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The Hybrid Model: Further Results.

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Summary. — In a previous paper we proposed a « hybrid » model for the large-energy, small-angle elastic scattering of hadrons, and used it to understand the structure in the angular distributions of proton-proton and proton-antiproton differential cross-sections. This model describes the scattering amplitude as the sum of an optical diffractive piece and a piece arising from the exchange of « absorbed » Regge poles; alternatively, it can be viewed as a specific prescription for calculating the effects of Regge cuts. In the present paper, we present some further results obtained from this hybrid model: we extrapolate our solutions for the pp amplitude to higher energies. We show how our model may be extended to deal with inelastic (or backward elastic) scattering, and consider processes which cannot be described by the single exchange of any known Regge trajectory, such as K^-p backward elastic scattering. We explain why Regge cuts do not affect the presence or location of the dip in the π^+p near-backward differential cross-section which is thought to arise from a nonsense zero of the nucleon trajectory. We predict that the differential cross-sections of elastic and of inelastic reactions should have the same t-dependence at large $|t|$, whether or not they do at small $|t|$.

1. – Introduction.

There has been a great deal of interest recently in the subject of possible unitarity corrections to strong-interaction scattering amplitudes. Some results of this interest have been the investigations of the properties of Regge cuts, the attempts to apply absorptive corrections to Regge-poles, and the calculation of multiple-scattering terms in composite models.

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Multiple-scattering corrections have been discussed within the framework of the Chou-Yang model [1], as well as within the quark model [2,3]. These corrections can be calculated by methods analogous to those developed by GLAUBER [4] for scattering by nuclei. The absorption model has been applied to Regge-pole amplitudes by several groups [5]; especially in attempts to explain the polarization observed in $\pi\mathcal{N}$ charge-exchange scattering. The justifications advanced for this procedure differ, but the prescription used is always equivalent to multiplying the partial-wave projection of the Regge pole by the elastic S-matrix. It has also been suggested that Regge cuts might provide corrections to Regge-pole amplitudes. From models [6], we can learn several things about Regge cuts: their position and signature, the types of singularity, and in some cases the signs of their discontinuities. Although this information is not enough to enable us to explicitly compute the contributions of these cuts to the scattering amplitude, it has proven sufficient to permit the prediction of certain features of the $\pi\mathcal{N}$ charge-exchange polarization [7] and of the pp elastic differential cross-section [8].

In a previous paper [9] (hereafter called I), we suggested a « hybrid » model which could relate these three approaches to unitarity corrections [10]; in the present paper we review, elaborate upon, and extend this model. In the hybrid model we identify the eikonal function with the sum of Regge-pole contributions. Formal expansion in powers of the eikonal then yields the Glauber multiple-scattering series; the single-scattering term contains the Regge poles which were used as input, and the multiple-scattering terms are Regge cuts. Furthermore, in I we took the Pomeranchukon to be a flat trajectory, whose residue function in proton-proton scattering was proportional to the square of

(1) T. T. CHOU and C. N. YANG: *A model of elastic high-energy scattering*, *Phys. Rev.* (to be published).

(2) V. FRANCO: *Phys. Rev. Lett.*, **18**, 1159 (1967); N. W. DEAN: CERN preprint, TH. 862 (1968).

(3) N. W. DEAN: CERN preprint, TH. 881 (1968).

(4) R. J. GLAUBER: in *Lectures in Theoretical Physics*, vol. **1** (New York, 1959), p. 315.

(5) See, for examples, G. COHEN-TANNOUDJI, A. MOREL and H. NAVELET: *Nuovo Cimento*, **48** A, 1075 (1967); and I. KIMEL and H. MIYAZAWA: *Absorption hard core and Regge cuts.*, University of Chicago preprint (1967).

(6) For example, V. N. GRIBOV: *Sov. Phys. JETP*, **26**, 414 (1968).

(7) C. B. CHIU and J. FINKELSTEIN: *Nuovo Cimento*, **48** A, 820 (1967); see also V.M. DELANY, D. J. GROSS, I. J. MUZINICH and V. L. TEPLITZ: *Phys. Rev. Lett.*, **18**, 149 (1967).

(8) A. A. ANSELM and I. T. DYATLOV: *Phys. Lett.*, **24** B, 479 (1967).

(9) C. B. CHIU and J. FINKELSTEIN: *A hybrid model for elastic scattering*, *Nuovo Cimento* (to be published), hereafter called I.

(10) Many of the ideas in I were proposed simultaneously—and independently—by R. C. ARNOLD: as *The next step in high-energy phenomenology*, ANL preprint (1968).

the proton-charge form factor, so that we recovered the Chou-Yang model [1] in the infinite-energy limit; however, most of the work presented in the present paper is independent of any specific model for the Pomeranchukon, and so would be equally valid in the eikonal formulation proposed earlier by ARNOLD [11].

In the next Section we elaborate further on the hybrid model. We also review the analysis of pp elastic scattering presented in I, and extrapolate the solution obtained there to higher energies. In Sect. **3** we discuss the extension of this model to inelastic processes, where to first order in the inelastic transition it corresponds exactly to the exchange of absorbed Regge poles. We point out that in this picture the differential cross-sections of elastic and of inelastic reactions should have the same t-dependence at large $|t|$ whether or not they do at small $|t|$. We argue that the application of absorptive corrections does not alter the successful predictions of Regge-pole theory; in particular, we show that absorption does not affect the prediction of a dip in the near-backward π^+p differential cross-section at the value of u at which the nucleon trajectory becomes nonsense. In Sect. **4** we present the further extension of our model to second order in inelastic transitions, which allows us to discuss reactions, such as $K^-p \to K^+\Xi^-$, which require exchange of more than one Regge pole.

2. – The hybrid model for elastic scattering.

For the moment, we ignore the presence of spin. Suppose we write the S-matrix in the impact parameter representation as

$$S(s, b) = \exp[2iB(s, b)]\,, \tag{1}$$

or, equivalently, the full amplitude as

$$A(s, t) = B(s, t) + \frac{i}{4\pi}\int \mathrm{d}t_1 \mathrm{d}t_2 B(s, t_1) B(s, t_2)\, \tau^{-\frac{1}{2}}(t, t_1, t_2)\theta(\tau) + \tag{2}$$
$$+ \text{ terms involving more than two } B\text{'s}\,.$$

Here τ is the triangle function; see the Appendix of ref. [12]. The amplitude A is normalized so that $\sigma^T = 4\pi \operatorname{Im} A(s, 0)$. Equation (1) or (2) may be considered to be a definition of B, which corresponds to the single-scattering

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[11] R. C. ARNOLD: *Phys. Rev.*, **153**, 1523 (1967).

[12] J. FINKELSTEIN and M. JACOB: *Absorptive corrections and Regge singularities*, *Nuovo Cimento* (to be published).

term of Glauber theory. In the hybrid model, we write

$$B(s,t) = P(t) + \sum_i R_i(s,t)\,, \tag{3}$$

where P is the «Pomeranchuk» contribution; we recover the Chou-Yang model ([1]) for pp scattering by letting P be proportional to the square of the proton electric form factor: $P(t) = iKF_1(t)$. The summation over $R_i \equiv \beta_i(t)s^{\alpha_i(t)-1}$ in eq. (3) then extends over Regge poles other than the Pomeranchukon; we refer to these as «proper» Regge poles.

Computations are particularly simple if, at fixed energy, B is expressed as a sum of exponentials in t. For example, if B contains two terms $\lambda_i \exp[t/a_i]$ and $\lambda_j \exp[t/a_j]$, there will be a contribution in the double scattering (*i.e.*, the second term on the right-hand side of eq. (2)), which is given simply by the rule

$$\text{Regge poles } \lambda_i \exp\left[\frac{t}{a_i}\right] \quad \text{and} \quad \lambda_j \exp\left[\frac{t}{a_j}\right] \Rightarrow \tag{4}$$
$$\Rightarrow \text{Regge cut } \left(1-\frac{1}{2}\delta_{ij}\right)\frac{\lambda_i\lambda_j a_i a_j}{2(a_i+a_j)}\exp\left[\frac{t}{(a_i+a_j)}\right].$$

A Regge pole $R_i = \gamma \exp[-i\pi\alpha(t)/2](s/s_0)^{\alpha(t)}$, with $\alpha(t) = \alpha_0 + \alpha' t$, can be written in the form $\lambda \exp[t/a]$, with $\lambda = \gamma \exp[-i\pi\alpha_0/2](s/s_0)^{\alpha_0}$ and $a^{-1} = \alpha'\big(\ln(s/s_0) - (i\pi/2)\big)$. The cut expression in (4) has precisely the same form as we used in ref. ([7]) as the simplest parametrization consistent with the supposed ([6]) properties of Regge cuts. We note that the phase (as well as the magnitude) of the cut contribution is specified by (4); this phase is controlled by the signature factors of the pole terms, which have been incorporated into the parameters a and γ.

The hybrid model may be thought of as a Reggeization of an optical model, or, alternatively, as an optical prescription for calculating the effects of cuts in a Regge model. Looked at in either way, it constitutes a guess, which we certainly cannot prove to be correct. To some, this guess may be plausible because of the fact that it corresponds to what one would obtain from a composite picture of hadrons—for example, the quark model, in either the strong or the weak binding limit—in which the components interact by exchange of Regge poles. To others, it may be plausible because the prescription for calculating cuts is so simple, and yet all of the intricate features of the cuts—signature, type of singularity etc.—that we expect on the basis of more sophisticated models ([6]) are reproduced. The test of this guess is, of course, by comparison with experimental data.

In I, we used an extremely simple form of this model to try to understand the general features of pp scattering. We took the dipole expression for the electric form factor which was used by DURAND and LIPES [13] to produce, by means of the multiple-scattering series, an asymptotic amplitude which has zeros at values of momentum transfer ($t \approx -1.3$ (GeV)^2 and -6 (GeV)^2) close to those at which the measured differential cross-sections show some indication of having structure [14]. We retained two proper trajectories, the ones on which lie the ω and the f^0, and approximated them by one exchange degenerate trajectory, with $\alpha(t) = \frac{1}{2} + t$, and exponential residue [15]. Thus we wrote

$$B(s,t) = \frac{iK\mu^8}{(\mu^2 - t)^4} + \frac{\Gamma}{s^{\frac{1}{2}}}\left(\frac{s}{s_0}\right)^t . \tag{5}$$

Following DURAND and LIPES, we set $\mu^2 = 1\ \text{GeV}^2$, and adjusted K so that we fit the total cross-section; that left us with two parameters, Γ and s_0, with which to reproduce the real part in the forward direction, the structure in the angular distribution, and all of the energy dependence.

Figure 1 shows the differential cross-sections predicted by this calculation together with data at $P_{\text{lab}} \simeq 12.4$ GeV/c [14,16] and $P_{\text{lab}} = 19.2$ GeV/c [17], for the values $\Gamma = -22$ (GeV)$^{-1}$, $s_0 = 4.5$ (GeV)^2. The data at 19.2 GeV/c are new, and were not used in determining the values of the parameters. Although our parametrization is too crude to give a very good fit, it does reproduce the general features of the energy dependence, as well as the structure around $t = -1.3$ (GeV)^2; there is also some slight structure, hard to see from the Figure, in the calculated curves near $t = -6$ (GeV)^2. In I we stressed that, in our calculation, the *real* part of the amplitude was largely responsible for this struc-

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([13]) L. DURAND III and R. LIPES: *Phys. Rev. Lett.*, **20**, 637 (1968); see also T. T. CHOU and C. N. YANG: *Phys. Rev. Lett.*, **20**, 1213 (1968).

([14]) J. V. ALLABY *et al.*: *CERN Topical Conference*, vol. 1 (1968), p. 580.

([15]) FRAUTSCHI and MARGOLIS (private communication) have recently analysed pp scattering in an eikonal model with a Pomeranchukon of nonzero slope, thus achieving an energy-dependent amplitude without invoking secondary trajectories. Since much of the structure can be understood in terms of the general properties of the multiple-scattering series (see for example, ref. ([8])), their calculation is somewhat similar to ours. Such a model, without secondary trajectories, predicts that the real part of the amplitude is positive in the forward direction and that the pp total cross-section is slowly increasing as the energy increases, as discussed in ref. ([9]).

([16]) J. OREAR, R. RUBINSTEIN, D. B. SCARL, D. H. WHITE, A. D. KRISCH, W. R. FRISKEN, A. L. READ and H. RUDERMAN: *Phys. Rev.*, **152**, 1162 (1967); and D. HARTING, P. BLACKALL, B. ELSNER, A. C. HELMHOLZ, W. C. MIDDELKOOP, B. POWELL, B. ZACHAROV, P. ZANELLA, P. DALPIAZ, M. N. FOCACCI, S. FOCARDI, G. GIACOMELLI, L. MONARI, J. A. BEANEY, R. A. DONALD, P. MASON, L. W. JONES and D. O. CALDWELL: *Nuovo Cimento*, **38**, 60 (1965).

([17]) Preliminary data from the CERN experiment at $P_{\text{lab}} = 19.2$ GeV/c; we thank Dr. A. N. DIDDENS and Dr. A. M. WETHERELL for this private communication.

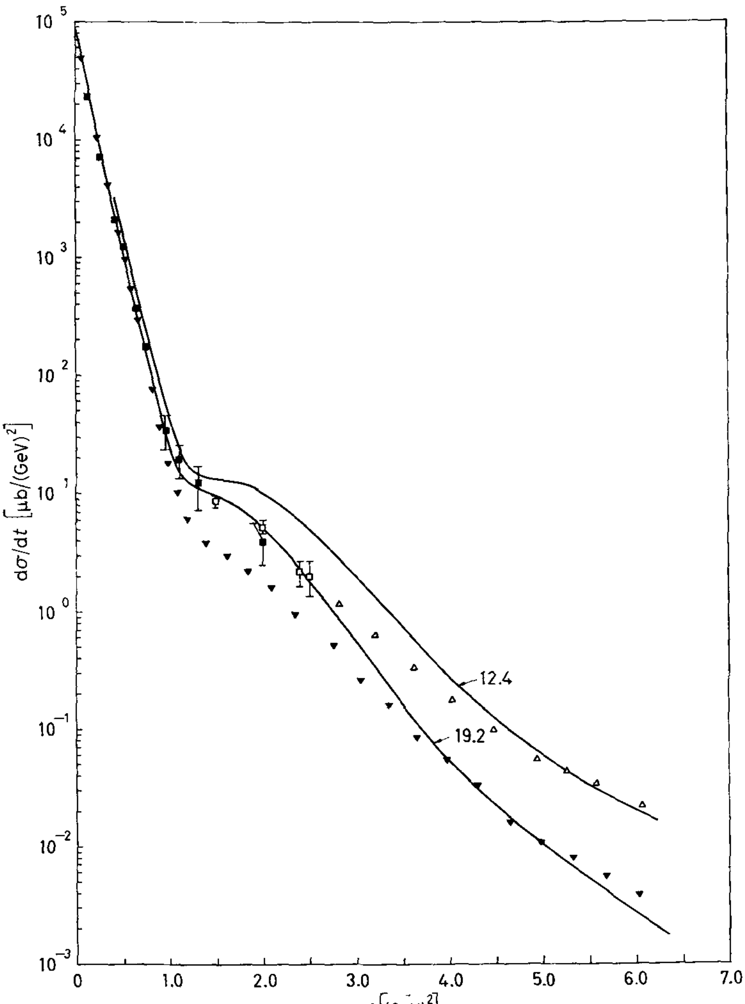

Fig. 1. – The pp elastic differential cross-section *vs.* $-t$. Calculated curves are at $P_{lab} = 12.4$ and 19.2 GeV/c. For details on data see ref. ([14,16,17]). □ 12.0 OREAR *et al.*, △ 12.1 ALLABY *et al.* (I); ■ 12.4 HARTING *et al.*; ▼ 19.2 ALLABY *et al.* (II).

ture, although at much higher energies the structure is primarily due to zeros of the imaginary part, as in the calculation by DURAND and LIPES ([13]).

In Fig. 2 we display the differential cross-sections predicted by our calculation at several higher values of the energy. These predictions are influenced by the fact that we have assumed the Pomeranchukon to be flat. However, a *small* slope for the Pomeranchukon probably would not make much difference, especially since the existence of multiple-scattering corrections tends to obscure more than ever the phenomenological differences that would be caused by a possible small slope for the Pomeranchukon. One very interesting

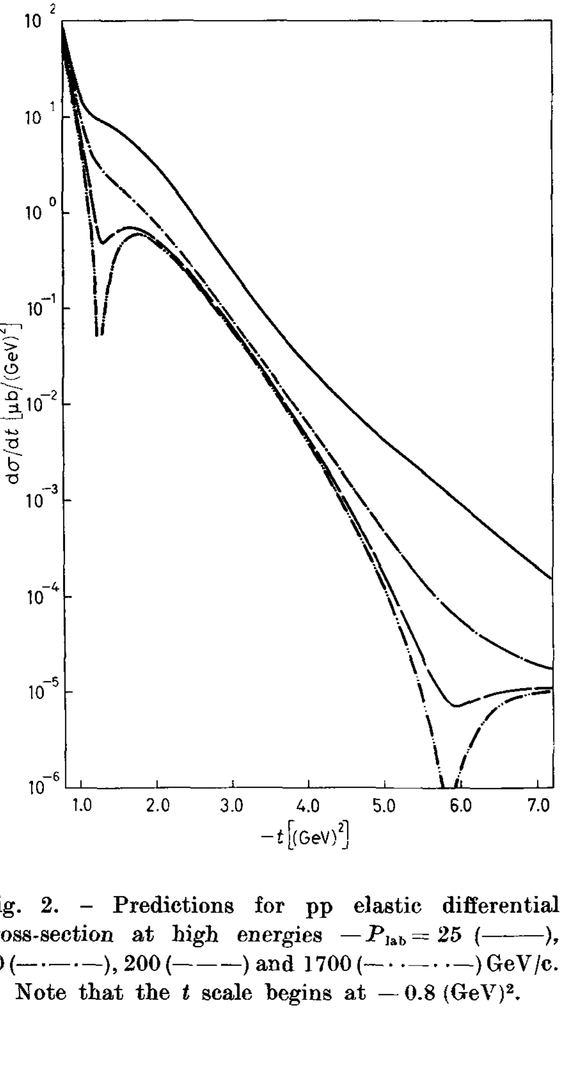

Fig. 2. – Predictions for pp elastic differential cross-section at high energies $-P_{lab} = 25$ (———), 70 (—·—·—), 200 (— — —) and 1700 (—··—··—) GeV/c. Note that the t scale begins at -0.8 (GeV)^2.

feature of this extrapolation is that the angular structure does not monotonically get sharper with increasing energy, as one might expect it would if the real part of the amplitude merely served to fill up the dips in the asymptotically constant imaginary part. Instead, there is an intermediate energy region in which the structure shifts from the real to the imaginary part, and only after that do the dips get sharper.

By changing the sign of the ω contribution, we were able, in I, to predict the $\bar{p}p$ differential cross-section with no free parameters. We found the crossover between the pp and $\bar{p}p$ differential cross-sections, and observed that the structure seen ([18]) at $-t = 0.5\,(GeV)^2$ at $P_{lab} = 5.9$ GeV/c should move slowly outward in t as P_{lab} increased. This last prediction has since then been confirmed by new $\bar{p}p$ data at $P_{lab} = 8$ and 16 GeV/c ([19]).

3. – The hybrid model for inelastic scattering.

3·1. *The absorption model.* – It has been pointed out by ARNOLD ([11]) that the eikonal formulation for elastic scattering leads naturally to the absorption model for inelastic scattering. In eqs. (1) and (3) we have written $S = \exp[2iP + 2iR]$, where R now stands for the proper Regge poles; to first

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([18]) R. RUBINSTEIN, *et al.*: *CERN Topical Conference*, vol. 1 (1968), p. 571.

([19]) D. BIRNBAUM, *et al.*: paper presented at the *Washington Meeting of the APS* (1968), and very kindly reported to us by Dr. A. N. DIDDENS.

order in R, this is $S = \exp[2iP][1 + 2iR]$. From this model for elastic scattering, we could compute, for example, the difference of the elastic π^+p and π^-p amplitudes, and so conclude that the π^-p charge-exchange amplitude is

$$A_{\pi^-p\to\pi^0 n} = \exp[2iP]R_\rho = R_\rho + 2iA_{\text{elastic}}R_\rho\,. \tag{6}$$

Equation (6) coincides with the expression that would be obtained by applying the absorption model to the ρ Regge pole. It has been pointed out [12] that this model will give the correct sign for the polarization in $\pi\mathcal{N}$ charge exchange; more detailed calculations [5] have shown that it can also give approximately the correct magnitude for the polarization, as well as a good fit to the differential cross-section. These successful calculations can now be regarded also as successes for the hybrid model.

Strictly speaking, our model only allows us to compute those inelastic amplitudes which can be expressed as differences of elastic amplitudes, but we may easily generalize to obtain the absorptive prescription for other processes. In our formalism, the simplest way is to treat eqs. (1), (2) and (3) as equations for matrix amplitudes which can connect different states. As long as we work to first order in inelastic transitions [20], this procedure is unambiguous; to this order, there is no question as to what states to include in the calculation, since any «intermediate» state must be connected by an elastic transition to either the initial or the final state. For charge-exchange scattering, this formulation clearly coincides with the one given previously, but it is sufficiently general to allow us to compute absorptive corrections in cases in which elastic scattering may be different in the initial and the final states.

The hybrid model can enable us to understand some general features of the angular distributions of inelastic processes. It is known that the slopes in momentum transfer of both elastic and inelastic differential cross-sections tend to decrease as the momentum transfer increases; in our model, this comes about mainly through multiple scattering. However, there is no reason to believe that the single-scattering terms (*i.e.*, the Regge pole) does not continue to fall rapidly with momentum transfer, especially if the Regge-trajectory function continues to decrease. Thus, it is likely that, at large $|t|$, the (inelastic) single-scattering term is falling much more rapidly than is the elastic amplitude. In this case, if we assume that the initial and final elastic amplitudes have more or less the same slope, it follows easily that the absorbed Regge pole has the same slope as does the elastic amplitude. Put another way, at large $|t|$, the dominant mechanism is the exchange of one proper trajectory together with many Pomeranchukons, in which case the slope of that one trajectory becomes irrelevant.

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[20] In the «n-th order» we include cuts involving exactly n proper trajectories, so that we are making an expansion in powers of $1/s$.

Thus we are led to predict that at large $|t|$ (but always small angle!) inelastic and elastic differential cross-sections have the same slope, whether or not they do at small $|t|$ ([21]). This behaviour is seen in the reactions $pp \to p\mathcal{N}^*(1520)$ and $pp \to p\mathcal{N}^*(1690)$: for $|t|$ larger than about $2\ \text{GeV}^2$, the differential cross-sections for these reactions and for pp elastic scattering are strikingly parallel ([17,22]). In these examples the inelastic process is itself pre sumably due to Pomeranchukon exchange (diffraction dissociation); nevertheless, at small $|t|$ the slopes are markedly different from the elastic slope. We would expect the differential cross-section for production of the $\mathcal{N}^*(1238)$ also to have the same slope (not the same energy dependence) at large $|t|$. At large $|t|$, the details of the production process are irrelevant; the slope is determined by absorption, and so is the same as the elastic slope.

3'2. *What about the Regge-pole model?* – It is clear that absorption can eliminate many of the difficulties of the Regge-pole model; for example, it can produce polarization where otherwise there would be none, and provide conspiracy without the need for conspiring trajectories. However, since the corrections we compute tend to be fairly large—typically around 25 % of the amplitude at $t=0$, and larger than that for $t \neq 0$—one might worry that this would also eliminate many of the triumphs of the Regge-pole model. We would like to argue that this is not so.

Probably the most important aspect of Regge theory is its prediction of energy dependence, in particular its prediction that the amplitude in the physical scattering region is bounded by $s^{\alpha(0)}$; this bound is no way disturbed by multiple-scattering corrections. Furthermore, since near the forward direction the tips of the cuts lie near the pole, their energy dependence will be quite similar. Thus we expect the energy dependence predicted by Regge theory to be maintained ([23]); this works in the case of $\pi\mathcal{N}$ charge exchange ([5]), and as we shall see it is also true in the example discussed below.

Multiple-scattering corrections will effect the t-dependence of amplitudes, but in most cases Regge theory does not specify t-dependence, since there is always an unknown residue function. Regge theory does predict dips when trajectories pass through nonsense points, and as we shall now show, at least in the example of π^+p near-backward elastic scattering, absorption does not affect this prediction.

([21]) This argument has previously been given by ARNOLD (ref. ([10])) for the case of small $|t|$, but extremely large s.

([22]) E. W. ANDERSON, E. J. BLESER, G. B. COLLINS, T. FUJII, J. MENES, F. TURKOT, R. A. CARRIGAN jr., R. M. EDELSTEIN, N. C. HIEN, T. J. MCMAHON and I. NADELHAFT: *Phys. Rev. Lett.*, **16**, 855 (1966).

([23]) For this reason, it is difficult to distinguish cuts from poles by the use of finite-energy sum rules.

The nucleon trajectory is thought to pass through $\alpha = -\frac{1}{2}$ at $-u \approx \approx 0.2\ (\text{GeV})^2$; at this value of u, the π^+p differential cross-section is expected [24], and indeed observed [18,25,26] to have a dip. Because of the expected zero in the nucleon-pole term, the absorptive correction, when evaluated near the backward direction, will be anomalously small, and this in turn will mean that the dip is not disturbed. The zero in the pole term means that the backward peak

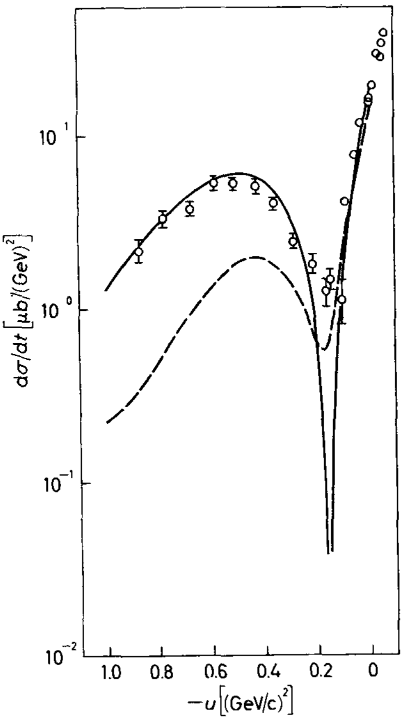

Fig. 3. – Differential cross-section for π^+p backward elastic scattering at $P_{\text{lab}} = = 5.9$ GeV/c. The solid curve represents the contribution of a nucleon Regge pole adjusted so that by itself it fits the data. The dashed curve shows the computed cross-section after the absorptive correction is applied. See Appendix A for the parameters used.

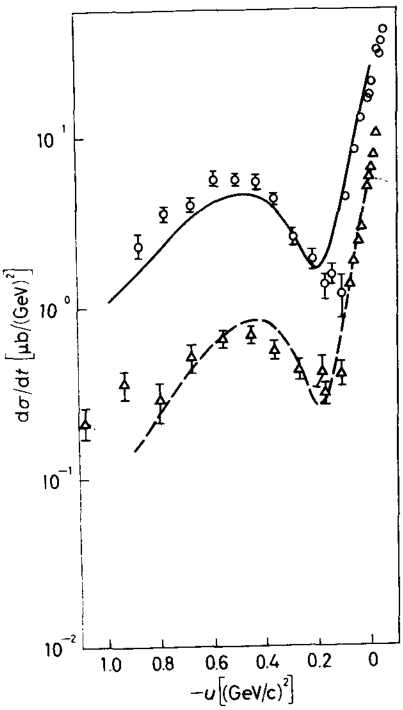

Fig. 4. – Differential cross-section for π^+p backward elastic scattering. The curves at 5.9 GeV/c (———) and 10 GeV/c (— — —) are obtained with the inclusion of basorptive effect and the readjustment of the pole residue function. Data points: o at 5.9 GeV/c, ▵ at 9.9 GeV/c. See ref. [18,25] for details.

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[24] C. B. Chiu and J. Stack: *Phys. Rev.*, **153**, 1575 (1967).

[25] A. Ashmore, C. J. S. Damerell, W. R. Frisken, R. Rubinstein, J. Orear, D. P. Owen, F. C. Peterson, A. L. Read, D. G. Ryan and D. H. White: *Phys. Rev. Lett.*, **19**, 460 (1967).

[26] This structure was first noticed by H. Brody, R. Lanza, R. Marshall, J. Niederer, W. Selove, M. Shoket and R. Van Berg: *Phys. Rev. Lett.*, **16**, 828 (1966).

is extremely steep; this makes the absorption small. Furthermore, since the pole term changes sign, there will be extensive cancellations within the integral defining the absorption, and so the absorption will be even smaller.

This point can easily be verified by direct calculation. The solid curve in Fig. 3 represents a nucleon-exchange Regge-pole term which was constructed so that it, by itself, fits the π^+p differential cross-section at $P_{lab} = 5.9$ GeV/c. When we apply absorptive corrections to this pole term—the dull details of this calculation, including the spinology, are presented in Appendix A—the result is the dashed curve shown in the same Figure. For small $|u|$ the two curves almost coincide; the position of the dip is not changed.

For larger $|u|$ the absorptive correction is somewhat more important, and so the secondary maximum is not reproduced correctly. We readjusted the pole-residue function—but *not* the trajectory function—so that the absorbed pole fit the differential cross-section at $P_{lab} = 5.9$ GeV/c, and then predicted the differential cross-section at 10 GeV/c. The results are shown in Fig. 4. This calculation indicates that, even in the presence of cuts, we can still have a good fit to the π^+p backward data, with the energy dependence as well as the position of the dip controlled by the trajectory function, and thus still correlated with the masses of the nucleon and its recurrences.

4. – Double charge exchange.

Another possible application of the hybrid model is in the analysis of reactions, such as $K^-p \to K^+\Xi^-$ scattering, K^-p backward elastic scattering, or $p\bar{p} \to \Sigma^-\bar{\Sigma}^+$ scattering, which do not permit the exchange of any single (known) Regge trajectory ([27]). We would expect the most important contributions to these reactions to come from simultaneous exchange of two trajectories, and so would have to extend our discussion to terms which are of second order in the proper trajectories ([20]). Conversely, these reactions are perhaps the only instances in which these second-order terms are experimentally accessible; certainly the results we have presented above have depended only on terms of zeroth or first order.

If these reactions are indeed dominated by Regge cuts, then their energy behaviour should be (letting α_1 and α_2 be the intercepts of the two trajectories exchanged)

$$\left.\frac{d\sigma}{dt}\right|_{t=0} = \frac{s^{2(\alpha_1+\alpha_2)-4}}{\ln (s/s_0)^2}\,. \tag{7}$$

If the intercept of the K*-trajectory is $\alpha(0) \approx 0$, then for reactions domi-

([27]) This application of the hybrid model was first suggested to us by Prof. L. VAN HOVE.

nated by double K*-exchange, this is $d\sigma/dt \sim s^{-4}/\ln s^2$. It would be extremely interesting if these predictions could be tested experimentally.

To proceed further, we would need some additional assumptions. The simplest picture of these reactions is that they proceed by two stages, with each of the two intermediate transitions governed by the exchange of an (absorbed) Regge pole; see Fig. 5 ([28]). Therefore we are obliged to specify what will be the « intermediate states ». This is an ambiguity which did not arise in our previous discussion of elastic and first-order inelastic transitions, and is perhaps a good indication that we are trying to push our formalism too far.

If we are willing to assume that multiparticle intermediate states are already contained inside the Regge poles, we have only to say what are the most important two-body states. Perhaps the best guess is to restrict ourselves to those states most closely related to the external states—say, those states whose particles lie within the same multiplets as the external particles (we might say that these are minimally inelastic intermediate states). For the reaction $K^-p \to K^+\Xi^-$ for example, we might use SU_3 multiplets, and so construct intermediate states out of either a π^0 or an η^0, and either a Λ^0 or a Σ^0, and no others. A calculation based on these assumptions is outlined in Appendix B.

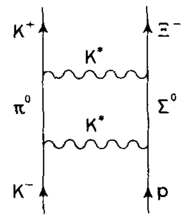

Fig. 5. – A contribution to $K^-p \to K^+\Xi^-$. See Appendix B for details.

To perform such a calculation seriously, one would need, as input, experimental information on strangeness-changing reactions that can proceed by exchanging a single Regge-pole—that is, one would need the single-scattering terms. For a given choice of the single-scattering terms, the calculation outlined in Appendix B turns out to give the same results as those already found by DEAN ([3]) from the quark model. This is not surprising since, in the hybrid model, Regge cuts are computed as if they were the multiple-scattering terms in a composite model.

Clearly, these ideas on double scattering are extremely speculative. They would seem to require a model with a certain group-theoretic structure, in order to select certain states as being less inelastic than others. It is no accident that it was when we took our intermediate states from the same SU_3 multiplets as the external states that we obtained agreement with Dean's (essentially spinless) quark model. On the other hand, the prediction of energy dependence [eq. (7)] does not rely on these speculations, and so is on a firmer footing.

([28]) We are aware that Fig. 5, if interpreted as a Feynman diagram, does not give a cut. This does not prevent us from speculating that we might obtain a reasonable approximation to the amplitude by an alternative interpretation of the Figure.

* * *

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APPENDIX A

Absorption of the nucleon-exchange amplitude.

The fact that a zero in the nucleon-pole term will cause the absorptive correction to be small could have been demonstrated in an (imaginary) spinless case, but it is not hard to include the effects of spin. We use standard notation for the $\pi\mathcal{N}$ problem; in particular $G_\pm(s,t)$ are the s-channel helicity amplitudes, with the normalization

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t}=\frac{\pi}{k^2 s}\left[|G_+|^2+|G_-|^2\right]. \tag{A.1}$$

Let $G^R_\pm$ be a Regge-pole contribution in the t-channel to $G_\pm$, and define

$$G^R_+=\cos\frac{\theta}{2}\,\bar{G}_+\,,\qquad G^R_-=\sin\frac{\theta}{2}\,\bar{G}_-\,. \tag{A.2}$$

Following ARNOLD [11], we define nonflip and flip eikonals χ_0 and χ_f by

$$\left\{\begin{aligned}\chi_0(s,b)&=\frac{1}{k^2}\int\limits_0^\infty x\,\mathrm{d}x J_0(xb)\bar{G}_+(s,-x^2)\,,\\ \chi_f(s,b)&=\frac{1}{2k^3}\int\limits_0^\infty x^2\,\mathrm{d}x J_1(xb)\bar{G}_-(s,-x^2)\,.\end{aligned}\right. \tag{A.3}$$

The cut term due to exchange of two trajectories (which we distinguish by superscripts i and j) is

$$\left\{\begin{aligned}G_+(s,t)&=(1-\tfrac{1}{2}\delta_{ij})ik^2\cos\frac{\theta}{2}\int\limits_0^\infty b\,\mathrm{d}b J_0(b\sqrt{-t})[\chi_0^i\chi_0^j+\chi_f^i\chi_f^j]\,,\\ G_-(s,t)&=(1-\tfrac{1}{2}\delta_{ij})ik^2\int\limits_0^\infty b\,\mathrm{d}b J_1(b\sqrt{-t})[\chi_0^i\chi_f^j+\chi_f^i\chi_0^j]\,.\end{aligned}\right. \tag{A.4}$$

If the Regge-pole terms $\bar{G}_\pm$ are chosen to be exponentials in t, then all the integrals in (A.3) and (A.4) can be done analytically. For the absorption calculation, it is convenient to imagine that the elastic amplitude is given by a single-pole term (*i.e.*, ignore multiple-scattering corrections to the elastic amplitude), and then to use (A.4). We have used the πp elastic data of ref. [29] to fit an exponential form to the elastic amplitude, which we assume to be purely imaginary and spin independent ($\chi_f = 0$).

We have assumed the nucleon trajectory to be given by $\alpha_{\mathcal{N}}(u) = -0.34 + u$. We parametrize the near-backward amplitude due to exchange of this trajectory as

$$\text{(A.5)} \qquad \left\{ \begin{aligned} \bar{G}_+ &= \frac{C_0 D_0}{\sqrt{s_0}}\, s\left(1 + \exp\left[-i\pi(\alpha - \tfrac{1}{2})\right]\right)\left(\frac{s}{s_0}\right)^{\alpha - \frac{1}{2}}, \\ \bar{G}_- &= \frac{C_0 \sqrt{s}}{\sqrt{s_0}}\left(1 + \exp\left[-i\pi(\alpha - \tfrac{1}{2})\right]\right)\left(\frac{s}{s_0}\right)^{\alpha - \frac{1}{2}}. \end{aligned} \right.$$

The parameter D_0 corresponds to $1/\sqrt{u_0}$ in ref. [24], where it was determined to be 1.18 GeV^{-1}. If we neglect the $\pi\mathcal{N}$ mass difference, we can construct an equivalent forward problem by interchanging $\bar{G}_+$ and $\bar{G}_-$, and writing t for u; we can then directly use (A.4).

With the following values of the parameters: $C_0 = 4.45$, and $s_0 = 0.5\ \text{GeV}^2$, the pole term alone fits the data at $P_{\text{lab}} = 5.9\ \text{GeV}/c$; this is the solid curve in Fig. 3. To make the *absorbed* pole better fit the data, we change only the value of s_0—to 0.9 GeV^2—and compute the curves shown in Fig. 4.

APPENDIX B

Calculation of $K^- p \to K^+ \Xi^-$.

Figure 5 is intended as a mnemonic for the following procedure: from the K^*-contribution to the amplitude for $K^- p \to \pi^0 \Sigma^0$, and to $\pi^0 \Sigma^0 \to K^+ \Xi^-$, calculate (two sets of) eikonals according to eq. (A.3), and then calculate a contribution to the amplitude for $K^- p \to K^+ \Xi^-$, according to eq. (A.4). The δ_{ij} in (A.4) is equal to one, since there is only one kind of trajectory involved.

In addition to the picture shown in Fig. 5, we should also consider the contributions of the intermediate states ($\pi^0\Lambda^0$), ($\eta^0\Lambda^0$), and ($\eta^0\Sigma^0$), calculated in the same way. Furthermore, we should include contributions from exchange of the trajectory of the $K^{**}(1400)$. Double exchange of the K^{**} can clearly be treated in the same way as double exchange of the K^*. In the SU_3 limit, there is no contribution from exchange of a K^* together with a K^{**}, since the amplitude is pure [27] in the t-channel, and so must have positive signature. (In the formalism given here, this is guaranteed by the fact, that, in addition to the picture shown in Fig. 5, there must be another kind of picture with the Regge poles crossed, and these two kinds of pictures would cancel if a K^* and a K^{**} were exchanged together. We can forget these crossed pictures if we remember not to exchange a K^* and a K^{**} together.) In principle, we should also consider absorptive effects, that is, there should be Pomeranchukons floating all around Fig. 5.

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[29] K. J. FOLEY, S. J. LINDENBAUM, W. A. LOVE, S. OZAKI, J. J. RUSSELL and L. C. L. YUAN: *Phys. Rev. Lett.*, 11, 425 (1963).

RIASSUNTO (*)

In un articolo precedente si è proposto un modello « ibrido » per lo scattering elastico a grandi energie e piccoli angoli degli adroni e lo si è usato per comprendere la struttura delle sezioni d'urto pione-protone e protone-antiprotone nelle distribuzioni angolari. Questo modello descrive l'ampiezza di scattering come la somma di una parte ottica diffrattiva ed una parte derivante dallo scambio di poli di Regge « assorbiti »; in modo alternativo, il modello può essere preso come una prescrizione specifica per il calcolo degli effetti dei tagli di Regge. In questo articolo si presentano alcuni ulteriori risultati ottenuti da questo modello ibrido: si estrapolano le nostre soluzioni per l'ampiezza di pp ad più alte energie. Si dimostra come questo modello possa essere esteso fino a comprendere scattering anelastici (oppure scattering elastici con prodotti finali all'indietro), ed a considerare processi che non possono essere descritti dal singolo scambio di alcuna traiettoria di Regge nota, come è nel caso dello scattering elastico all'indietro K^-p. Si spiega perchè i tagli di Regge non influiscano sulla presenza o la collocazione della pendenza nella sezione d'urto differenziale per urti quasi all'indietro di π^+p, che si pensa derivi da uno zero senza senso della traiettoria del nucleone. Si predice che le sezioni d'urto differenziali delle reazioni elastiche e anelastiche dovrebbero avere la stessa dipendenza da t per valori di $|t|$ grandi, qualunque sia il loro comportamento per $|t|$ piccoli.

(*) *Traduzione a cura della Redazione.*

Гибридная модель: Дальнейшие результаты.

Резюме (*). — В предыдущей статье мы предложили « гибридную » модель для малоуглового упругого рассеяния адронов при выских энергиях, и использовали ее для того, чтобы понять структуру угловых распределений для протон-протонных и протон-антипротонных дифференциальных поперечных сечений. Эта модель описывает амплитуду рассеяния, как сумму оптической диффракционной части и части, возникающей от обмена « поглощенным » полюсом Редже; с другой стороны, эта модель может быть рассмотрена, как определенный рецепт для вычисления эффектов разрезов Редже. В настоящей работе мы приводим некоторые дальнейшие результаты, полученные из этой гибридной модели: мы экстраполируем наши решения для pp амплитуды для высоких энергий. Мы показываем, как наша модель может быть расширена для рассмотрения неупругого (или обратного упругого) рассеяния, и изучения процессов, которые не могут быть описаны с помощью отдельного обмена любой известной траекторией Редже, такого как K^-p обратное уиругое рассеяние. Мы объясняем, почему разрезы Редже не влияют на присутствие или локализацию провала в π^+p дифференциальном поперечном сечении вблизи направления назад, что, думается, возникает от бессмысленного нуля нуклонной траектории. Мы предсказываем, что дифференциальные поперечные сечения для упругих и неупругих реакций должны иметь ту же самую зависимость от t при больших $|t|$, вне зависимости от того, имели они или нет одинаковую зависимость при малых t.

(*) *Переведено редакцией.*