')}{\nu'} \tag{19}$$

[19] Some of the possible candidates for the N_β resonances may be found in the Rosenfeld table (Ref. 15).

[20] R. Gatto, Phys. Rev. Letters **18**, 803 (1967); L. A. P. Balázs and J. M. Cornwall, Phys. Rev. **160**, 1313 (1967). These authors have also discussed the sum rule for the $B^{(+)}(\nu)$. They confined their attention to low-energy resonances.

[21] We define $f^{(+)}(\nu)\equiv[A^{(+)}(\nu)+\nu B^{(+)}(\nu)]/4\pi$.

with $\nu_B = -\mu^2/2m$. If there are no other singularities except the P and P' poles above $J=0$, the left-hand side of Eq. (19) vanishes faster than ν^{-1}. Thus, we obtain

$$0 = f^{(+)\prime}(0) - \frac{g_r^2}{4\pi}\frac{1}{m} - \frac{2}{\pi}\int_\mu^\infty d\nu' \, \mathrm{Im}\frac{f^{(+)\prime}(\nu')}{\nu'}. \quad (20)$$

This relation can be easily shown to be equivalent to the sum rule for the πN s-wave scattering length which was used to deduce the P' pole[11] (see Appendix).

(B) We next come to the superconvergence sum rule for $\nu f^{(+)\prime}(\nu)$ under the assumption that there are no singularities with the vacuum quantum numbers except that the P and P' poles above $J=-2$. Then, we are led to the following sum rule[2]:

$$2\pi^2 f^2 \frac{\mu}{m} + \frac{1}{2\mu}\int_\mu^{\nu_A} d\nu \, k\nu \{\sigma_{\pi^- p}(\nu) + \sigma_{\pi^+ p}(\nu)\}$$
$$= \sum_{P(i)=P,P'} C_{P(i)} \frac{\alpha_{P(i)}}{\alpha_{P(i)}+2}\left(\frac{\nu_A}{\mu}\right)^{\alpha_{P(i)}+2}. \quad (21)$$

The main point of difference between Eqs. (20) and (21) is that Eq. (21) was derived under much stronger assumptions than Eq. (20). No singularities for $0 > J > -2$ were required in addition to case (A). Theoretically, cuts[22] and a fixed pole[23] at $J=-1$ could possibly be expected. Let us check this experimentally using the above sum rules [Eqs. (20) and (21)]. The best parametrization for the P and P' poles is, according to Scanio,[24] $\alpha_{P'} = 0.69$, $C_P = 0.810$, and $C_{P'} = 2.63$. Substituting these values in Eq. (21), we find it to hold within an error of 1%. Therefore, experiments are consistent with weak-cut discontinuities, if they exist.

V. CONCLUDING REMARKS

As was discussed in Secs. I and IV, where the absorptive part of the amplitudes $f^{(\pm)}(\nu)$ was expressed in terms of total cross sections, the sum rules have been shown to be quite consistent with the Regge-pole dominance. The sum rule for $B^{(+)}(\nu)$ would be useful as a constraint imposed on the helicity-flip Regge-pole parameters since the integral converges rapidly and is saturated by low-energy resonances. Investigation of the sum rule for $B^{(-)}(\nu)$, which converges rather slowly, has led us to suggest a set of resonances on the N_β, Δ_γ, and N_δ baryon trajectories. We hope that a more careful search for other baryon resonances as well as resonance parameters (especially widths and elasticities) for all resonances, will be made in the near future. If the sum rule for $B^{(-)}(\nu)$ still does not hold, even taking into account the above effects, then one will be forced to introduce singularities above $J=-1$ in addition to the ρ pole.

APPENDIX: EQUIVALENCE OF EQ. (20) WITH THE SUM RULE FOR THE s-WAVE SCATTERING LENGTH

In order to express $f^{(+)\prime}(0)$ in terms of the physical quantities, we put

$$f^{(+)\prime}(0) = f^{(+)}(0) - \sum_{P(i)=P,P'} f_{P(i)}(\nu)$$
$$= f^{(+)}(0) + \sum_{P(i)=P,P'} \frac{\beta_{P(i)}}{\sin\pi\alpha_{P(i)}} P_{\alpha_{P(i)}}(0). \quad (\mathrm{A1})$$

The value $f^{(+)}(0)$ can be expressed, using the subtracted dispersion relation, as follows:

$$f^{(+)}(0) = f^{(+)}(\mu) + \frac{f^2}{m}\frac{\mu^4}{(\mu^2-\nu_B^2)\nu_B^2}$$
$$- \frac{\mu^2}{4\pi^2}\int_\mu^\infty \frac{d\nu}{k\nu}\{\sigma_{\pi^- p}(\nu) + \sigma_{\pi^+ p}(\nu)\}. \quad (\mathrm{A2})$$

Substituting Eqs. (A1) and (A2) into Eq. (20), we obtain

$$f^{(+)}(\mu) + \frac{g_r^2}{4\pi}\frac{\nu_B^2}{m(\mu^2-\nu_B^2)} + \sum_{P(i)=P,P'} \frac{\beta_{P(i)}}{\sin\pi\alpha_{P(i)}} P_{\alpha_{P(i)}}(0)$$
$$= \frac{1}{4\pi^2}\int_\mu^\infty d\nu \left[\frac{\nu}{k}\{\sigma_{\pi^- p}(\nu) + \sigma_{\pi^+ p}(\nu)\}\right.$$
$$\left. - \sum_{P(i)=P,P'} 4\pi\beta_{P(i)} \frac{1}{\nu} P_{\alpha_{P(i)}}\left(\frac{\nu}{\mu}\right)\right]. \quad (\mathrm{A3})$$

This is equivalent to the previous sum rule[11] for the s-wave scattering length.

[22] S. Mandelstam, Nuovo Cimento **30**, 1148 (1963).
[23] C. E. Jones and V. L. Teplitz, Phys. Rev. **159**, 1271 (1967); S. Mandelstam and L. L. Wang, *ibid.* **160**, 1490 (1967).
[24] J. J. G. Scanio, Phys. Rev. **152**, 1337 (1967).