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TOPICS ON REGGE-POLE THEORY
OF HIGH-ENERGY SCATTERING

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Lectures given in the
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G E N E V A
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PREFACE

The present report contains, with minor additions, the material presented in a lecture course at CERN in the Spring 1968 under the title "Ups and Downs of Regge Poles". A few selected topics were treated in order to illustrate the successes and complications encountered in the description of high-energy scattering processes of hadrons in the framework of Regge-pole theory. The report retains the subdivision of the course in successive lectures.

The lecture notes were taken and drafted by G. Cohen-Tannoudji and W. Drechsler, to whom I want to express my deep appreciation for their excellent and diligent work.

L. Van Hove

CONTENTS

	<u>Page</u>
1. INTRODUCTION	1
2. KINEMATICAL FORMULAE	2
3. CROSSING RELATIONS	5
4. WATSON-SOMMERFELD TRANSFORMATION AND REGGE POLES	12
5. ASYMPTOTIC BEHAVIOUR	20
6. BEHAVIOUR OF REGGE AMPLITUDES FOR POSITIVE t	22
7. THREE EXAMPLES	23
7.1 $\pi^- p \rightarrow \pi^0$ reaction	23
7.2 Backward meson-nucleon scattering	26
7.3 Elastic-like scattering	28
8. FURTHER INTERPRETATION OF THE SIGNATURE	29
9. COMPLICATIONS DUE TO MASS DIFFERENCE AND SPINS	32
9.1 Scattering of spinless particles with different masses	32
9.2 Complications arising from non-vanishing spins of the external particles	36
10. MULTIPLE SCATTERING EFFECTS AND REGGE CUTS	42
11. CONSPIRACY IN PION PHOTOPRODUCTION	53
12. VECTOR MESON PRODUCTION	68
13. NUCLEON-NUCLEON SCATTERING	71
14. REMARK ON FERMION TRAJECTORIES	72
<u>REFERENCES</u>	75

1. INTRODUCTION

In this series of lectures we try to discuss the basic features of the Regge-pole theory and the more recent problems occurring when experimental data are analysed in terms of this model. Although the method of analysis for high-energy processes provided by the Regge-pole model has had continuous success in the last years, there are clear indications of a number of complications. Before we list the points in support of the theory as well as the difficulties, let us touch very briefly upon the historical development.

The theoretical introduction of poles in the angular momentum plane goes back to the fundamental work of Regge¹⁾. After this work, which was done in connection with non-relativistic potential scattering, the usefulness of the Regge-pole idea for relativistic processes was suggested by Chew and Frautschi²⁾, and by Gribov and Pomeranchuk³⁾. Some pedagogical material for introductions into the subject can be found, among others, in the following references: Chan⁴⁾, Svensson⁵⁾, and Van Hove⁶⁾.

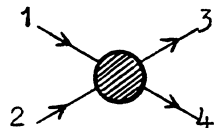
Let us now go through a short list of general features of the Regge-pole model after five years of development.

- 1) Relation between two-body collisions and exchange of known particles: if such exchange is possible, the collision has an appreciable cross-section; otherwise the cross-section is experimentally much smaller.
- 2) Compatibility with the requirement of relativistic invariance and analyticity properties of the S-matrix elements in momentum transfer and energy.
- 3) Diffraction properties and absorption corrections, suggested by the data but complicated when written down in Regge-pole formalism.
- 4) A large number of refinements of the model are needed when confronted with experimental data.
- 5) The Regge-pole model shows a great many mathematical complications resulting from non-vanishing spins and unequal masses of the external particles.

Point (1) states that through the Regge-pole model one has a successful means of studying two-body reactions at high energy and small momentum transfer via an exchange mechanism. The exchanged object is described in a reggeized fashion, which means that it is given a continuously varying spin. It corresponds to a family of known particles of definite quantum numbers but with various spin values. Point (2) states that the mathematical features of the model are compatible with the general principles of Lorentz invariance and analyticity. Points (3), (4), and (5) mention the difficulties of the present model. The relation to the unitarity requirements of the S-matrix remains a problem. Also the representation of elastic diffraction-like processes in terms of the Regge model is unclear. For inelastic processes it is known that absorptive corrections play a role, which also is not easily incorporated into the model. To point (5) one should add that at present it is unknown whether all the complications coming from non-vanishing spins and unequal masses are indeed physical or are merely a property of the mathematical formalism used. One could sum up by saying that the Regge-pole model is not a theory with a high predictive power, but it is a refined framework to relate collisions -- especially of inelastic type -- to exchange processes.

2. KINEMATICAL FORMULAE

After this introduction we list a few formulae of kinematical nature. We restrict ourselves to two-body collisions as the best testing-ground of the model:



$$1 + 2 \rightarrow 3 + 4 \quad (I)$$

The notation will be as follows:

$$p_i = (p_i^1, p_i^2, p_i^3, p_i^4) \quad \text{momentum of particle } i = 1, 2, 3, 4$$

with $p_i^4 = ip_i^0$, p_i^0 being the energy

$$p_i^2 = - \sum_{\mu=1}^4 (p_i^\mu)^2 = (p_i^0)^2 - (p_i^1)^2 - (p_i^2)^2 - (p_i^3)^2 = m_i^2 ,$$

where m_i is the mass of particle i . Analogously the scalar product of two four-vectors is

$$(p \cdot q) = - \sum_{\mu=1}^4 (p^\mu q^\mu) .$$

Energy-momentum conservation means $p_1 + p_2 = p_3 + p_4$. In addition to the momenta p_i , one needs a spin index λ_i for each particle to describe reaction (I). We define, as usual,

$$\begin{aligned} s &= (p_1 + p_2)^2 = \text{square of c.m. energy} \\ t &= (p_1 - p_3)^2 = \text{square of momentum transfer from particle 1 to 3} \\ u &= (p_1 - p_4)^2 = \text{square of momentum transfer from particle 1 to 4.} \end{aligned}$$

These three Lorentz invariant variables are restricted by the relation

$$s + t + u = \sum_{i=1}^4 m_i^2 .$$

A particular process is described through the covariant scattering amplitude $T(p_1 \lambda_1, p_2 \lambda_2; p_3 \lambda_3, p_4 \lambda_4)$, which is related to the S-matrix element by

$$\begin{aligned} &\langle p_3 \lambda_3, p_4 \lambda_4 | S - 1 | p_1 \lambda_1, p_2 \lambda_2 \rangle = \\ &\frac{i \prod_{\alpha=0}^3 \delta(p_1^\alpha + p_2^\alpha - p_3^\alpha - p_4^\alpha)}{\sqrt{p_1^0 p_2^0 p_3^0 p_4^0}} T(p_1 \lambda_1, p_2 \lambda_2; p_3 \lambda_3, p_4 \lambda_4) . \end{aligned} \quad (1)$$

Here the single-particle states are normalized in the following way:

$$\langle p_i' \lambda_i' | p_i \lambda_i \rangle = \delta_{\lambda_i' \lambda_i} \prod_{k=1}^3 \delta(p_i'^k - p_i^k) . \quad (2)$$

From Eq. (1) there follows for the differential cross-section for given polarizations of all particles:

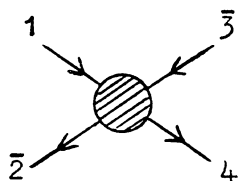
$$\frac{d\sigma}{dt} = \frac{4\pi^3}{sk_s^2} |T(p_1 \lambda_1, p_2 \lambda_2; p_3 \lambda_3, p_4 \lambda_4)|^2 . \quad (3)$$

The last kinematical formula we would like to mention is the optical theorem which expresses the fact that any kind of scattering results in elastic shadow-scattering:

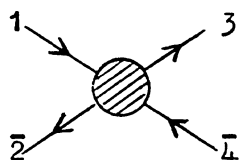
$$\sigma_T(\lambda_1, \lambda_2) = \frac{8\pi}{k_s \sqrt{s}} \operatorname{Im} T_{e1}(p_1 \lambda_1, p_2 \lambda_2; p_1 \lambda_1, p_2 \lambda_2) . \quad (4)$$

Here and in Eq. (3), $k_s = (1/2\sqrt{s}) [(s-(m_1 + m_2)^2)][s-(m_1 - m_2)^2]$ is the c.m. momentum of the incoming particles. σ_T is the total cross-section, and T_{e1} the amplitude for elastic forward scattering without change of spin, both for fully polarized particles. A detailed derivation of Eqs. (3) and (4) can be found elsewhere⁷⁾.

Let us come now to a property which is crucial to high-energy scattering and Regge-pole theory: the crossing property. This property which has first been discovered in quantum electrodynamics is assumed to be valid in all forms of S-matrix theories. It corresponds to the relation which exists between a phenomenon where a particle is absorbed and the one where the corresponding antiparticle is created. In terms of two-body processes, the crossing property is a relation between reaction (I) and the following two reactions:



$$1 + \bar{3} \rightarrow \bar{2} + 4 \quad (II)$$



$$1 + \bar{4} \rightarrow 3 + \bar{2} \quad (III)$$

(By "crossing" pairs of particles one can obtain further reactions, but these are related to (I), (II), and (III) by C, T, or CT invariance.) Let us call $T^{(s)}$, $T^{(t)}$, and $T^{(u)}$ the scattering amplitudes describing the reactions (I), (II), and (III), respectively, where the superscripts are chosen because, as we shall see, s, t, or u is the respective c.m. energy

of the process. The actual statement of the crossing property is that if $T(s)$, $T(t)$, and $T(u)$ are analytic functions of the four-momenta, they are related through analytic continuation. So it is one analytic function T which determines all three reactions when continuation is performed to the respective physical regions of the variables s , t , and u .

LECTURE II

3. CROSSING RELATIONS

Let us now try to be more explicit and to sketch how the crossing property can be derived from relativistic quantum field theory^{*)}. The field operator $\Phi_j(x)$ (x stands for the space-time four-vector) which describes particle j ($j = 1, \dots, 4$) is supposed to tend to asymptotic field operators when the time goes to $\pm \infty$:

$$\Phi_j(x) \underset{x_0 \rightarrow -\infty}{\rightarrow} \Phi_j^{\text{in}}(x), \quad \Phi_j(x) \underset{x_0 \rightarrow +\infty}{\rightarrow} \Phi_j^{\text{out}}(x).$$

The asymptotic field operators Φ_j^{in} , Φ_j^{out} describe the incoming and outgoing particles, respectively. They can be written in terms of creation and annihilation operators:

$$\Phi_j^{\text{in/out}}(x) = \sum_{k,\lambda} \left(a_{j,\vec{k},\lambda}^{\text{in/out}} u_{j,\vec{k},\lambda} e^{ikx} + b_{j,\vec{k},\lambda}^{\text{in/out}} v_{j,\vec{k},\lambda} e^{-ikx} \right), \quad (5)$$

where $u_{j\vec{k}\lambda}$ is a spin wave function describing the spin orientation state λ of a particle j when it has three-momentum \vec{k} and energy $k^0 = +\sqrt{|\vec{k}|^2 + m^2}$ (for a spin $1/2$ particle $u_{j\vec{k}\lambda}$ would be a Dirac spinor, for a spin 1 particle it would be a polarization four-vector, and for a spin $3/2$ it would be a Rarita-Schwinger spinor). In the same way $v_{j\vec{k}\lambda}$

*) A simple mathematical description of the crossing property, in which the required analyticity is not proved but is assumed, can be found in a paper by Svensson⁸⁾, and also by Logunov et al.⁹⁾. The proof from axiomatic field theory, of sufficiently large analyticity domains to perform the crossing, is given by Bros et al.¹⁰⁾.

is a spin wave function describing the spin orientation of particle \bar{j} , the antiparticle of particle j . $a_{j\vec{k}\lambda}^{\text{in}}$ is a creation operator which creates an incoming or outgoing particle j in the state defined by \vec{k} and λ , whereas $b_{j\vec{k}\lambda}^{\text{out}}$ is an annihilation operator which annihilates \bar{j} , the antiparticle of j in the state defined by \vec{k} and λ .

In order to define a quantum field theory, one needs to postulate commutation or anticommutation rules for the field operators; in particular the rule

$$\left[\Phi_j(x), \Phi_{j'}(x') \right]_{\pm} = 0$$

if $x-x'$ is a space-like four vector. The "+" subscript refers to the anticommutator (to be used for fermion fields) and the "-" subscript to the commutator (boson fields). About this important property, which expresses the locality of the interactions, we shall say only that it is at the origin of the derivations of the analyticity properties used below for the scattering amplitudes.

Using now the so-called "reduction formulae" of Lehmann, Symanzik and Zimmerman¹¹⁾, one can express each scattering amplitude in terms of the expectation value in the vacuum state $|0\rangle$ of a product of operators $\Phi(x)$, one for each particle involved. For the s-channel reaction $1+2 \rightarrow 3+4$, one can write (we limit ourselves to spinless particles and drop some irrelevant coefficients)

$$T^{(s)}(p_1, p_2, p_3, p_4) = \dots \int d_4x_1 d_4x_2 d_4x_3 d_4x_4 e^{-ip_1x_1} e^{-ip_2x_2} e^{ip_3x_3} e^{ip_4x_4} \cdot K_1 K_2 K_3 K_4 \langle 0 | T[\Phi_1(x_1) \Phi_2(x_2) \Phi_3(x_3) \Phi_4(x_4)] | 0 \rangle \quad (6)$$

where the K_j are the Klein-Gordon operators $\partial^2/\partial x_j^2 - m_j^2$, and where T denotes an ordering operator for the product $\Phi_1 \Phi_2 \Phi_3 \Phi_4$, putting it in the order of decreasing times

$$T[\Phi_1(x_1) \Phi_2(x_2) \Phi_3(x_3) \Phi_4(x_4)] = \Phi_{j_1}(x_1) \Phi_{j_2}(x_2) \Phi_{j_3}(x_3) \Phi_{j_4}(x_4)$$

when $x_{j_1}^0 > x_{j_2}^0 > x_{j_3}^0 > x_{j_4}^0$.

One can more or less understand Eq. (6) as follows. The Fourier integral $\int d_4 x_1 e^{-ip_1 x_1}$ picks out of $\Phi_1(x_1)$ the Fourier component e^{ikx} with $k = p_1$. Its contribution to Eq. (6) comes from the asymptotic part of Φ_1 toward $x_1^0 \rightarrow -\infty$, which is Φ_1^{in} . It picks out of Eq. (5) the term with the creation operator $a_{1\vec{p}_1}^{\text{in}\dagger}$; it thus creates the incident particle 1. Similarly, $\int d_4 x_2 e^{-ip_2 x_2}$ picks out the $a_{2\vec{p}_2}^{\text{in}\dagger}$ of Φ_2^{in} and creates 2. $\int d_4 x_3 e^{+ip_3 x_3}$ gets its contribution from the asymptotic part Φ_3^{out} of Φ_3 ($x_3^0 \rightarrow +\infty$), picking out the term with the annihilation operator $b_{3\vec{p}_3}$ which annihilates the outgoing particle 3, and similarly for 4.

In the same way, the scattering amplitude for the t-channel reaction $1+\bar{3} \rightarrow \bar{2}+4$ can be written [with the same irrelevant coefficients as in $T^{(s)}$]

$$T^{(t)}(p_1, p_3, p_2, p_4) = \dots \int d_4 x_1 d_4 x_2 d_4 x_3 d_4 x_4 e^{-ip_1 x_1} e^{-ip_3 x_2} e^{ip_2 x_2} e^{ip_4 x_4} \cdot K_1 K_2 K_3 K_4 \langle 0 | T[\Phi_1(x_1) \Phi_1(x_2) \Phi_3(x_3) \Phi_4(x_4)] | 0 \rangle . \quad (7)$$

The changes with respect to Eq. (6) are that $\int d_4 x_2 e^{ip_2 x_2}$ now picks out the b_2^{out} part of Φ_2^{out} , which annihilates the outgoing particle $\bar{2}$, and $\int d_4 x_3 e^{-ip_3 x_3}$ takes the $a_3^{\text{in}\dagger}$ part of Φ_3^{in} which creates the incoming particle $\bar{3}$.

Equations (6) and (7) suggest immediately that $T^{(s)}$ and $T^{(t)}$ are given by a common function, namely

$$F(q_1, q_2, q_3, q_4) = \dots \int d_4 x_1 d_4 x_2 d_4 x_3 d_4 x_4 e^{iq_1 x_1} e^{iq_2 x_2} e^{iq_3 x_3} e^{iq_4 x_4} \cdot K_1 K_2 K_3 K_4 \langle 0 | T[\Phi_1(x_1) \Phi_1(x_2) \Phi_3(x_3) \Phi_4(x_4)] | 0 \rangle . \quad (8)$$

Indeed, Eqs. (6) and (7) now read formally,

$$T^{(s)}(p_1, p_2, p_3, p_4) = F(-p_1, -p_2, p_3, p_4) \quad (6')$$

$$T^{(t)}(p_1, p_3, p_2, p_4) = F(-p_1, p_2, -p_3, p_4) \quad (7')$$

These relations are only meaningful, however, if the domain of analyticity of F in q_1, \dots, q_4 is sufficiently large to extend from the values taken by these four-vectors in Eq. (6') to those taken in Eq. (7'). These values are always different, as is seen from the fact that particle energies such as $p_2^0, p_2^0, p_3^0, p_3^0$ are always positive. From the axioms of local field theory, especially the commutation or anticommutation rules, one knows that F has indeed the necessary analyticity domain; the general case of this theorem has been proved by Bros, Epstein and Glaser¹⁰⁾.

The traditional formulation of the crossing property is now read from Eqs. (6') and (7'). It is the following:

$$\begin{aligned}
 T^{(s)}(p_1, p_2, p_3, p_4) &= \left[T^{(t)}(p_1, p_3, p_2, p_4) \right] \text{ continued} \\
 &\text{when the analytic continuation is done in } p_2 \text{ and } p_3 \text{ to} \\
 p_2^0 &= -p_2^0 \text{ and } p_3^0 = -p_3^0.
 \end{aligned}
 \tag{9}$$

Of course the analytic continuation must be done in a proper way; it must avoid all the possible cuts and preserve four-momentum conservation $p_1 + p_3 = p_2 + p_4$. Analyticity holds in the upper-half energy planes, the cuts being on the real energy axes and the physical values of $T^{(t)}$ being obtained on the upper lips, i.e. for

$$\begin{aligned}
 p_2^0 &= \text{Re } p_2^0 + i0 \\
 p_3^0 &= \text{Re } p_3^0 + i0.
 \end{aligned}$$

This agrees with the fact that in Eq. (7) the x_2 integral picks out the outgoing contribution ($x_2^0 \rightarrow +\infty$), so that $e^{ip_2^0 x_2}$ is a converging factor. Similarly the x_3 integral picks out the incoming contribution ($x_3^0 \rightarrow -\infty$).

The analytic continuation mentioned in Eq. (9) is most easily done by keeping the imaginary parts of p_2^0 and p_3^0 positive. But it then leads to values

$$\begin{aligned}
 p_2^0 &= -\text{Re } p_2^0 - i0 \\
 p_3^0 &= -\text{Re } p_3^0 - i0
 \end{aligned}$$

which are below the real p_2^0, p_3^0 axes. Now $T^{(s)}$ has similar analyticity properties to $T^{(t)}$; it is analytic in the upper-half energy planes of p_2^0, p_3^0 with cuts on the real axes, and the physical values of $T^{(s)}$ are obtained on the upper lips, i.e. for

$$p_2^0 = \text{Re } p_2^0 + i0$$

$$p_3^0 = \text{Re } p_3^0 + i0 .$$

Equation (9), on the other hand, when the continuation is done in the upper-half p_2^0, p_3^0 planes, leads us to the lower lips.

One can, however, go from lower to upper lip by using the following property (all p_j^0 are taken real except p_2^0 and p_3^0):

$$T^{(s)}(p_1 p_2 p_3 p_4) = [T^s(p_1 p_2^* p_3^* p_4)]^*$$

If the value of $T^{(s)}$ in the right-hand side is taken on the lower lip, $T^{(s)}$ on the left is on the upper one because of the replacements $p_{2,3}^* \rightarrow p_{2,3}$. The proof of this relation uses PT invariance (P = space inversion, T = time reversal). We sketch the argument in a simple form. For any matrix element $\langle \psi | 0 | \varphi \rangle$ of any operator 0, the PT transformed quantities

$$|\varphi' \rangle = \text{PT}|\varphi \rangle , \quad |\psi' \rangle = \text{PT}|\psi \rangle , \quad 0' = (\text{PT}) 0 (\text{PT})^{-1}$$

obey the relation

$$\langle \psi | 0 | \varphi \rangle = \langle \psi' | 0' | \varphi' \rangle^* .$$

The complex conjugation stems from the fact that PT is antiunitary [for details, see e.g. L. Van Hove, CERN Report 67-27 (1967), p. 28]. If 0 is a function of operators Q with complex coefficients α :

$$0 = F(Q, \alpha) ,$$

then one obtains for the transformed O' :

$$O' = F(Q', \alpha^*),$$

α^* occurring instead of α because of the antiunitarity of PT. The S-matrix written in terms of a Hamiltonian operator H has such a form, α being a complex energy parameter related to $p_{2,3}^0$. PT invariance means $H' = H$, so that the transformed S' of S is obtained by the replacement $\alpha \rightarrow \alpha^*$, i.e. $p_{2,3} \rightarrow p_{2,3}^*$. Hence

$$\langle \psi | S(\alpha) | \varphi \rangle = \langle \psi' | S(\alpha^*) | \varphi' \rangle^*.$$

This proves our above relation for $T^{(s)}$.

We can now write a new form for the crossing relation (9), valid for $p_{2,3}^0 = \text{Re } p_{2,3}^0 + i0$. It is

$$T^{(s)}(p_1, p_2, p_3, p_4) = [T^{(t)}(p_1, p_{\bar{3}}, p_{\bar{2}}, p_4)]_{\text{continued}}^* \quad (9')$$

where the analytic continuation is done in $p_{\bar{2}}, p_{\bar{3}}$ with $\text{Im } p_{\bar{2}, \bar{3}}^0 > 0$, up to the values $p_{\bar{2}} = -p_2^*$, $p_{\bar{3}} = -p_3^*$; the complex conjugation [...] is taken after the analytic continuation is carried out.

We remark briefly on the fact that under the substitution $p_2 \rightarrow -p_{\bar{2}}, p_3 \rightarrow -p_{\bar{3}}$ involved in crossing (we now neglect small imaginary parts), the variables s, t, u defined above become

$$s = (p_1 + p_2)^2 \rightarrow (p_1 - p_{\bar{2}})^2$$

$$t = (p_1 - p_3)^2 \rightarrow (p_1 + p_{\bar{3}})^2$$

$$u = (p_1 - p_4)^2 \text{ remains unchanged.}$$

t becomes the (c.m. energy)² of the reaction $1+\bar{3} \rightarrow \bar{2}+4$; this is why one usually calls it the "t-channel reaction", $1+2 \rightarrow 3+4$ being the "s-channel reaction".

We now say a few words on the spin complications. In presence of spins, Eq. (9) must be replaced by a matrix equation:

$$T^{(s)}(p_1, \lambda_1, p_2, \lambda_2, p_3, \lambda_3, p_4, \lambda_4) = \sum_{\lambda'_1, \lambda'_2, \lambda'_3, \lambda'_4} C(p_i, \lambda_i, \lambda'_i) [T^{(t)}(p_1, \lambda'_1, p_3, \lambda'_3, p_2, \lambda'_2, p_4, \lambda'_4)]$$

with $[T^{(t)}]$ analytically continued to $p_2 = -p_2$ and $p_3 = -p_3$. The "crossing matrix" $C(p_i, \lambda_i, \lambda'_i)$ is a more or less complicated matrix depending on the way in which the spin orientations are defined. In most cases, the amplitudes $T^{(t)}$ have a lot of singularities in the four-momenta, which are known not to appear in $T^{(s)}$ [kinematical singularities of $T^{(t)}$]. These singularities of the various $T^{(t)}$ will then be forced to cancel out against each other in the linear combination $\sum_{\lambda} C T^{(t)}$ generated by the crossing matrix [kinematical constraints for $T^{(t)}$]. One of the troubles of Regge-pole theory is that, in order to describe $T^{(s)}$, it gives a theoretical model for $T^{(t)}$. This model is forced to embody the cumbersome kinematical singularities and constraints of $T^{(t)}$ with the sole purpose of cancelling them out when one goes over to the physically relevant amplitude $T^{(s)}$. We shall return to these problems later.

Similarly to the crossing relation between $T^{(s)}$ and $T^{(t)}$, there is such a relation between $T^{(s)}$ and $T^{(u)}$, and one between $T^{(t)}$ and $T^{(u)}$, where $T^{(u)}$ refers to the "u-channel reaction" $1+\bar{4} \rightarrow 3+\bar{2}$ with (c.m. energy)² = u. In the spinless case and in the form analogous to Eq. (9') they read

$$T^{(s)}(p_1, p_2, p_3, p_4) = [T^{(u)}(p_1, p_4, p_3, p_2)]^* \text{ cont. to } p_2 = -p_2^*, p_4 = -p_4^* \quad (9'')$$

$$T^{(u)}(p_1, p_4, p_3, p_2) = [T^{(t)}(p_1, p_3, p_2, p_4)]^* \text{ cont. to } p_3 = -p_3^*, p_4 = -p_4^* \quad (9''')$$

It should also be remarked that, beyond certain coefficients which are relativistically invariant combinations of four-momenta and spin wave functions, the amplitudes $T^{(s)}$, $T^{(t)}$, and $T^{(u)}$ are functions of s and t

only (remember that $u = \sum_{j=1}^4 m_j^2 - s - t$). Regarding analyticity, $T(s)$ is, at fixed t and also at fixed u , analytic in s in the upper-half s -plane; its cuts are on the real s -axis and its physical values are obtained on the upper lip. [Indeed, $s = (p_1 + p_2)^2 = m_1^2 + m_2^2 + 2p_1^0 p_2^0 - 2\vec{p}_1 \cdot \vec{p}_2$ gets a positive imaginary part if p_2^0 has one.] $T(t)$ has similar properties in t at fixed s and at fixed u , and so has $T(u)$ in u at fixed s and at fixed t . The analytic continuations of $T(t)$ in Eqs. (9') and (9'') are in the upper-half t -plane at fixed u and fixed s , respectively; the continuation of $T(u)$ in Eq. (9'') is in the upper-half u -plane at fixed t .

Although we will not need it for the present lectures, we make a final remark on what happens when all four particles are crossed in the reaction $1+2 \rightarrow 3+4$ (up to now only two particles were crossed, the others remaining untouched). The reaction then becomes $\bar{3}+\bar{4} \rightarrow \bar{1}+\bar{2}$, which is the CPT transform of $1+2 \rightarrow 3+4$ (PT defined as before, C is the charge conjugation). Its scattering amplitude $\bar{T}(s)$ is obtained from the one $T(s)$ of $1+2 \rightarrow 3+4$ by analytic continuation in all p_j to the values

$$p_1 = p_{\bar{1}}, \quad p_2 = -p_{\bar{2}}, \quad p_3 = -p_{\bar{3}}, \quad p_4 = -p_{\bar{4}}.$$

Hence $s = (p_1 + p_2)^2$ becomes $(p_{\bar{1}} + p_{\bar{2}})^2$ which is again the (c.m. energy)², and $t = (p_1 - p_3)^2$ becomes $(p_{\bar{1}} - p_{\bar{3}})^2$, again the (four-momentum transfer)². Leaving out spin complications, $T(s)$ and $\bar{T}(s)$ are functions of s and t only. Analytic continuation then gives (the argument does not require use of PT invariance)

$$\bar{T}(s)(s, t) = T(s)(s, t)$$

which states CPT invariance of the scattering. This is a special case of the well-known CPT theorem which states that any relativistic, local field theory is CPT invariant.

LECTURE III

4. WATSON-SOMMERFELD TRANSFORMATION AND REGGE POLES

After the foregoing discussion of the crossing principle we now turn to the main mechanism which will lead to the typical Regge behaviour of

an amplitude for large energies and small momentum transfers. The mathematical trick which is used to transform the scattering amplitude and obtain its asymptotic behaviour was first discussed in the mathematical literature by Nicholson^{'2)} in 1907 and by Poincaré^{'3)} in 1910. It was re-discovered in 1918 by Watson^{'4)}, and used later by Sommerfeld^{'5)} to treat the problem of propagation of radio waves on the surface of the earth. [These historical remarks are taken from the book by Frautschi^{'6)}.]

We shall now discuss this so-called Watson-Sommerfeld transformation by taking the simplest case of relativistic scattering, namely for spinless equal-mass particles. The amplitude for the s-channel reaction $1+2 \rightarrow 3+4$ is called $T^{(s)}(s,t)$, with

$$s = (p_1 + p_2)^2 = 4(k_s^2 + m^2)$$

and

$$t = (p_1 + p_3)^2 = -2k_s^2(1 - \cos \Theta_s) ,$$

k_s denotes the c.m. momentum and Θ_s the c.m. scattering angle between particle 1 and 3, both referring to the s-channel. Analogously, $T^{(t)}(s,t)$ describes the t-channel reaction $1+\bar{3} \rightarrow \bar{2}+4$, with

$$s = (p_1 - p_{\bar{2}})^2 = -2k_t^2(1 - \cos \Theta_t)$$

and

$$t = (p_1 + p_{\bar{3}})^2 = 4(k_t^2 + m^2) ,$$

where k_t denotes the c.m. momentum, and Θ_t the scattering angle from particle 1 to particle $\bar{2}$. As was discussed extensively above, the amplitudes $T^{(s)}(s,t)$ and $T^{(t)}(s,t)$ are related by crossing. Using a path for the analytic continuation to $(\vec{p}_2, p_2^0) = -(\vec{p}_2, p_2^0)^*$ and $(\vec{p}_3, p_3^0) = -(\vec{p}_3, p_3^0)^*$ which lies wholly in the upper-half s-plane (which means that $\text{Im } p_2^0, \text{Im } p_3^0, \text{Im } p_2^0, \text{ and } \text{Im } p_3^0$ are all positive), $T^{(s)}(s,t)$ is related to the complex conjugate of $T^{(t)}(s,t)$

$$T^{(s)}(s,t) = \left(T^{(t)}(s,t) \right)^* \text{ cont. anal. in } p_{\bar{2}} \text{ and } p_{\bar{3}} \text{ to } p_{\bar{2}} = -p_2^* \text{ and } p_{\bar{3}} = -p_3^* .$$

We now consider the partial wave expansion in the t-channel:

$$T^{(t)}(s,t) = \sum_{\ell=0}^{\infty} (2\ell+1)\zeta_{\ell}(t)P_{\ell}(\cos \Theta_t) \quad ; \quad \cos \Theta_t = \frac{s + \frac{1}{2}t - 2m^2}{\frac{1}{2}t - 2m^2}, \quad (10)$$

where the $\zeta_{\ell}(t)$ are the partial wave amplitudes. The function (10) has to be continued analytically into the region where it represents $T^{(s)}(s,t)$. We are especially interested in an expression valid for large energies and small momentum transfers in the s-channel. We therefore want to continue analytically from the physical region of the t-channel ($t > 4m^2$; $-4k_t^2 < s < 0$) to large, positive values of s, and to finite, negative values of t. For this domain of s and t the argument of the Legendre function in Eq. (10) becomes large and negative:

$$\cos \Theta_t \xrightarrow[s \rightarrow \infty]{t < 0, \text{ finite}} -\infty.$$

Whereas the $\zeta_{\ell}(t)$ in Eq. (10) change only a little in the analytic continuation involved in t, one has to consider a large continuation for the Legendre functions; this turns out to be manageable because the latter functions are well known. Because of the relation

$$P_{\ell}(z) \underset{z \rightarrow \infty}{\sim} z^{\ell},$$

the convergence of the series (10) is spoilt during the analytic continuation. The mathematical trick now is to abandon the series expansion (10) and to replace the sum by a complex contour integral in the ℓ -plane. The idea is then to move this contour in order to obtain a representation of the original partial wave sum with much better convergence properties and a wider domain of validity in $\cos \Theta_t$. This last step is known as the Watson-Sommerfeld transformation. It appears, however, that the whole procedure has to be done in two steps, treating even and odd partial waves separately.

We now express the sum (10) by the following integral taken along the contour C in the complex l -plane shown in Fig. 1:

$$T^{(t)}(s,t) = \frac{1}{2i} \int_C (2l+1) \zeta_l(t) \frac{P_l(-\cos \theta_t)}{\sin \pi l} dl . \quad (11)$$

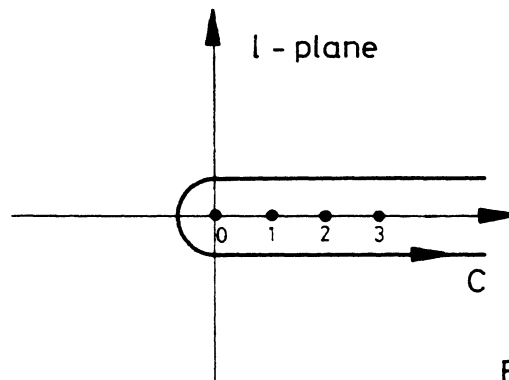
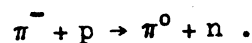


FIG.1

It is assumed here that the $\zeta_l(t)$ have no singularities in l near the positive real axis. So one reobtains the original partial wave sum by taking the residues of the function $1/\sin \pi l$ at $l = 0, 1, 2, \dots$, and using the relation $P_l(-\cos \theta_t) = (-1)^l P_l(\cos \theta_t)$ valid at these points.

The crucial requirement in going from Eq. (10) to Eq. (11) is that $\zeta_l(t)$ is an analytic function in l which interpolates the physical partial wave amplitudes ζ_l for $l = 0, 1, 2, \dots$. It was shown by Regge in the framework of non-relativistic potential scattering that for a certain class of potentials the partial wave amplitudes can indeed be uniquely continued to complex values of l , and that they have the property of allowing a Watson-Sommerfeld transformation on the complete scattering amplitude. It is not possible to establish a similar property of the partial wave amplitudes in the relativistic case. But the analytic continuation away from the physical l -values is again possible provided one considers two interpolations, one for even l -values (the so-called "positive signature" partial waves), and another for odd l -values ("negative signature" partial waves). A transparent example for the impossibility of obtaining one interpolation for all physical partial wave amplitudes is given by the following s -channel reaction



When analysed in the t-channel where the reaction is $\pi^- + \pi^0 \rightarrow \bar{p} + n$, it follows from the requirements of Bose statistics for the initial $\pi\pi$ state, and from isospin invariance of strong interaction (only I=1 states contribute), that all even partial wave amplitudes vanish and only the odd waves contribute. The question now arises whether there exists one analytic function $\zeta_\ell(t)$ such that at the physical points one has:

$$\zeta_0, \zeta_2, \zeta_4 \dots = 0$$

$$\zeta_1, \zeta_3, \zeta_5 \dots \neq 0 .$$

It follows from Carlson's theorem that such a function does not exist. This theorem states the following:

$$\begin{aligned} & \text{if } f(z) \text{ is holomorphic for } \operatorname{Re} z \geq 0 \\ & \text{and } f(z)/e^{\lambda|z|} \rightarrow 0 \text{ for } z \rightarrow \infty, \quad \lambda < \pi . \\ & \text{and further } f(n) = 0 \quad \text{for } n = 0, 1, 2, \dots , \\ & \text{then } f(z) \equiv 0 . \end{aligned}$$

For this and other relevant mathematical facts, we refer to Squires¹⁷⁾.

It follows from this discussion that even and odd signature partial waves are physically different, thus reflecting the parity dependence of the interaction in the t-channel. One therefore has to determine the interpolating function separately for even and odd orbital angular momentum. Then the same theorem guarantees the uniqueness of the interpolation in each case. Explicitly the procedure is as follows. The amplitude $T^{(t)}(s,t)$ is separated in positive and negative signature parts:

$$T^{(t)}(s,t) = T_+^{(t)}(s,t) + T_-^{(t)}(s,t) , \quad (10')$$

with

$$T_+^{(t)}(s,t) = \sum_{\ell=\text{even}} (2\ell+1)\zeta_\ell(t)P_\ell(\cos \Theta_t) ,$$

and

$$T_-^{(t)}(s,t) = \sum_{\ell=\text{odd}} (2\ell+1)\zeta_\ell(t)P_\ell(\cos \Theta_t) .$$

We now define for general l the functions $\zeta_l^\pm(t)$, where $\zeta_l^+(t)$ is the analytic interpolation of $\zeta_l(t)$ for $l = 0, 2, 4, \dots$, and $\zeta_l^-(t)$ is the analytic interpolation of $\zeta_l(t)$ for $l = 1, 3, 5, \dots$, and obtain as in the case of Eq. (11)

$$T_{\pm}^{(t)}(s, t) = \frac{1}{2i} \int_C (2l+1) \zeta_l^\pm(t) \frac{1}{2} \frac{[P_l(-\cos \Theta_t) \pm P_l(\cos \Theta_t)]}{\sin \pi l} dl. \quad (12)$$

In Eqs. (11) and (12) one has, of course, to insert an interpolating expression for the Legendre functions. This is provided by expressing the P_l in terms of hypergeometric functions. For complex l one still finds the relation

$$P_l(-\cos \Theta_t) \sim (-\cos \Theta_t)^{\operatorname{Re} l + i \operatorname{Im} l}$$

for $\operatorname{Re} l \geq -\frac{1}{2}$ and large $\cos \Theta_t$. This follows from the equation (l real or complex):

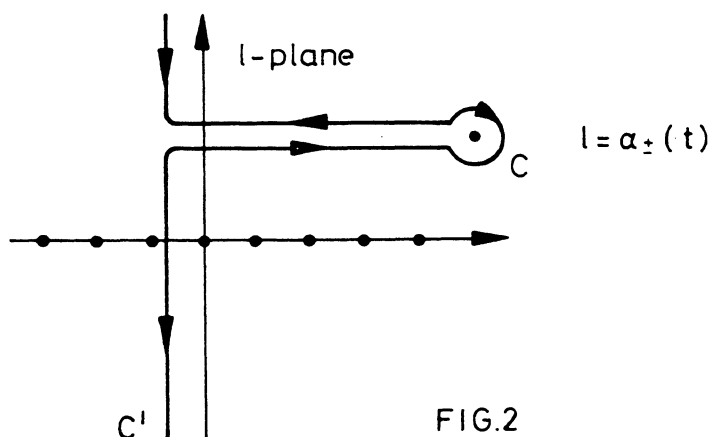
$$P_l(z) \underset{z \rightarrow \infty}{\sim} \frac{\Gamma\left(l + \frac{1}{2}\right)}{\Gamma(l+1)} \frac{(2z)^l}{\sqrt{\pi}} F\left(-\frac{l}{2}, -\frac{l-1}{2}; -l + \frac{1}{2}; \frac{1}{z^2}\right) + \frac{\Gamma\left(-l - \frac{1}{2}\right)}{\Gamma(-l)} \frac{(2z)^{-l-1}}{\sqrt{\pi}} F\left(\frac{l+1}{2}, \frac{l}{2} + 1; l + \frac{3}{2}; \frac{1}{z^2}\right),$$

when the hypergeometric function F in the right-hand side tends to 1 for $z \rightarrow \infty$. To obtain a convergent expression for the amplitude continued in s and t , one therefore wants to have $\operatorname{Re} l$ small or negative (but $\operatorname{Re} l$ not smaller than $-\frac{1}{2}$).

We now assume that the asymptotic properties in l of the functions $\zeta_l^\pm(t)$ are such that a Watson-Sommerfeld transformation of $T_{\pm}^{(t)}(s, t)$ is possible, i.e. that one can open up the contour C of Fig. 1 toward the left until it runs parallel to the imaginary axis, and that the infinite half circles give no contribution (because of the asymptotic behaviour of P_l , the contour can best be deformed onto $\operatorname{Re} l = -\frac{1}{2}$). Since the

function $P_\ell/\sin \pi\ell$ has no complex singularities in ℓ , one will then pick up singularities of $\zeta_\ell^\pm(t)$. Let us first assume that $\zeta_\ell^\pm(t)$ has a pole at $\ell = \alpha_\pm(t)$. If $\zeta_\ell^\pm(t)$ has a number of such poles, each will contribute, but the one which lies farthest to the right, i.e. the term with the biggest real part of ℓ , will give the leading behaviour of the amplitude for large s . Such poles of ζ_ℓ^\pm are called Regge poles. The crucial point is that $\zeta_\ell^\pm(t)$ does not have an accumulation of singularities with $\text{Re } \alpha(t) \rightarrow \infty$, since then the Watson-Sommerfeld transformation would not lead to a representation of the scattering amplitude with improved asymptotic behaviour as compared to the original partial wave sum (10). However, it follows from the Mandelstam representation that the Regge poles cannot penetrate infinitely far into the right-half ℓ -plane.

The contour C of Fig. 1 will be deformed in such a way that it coincides with the path C' of Fig. 2



Calling $\beta^\pm(t)$ the residue of $\zeta_\ell^\pm(t)$ at the pole $\ell = \alpha_\pm(t)$, one finally obtains for the s -channel amplitude at large s when only the rightmost Regge pole in the ℓ -plane is considered:

$$\begin{aligned} \left(T^{(s)}(s, t) \right)^* &= \left(T^{(t)}(s, t) \right) \begin{array}{c} \text{continued} \\ s \rightarrow \infty; t < 0, \text{ finite} \end{array} \\ &\sim -\pi(2\alpha_\pm + 1)\beta^\pm(t) \frac{\frac{1}{2} \left[P_{\alpha_\pm}(-\cos \Theta_t) \pm P_{\alpha_\pm}(\cos \Theta_t) \right]}{\sin \pi\alpha_\pm}. \end{aligned} \quad (13)$$

If there are several Regge poles, each will contribute a term of the above form.

If the $\zeta_l^+(t)$ has a branch-point at $l = \alpha_{\pm}^C(+)$ and a Regge cut extending from it towards the left, one arrives, after deforming the contour C , at a path shown in Fig. 3. Denoting with $\beta_{\alpha}^+(t)$ the discontinuity across the cut starting at $\alpha_{\pm}^C(t)$, its contribution to the s -channel amplitude is written:

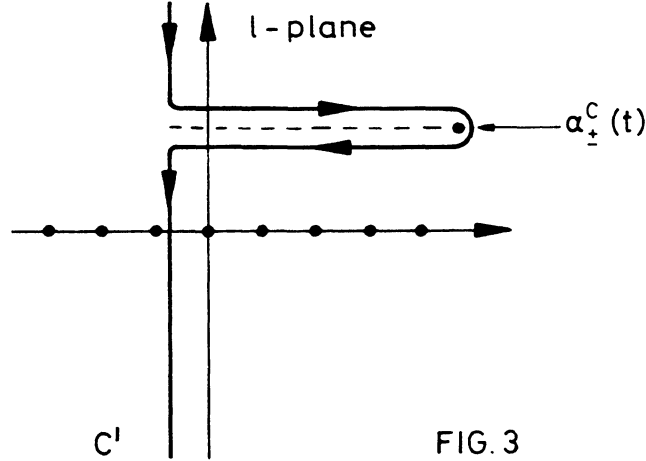


FIG. 3

$$\begin{aligned} \left(T^{(s)}(s, t) \right)^* &= \left(T^{(t)}(s, t) \right) \quad \text{continued} \\ &\quad s \rightarrow \infty; t < 0, \text{ finite} \\ &\sim \frac{1}{2i} \int_{\text{out}} d\alpha (2\alpha + 1) \beta_{\alpha}^+(t) \frac{\frac{1}{2} \left[P_{\alpha}(-\cos \Theta_t) \pm P_{\alpha}(\cos \Theta_t) \right]}{\sin \pi \alpha}. \end{aligned} \quad (14)$$

Here and in Eq. (13) we have neglected the background integral which is that part of the contour C' in Figs. 2 and 3 running parallel to the imaginary axis at $\text{Re } l = -\frac{1}{2}$. This is justified because only the right-most singularities matter for large s . Thus it is really the region of the Regge cut nearest to the branch-point which contributes significantly.

We remark that a Regge cut can always be approximated by a sum over many Regge poles: this is indeed seen by approximating the integral in Eq. (14) by a sum over discrete α values. Such an approximation becomes increasingly poorer for increasing s . In practice, however, one has at small t for $\cos \Theta_t$ an order of magnitude $|\cos \Theta_t| \sim s \approx 2p_{\text{lab}}$ in GeV/c. This allows one to estimate how sensitive the amplitude is to the precise values of α , because a shift $\Delta\alpha$ of α contributes a factor $|\cos \Theta_t|^{\Delta\alpha}$. If experiments cover a range $z_1 \leq |\cos \Theta_t| \leq z_2$, the uncertainty $\Delta\alpha$ on α will

appear in the form $(z_2/z_1)^{\Delta\alpha}$. At present we have often $z_2/z_1 \sim 2$ or 3 . The uncertainty $\Delta\alpha$ therefore often remains at or above 0.1 ($3^{0.1} \approx 1.1$). Hence present experiments cannot distinguish between cuts and groups of poles separated by $\Delta\alpha \sim 0.1$.

We finally derive from Eqs. (13) and (14) the following truly asymptotic behaviour when only the leading term for $s \rightarrow \infty$ is kept :

$$T^{(s)}(s,t) \sim s^{\alpha_{\pm}(t)} \quad \text{for a Regge pole ,} \quad (15)$$

$$T^{(s)}(s,t) \sim \frac{s^{\alpha_{\pm}^c(t)}}{(\log s)^{\gamma}} \quad \text{for a Regge cut .} \quad (16)$$

In the last relation γ is a number depending on the nature of the branch-point $\alpha_{\pm}^c(t)$.

LECTURE IV

5. ASYMPTOTIC BEHAVIOUR

For phenomenological purposes it is very important to know the behaviour of expressions (13) and (14) when s is large [expressions (15) and (16) are only incomplete approximations neglecting all coefficients]. We are going to show that in the Regge theory one can say more than is contained in these latter expressions; in particular, that the phases of the amplitudes can be predicted. For that, one needs first to know the behaviour of $P_{\alpha}(z)$ when its argument z is large. $P_{\alpha}(z)$ is the analytic interpolation of the Legendre polynomials $P_{\ell}(z)$ to complex $\ell = \alpha$; it can be written in terms of hypergeometric functions, which allows one to find its asymptotic behaviour. As already mentioned, one finds that

$$\text{for} \quad \text{Re } \alpha \geq -\frac{1}{2},$$

$$\text{and} \quad |z| \rightarrow \infty,$$

$$P_{\alpha}(z) \simeq \frac{2^{\alpha}}{\sqrt{\pi}} \frac{\Gamma\left(\alpha + \frac{1}{2}\right)}{\Gamma(\alpha + 1)} z^{\alpha}.$$

[For $\text{Re } \alpha < -\frac{1}{2}$, one has on the contrary $P_{\alpha}(z) \propto z^{-1-\alpha}$.]

The main problem is to find the right determination of z^α . This is easy for $z = -\cos \Theta_t$ but more delicate for $z = \cos \Theta_t$. Remember indeed that

$$\cos \Theta_t = \frac{s + \frac{1}{2}t - 2m^2}{\frac{1}{2}t - 2m^2}$$

is a large negative number for s large positive and t negative; one has then to choose between the two determinations $|\cos \Theta_t|^\alpha e^{i\pi\alpha}$ or $|\cos \Theta_t|^\alpha \times e^{-i\pi\alpha}$ of $(\cos \Theta_t)^\alpha$. In other words, one must know at which determination of the multivalued functions $(\cos \Theta_t)^\alpha$ one arrives, once the analytic continuation implied in crossing is performed. To answer this question it is sufficient to know the sign of the small imaginary part of $\cos \Theta_t$. Applying the crossing formula (9') one finds that before one takes the complex conjugate of $T(t)$ one arrives at a point $t = (p_1^* - p_3^*)^2$ and $s = (p_1^* + p_2^*)^2$. (We consider here that also p_i^0 has a small positive imaginary part which was neglected in the previous section.) For t , one has $\text{Re } t < 0$ and $\text{Im } t$ can be taken as zero, since $\text{Im } p_1^0$ and $\text{Im } p_3^0$ can be taken as equal. For s one has $\text{Re } s > 0$ and large, whereas $\text{Im } s < 0$. Hence one obtains that $\cos \Theta_t$ has a small positive imaginary part: one must then write $(\cos \Theta_t)^\alpha = |\cos \Theta_t|^\alpha e^{+i\pi\alpha}$. Taking the complex conjugate of $T(t)$, $\cos \Theta_t$ gets a negative imaginary part as it should because now $\text{Im } s > 0$. One then finds the following contribution of a Regge pole to the asymptotic behaviour for $T(s)$:

$$T^{(s)}(s, t) \simeq \left[- \frac{\sqrt{\pi} 2^{\alpha_\pm} \Gamma\left(\alpha_\pm + \frac{1}{2}\right)}{\Gamma(\alpha_\pm + 1)} (2\alpha_\pm + 1) \beta_\pm(t) \right]^* \frac{1 \pm l^{-i\alpha_\pm^*}}{\sin \pi\alpha_\pm^*} \left| \frac{s + \frac{t}{2} - 2m^2}{\frac{t}{2} - 2m^2} \right|^{\alpha_\pm^*} \quad (17)$$

(We have not written the equivalent formula for a Regge cut which is quite similar apart from the fact that one has to perform an integral on the cut.) Now it can be shown that $\alpha(t)$ and $\beta(t)$ are real when t is smaller than the t channel threshold, a fortiori for t negative. (We shall come back later to this important property.) That is why in Eq. (17) one can drop all complex conjugation symbols. Moreover, since the only term which can provide a non-vanishing phase is $(1 \pm e^{-i\pi\alpha})$, one sees that in the Regge-pole model the

asymptotic behaviour of the modulus and of the phase of the scattering amplitudes are rigidly correlated. It can be shown that very general analyticity properties imply that such a correlation must be valid. It is one of the most appealing features of the Regge-pole theory that it provides it automatically (cf. point 2 of the introduction).

6. BEHAVIOUR OF REGGE AMPLITUDES FOR POSITIVE t

Up to now we have been interested only in the small-angle scattering region of the s-channel (s large, t negative). However, in the Regge-pole theory there exists another region of interest in connection with point 1 of the introduction: we had said that the Regge-pole model allows one to correlate the high-energy behaviour of scattering amplitudes with the occurrence in the crossed channel of particles or resonances having a definite set of internal quantum numbers (baryon number, charge, isospin, strangeness). In this respect it is interesting to see if a Regge-type amplitude is suitable for describing the occurrence of such particles in the t-channel.

Let us suppose that for $t = t_0 > 0$ the real part of α_{\pm} is equal to some positive integer ℓ_0 (ℓ_0 even for α_{+} , odd for α_{-}), that is $\alpha_{\pm}(t_0) = \ell_0$. Near $t = t_0$, the physical partial wave amplitude in the t-channel can be written, for $\ell = \ell_0$, as:

$$\zeta_{\ell_0}^{\pm}(t) \approx \frac{\beta_{\pm}(t)}{\ell_0 - \alpha_{\pm}(t)}. \quad (18)$$

Consider the two following cases:

i) t_0 is smaller than the t-channel threshold

$\zeta_{\ell_0}^{\pm}(t)$ exhibits a pole which is interpreted as a bound state of the $1 + \bar{3}$ system (and also of $\bar{2} + 4$), the mass being equal to $\sqrt{t_0}$ and the spin ℓ_0 . It should also be noted this interpretation allows one to understand why the Regge trajectory $\alpha_{\pm}(t)$ and the residue $\beta_{\pm}(t)$ have to be real for t smaller than the t-channel threshold. Indeed, in quantum mechanics of spinless particles, a bound state has always a real energy and, if non-degenerate, it has a real radial wave function (these properties hold if the Hamiltonian

is hermitian). Hence the solution t_0 of $\alpha_{\pm}(t_0) = l_0$ is real, and $\beta_{\pm}(t_0)$ is also real because it is just the product of the wave function of the $1 + \bar{3}$ system by the wave function of the $\bar{2} + 4$ system. One can extend this reasoning to real non-integral values of l by considering a radial wave equation for such l . One concludes from such non-rigorous arguments that $\alpha_{\pm}(t)$ and $\beta_{\pm}(t)$ must be real for t below the t -channel threshold (more rigorous proofs exist for these properties¹⁸).

ii) t_0 is greater than the t channel threshold

$\alpha_{\pm}(t_0)$ may have an imaginary part, and one recognizes in expression (18) a Breit-Wigner type formula which one indeed interprets as the occurrence of a resonance with a mass equal to $\sqrt{t_0}$, a spin equal to l_0 , and a width related to $\text{Im } \alpha_{\pm}(t_0)$.

7. THREE EXAMPLES

At this point it is instructive to look for examples of actual scattering phenomena which illustrate the "ups and downs" of the Regge pole theory just described. We shall refer to cases with spins and unequal masses although we have not worked out the theory for such cases. As mentioned later, these cases lead to considerable formal complications in the mathematics, but they retain the essential physical implications: powers $s^{\alpha_{\pm}(t)}$ in the amplitudes, reality of $\alpha_{\pm}(t)$, and identification of $l_0 = \alpha_{\pm}(t_0)$ with bound state or resonance of mass $\sqrt{t_0}$ and spin l_0 in the t -channel.

7.1 $\pi^- p \rightarrow \pi^0 n$ reaction

The π -nucleon charge exchange reaction $\pi^- p \rightarrow \pi^0 n$ (the particles are labelled in the order $1 + 2 \rightarrow \bar{3} + 4$) is interesting because the quantum numbers which are allowed to occur in the t -channel reaction ($1 + \bar{3} \rightarrow \bar{2} + 4$ or $\pi^- \pi^0 \rightarrow \bar{p}n$) are restricted: $I = 1$, $G = +1$, l odd which means odd signature. In fact, only one known particle is a candidate for bearing these quantum numbers: it is the ρ -meson. One therefore hopes that a single Regge pole must describe $T^{(s)}$ for large s . Applying formula (17) one therefore expects the differential cross-section to behave as

$$\frac{d\sigma}{dt} = \dots |T^{(s)}|^2 \frac{1}{s^2} \simeq s^{2\alpha_{\rho}(t)-2} f(t).$$

One uses the experimental data¹⁹⁾ from 6 to 18 GeV/c in the following way:

- i) From the energy dependence of the forward cross-section $(d\sigma/dt)_{t=0}$ one deduces $\alpha_\rho(0)$. One finds $\alpha_\rho(0) = 0.57$ with an error of perhaps 10% [accurate error statements have little meaning because at least at the lower energies (and even higher, see the discussion of polarization) the assumption of a single pole contributing can only be approximately valid].
- ii) From $(d\sigma/dt)/(d\sigma/dt)_{t=0}$ (Fig. 4) one estimates $\alpha_\rho(t)$ for $t < 0$ by studying the s -dependence at fixed t . The data are fully compatible with a power law $s^{2\alpha_\rho(t)-2}$ with real $\alpha_\rho(t)$.

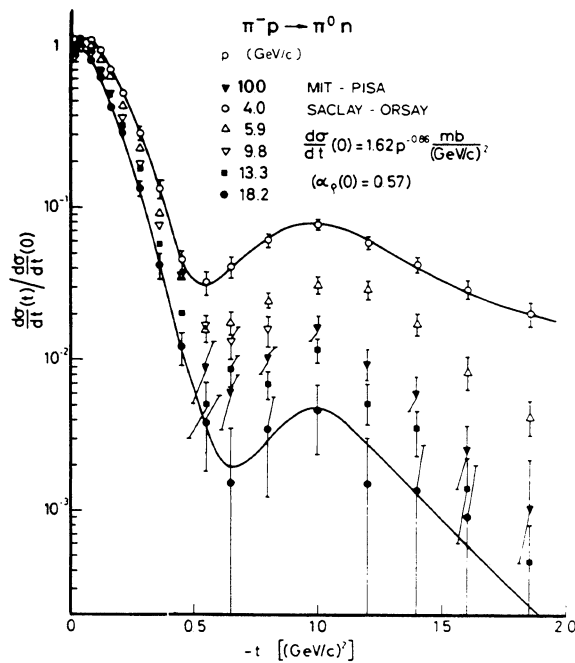


FIG.4

[Taken from P. Sonderegger et al., Physics Letters 20, 75 (1966).]

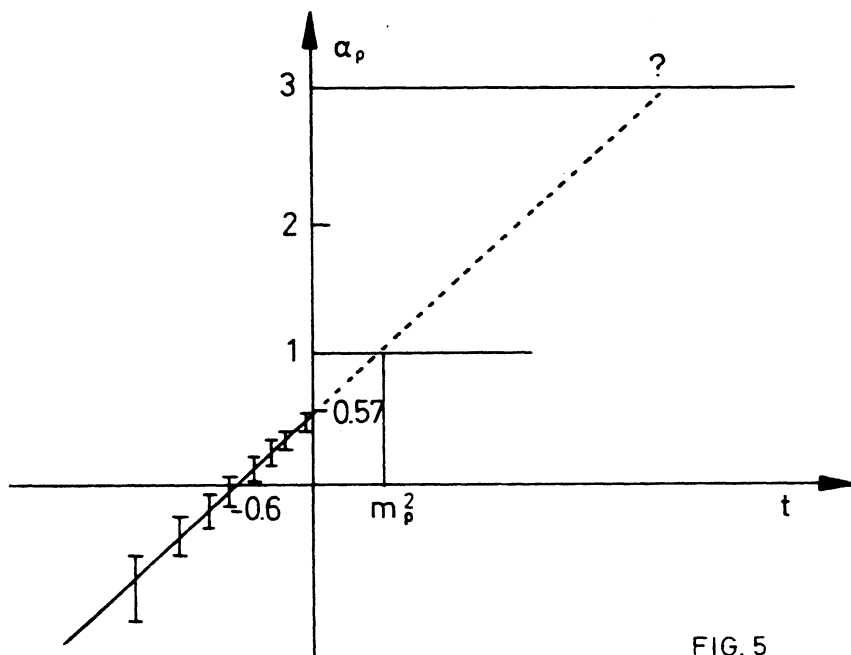


FIG. 5

The results are shown roughly in the so-called "Chew-Frautschi plot" of Fig. 5: one finds that the function $\alpha_\rho(t)$ can be fitted by a straight line (although the error bars become larger for larger $|t|$) with an intercept equal to 0.57 and a slope of the order of $1 (\text{GeV}/c)^{-2}$. The interesting point is that the extrapolation of this straight line (dots in Fig. 5) crosses the line $\alpha_\rho = 1$ at a value of t which is fairly close to m_ρ^2 : this is in full agreement with the considerations of the previous section. For $t > 4m_\pi^2$, the line of Fig. 5 is supposed to represent $\text{Re } \alpha_\rho$ since then α_ρ can become imaginary. We have here the most striking success of Regge-pole theory.

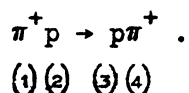
Unfortunately, this same reaction provides a severe limitation to this success. It is related to polarization. To describe the reaction $\pi^- p \rightarrow \pi^0 n$ one needs two independent amplitudes: the non-spin flip amplitude f , and the spin flip amplitude g . One can show that formula (17) applies for both of these amplitudes. In particular, they both contain the same factor $1 - e^{-i\pi\alpha_\rho(t)}$. Since the residues and the trajectory are real for

negative t , f and g have the same phase if the ρ trajectory is the only one to contribute. Thus the polarization of the recoil neutron, which is proportional to $\text{Im } fg^*$, is predicted to be identically equal to zero in a pure Regge-pole model with ρ exchange. This polarization has been measured²⁰⁾ using a polarized proton target, and it turned out to be significantly different from zero and to decrease slowly when the energy increases [its average over $0.05 \lesssim -t \lesssim 0.3$ (GeV/c)² is $16 \pm 3\%$ at 6 GeV/c and $12 \pm 2\%$ at 11 GeV/c]. This is a "down"! The only way to account for the polarization is to add to the Regge pole α_ρ an extra singularity in the l -plane; it should affect $d\sigma/dt$ very little but should give the polarization. It is not very attractive to postulate a second pole because one does not know any particle with the same quantum numbers as those of the ρ meson. In fact it seems more natural to assume the existence of a non-negligible Regge cut term. Indeed, as we shall see, any reasonable theory for high-energy scattering easily generates cuts which are in about the same position as the poles, and which are not associated with particles.

7.2 Backward meson-nucleon scattering

The Regge-pole model seems to be also successful in reproducing the data in the backward scattering region of elastic meson-nucleon collisions.

Consider the s-channel reaction



A small momentum transfer between particles 1 and 3 corresponds to backward π^+p elastic scattering. The t-channel reaction is $\pi^+\bar{p} \rightarrow \bar{p}\pi^+$, its charge conjugate is $\pi^-p \rightarrow p\pi^-$. These reactions carry baryon number -1 and +1, respectively (i.e. the original s-channel reaction is characterized by baryon exchange). Hence baryonic Regge poles can occur. Although there are some complications due to spins, a Regge theory for baryonic states can be developed. Figure 6 is a Chew-Frautschi plot for two families of nucleon resonances (Δ : isospin $\frac{3}{2}$, N: isospin $\frac{1}{2}$) in which the spins of the resonances are plotted versus the squared masses. An amazing

regularity, suggesting straight-line Regge trajectories, appears in this plot:

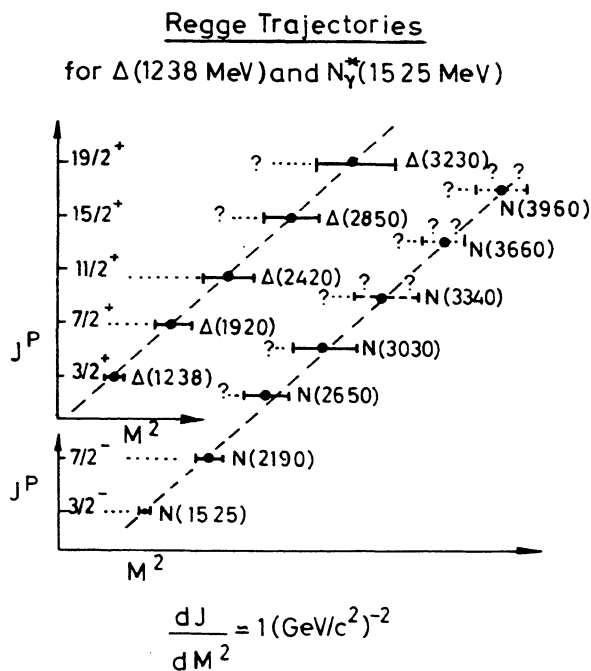


FIG. 6

As indicated by the question marks, the spins and parities of the heavier resonances are not yet known (they seem very hard to determine; also some widths are not yet known). However, the mass values are such that, in each case, one reasonable spin assignment is clearly suggested by the linear trajectory. Surprisingly one has found fewer resonances on the trajectory of the nucleon itself (not shown in Fig. 6) but it has about the same slope as the ones indicated and as the ρ trajectory $(\sim 1 (\text{GeV}/c)^2)$. The other interesting point is that the continuations of these straight lines to $t = M^2 \leq 0$ are in good agreement with the power laws $d\sigma/dt \propto s^{2\alpha-2}$ found experimentally for backward $\pi^\pm p \rightarrow p \pi^\pm$ scattering (the contributions of the various possible Regge poles must be added using isospin rules). This is again a good success of Regge-pole theory. It is at present only qualitative because the scattering data are not good enough to determine the t variation of α . It should also be said that the Regge formalism for half integer spin particles such as baryons is much more complicated than for mesons.

7.3 Elastic-like scattering

The third example we shall consider concerns an important class of reactions: these are the reactions which proceed without any exchange of internal quantum numbers (internal quantum numbers are baryon number, isospin, strangeness, G-parity). In the usual terminology one says that such reactions proceed via the exchange of the "vacuum quantum numbers".

The elastic scattering belongs to this class of reactions. It also contains other reactions, which are inelastic, but for which the exchanged quantum numbers can be those of the vacuum, e.g.

$$\pi^- p \rightarrow A_1 p$$

$$\pi^- p \rightarrow \pi^- N^*$$

$$pp \rightarrow pN^*$$

$$pp \rightarrow N^*N^*$$

(where N^* stands for a nucleon resonance with the same internal quantum numbers as the nucleon itself). Such inelastic reactions are often called "diffractive dissociation processes". Elastic scattering and diffractive dissociation processes can be given the joint name of elastic-like or diffractive-like reactions.

For all elastic-like reactions it seems experimentally that $d\sigma/dt$ becomes energy-independent at high energy. Writing as usual $d\sigma/dt \propto s^{2\alpha_P(t)-2}$ for large s suggests the existence of a single Regge trajectory $\alpha_P(t)$ bearing the quantum numbers of the vacuum. This trajectory, called the Pomeron or vacuum trajectory $\alpha_P(t)$, is rather different from the other ones. The constant cross-section indeed gives $\alpha_P(t) \simeq 1$ for the t interval measured, $0 \leq -t \lesssim 0.5$ (GeV/c)². The precision is not very good, however. The signature is supposed to be positive, the main reason being that $\text{Re } T \ll \text{Im } T$ at $t = 0$ for elastic reactions (note also that a negative signature trajectory taking the value 1 for $t = 0$ would imply the existence of a strongly interacting massless particle of spin 1). In this picture, the fact that elastic-like reactions seem to be the only ones which survive at very high energy is accounted for by noting that all trajectories except the Pomeron one have an intercept appreciably smaller than one. The known intercepts are, indeed, $\lesssim 0.5$.

$\alpha_p(0)$ is experimentally one, with an error of perhaps 0.05 to the lower side. Since no total cross-section increases with energy, the error on the upper side is much smaller [$\alpha_p(0) > 1$ would also violate the so-called Froissart bound deduced theoretically from general analyticity assumptions^{21, 22}].

The slope of $\alpha_p(t)$ near $t = 0$ is even less well known: the lack of shrinkage of elastic diffraction peaks indicates that this slope is small. Indeed one generally finds

$$\alpha_p(t) = 1 + \epsilon t$$

with roughly

$$\epsilon \simeq 0 \pm 0.3 .$$

The question whether for $t > 0$ the Pomeranchuk trajectory actually carries a 2^+ resonance with $B = I = S = 0$, $G = +1$ (the f_0 or f_0' mesons for instance), is still an open one.

The lack of precise information about the Pomeranchuk trajectory can be understood from the two following reasons: on the one hand, the conservation laws never forbid the exchange of other trajectories than α_p in diffraction-like reactions (e.g. the ρ trajectory can also be exchanged in πN elastic scattering). So the interpretation of the data is never as simple as, for example, for the π -nucleon charge exchange reaction. On the other hand, as mentioned in the Introduction (cf. point 3) one does not yet understand how the complicated shadow scattering mechanism, which certainly must operate, can be reproduced by the exchange of a vacuum trajectory.

8. FURTHER INTERPRETATION OF THE SIGNATURE

When performing the analytic interpolation of partial wave amplitudes in the t -channel we have found it useful to separate the t -channel amplitude $T^{(t)}(s, t)$ into two parts:

$$T^{(t)}(s, t) = T_+^{(t)}(s, t) + T_-^{(t)}(s, t) ,$$

where $T_+^{(t)}(s,t)$ [or $T_-^{(t)}(s,t)$, respectively] is the sum over even (or odd, respectively) partial wave amplitudes:

$$T_+^{(t)}(s,t) = \sum_{l \text{ even}} (2l+1) \zeta_l^+ P_l(\cos \Theta_t) ,$$

$$T_-^{(t)}(s,t) = \sum_{l \text{ odd}} (2l+1) \zeta_l^- P_l(\cos \Theta_t) .$$

After the analytic continuation involved in crossing, one is then led to an analogous decomposition of the s-channel amplitude [both $T_+^{(t)}$ and $T_-^{(t)}$ can be analytically continued]:

$$T^{(s)}(s,t) = T_+^{(s)}(s,t) + T_-^{(s)}(s,t) \quad (19)$$

with $T_+^{(s)}$ = analytic continuation of $T_+^{(t)}$
and $T_-^{(s)}$ = analytic continuation of $T_-^{(t)}$.

Our purpose now is to give an interpretation of this decomposition in terms of the s- and u-channel reactions. We are going to show that the u-channel amplitude can be expressed simply in terms of $T_+^{(s)}$ and $T_-^{(s)}$; namely

$$T^{(u)}(u,t) = T_+^{(s)}(u,t) - T_-^{(s)}(u,t) . \quad (20)$$

We recall that the t-channel reaction is $1 + \bar{4} \rightarrow 3 + \bar{2}$, whilst $1 + 2 \rightarrow 3 + 4$ and $1 + \bar{3} \rightarrow \bar{2} + 4$ are the s- and t-channel ones.

In order to prove Eq. (20) we come back to the explicit formulation of $s \leftrightarrow t$ and $u \leftrightarrow t$ crossing properties. We write it as (p_1, p_4 , and $p_{\bar{2}}$ are treated as real)

$$T^{(s)}(p_1, p_2, p_3, p_4) = \left[T^{(t)}(p'_1, p'_3, p'_2, p'_4) \right]^* \text{ continued to:}$$

$$p'_1 = p_1, \quad p'_2 = -p_2^*, \quad p'_3 = -p_3^*, \quad p'_4 = p_4, \quad (21)$$

$$T^{(u)}(p_1, p_4, p_3, p_{\bar{2}}) = \left[T^{(t)}(p'_1, p'_3, p'_2, p'_4) \right]^* \text{ continued to:}$$

$$p'_1 = p_1, \quad p'_2 = p_{\bar{2}}, \quad p'_3 = -p_3^*, \quad p'_4 = -p_4^* . \quad (22)$$

Comparing these two equations one sees that in Eq. (21) p'_4 can be kept constant but p'_2 must be continued, whereas in Eq. (22) p'_4 is continued and p'_2 kept constant. In this respect, the roles of particles $\bar{2}$ and 4 in $T^{(t)}$ are interchanged. Let us interchange p'_2 and p'_4 in Eq. (22); it now reads

$$T^{(u)}(p_1 p_4 p_3 p_2) = \left[T^{(t)}(p'_1 p'_3 p'_4 p'_2) \right]^* \text{ continued to:}$$

$$p'_1 = p_1, p'_2 = -p_4^*, p'_3 = -p_3^*, p'_4 = p_2 \quad (22')$$

i.e. p'_4 can now be kept constant but p'_2 is continued. In the t-channel amplitude the interchange of p_2 and p_4 corresponds to the change of Θ_t into $\pi - \Theta_t$, that is $\cos \Theta_t$ into $-\cos \Theta_t$. Thus $T_+^{(t)}$ is symmetric under this transformation, while $T_-^{(t)}$ is antisymmetric. Equations (21) and (22') can then be written in the following way:

$$T^{(s)}(p_1 p_2 p_3 p_4) = \left[T_+^{(t)}(p'_1 p'_3 p'_2 p'_4) + T_-^{(t)}(p'_1 p'_3 p'_4 p'_2) \right]^* \text{ cont. in } p'_2 \text{ and } p'_3,$$

$$T^{(u)}(p_1 p_4 p_3 p_2) = \left[T_+^{(t)}(p'_1 p'_3 p'_2 p'_4) - T_-^{(t)}(p'_1 p'_3 p'_4 p'_2) \right]^* \text{ cont. in } p'_2 \text{ and } p'_3.$$

The analytic continuations are to be done as indicated in Eqs. (21) and (22'), respectively. They can be done separately for both signature t-channel amplitudes, which leads to Eqs. (19) and (20) [note that the difference between the continuations in Eqs. (21) and (22') just means that s and u are interchanged between the right-hand sides of Eqs. (19) and of (20)].

One sees in Eqs. (19) and (20) that the signature refers to a symmetry property of the scattering amplitude for $s \leftrightarrow u$ crossing, and not to a kind of internal quantum number exchanged in the t-channel. In the spinless case considered here, the signature which is equal to $(-1)^l$ in the t-channel is identical to the parity of the exchanged particle, but this is not true in the general spin case. Although one can define the signature of a particle [it is usually defined as $(-1)^{J-v}$ where J is the spin of the particle and $v=0$ for a boson and $1/2$ for a fermion] it is more relevant to speak of the "signature of a reaction".

We now show briefly how Eqs. (19) and (20) allow one to prove that the residue function $\beta_{\pm}(t)$ in Eq. (17) is indeed real for $t < 0$. Equations

(19) and (20) mean that $T_+^{(s)}$ is even and $T_-^{(s)}$ odd under $s \leftrightarrow u$ crossing. In other words,

$$T_{\pm}^{(s)}(s,t) = \pm \left[T_{\pm}^{(s)}(u,t) \right]_{\text{cont.}}^* .$$

The continuation is done as indicated in Eq. (9''). When done in u , it goes to $u = -s^* - t + 4m^2$ with $\text{Im } u > 0$, and t is kept fixed.

Substituting for $T_{\pm}^{(s)}$ the leading term given by Eq. (17), and using reality of $\alpha_{\pm}(t)$, one gets

$$\begin{aligned} \left[1 \pm e^{-i\pi\alpha_{\pm}(t)} \right] \beta_{\pm}(t) \left| \frac{s + \frac{t}{2} - 2m^2}{\frac{t}{2} - 2m^2} \right|^{\alpha_{\pm}(t)} &= \\ &= \pm \left[1 \pm e^{-i\pi\alpha_{\pm}(t)} \right]^* \beta_{\pm}^*(t) \left| \frac{s + \frac{t}{2} - 2m^2}{\frac{t}{2} - 2m^2} \right|^{\alpha_{\pm}(t)} \left[e^{+i\pi\alpha_{\pm}(t)} \right]^* , \end{aligned}$$

the last phase coming from the analytic continuation of $u + (t/2) - 2m^2$ to the value $-s^* - (t/2) + 2m^2$. Hence, as stated previously,

$$\beta_{\pm}(t) = \beta_{\pm}^*(t) .$$

9. COMPLICATIONS DUE TO MASS DIFFERENCES AND SPINS

For the sake of simplicity, we have restricted ourselves in the foregoing to the academic case of the scattering of spinless equal mass particles. In order to get a model suitable for realistic reactions, it is necessary to examine carefully the complications arising from kinematics. We first study the case of different masses.

9.1 Scattering of spinless particles with different masses

In this case the simplicity of kinematical formulae in the t -channel ($1 + \bar{3} \rightarrow \bar{2} + 4$) is lost as soon as $m_1 \neq m_3$ and/or $m_2 \neq m_4$. The incoming c.m. momentum p_t is distinct from the outgoing one p_t' . From

$$t^{1/2} = (p_t^2 + m_1^2)^{1/2} + (p_t^2 + m_3^2)^{1/2} = (p_t'^2 + m_2^2)^{1/2} + (p_t'^2 + m_4^2)^{1/2} ,$$

one derives

$$p_t = \frac{\sqrt{t - (m_1 + m_3)^2} \sqrt{t - (m_1 - m_3)^2}}{2\sqrt{t}} \quad (23)$$

and

$$p'_t = \frac{\sqrt{t - (m_2 + m_4)^2} \sqrt{t - (m_2 - m_4)^2}}{2\sqrt{t}} .$$

Of course, p_t and p'_t vanish with square root branch-points at the thresholds $t = (m_1 + m_3)^2$ and $t = (m_2 + m_4)^2$. But for $m_1 \neq m_3$, $m_2 \neq m_4$ the relativistic kinematics introduces other branch-point singularities: at $t = 0$ and at the so-called "pseudo-thresholds" [$t = (m_1 - m_3)^2$ and $t = (m_2 - m_4)^2$]. From

$$s = (p_1 - p_2)^2 = m_1^2 + m_2^2 - 2\sqrt{p_t^2 + m_1^2} \sqrt{p_t'^2 + m_2^2} + 2p_t p_t' \cos \Theta_t ,$$

$$u = (p_1 - p_4)^2 = m_1^2 + m_4^2 - 2\sqrt{p_t^2 + m_1^2} \sqrt{p_t'^2 + m_4^2} - 2p_t p_t' \cos \Theta_t ,$$

one deduces

$$\cos \Theta_t = \frac{1}{4p_t p_t'} \left[s - u + \frac{(m_1^2 - m_3^2)(m_2^2 - m_4^2)}{t} \right] . \quad (24)$$

In addition to involving the singularities of p_t , p'_t , it has the t^{-1} term in the square bracket when $m_1 \neq m_3$ and $m_2 \neq m_4$. All these kinematical singularities greatly complicate the t -channel considerations and formulae required in Regge-pole theory, but they must be cancelled in some way at the end when crossing over to the s -channel, because $T^{(s)}$ is certainly free of them. The cancellation mechanisms are unfortunately complicated. We shall only indicate their general nature.

When one follows the "Reggeization" procedure one starts from the partial wave expansion of the t -channel amplitude:

$$T_{\pm}^{(t)} = \sum_{\ell} (2\ell + 1) \zeta_{\ell}^{\pm} P_{\ell}(\cos \Theta_t) . \quad (25)$$

The first difficulty one meets deals with the singular denominator $p_t p'_t$ of $\cos \Theta_t$. This difficulty can be got rid of for spinless particles by

a reasonable assumption about the "threshold behaviour" of partial wave amplitudes: one has good theoretical reasons to believe that ζ_ℓ^\pm can be written (see next lecture for the main argument):

$$\zeta_\ell^\pm = p_t^\ell p_t'^\ell \tilde{\zeta}_\ell^\pm, \quad (26)$$

where the reduced partial wave amplitude $\tilde{\zeta}_\ell^\pm$ is free from the singularities of p_t and p_t' and in particular remains finite when p_t or $p_t' \rightarrow 0$ (the factor $p_t^\ell p_t'^\ell$ describes the centrifugal barrier effect). Inserting Eq. (26) in Eq. (25) and doing the analytic interpolation in ℓ implied in the Regge procedure, one is led to make the exponent ℓ of p_t and p_t' complex as well. Picking out the asymptotic contribution of a Regge pole in $\tilde{\zeta}_\ell^\pm$

$$\tilde{\zeta}_\ell^\pm = \frac{\tilde{\beta}_\pm(t)}{\ell - \alpha_\pm(t)},$$

one gets the following combination

$$\dots \tilde{\beta}_\pm(t) p_t^{\alpha_\pm} p_t'^{\alpha_\pm} (\cos \Theta_t)^{\alpha_\pm},$$

where $\tilde{\beta}_\pm(t)$, the so-called reduced residue, is finite and regular at the zeros of p_t , p_t' . The denominator of $\cos \Theta_t$ is then cancelled out, and one finds

$$\dots \tilde{\beta}_\pm(t) \left[s - u + \frac{(m_1^2 - m_3^2)(m_2^2 - m_4^2)}{t} \right]^{\alpha_\pm}, \quad (27)$$

which has no singularities of thresholds nor of pseudothresholds.

From expression (27) one sees that there remains a problem at $t = 0$. Expanding Eq. (27) in inverse powers of $s-u$ one finds:

$$\tilde{\beta}_\pm(t) \left[(s-u)^{\alpha_\pm} + (s-u)^{\alpha_\pm-1} \alpha_\pm \frac{(m_1^2 - m_3^2)(m_2^2 - m_4^2)}{t} + \frac{0 \left\{ (s-u)^{\alpha_\pm-2} \right\}}{t^2} + \dots \right] \quad (27')$$

with increasing singularities at $t = 0$. Some comments are now in order:

- i) We have only expanded the highest order terms of $P_\alpha(\cos \Theta_t)$. In order to get an expansion of the full amplitude one would also have to

consider the lower order terms, which begin with $(\cos \theta_t)^{\alpha_{\pm} - 2}$. Their treatment is analogous.

ii) The expansion one needs goes to finite order only: let N be the integer such that $\alpha_{\pm} - N \geq -1/2$ and that $\alpha_{\pm} - N - 1 < -1/2$; then the expansion needed will contain $N + 1$ terms only, since the terms in $(s-u)^{\alpha_{\pm} - n}$ where $n > N$ are negligible as compared to the background integral (the integral over $\text{Re } l = -1/2$), which is already neglected. In practice $\alpha_{\pm} \leq 1$, hence N is at most 1.

We do not want to give the explicit form of all the terms of the expansion; we shall simply exhibit the analyticity properties of these terms near $t = 0$. One gets essentially:

$$\dots \tilde{\beta}_{\pm}(t) \left[(s-u)^{\alpha_{\pm}} + (s-u)^{\alpha_{\pm}-1} \frac{A_1(s,t)}{t} + \dots + (s-u)^{\alpha_{\pm}-N} \frac{A_N(s,t)}{t^N} \right], \quad (28)$$

where $A_1(s,t), \dots, A_N(s,t)$ are different from zero at $t = 0$. Expansion (28) exhibits poles at $t = 0$ which, from general analyticity requirements, are not allowed to appear in the full amplitude. There exist two possible ways to get rid of this trouble:

a) One can suppose that the reduced residue $\tilde{\beta}_{\pm}(t)$ vanishes at $t = 0$ as t^N . This would imply that the leading term of the amplitude [i.e. the term in $(s-u)^{\alpha_{\pm}}$] vanishes at $t = 0$, giving a dip in the forward direction, and such dips are not observed in reactions for which $t = 0$ is inside or near the s -channel physical region.

b) One postulates the existence of other Regge trajectories, which together with the original one, manage to restore the required analyticity at $t = 0$. About these extra Regge trajectories, called the "daughter trajectories"²³⁾, one can say at once that their intercept at $t = 0$ must differ by integers from that of the original or "parent" trajectory α_{\pm} [simply because they have to cancel singularities in the terms which behave like $(s-u)^{\alpha_{\pm}-k}$ $k = 1, 2$]. Their residues should have singularities at $t = 0$ to cancel those from the original trajectory. They should have opposite signature to the original one for k odd, and the same signature as the original one for k even, in order to produce the right phase factor at $t = 0$. We insist on the fact that such a complicated cancellation

mechanism is needed in order to satisfy the analyticity requirements, if one wants to avoid extra zeros which are not experimentally observed. It is still an open question whether these auxiliary trajectories, which are introduced from purely mathematical considerations, have for $t > 0$ something to do with particles. The common philosophy in Regge phenomenology is to say that there exist daughter trajectories which do not necessarily give rise to observed particles, but which allow one to write the usual Regge formulae without singularities or extra zeros.

LECTURE VI

9.2 Complications arising from non-vanishing spins of the external particles

Having discussed in Section 9.1 the origin of the troubles which arise at $t = 0$ for the scattering of unequal mass particles, which has led to the introduction of "daughter trajectories" in order to get rid of unwanted singularities at $t = 0$, we now indicate the source of the kinematical singularities due to the spins of the colliding particles.

We again refer to the s-channel reaction $1 + 2 \rightarrow 3 + 4$, and the t-channel reaction $1 + \bar{3} \rightarrow \bar{2} + 4$. The initial and final momenta in the t-channel defined in Eq. (23) will, for the purpose of the following discussion, be written

$$P_t = \frac{T_N T_P}{2\sqrt{t}}$$

(23')

and

$$P'_t = \frac{T'_N T'_P}{2\sqrt{t}},$$

where we have introduced the normal threshold factors $T_N = \sqrt{t - (m_1 + m_3)^2}$ and $T'_N = \sqrt{t - (m_2 + m_4)^2}$, as well as the pseudo-threshold factors $T_P = \sqrt{t - (m_1 - m_3)^2}$ and $T'_P = \sqrt{t - (m_2 - m_4)^2}$. Notice that for equal masses in the initial and final states in the t-channel ($m_1 = m_3$; $m_2 = m_4$), the pseudo-thresholds coincide with $t = 0$, thus cancelling the \sqrt{t} factors in the denominator of Eq. (23'); then the difficulties encountered in Section 9.1 also disappear, see Eq. (24). The scattering angle Θ_t for the scattering from particle 1 to particle $\bar{2}$ in the t-channel is given

by Eq. (24), which we repeat here in the form

$$p_t p'_t \cos \Theta_t = \frac{1}{4} \left[s - u + \frac{(m_1^2 - m_3^2)(m_2^2 - m_4^2)}{t} \right]. \quad (24')$$

It is indeed this combination $p_t p'_t \cos \Theta_t$ which appears in the formulae for the spinless case and avoids here all the difficulties at thresholds and pseudo-thresholds. To see this, one has to consider the centrifugal barrier effects inherent in the partial wave amplitudes $\zeta_\ell(t)$. Let us give here the argument leading to Eq. (26) in absence of spins.

It can be shown that the centrifugal barrier effect -- as in the case of non-relativistic quantum mechanics -- is governed by the orbital angular momentum of the state in which the scattering takes place. To determine the threshold behaviour of the partial wave amplitudes we start from Eq. (10) and project out the $\zeta_\ell(t)$ using the formula

$$\zeta_\ell(t) = \frac{1}{2} \int_{-1}^{+1} T^{(t)}(s,t) P_\ell(z) dz. \quad (29)$$

To proceed further, we derive the so-called Froissart-Gribov formula for the partial wave amplitudes (it is the formula which allows the analytic continuation to complex l as discussed in Section 4). We assume that $T^{(t)}(s,t)$ obeys the following fixed- t dispersion relation:

$$T^{(t)}(s,t) = \frac{1}{\pi} \int_{s_0}^{\infty} \frac{A_s(s',t)}{s' - s} ds' + \frac{1}{\pi} \int_{u_0}^{\infty} \frac{A_u(u',t)}{u' - u(s,t)} du'. \quad (30)$$

A_s and A_u represent the absorptive parts of $T^{(t)}(s,t)$ in the s and u channel, respectively, and s_0 and u_0 are the corresponding thresholds. Inserting now the dispersion relation (30) into Eq. (29) and using the formula

$$\frac{1}{2} \int_{-1}^{+1} \frac{P_\ell(z)}{z' - z} dz = Q_\ell(z'), \quad (31)$$

which for positive integer l defines the Legendre function of the second kind $Q_\ell(z)$, in terms of the Legendre function of the first kind $P_\ell(z)$,

we arrive [after using the relation $P_\ell(-z) = (-1)^\ell P_\ell(z)$ for the contribution originating from the second integral in Eq. (30)] at the Froissart-Gribov formula for $\zeta_\ell(t)$:

$$\zeta_\ell(t) = \frac{1}{\pi} \int_{s_0}^{\infty} A_s(s,t) Q_\ell \left(\frac{2s+t - \sum_1^4 m_k^2 + \frac{(m_1^2 - m_3^2)(m_2^2 - m_4^2)}{t}}{4p_t p_t'} \right) \frac{ds}{4p_t p_t'} + (-1)^\ell \frac{1}{\pi} \int_{u_0}^{\infty} A_u(u,t) Q_\ell \left(\frac{2u+t - \sum_1^4 m_k^2 - \frac{(m_1^2 - m_3^2)(m_2^2 - m_4^2)}{t}}{4p_t p_t'} \right) \frac{du}{4p_t p_t'}. \quad (32)$$

As was mentioned in Section 4, it is necessary to consider even and odd ℓ values separately in order to allow a unique analytic extension to complex ℓ values. This separation is easily seen in Eq. (32); it eliminates the $(-1)^\ell$ factor. The signed partial wave amplitudes considered before are given by

$$\zeta_\ell^\pm(t) = I_s(t) \pm I_u(t), \quad (33)$$

where $I_s(t)$ and $I_u(t)$ are defined by the s and u integrals in Eq. (32), respectively.

Returning now to the behaviour of $\zeta_\ell^\pm(t)$ at thresholds and pseudo-thresholds we make use of the formula

$$Q_\ell(z) \underset{z \rightarrow \infty}{\sim} z^{-\ell-1}.$$

For t near a threshold or a pseudo-threshold (where p_t or p_t' vanishes) one then obtains for the first integral in Eq. (33)

$$I_s(t) \simeq (4p_t p_t')^\ell \frac{1}{\pi} \int_{s_0}^{\infty} A_s(s,t) \left[2s+t - \sum_1^4 m_k^2 + \frac{(m_1^2 - m_3^2)(m_2^2 - m_4^2)}{t} \right]^{-\ell-1} ds \quad (34)$$

and a similar expression for $I_u(t)$. The integral in this equation is regular at threshold and pseudo-threshold, thus showing that $I_s(t)$ [and

similarly $I_u(t)$] vanishes there like $(p_t \cdot p'_t)^\ell$. From this discussion one arrives at the threshold behaviour of the signed partial wave amplitudes for the scattering of spinless particles:

$$\zeta_\ell^\pm(t) = p_t^\ell p'_t{}^\ell \tilde{\zeta}_\ell^\pm(t) . \quad (35)$$

Here the reduced partial wave amplitudes $\tilde{\zeta}_\ell^\pm(t)$ are regular at thresholds and pseudo-thresholds. Notice that Eq. (35) is valid also for arbitrary complex ℓ .

Using this result the contribution of a partial wave to Eq. (10) for large $\cos \Theta_t$ (disregarding signature for the moment) can be written as

$$(2\ell + 1) \zeta_\ell(t) P_\ell(\cos \Theta_t) \simeq (2\ell + 1) \tilde{\zeta}_\ell(t) (p_t p'_t \cos \Theta_t)^\ell . \quad (36)$$

From Eq. (24') it is clear that this is a perfectly regular expression at thresholds and pseudo-thresholds. It has only the singularity at $t = 0$ for $m_1 \neq m_3$ and $m_2 \neq m_4$, which was discussed above in Section 9.1.

We now go over to the case of general spins of the external particles. The situation is changed because in this case one cannot combine the factors p_t , p'_t , and $\cos \Theta_t$ as in Eq. (36) and avoid all factors which are singular at thresholds and pseudo-thresholds. This is due to the fact that one now has to add the orbital angular momentum ℓ and the particle spins in the initial and final states to obtain the total angular momentum, j . It turns out that the power of $\cos \Theta_t$ is given by j , whereas the centrifugal barrier effects are determined by the orbital angular momentum which now is in general different in the initial and final states, and moreover different from j . Hence there will appear square root singularities due to threshold and pseudo-threshold factors T_N , $T_{N'}$, T_P , and $T_{P'}$. Instead of Eq. (35) one will have in the general spin case the behaviour (disregarding signature)

$$\zeta_{\lambda_1 \lambda_3, \lambda_2 \lambda_4}^j(t) = T_N^{\ell_N} T_P^{\ell_P} T_{N'}^{\ell'_N} T_{P'}^{\ell'_P} \tilde{\zeta}_{\lambda_1 \lambda_3, \lambda_2 \lambda_4}^j(t) , \quad (37)$$

where $\tilde{\zeta}^j$ is regular at thresholds and pseudo-thresholds, and the exponents ℓ_N , ℓ_P , ℓ'_N , and ℓ'_P are in general different from each other and from j . The

$\zeta_{\lambda_1 \lambda_3, \lambda_2 \lambda_4}^j(t)$ appearing in Eq. (37) are the partial wave helicity amplitudes which occur in the Jacob and Wick expansion²⁴⁾ of the helicity amplitude $T_{\lambda_1 \lambda_3, \lambda_2 \lambda_4}^{(t)}(s, t)$:

$$T_{\lambda_1 \lambda_3, \lambda_2 \lambda_4}^{(t)}(s, t) = \sum_{j=\lambda_{\max}}^{\infty} (2j+1) \zeta_{\lambda_1 \lambda_3, \lambda_2 \lambda_4}^j(t) d_{\lambda \mu}^j(\cos \Theta_t). \quad (38)$$

Here $\lambda = \lambda_1 - \lambda_3$, $\mu = \lambda_2 - \lambda_4$, and $d_{\lambda \mu}^j(\Theta_t)$ are the Wigner rotation functions²⁵⁾. A convenient representation of the functions $d_{\lambda \mu}^j(z)$, in terms of hypergeometric functions, which can be extended also to complex values of j , is given by the following formula:

$$d_{\lambda \mu}^j(z) = \text{sign}(\lambda, \mu) \Phi_{\lambda \mu}(j) \left[\sqrt{\frac{1-z}{2}} \right]^a \left[\sqrt{\frac{1+z}{2}} \right]^b \frac{1}{\Gamma(a+1)} \times \\ \times F\left(-j + \frac{1}{2}(a+b), j+1 + \frac{1}{2}(a+b); a+1; \frac{1-z}{2}\right), \quad (39)$$

where $a = |\lambda - \mu|$, $b = |\lambda + \mu|$, $\lambda_{\max} = \max(|\lambda|, |\mu|)$,

$$\Phi_{\lambda \mu}(j) = \left[\frac{\Gamma\left(j+1 + \frac{1}{2}(a+b)\right) \Gamma\left(j+1 + \frac{1}{2}(a-b)\right)}{\Gamma\left(j+1 - \frac{1}{2}(a+b)\right) \Gamma\left(j+1 - \frac{1}{2}(a-b)\right)} \right]^{\frac{1}{2}},$$

$$\text{and sign}(\lambda, \mu) = \begin{cases} 1 & \text{for } \lambda - \mu \geq 0 & \text{and } \lambda + \mu \geq 0 \\ & & \text{and } \lambda - \mu \leq 0 \\ & & \text{and } \lambda + \mu \leq 0 \\ (-1)^{\lambda-\mu} & \text{for } \lambda - \mu \leq 0 & \text{and } \lambda - \mu \leq 0 \\ & & \text{and } \lambda + \mu \geq 0 \\ & & \text{and } \lambda + \mu \leq 0 \end{cases}.$$

The factor $\text{sign}(\lambda, \mu)$ reflects the relations

$$d_{\lambda, \mu}^j(z) = (-1)^{\lambda-\mu} d_{\mu, \lambda}^j(z) = (-1)^{\lambda-\mu} d_{-\lambda, -\mu}^j(z) = d_{-\mu, -\lambda}^j(z).$$

Notice that $d_{0,0}^j(z) = P_j(z)$.

The formula (39) is valid for $|1-z| < 2$. In order to obtain a representation valid for large z one has to continue the hypergeometric function in the following way:

$$F(A, B; C; \xi) \rightarrow F(A', B'; C'; 1/\xi) .$$

This leads to a representation of the $d_{\lambda\mu}^j(z)$ similar to the one for the $P_\ell(z)$ given in Section 4 (notice that in the present case a transformation leading to a hypergeometric function depending on z^2 is no longer possible). With $\lambda_{\max} = \max(|\lambda|, |\mu|) = \frac{1}{2}(a+b)$ we write this formula

$$\begin{aligned} d_{\lambda\mu}^j(z) = & \text{sign}(\lambda, \mu) \Phi_{\lambda\mu}(j) \left[\sqrt{\frac{1-z}{2}} \right]^a \left[\sqrt{\frac{1+z}{2}} \right]^b \times \\ & \times \left\{ \frac{\Gamma(2j+1)}{\Gamma(j+1+\lambda_{\max})\Gamma(j+1-\lambda_{\max}+a)} \left(\frac{z-1}{2}\right)^{j-\lambda_{\max}} \right. \\ & \cdot F\left(-j+\lambda_{\max}, -j+\lambda_{\max}-a; -2j; \frac{2}{1-z}\right) + \\ & + \frac{\Gamma(-2j-1)}{\Gamma(-j+\lambda_{\max})\Gamma(-j-\lambda_{\max}+a)} \left(\frac{z-1}{2}\right)^{-j-1-\lambda_{\max}} \\ & \left. \cdot F\left(j+1+\lambda_{\max}, j+1+\lambda_{\max}-a; 2j+2; \frac{2}{1-z}\right) \right\} . \quad (40) \end{aligned}$$

This representation is valid for $|1-z| > 2$, $|\arg(z-1)| < \pi$, and real or complex j . For large z and $\text{Re } j \geq -\frac{1}{2}$ only the first term survives (the F-function tends to 1), and one obtains the asymptotic behaviour

$$d_{\lambda\mu}^j(z) \underset{\substack{|z| \rightarrow \infty \\ \text{Re } j \geq -1/2}}{\sim} z^j .$$

Returning now to Eq. (38) we see that for large $\cos \theta_t$ the power in j coming from the rotation function is no longer equal to the powers ℓ_N, \dots , etc. which determine the centrifugal barrier effect [Eq. (37)]. Hence one retains some of the square root singularities of the t-channel amplitude. One knows, however, that such singularities are not present in the s-channel amplitude. The question now arises: how do they disappear? In order to

answer this question one has to investigate all the possible constraints of the t-channel amplitudes. It turns out that due to the crossing relation which relates linearly the t-channel helicity amplitudes to those of the s-channel, one arrives at an analytic behaviour in the s-channel²⁶⁾. Later when we discuss the conspiracy problem we will circumvent this difficulty by using invariant amplitudes free of kinematic singularities.

This concludes our discussion of the formal difficulties originating from unequal masses and non-vanishing spins. We insist on their formal nature: they have no observable implication for the s-channel reactions, and it would be quite misleading to invoke them in order to "explain", for example, some features of the observed t-dependence of $T^{(s)}$. The formal nature is also confirmed by the fact that these difficulties are completely absent when an amplitude $T^{(s)}$ with Regge behaviour for $s \rightarrow \infty$ is constructed in a field-theoretical way using Feynman diagrams with propagators and vertex functions which satisfy the normal analyticity properties. The simplest model of this type is a single-particle exchange model (see Fig. 7) where the particle exchanged (particle 5) has an infinite spectrum of mass M and spin J carried by an infinitely rising Regge trajectory $\alpha(M^2) = J$ (for even signature mesons $J = 0, 2, \dots$; for odd ones, $J = 1, 3, \dots$)²⁷⁾. When the external particles 1, 2, 3, 4 are given general masses and spins, the Regge daughter compensation of singularities at $t = 0$ and the compensation of all t-channel threshold and pseudo-threshold singularities are automatic in the sense that these singularities are simply absent in $T^{(s)}$ 2a)

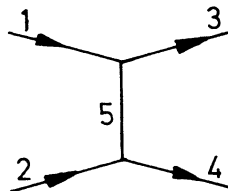


FIG. 7

10. MULTIPLE SCATTERING EFFECTS AND REGGE CUTS

In the present section we study a scattering process which is characterized by a number of successive interactions. Let us consider

the simplest situation of two successive interactions as indicated in Fig. 8. The two incoming particles 1 and 2 scatter at A either through an elastic-like process or through the exchange of internal quantum numbers in the t-channel (which, for example, could be described by a Pomeron pole or another Regge pole)

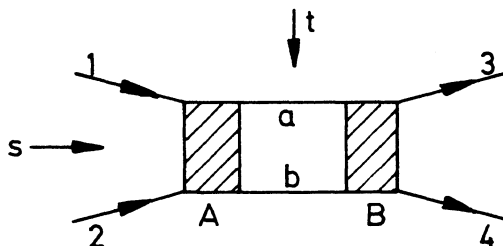


FIG.8

The intermediate particles a and b are considered to be real, i.e. with positive energy and on the mass shell. After a second interaction at B they produce the final particles 3 and 4. This idea of multiple scattering, which is of course fundamental in atomic physics, has been successfully applied to the scattering of high-energy nucleons and pions by complex nuclei. The same principle could also be applied to single hadron scattering in the sense that hadron-hadron collisions could involve double or multiple scattering effects; obvious examples are obtained in the quark model. Another kind of subdivision which seems to be useful in explaining the experimental results is the one where one has a first interaction at A with exchange of definite quantum numbers (not necessarily those of the vacuum) followed by a final-state interaction of diffractive type at B which corresponds to an absorptive correction (or diffraction in A and exchange in B).

Now the question arises: how do such rescattering effects look in the framework of the Regge model? We again take for simplicity spinless particles. The s-channel partial wave decomposition will be written as

$$T^{(s)}(s, t) = \frac{i\sqrt{s}}{4\pi k_s} \sum_{\ell=0}^{\infty} (2\ell + 1) \eta_{\ell} P_{\ell}(\cos \Theta_s), \quad (41)$$

θ_s is the angle of scattering from particle 1 to particle 3. The η_ℓ in Eq. (41) have the following relation to the S-matrix:

$$\langle (3+4)_\ell | S | (1+2)_\ell \rangle = S_\ell = e^{2i\delta_\ell} = 1 - \eta_\ell ; \quad |S_\ell| \leq 1 .$$

Imaginary phase shifts δ_ℓ correspond to real $\eta_\ell > 0$ such that $1 - \eta_\ell$ is below 1.

If one has two successive scatterings one would describe this in the following way for the S-matrix

$$\begin{array}{l} (1) \text{ scattering A: } S_\ell^{(1)} \\ (2) \text{ scattering B: } S_\ell^{(2)} \end{array} \rightarrow \text{for the total scattering } S_\ell = S_\ell^{(1)} S_\ell^{(2)} . \quad (42)$$

In other words, for real intermediate particles one has for each ℓ multiplicativity of S-matrix elements, i.e. additivity of the phase shifts. In terms of the η_ℓ , Eq. (42) reads

$$\begin{aligned} S_\ell^{(1)} &= 1 - \eta_\ell^{(1)} ; \quad S_\ell^{(2)} = 1 - \eta_\ell^{(2)} \\ S_\ell &= 1 - \eta_\ell = 1 - \eta_\ell^{(1)} - \eta_\ell^{(2)} + \eta_\ell^{(1)} \eta_\ell^{(2)} . \end{aligned} \quad (43)$$

Hence, equivalently to Eq. (42) one can write

$$\eta_\ell = \eta_\ell^{(1)} + \eta_\ell^{(2)} - \eta_\ell^{(1)} \eta_\ell^{(2)} . \quad (44)$$

Equation (44) describes the total scattering as the sum of three terms: two single scatterings [scattering in A without interaction in B, $(\eta_\ell^{(1)})$, and scattering in B without interaction in A, $(\eta_\ell^{(2)})$], and a double scattering $(\eta_\ell^{(1)} \eta_\ell^{(2)})$.

How does all this look in the variable t suitable for studying the asymptotic behaviour of $T^{(s)}$ when the individual scatterings are controlled by Regge poles²? Assume

$$T_1^{(s)}(s, t_1) = \frac{i\sqrt{s}}{4\pi k_s} \sum_{\ell=0}^{\infty} (2\ell+1) \zeta_{\ell}^{(1)} P_{\ell}[\cos \Theta_s^{(1)}] \simeq f_1(t_1) s^{\alpha_1(t_1)}$$

$$T_2^{(s)}(s, t_2) = \frac{i\sqrt{s}}{4\pi k_s} \sum_{\ell=0}^{\infty} (2\ell+1) \zeta_{\ell}^{(2)} P_{\ell}[\cos \Theta_s^{(2)}] \simeq f_2(t_2) s^{\alpha_2(t_2)}, \quad (45)$$

where t_1 and t_2 are the individual momentum transfers in A and B. The question is: what is the resulting s -dependence of the total amplitude $T^{(s)}$ which takes double scattering into account? We shall see below that the double scattering term corresponds to a Regge cut, and we shall determine its position in the angular momentum plane.

Before we do this in the framework of the double scattering formalism, let us mention very briefly that there have been also other more sophisticated approaches to the problem of cuts in the angular momentum plane. The basic contribution is due to Mandelstam²⁷⁾. In his approach a certain class of Feynman diagrams are investigated, thus allowing also the intermediate particles to become virtual (remember that in the re-scattering diagram of Fig. 8 the intermediate particles a and b were real). The result of this more complicated analysis is that only those Feynman graphs which have a third double spectral function with respect to the t -channel reaction ($\rho_{su} \neq 0$ in Mandelstam's notation) will lead to cuts in the angular momentum plane. The simplest such graph contains two "crosses" and is of the type shown in Fig. 9. The wavy lines represent

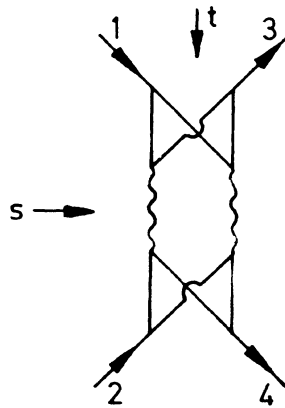


FIG. 9

Regge poles and would correspond to the A and B scatterings of Fig. 8. The crosses should simulate approximately the fact that in Fig. 8 particles a and b should propagate "forward in time" and, at least approximately, as real particles. It turns out that the location of the branch points when studied in this Mandelstam approach is exactly the same as the one we shall obtain in the much simpler double scattering approach.

Let us now pursue our analysis and determine the singularities in the angular momentum plane of an amplitude containing double scattering terms.

For the initial and final momenta in the s-channel

$$k_s = \frac{1}{2\sqrt{s}} \sqrt{s - (m_1 + m_2)^2} \sqrt{s - (m_1 - m_2)^2}$$

$$k'_s = \frac{1}{2\sqrt{s}} \sqrt{s - (m_3 + m_4)^2} \sqrt{s - (m_3 - m_4)^2} ,$$

we have for large s

$$k_s \simeq \frac{\sqrt{s}}{2} \left[1 - \frac{m_1^2 + m_2^2}{s} \dots \right]$$

$$k'_s \simeq \frac{\sqrt{s}}{2} \left[1 - \frac{m_3^2 + m_4^2}{s} \dots \right] ,$$

and similarly for $\cos \theta_s$ for finite t and large s

$$\cos \theta_s \simeq 1 + \frac{2t}{s} \dots .$$

From the last expression it follows that $\cos \theta_t$ is close to 1 for large s and finite t.

Using the formula

$$P_\ell \left(1 + \frac{2t}{s} \right) \rightarrow J_0 \left((2\ell + 1) \sqrt{\frac{-t}{s}} \right)$$

for $s \rightarrow \infty$, $\ell \rightarrow \infty$ with $(2\ell + 1) \sqrt{-t/s}$ finite, where $J_0(x)$ is the zero-th order Bessel function, we rewrite Eq. (41) and similarly Eqs. (45) in terms of the impact parameter $b = (2\ell + 1)/\sqrt{s} \simeq \ell/k_s$:

$$T^{(s)}(s, t) = \frac{is}{4\pi} \int_0^{\infty} b\eta(b)J_0(b\sqrt{-t}) db \quad (46)$$

$$T_j^{(s)}(s, t_j) = \frac{is}{4\pi} \int_0^{\infty} b\eta^{(j)}(b)J_0(b\sqrt{-t}) db ; \quad j = 1, 2 .$$

In the transition to (46) we have made the replacement $\Sigma_{\ell} \rightarrow 1/\Delta b \int_0^{\infty} db$ with $\Delta b = 2/\sqrt{s}$ (remember that $\Delta l = 1$), and we have set $\eta_{\ell} = \eta[(2\ell + 1)/\sqrt{s}] \simeq \eta(b)$ and similarly for $\eta^{(j)}$. The expression (46) can be further transformed using a two-dimensional vector notation (\vec{q}, \vec{q}_j : two-dimensional momentum transfer vectors of lengths $\sqrt{-t}, \sqrt{-t_j}$, i.e. $|\vec{q}|^2 = -t, |\vec{q}_j|^2 = -t_j$, and \vec{b} : two dimensional impact parameter vector)

$$T^{(s)}(s, t) = \frac{is}{8\pi^2} \iint \eta(b) e^{i\vec{b}\vec{q}} d_2 b \quad (47)$$

$$T_j^{(s)}(s, t_j) = \frac{is}{8\pi^2} \iint \eta^{(j)}(b) e^{i\vec{b}\vec{q}_j} d_2 b ; \quad j = 1, 2 .$$

The vectors \vec{q}, \vec{q}_j are best defined as $\vec{q} = \vec{p}_1 - \vec{p}_3|_{\text{c.m.s.}}$, $\vec{q}_1 = \vec{p}_1 - \vec{p}_a|_{\text{c.m.s.}}$, and $\vec{q}_2 = \vec{p}_a - \vec{p}_3|_{\text{c.m.s.}}$, and are easily proved to be perpendicular to the incident direction for t, t_1, t_2 fixed and large s . By definition we take \vec{b} also in the plane orthogonal to the direction of motion. The equivalence of Eqs. (46) and (47) -- after introducing polar coordinates in Eq. (47) -- follows from the following integral representation of the Bessel function:

$$J_0(b\sqrt{-t}) = \frac{1}{2\pi} \int_0^{2\pi} e^{ib\sqrt{-t} \cos \phi} d\phi .$$

The advantage of the representation (47) is that it gives a direct wave picture of the scattering process. We shall use this form to calculate the double scattering term and determine to what kind of singularities in the angular momentum plane it corresponds.

10. MULTIPLE SCATTERING EFFECTS AND REGGE CUTS (continued)

We shall now use the impact parameter formulation of scattering as given by Eq. (47). For two successive scatterings, as depicted in Fig. 10, we write according to Eq. (44)

$$\eta(b) = \eta^{(1)}(b) + \eta^{(2)}(b) - \eta^{(1)}(b)\eta^{(2)}(b) . \quad (44')$$

Before studying this for Regge-pole scattering, we make a few remarks on Eq. (44').

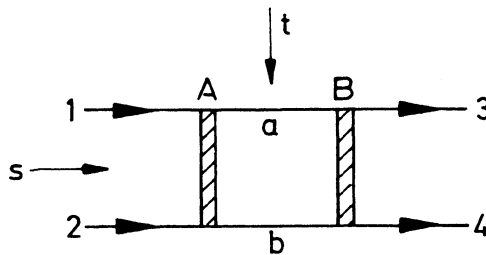


FIG.10

For purely diffractive scattering the amplitude $T_j^{(s)}(s,t)$ becomes imaginary which, according to Eq. (46), corresponds to real values of $\eta^{(j)}(b)$. This, by the way, was the reason for having extracted a factor i in Eq. (41) above. If now one of the interactions, say (2), is elastic ($a = 3$ and $b = 4$) and purely diffractive, while the other, say (1), describes an exchange reaction ($3 \neq 1$ and/or $2 \neq 4$), then the complete exchange amplitude from Eq. (44') is $\eta^{(1)}(1 - \eta^{(2)})$. Since $\eta^{(2)}(b) > 0$, the so-called absorptive correction $-\eta^{(1)}\eta^{(2)}$ has opposite sign as compared to the single scattering $\eta^{(1)}$. This is a general feature of multiple scattering. It means a decrease of the amplitude when rescattering corrections are included.

Another consideration based on Eq. (44') and Fig. 10 concerns the situation where one has a large number of successive scatterings (Fig. 11) each of which is supposed to be small. The total S-matrix element corresponding to n successive scatterings is given by

$$S(b) = \prod_{i=1}^n S_i(b) .$$

With $S_{\pm}(b) = 1 - \delta\eta(b)$, assuming $\delta\eta(b)$ small and the same for all individual

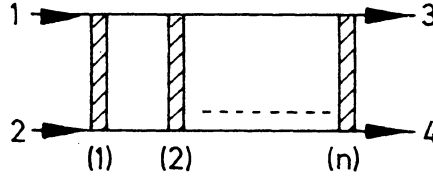


FIG.11

scatterings, one has

$$1 - \eta(b) = [1 - \delta\eta(b)]^n . \quad (48)$$

We now consider the case where n is large and $\delta\eta(b)$ is small such that $n \delta\eta(b) = \eta_s(b)$ is finite. Then Eq. (48) becomes

$$1 - \eta(b) = S(b) = e^{-\eta_s(b)} . \quad (49)$$

Equation (49) expresses the well-known fact that the total S-matrix for an unlimited number of very weak, successive scatterings is given by an exponential function with the single scattering as exponent.

After these remarks let us now proceed with our analysis of the double scattering diagram in Regge-pole theory. We write for the total amplitude corresponding to the diagram of Fig. 10 using Eq. (44'):

$$T^{(s)}(s, t) = T_1^{(s)}(s, t) + T_2^{(s)}(s, t) + T_{1,2}^{(s)}(s, t) , \quad (50)$$

where the single scattering terms $T_1^{(s)}$ and $T_2^{(s)}$ are given by Eq. (47), and the double scattering term $T_{1,2}^{(s)}$ is given by

$$T_{1,2}^{(s)}(s, t) = - \frac{is}{8\pi^2} \iint \eta^{(1)}(b) \eta^{(2)}(b) e^{i\vec{b}\vec{q}} d_2 b . \quad (51)$$

In order to write the right-hand side of Eq. (51) as a convolution integral of $T_1^{(s)}$ and $T_2^{(s)}$, we change the variable b in Eq. (51) and call it b_2 , and introduce together with the two-dimensional δ -function

$$\delta_2(b_1 - b_2) = \frac{1}{4\pi^2} \iint e^{i(\vec{b}_1 - \vec{b}_2)\vec{q}_1} d_2 q_1$$

a further integration over b_1 . The result is

$$T_{12}^{(s)}(s, t) = - \frac{is}{8\pi^2} \int \frac{1}{4\pi^2} \int \eta^{(1)}(b_1) e^{i\vec{b}_1 \cdot \vec{q}_1} d_2 b_1 \int \eta^{(2)}(b_2) e^{i\vec{b}_2 \cdot (\vec{q} - \vec{q}_1)} d_2 b_2 d_2 q_1 . \quad (52)$$

This expression is equivalent to

$$T_{12}^{(s)}(s, t) = \frac{i}{s\pi} \int T_1^{(s)}(s, -|\vec{q}_1|^2) T_2^{(s)}(s, -|\vec{q} - \vec{q}_1|^2) d_2 q_1 . \quad (53)$$

This last equation [Eq. (53)] enables us to study the Regge cut which is produced by the successive exchange of two Regge poles.

To obtain the Regge singularity of the double scattering term (53) in the full amplitude (50) we now insert the Regge-pole behaviour

$$T_j^{(s)}(s, t_j) = f_j(t_j) s^{\alpha_j(t_j)}$$

into Eq. (53) and obtain an integral over a continuum of powers of s

$$T_{12}^{(s)}(s, -|\vec{q}|^2) = \frac{i}{\pi} \int f_1(-|\vec{q}_1|^2) f_2(-|\vec{q} - \vec{q}_1|^2) s^{\alpha_1(-|\vec{q}_1|^2) + \alpha_2(-|\vec{q} - \vec{q}_1|^2) - 1} d_2 q_1 . \quad (54)$$

Comparing this expression with the Regge-cut form of Eq. (14) which reads for large s

$$T(s) \sim \int_{A(t)}^{\alpha_c(t)} f(\alpha, t) s^\alpha d\alpha , \quad (14')$$

we see that the double scattering term (54) corresponds to a Regge-cut contribution to the total amplitude (50). The distinctive feature of such a cut contribution is that it is not governed by a single power of s , but by a continuous superposition of powers of s up to a maximum $\alpha_c(t)$ which determines the branch point at the right end of the cut. From Eq. (54)

$$\alpha_c(t) = \max_{(\vec{q}_1)} \{ \alpha_1(-|\vec{q}_1|^2) + \alpha_2(-|\vec{q} - \vec{q}_1|^2) - 1 \} . \quad (55)$$

This expression gives the position of the branch point as a function of the position of the poles α_1 and α_2 . The vectors \vec{q} , \vec{q}_1 , and $\vec{q} - \vec{q}_1$ form a

triangle. Let us assume that α_1 and α_2 are non-decreasing functions of their arguments. Then it is clear that one has to minimize the momentum transfers in α_1 and α_2 -- corresponding to a flat triangle, i.e. \vec{q}_1 and $\vec{q}-\vec{q}_1$ in the same direction-- in order to find the maximum in Eq. (55). Introducing $t_j = -|\vec{q}_j|^2$, one can rewrite Eq. (55) as

$$\alpha_c(t) = \max \{ \alpha_1(t_1) + \alpha_2(t_2) - 1 \} , \quad (55')$$

subject to the condition $\sqrt{-t_1} + \sqrt{-t_2} = \sqrt{-t}$, with all square roots positive. The solution of this problem for linear trajectories

$$\alpha_j(t_j) = \alpha_j^0 + \alpha_j' t_j$$

is the following linear trajectory for the branch point:

$$\alpha_c(t) = \alpha_c^0 + \alpha_c' t = \alpha_1^0 + \alpha_2^0 - 1 + \frac{\alpha_1' \alpha_2'}{\alpha_1' + \alpha_2'} t . \quad (56)$$

We see from Eq. (56) that the intercept of the branch-point trajectory is given by

$$\alpha_c^0 = \alpha_1^0 + \alpha_2^0 - 1 \quad (57)$$

and its slope is given by

$$\alpha_c' = \frac{\alpha_1' \alpha_2'}{\alpha_1' + \alpha_2'} . \quad (58)$$

Let us rewrite the Eqs. (57) and (58) to exhibit the additivity properties of the intercepts and the slopes of the individual pole contributions in the expression for the corresponding quantities of the cut

$$(\alpha_c^0 - 1) = (\alpha_1^0 - 1) + (\alpha_2^0 - 1) , \quad (57')$$

$$\frac{1}{\alpha_c'} = \frac{1}{\alpha_1'} + \frac{1}{\alpha_2'} . \quad (58')$$

The Eqs. (57') and (58') are easily extended to the situation where more than two Regge poles are exchanged.

If one of the poles is the Pomeranchuk pole ($\alpha_P^0 = 1$) then the intercept of the cut is equal to that of the other pole, but the slope of the

cut is mostly determined by the Pomeranchuk, since α'_P is small or even zero and hence will dominate the right-hand side of Eq. (58'). Let us consider, for instance, the contribution of the ρP cut due to exchange of the ρ meson and Pomeranchuk trajectories in the case of the $\bar{\pi} p$ charge-exchange reaction. The s -dependence of the contribution from the ρP cut to $d\sigma/dt$ will be the same in the forward direction as the usual ρ -pole contribution, namely (except for a logarithmic factor)

$$\left. \frac{d\sigma}{dt} \right|_{t=0} \sim s^{2(\alpha'_\rho - 1)}.$$

It is different for all $t < 0$, and the resulting phase factor $(1 - e^{-i\pi\alpha})$ will also be different. Hence there will be an interference of the ρ -pole and ρP -cut contributions leading to a non-vanishing polarization of the final nucleon. As mentioned in Section 7, such a polarization has been found experimentally. Away from $t = 0$ the ρP cut lies above the ρ trajectory. It would thus dominate over the ρ pole at sufficiently large s , but this would occur only at very high energies if the cut is coupled much more weakly than the ρ pole.

It is also interesting to consider multiple Pomeranchuk exchange in diffraction-like scattering. Then the n -times iterated P -pole -- corresponding to the multiple scattering diagram of Fig. 11 -- dominates at large s and $t < 0$ all graphs with a smaller number of P exchanges. This is due to the fact that at $t = 0$ the P pole, the PP cut, the PPP cut, etc., are all superimposed, and that if the slope of P is α'_P then the slope of the cut due to nP exchange is $\alpha'_{nP} = \alpha'_P/n$. This is another aspect of the fact that the P trajectory has good reasons for being quite a complicated object.

If one considers the exchange of two Regge poles carrying quantum numbers different from those of the vacuum, then the intercept of the corresponding cut lies below those of the pole trajectories. Thus at $t = 0$ the asymptotically leading contributions are still given by the single scattering terms. However, at some negative value of t , the double scattering contribution may take over and dominate the large s behaviour since the cut has a smaller slope. For processes like double charge or strangeness exchange, or similar reactions which are not possible through single Regge-pole exchange, one expects the whole reaction to be due to

double exchange, i.e. to a Regge cut. This is one of the many reasons why a systematic experimental study of these reactions is of the greatest interest (also for other theories of high-energy scattering, like the one based on the quark model, these reactions play an essential role).

The foregoing discussion is of course not a proof that Regge cuts really exist, but the theoretical and experimental evidence for their existence is just as good as for the Regge poles themselves, so that their study is equally important.

We end by a remark on the assumption which was made throughout our discussion of double scattering, namely that the particles a and b in the intermediate state between the interactions A and B (see Fig. 10) are real and not virtual. This assumption is the simplest one possible and allows us to describe the scatterings A and B by their respective S-matrix elements, as we did when we represented them by Regge-pole expressions. The assumption need not be true, however, and it can be replaced by others for which various choices are possible, corresponding to various ways of treating particles a and b when they are virtual (i.e. off their mass shell). A well-known example of alternative assumption is the one used in Sopkovich's theory of absorptive corrections to exchange reactions, as extensively studied by Gottfried, Jackson, and many others³⁰). Sopkovich's absorptive model is closely related to the distorted wave Born approximation often used in nuclear reaction theory. In absence of any real theory of strong interactions, only experiment can decide which assumption is the best, a fact that adds further interest to the study of reactions where double or multiple scattering effects are expected to be important.

11. CONSPIRACY IN PION PHOTOPRODUCTION

LECTURES VIII and IX

One has called a "conspiracy" the possibility that at high energies several Regge poles coincide at a particular value of the momentum transfer and thereby produce a perfectly regular behaviour of the amplitude at that point, although the various conspiring poles -- when taken separately -- are singular. We will be especially concerned with the forward direction, corresponding to $t = 0$ for elastic processes, and

approaching the value $t = 0$ for large s in the case of a general reaction involving particles with arbitrary masses. The most important and interesting case of conspiracy in the forward direction comes up in reactions where contributions corresponding to the exchange of opposite parities occur with the same strength, although they belong to different Regge trajectories.

The problem occurs independently of the Regge-pole model. We therefore discuss it first by analysing any exchange mechanism in terms of spin j and parity P of the object exchanged.

In Fig. 12 we consider the exchange in the t -channel of a dynamical system (it need not be a single particle) with spin j and parity P . For $P = (-1)^j$ we speak of an exchanged system with natural parity; for $P = -(-1)^j$ we speak of one with unnatural parity. The "naturalness" of the parity is thus defined by $P' = P(-1)^j$, being $+1$ for natural parity and -1 for unnatural parity

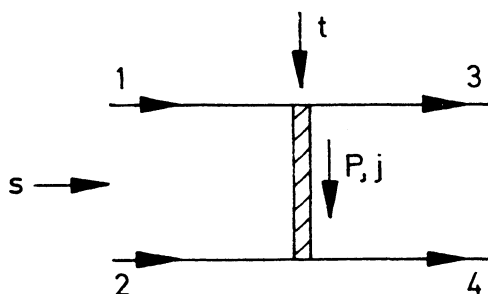


FIG. 12

We shall, in this section, treat in detail the process $\gamma + p \rightarrow \pi^+ + n$ near the forward direction where conspiracy has been observed experimentally. Our first aim will be to study carefully, in the photoproduction of positive pions, the effect of exchanging a definite $P' = \pm 1$ in the t -channel. We consider an amplitude corresponding to the diagram shown in Fig. 13 belonging to either $P' = +1$ or $P' = -1$

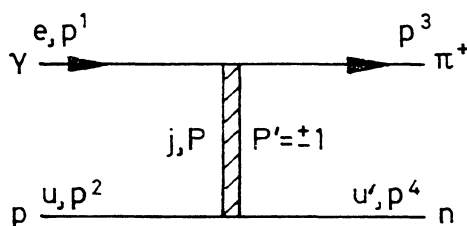


FIG. 13

We shall see that for natural as well as unnatural parity exchange the amplitude corresponding to Fig. 13 has a dip in the forward direction at high energies. If, however, one gives up the idea that at high energies the photoproduction of charged pions is governed by the exchange of a definite P' , and instead assumes that this process is described by the exchange of both natural as well as unnatural parity contributions of equal strength (in fact singular at $t = 0!$), one can obtain a finite, non-zero, forward cross-section at high energies. This co-operation among the $P' = +1$ and $P' = -1$ amplitudes in π^+ photoproduction would then explain the experimentally observed forward peak.

Before we study the amplitudes corresponding to the diagram of Fig. 13 with definite P' , let us assemble a number of standard formulae for the description of photoproduction which will be needed in the following discussion.

The T-matrix element for the process $\gamma + p \rightarrow \pi^+ + n$ is given by the following relativistic invariant decomposition

$$T(s,t) = \sum_i M_i A_i(s,t), \quad (59)$$

where the M_i are kinematical factors containing the polarization vector e_μ of the photon, and the initial and final Dirac spinors u and u' of the nucleons (see Fig. 13). The general structure of the M_i is

$$M_i = \bar{u}' O_\mu^i u e_\mu,$$

where the operators O_μ^i are some set of 4×4 matrices constructed from the Dirac matrices and the independent momenta. For later use let us state also the 16 Dirac covariants which are needed at the lower vertex in Fig. 13 involving the nucleons: $\bar{u}'u$ (scalar), $\bar{u}'\gamma_\mu u$ (vector), $i\bar{u}'\gamma_5\gamma_\mu u$ (pseudovector), $\bar{u}'\gamma_5 u$ (pseudoscalar), $\bar{u}'\sigma_{\mu\nu} u = \bar{u}'(1/2i) \times [\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu]u$ (tensor).

The M_i are constructed in such a way that the invariant functions $A_i(s,t)$ which multiply them in Eq. (59) have only dynamical singularities and obey a Mandelstam representation. In particular, the $A_j(s,t)$ cannot

be singular in the forward direction at high energies, i.e. at $t = 0$, since there is no strongly interacting particle of zero mass.

It is easy to convince oneself that the number of independent amplitudes in pion photoproduction is four. There are two (transverse) polarization states for the incoming photon, one for the outgoing pion, and two possible spin orientations for both the initial and the final nucleon, thus allowing a total of eight different arrangements which reduce to four due to parity invariance (reflection with respect to the plane of collisions). One possible choice of the four kinematical factors M_i is³¹⁾

$$\begin{aligned}
 M_1 &= \bar{u}' \sigma_{\rho\sigma} u \epsilon_{\rho\sigma\mu\nu} e_\mu p_\nu^1 \\
 M_2 &= 2i \bar{u}' \gamma_5 u p_\mu^2 p_\nu^4 (e_\mu p_\nu^1 - e_\nu p_\mu^1) \\
 M_3 &= \bar{u}' \gamma_5 \gamma_\mu u p_\nu^2 (e_\mu p_\nu^1 - e_\nu p_\mu^1) \\
 M_4 &= \bar{u}' \gamma_5 \gamma_\mu u p_\nu^4 (e_\mu p_\nu^1 - e_\nu p_\mu^1) .
 \end{aligned} \tag{60}$$

Here $\epsilon_{\rho\sigma\mu\nu}$ is the fully antisymmetric tensor ($\epsilon_{1234} = 1$). The matrices (60) are pseudoscalars corresponding to the fact that the produced pion is pseudoscalar. They are gauge invariant, which means that they all vanish when the photon polarization vector e_μ is replaced by the photon momentum p_μ^1 . By going over from the Dirac spinors u, u' to the non-relativistic Pauli spinors χ, χ' in Eq. (60), it is easy to show that in the forward direction for $s \rightarrow \infty$ (in which case the nucleon recoil becomes zero in the laboratory system) the four M_i reduce to a single one proportional to $\chi' \vec{\sigma} \chi$, where $\vec{\sigma} = (\sigma_1, \sigma_2, \sigma_3)$ represent the well-known 2×2 non-relativistic spin matrices of Pauli. There is indeed only one amplitude which describes pion photoproduction in the forward direction (this is an obvious consequence of rotation and parity invariance). The forward direction (c.m. scattering angle $\Theta_s = 0$) corresponds to $t = 0$ at high energies as can be seen from the following two formulae (the first one being relevant to $\gamma + p \rightarrow \pi^+ + n$):

$$\begin{matrix} m_1 \neq m_3 \\ m_2 = m_4 \end{matrix} : t = \frac{s}{2} (\cos \Theta_s - 1) \left[1 + O\left(\frac{1}{s}\right) \right] - \frac{m_2^2 (m_3^2 - m_1^2)^2}{s^2} + O\left(\frac{1}{s^3}\right), \quad (61)$$

$$\begin{matrix} m_1 \neq m_3 \\ m_2 \neq m_4 \end{matrix} : t = \frac{1}{2} (\cos \Theta_s - 1) \left[s - m_1^2 + m_2^2 - m_3^2 - m_4^2 + \frac{1}{s} (m_1^2 + m_2^2)(m_3^2 + m_4^2) - \frac{2}{s} (m_1^2 m_2^2 + m_3^2 m_4^2) \right] + \frac{1}{s} (m_1^2 - m_3^2)(m_4^2 - m_2^2) + O\left(\frac{1}{s^2}\right). \quad (62)$$

After these introductory remarks we now return to the discussion of the conspiracy problem in pion photoproduction. As was mentioned above, we are interested in the behaviour of the amplitude corresponding to the diagram of Fig. 13 for natural as well as unnatural parity exchange. From the preceding discussion we know that in the forward direction the number of independent amplitudes reduces to one. The crucial point is that this single amplitude mixes $P' = 1$ and $P' = -1$. To show this we study the amplitude in more detail so as to determine its behaviour in the forward direction under the assumption that the exchanged object has natural parity (case a) or unnatural parity (case b).

Case a: $P' = +1$

We know from field theory that the propagator for the exchanged system of integral spin j in Fig. 13 is a tensor constructed from $\delta_{\mu\nu}$ and the four-momentum $q_\mu = p_\mu^1 - p_\mu^3 = p_\mu^4 - p_\mu^2$ carried by the exchanged system. It has the shape $\mathcal{P}_{\mu_1 \dots \mu_j, \nu_1 \dots \nu_j}$. It is fully symmetric in μ_1, \dots, μ_j , also fully symmetric in ν_1, \dots, ν_j , and symmetric for exchange of all μ 's with all ν 's.

A familiar example is the propagator for $j = 1$ given by

$$\delta_{\mu\nu} + \frac{q_\mu q_\nu}{t}, \quad t = -q_\mu q_\mu$$

(remember that we use the metric with $\mu = 1, 2, 3, 4$, and $q_4 = iq_0$). In the general case \mathcal{P} can simply be constructed in terms of the Legendre polynomial $P_j(x)$ by the following identity

$$\begin{aligned} a_{\mu_1} \dots a_{\mu_j} \mathcal{P}_{\mu_1 \dots \mu_j, \nu_1 \dots \nu_j} b_{\nu_1} \dots b_{\nu_j} &= [(a \cdot a)_q (b \cdot b)_q]^{j/2} \times \\ &\times P_j\left(\frac{(a \cdot b)_q}{[(a \cdot a)_q (b \cdot b)_q]^{1/2}}\right) \end{aligned}$$

with the definition

$$(a \cdot b)_q = a_\mu \left(\delta_{\mu\nu} + \frac{q_\mu q_\nu}{t} \right) b_\nu,$$

and similarly for $(a \cdot a)_q$ and $(b \cdot b)_q$.

The propagator $\mathcal{P}_{\mu_1 \dots \mu_j, \nu_1 \dots \nu_j}$ has to be contracted with a vertex function $V_{\mu_1 \dots \mu_j}^{13}$ at the upper vertex, and a vertex function $V_{\nu_1 \dots \nu_j}^{24}$ at the lower vertex. Both vertex functions have to be relativistic tensors which are fully symmetric with respect to permutation of their indices.

To construct $V_{\mu_1 \dots \mu_j}^{13}$ we have at our disposal the photon polarization vector e_μ and the momenta p_μ^1 and p_μ^3 ; to construct $V_{\nu_1 \dots \nu_j}^{24}$ we have p_ν^2 and p_ν^4 and the Dirac spinors u and u' . Moreover, $V_{\nu_1 \dots \nu_j}^{13}$ has to vanish for the substitution $e_\mu \rightarrow p_\mu^1$ (gauge invariance) and has to behave like a relativistic pseudotensor. The last-mentioned property is due to the fact that we are concerned here with the exchange of an object having $P' = +1$. Since $P'_\gamma = +1$ and $P'_\pi = -1$, the vertex function couples two natural parity states to an unnatural parity state, and thus has to be a pseudotensor. On the contrary, $V_{\nu_1 \dots \nu_j}^{24}$ has to be a relativistic tensor since it couples a boson with $P' = +1$ to two fermions with relative $P' = +1$.

To obtain a pseudotensor for $V_{\mu_1 \dots \mu_j}^{13}$ one has to combine e , p^1 , and p^3 using the ϵ -tensor, in this way giving automatically a gauge invariant expression:

$$V_{\mu_1 \dots \mu_j}^{13} = \left\{ \epsilon_{\mu_1 \nu \rho \sigma} e_\nu p_\rho^1 p_\sigma^3 (p_{\mu_2}^1 + p_{\mu_2}^3) \dots (p_{\mu_j}^1 + p_{\mu_j}^3) \right\}_{\text{sym. in } \mu_1 \dots \mu_j}. \quad (63)$$

The curly brackets on the right-hand side should indicate that the expression has to be symmetrized with respect to the indices $\mu_1 \dots \mu_j$. The reason why the sum of the momenta p^1 and p^3 appears, and not any other linear combination, will be explained below.

To construct the tensor $V_{\nu_1 \dots \nu_j}^{24}$ corresponding to the coupling of the spin j object with natural parity to the nucleons, we observe that

among the 16 Dirac covariants the $\bar{u}'\gamma_5 u$ cannot appear (otherwise there would be a $\epsilon_{\mu\nu\rho\sigma}$ to be contracted with three momenta, which gives 0 because only two independent momenta p^2 and p^4 are available). The pseudovector, appearing as $\epsilon_{\nu\rho\sigma\tau} i\bar{u}'\gamma_5\gamma_\rho u p_\sigma^2 p_\tau^4$, and the tensor $\bar{u}'\sigma_{\nu\rho} u$, appearing contracted with p_ρ^2 or p_ρ^4 , can be transformed by using the Dirac equations $(p_\nu^2\gamma_\nu - im)u = 0$ and $\bar{u}'(p_\nu^4\gamma_\nu - im) = 0$, and they will reduce to scalar and vector expressions. From this discussion it follows that $V_{\nu_1 \dots \nu_j}^{24}$ will be given by any linear combination of the following two expressions

$$V_{\nu_1 \dots \nu_j}^{24} = \left\{ \begin{array}{l} \bar{u}' u(p_{\nu_1}^2 + p_{\nu_1}^4) \dots (p_{\nu_j}^2 + p_{\nu_j}^4) \\ \left\{ \bar{u}' \gamma_{\nu_1} u(p_{\nu_2}^2 + p_{\nu_2}^4) \dots (p_{\nu_j}^2 + p_{\nu_j}^4) \right\}_{\text{sym. in } \nu_1 \dots \nu_j} \end{array} \right. \quad (64)$$

Let us now explain why we have written $(p^1 + p^3)$ and $(p^2 + p^4)$ in Eq. (63) and Eq. (64), respectively. Let us discuss this choice for the case of spin j exchange in a reaction involving only spinless particles (see Fig. 14)

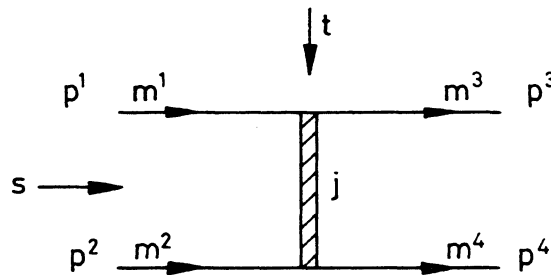


FIG. 14

The amplitude for the diagram shown in Fig. 14 is given by

$$V_{\mu_1 \dots \mu_j}^{13} \mathcal{F}_{\mu_1 \dots \mu_j, \nu_1 \dots \nu_j} V_{\nu_1 \dots \nu_j}^{24}$$

This expression must lead to a Legendre polynomial $P_j(\cos \Theta_t)$ because it corresponds to an orbital angular momentum j in the t channel. If it is now claimed that V^{13} and V^{24} have to be constructed with the sum of the respective momenta, i.e.

$$V_{\mu_1 \dots \mu_j}^{13} = (p_{\mu_1}^1 + p_{\mu_1}^3) \dots (p_{\mu_j}^1 + p_{\mu_j}^3) \quad (65)$$

$$V_{\nu_1 \dots \nu_j}^{24} = (p_{\nu_1}^2 + p_{\nu_1}^4) \dots (p_{\nu_j}^2 + p_{\nu_j}^4)$$

then, for $j = 1$, this would lead to the following amplitude

$$\begin{aligned} V_{\mu}^{13} \left(\delta_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{t} \right) V_{\nu}^{24} &= (p_{\mu}^1 + p_{\mu}^3) \left(\delta_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{t} \right) (p_{\nu}^2 + p_{\nu}^4) \\ &= - \left(s - u + \frac{1}{t} (m_1^2 - m_3^2)(m_2^2 - m_4^2) \right). \end{aligned} \quad (66)$$

According to Eq. (24') the last expression is exactly proportional to $\cos \Theta_t$, that is $P_1(\cos \Theta_t)$. Using another combination of momenta in Eq. (65) would introduce other Legendre polynomials of order lower than j (for example, an additional constant for $j = 1$), and hence would not correspond to pure spin j exchange.

We now show that the expression (63) will force the amplitude to vanish for $t = 0$, thus leading to a dip in the forward direction at high energies. Let us first notice that we can write in Eq. (63)

$$\epsilon_{\mu_1 \nu \rho \sigma} e_{\nu} p_{\rho}^1 p_{\sigma}^3 = - \epsilon_{\mu_1 \nu \rho \sigma} e_{\nu} p_{\rho}^1 q_{\sigma} \quad (67)$$

Remembering Eq. (61) we know that $t = -q^2$ behaves for large s and in the forward direction as $1/s^2$. Thus q_{σ} becomes small in the forward direction for large s . Let us study the forward direction kinematics in the laboratory system (Fig. 15)

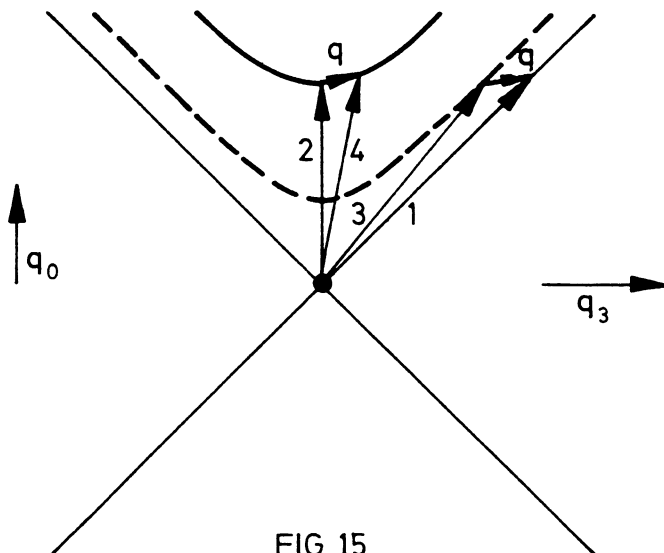


FIG. 15

The full hyperbola represents the mass shell of the nucleon, the dashed one the mass shell of the pion. It is seen from this figure that the pion is slowed down somewhat as compared to the photon (which is evident since the pion is a massive particle). On the other hand, the final nucleon 4 gets a recoil of the same order. It is clear from Fig. 15 that q_3 vanishes with increasing s like $1/s$, whereas q_1 and q_2 are finite. From this discussion it follows that a vertex function of the type (63) will necessarily lead to a dip in the forward direction.

Case b: $P' = -1$

Considering now the exchange of an object with unnatural parity we can, after the foregoing discussion, write down the vertex functions immediately, taking into account that now $V_{\mu_1 \dots \mu_j}^{13}$ has to be a relativistic tensor, whereas $V_{\nu_1 \dots \nu_j}^{24}$ has to be a relativistic pseudotensor. The expressions are

$$V_{\mu_1 \dots \mu_j}^{13} = \left\{ [e_{\mu_1} (p^1 \cdot p^3) - p_{\mu_1}^1 (e \cdot p^3)] (p_{\mu_2}^1 + p_{\mu_2}^3) \dots (p_{\mu_j}^1 + p_{\mu_j}^3) \right\} \text{sym. in } \mu_1 \dots \mu_j \quad (68)$$

$$V_{\nu_1 \dots \nu_j}^{24} = \begin{cases} \bar{u}' \gamma_5 u (p_{\nu_1}^2 + p_{\nu_1}^4) \dots (p_{\nu_j}^2 + p_{\nu_j}^4) \\ \left\{ i \bar{u}' \gamma_5 \gamma_{\nu_1} u (p_{\nu_2}^2 + p_{\nu_2}^4) \dots (p_{\nu_j}^2 + p_{\nu_j}^4) \right\} \text{sym. in } \nu_1 \dots \nu_j \end{cases} \quad (69)$$

In Eq. (68) we can now again replace p^3 by $-q$

$$e_{\mu_1}(p^1 \cdot p^3) - p_{\mu_1}^1(e \cdot p^3) = - [e_{\mu_1}(p^1 \cdot q) - p_{\mu_1}^1(e \cdot q)] . \quad (70)$$

The equivalence expressed in Eq. (70) is due to the vanishing mass of the photon $p^1 \cdot p^1 = 0$, and to the Lorentz condition $e \cdot p^1 = 0$. The same argument as above now again leads to the conclusion that the amplitude corresponding to unnatural parity exchange produces a dip in the forward direction.

This now leads us to the result--regardless of whether we treat the Reggeized version of the exchange, or any other model specifying the exchanged P' --that if the exchange is dominantly $P' = 1$ or $P' = -1$ then there must be a dip in the forward direction of the differential cross-section at high energies. If, however, one wants to reproduce a peak in the forward direction as found experimentally³²⁾, one must consider an interplay of both $P' = 1$ and $P' = -1$ contributions, each of which turn out to be singular at $t = 0$. The total contribution

$$T = T^{(P'=1)} + T^{(P'=-1)}$$

is, of course, regular at $t = 0$. Such a situation was called above a 'conspiracy'.

The question now arises: how can one construct an amplitude which is finite at $t = 0$ for $s \rightarrow \infty$ and which gives both $P' = 1$ and $P' = -1$ contributions? Until now we have used for the nucleon vertex function in Case a ($P' = 1$) only the scalar and vector Dirac covariants, and in Case b ($P' = -1$) only the pseudoscalar and pseudovector Dirac covariants. The antisymmetric tensor $\bar{u}' \sigma_{\mu\nu} u$ for the nucleon vertex has never been used until now. The reason is simple: it contains both $P' = 1$ and $P' = -1$ parts and hence is the candidate to be used for constructing the amplitude we are looking for. Using this Dirac covariant for the nucleon vertex, a gauge invariant and pseudoscalar expression for an amplitude giving both $P' = 1$ and $P' = -1$ contributions is given by

$$\bar{u}' \sigma_{\mu\nu} u \epsilon_{\mu\nu\rho\sigma} e_{\rho} p_{\sigma}^1 A(s, t) . \quad (71)$$

This can be decomposed into $P' = 1$ and $P' = -1$ contributions of the type derived above:

$$T^{(P'=+1)} = [\bar{u}' u(p_\mu^2 + p_\mu^4) A_1(s, t) + \bar{u}' \gamma_\mu u A_2(s, t)] \epsilon_{\mu\nu\rho\sigma} e_\nu p_\rho^1 p_\sigma^3 . \quad (72)$$

$$T^{(P'=-1)} = [\bar{u}' \gamma_5 u(p_\mu^2 + p_\mu^4) A_3(s, t) + i \bar{u}' \gamma_5 \gamma_\mu u A_4(s, t)] [e_\mu (p^1 \cdot p^3) - p_\mu^1 (e \cdot p^3)] . \quad (73)$$

We shall show that in separating expression (71) into the contributions (72) and (73), a singularity at $t = 0$ will appear in the coefficients A_1, \dots, A_4 .

LECTURE X

11. CONSPIRACY IN PION PHOTOPRODUCTION (continued)

The separation of the tensor term (71) into $P' = +1$ and $P' = -1$ contributions can be done, using the four-momentum transfer $q_\mu = p_\mu^1 - p_\mu^3 = p_\mu^4 - p_\mu^2$, ($q^2 = -t$). Let

$$\bar{u}' \sigma_{\mu\nu} u q_\nu = v_\mu \quad (\text{vector})$$

$$\bar{u}' \sigma_{\mu\nu} u \epsilon_{\mu\nu\rho\sigma} q_\sigma = a_\rho , \quad (\text{pseudovector})$$

v and a are orthogonal to q :

$$v_\mu q_\mu = a_\mu q_\mu = 0 .$$

From these quantities it is easy to reconstruct the antisymmetric tensor term; one finds

$$\bar{u}' \sigma_{\mu\nu} u = \frac{1}{q^2} \left(v_\mu q_\nu - v_\nu q_\mu + \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} a_\rho q_\sigma \right) . \quad (74)$$

Inserting now Eq. (74) in Eq. (71), one finds the splitting of the tensor term into natural and unnatural parity contributions. To carry out the splitting after this substitution, one must use the following relations which allow one to reduce the number of γ matrices and ϵ tensors:

- the Dirac equation:

$$\begin{cases} p^2 \gamma u = i m_2 u \\ \bar{u}' p^4 \gamma = i m_4 \bar{u}' \end{cases}$$

- the identity:

$$\bar{u}' \gamma_\mu \gamma_\nu u \epsilon_{\mu\nu\rho\sigma} = - \bar{u}' \gamma_5 \gamma_\rho \gamma_\sigma u .$$

The net result is

$$\bar{u}' \sigma_{\mu\nu} u \epsilon_{\mu\nu\rho\sigma} e_\rho p'_\sigma A(s,t) = \begin{cases} \frac{1}{q^2} [2m\bar{u}' \gamma_\mu u + i(p_\mu^2 + p_\mu^4)] \epsilon_{\mu\nu\rho\sigma} p_\nu^3 p_\rho^1 e_\sigma A(s,t) + & (P' = +1) \\ + \frac{2i}{q^2} \bar{u}' \gamma_5 u (p_\mu^2 + p_\mu^4) [p_\mu^1 (e \cdot p^3) - e_\mu (p^1 \cdot p^3)] A(s,t) & (P' = -1) . \end{cases} \quad (75)$$

It shows clearly that a compensation mechanism (i.e. a conspiracy) operates between the $P' = +1$ and $P' = -1$ terms. It cancels the $1/q^2$ singularity which appears in each term and is absent in the tensor coupling on the left-hand side.

Before discussing this conspiracy mechanism in the Regge framework, a comment is in order about the absence of $\gamma_5 \gamma_\mu$ terms in the $P' = -1$ contribution [see Eq. (69)]. It can be understood as a consequence of G-parity conservation. Defining a quantity, analogous to P' ,

$$G' = G(-1)^{I+j}$$

(where G is the G-parity of the exchanged object, I its isospin, and j its spin), one can show, from the charge conjugation properties of Dirac covariants, that $\bar{u}' u$, $\bar{u}' \gamma_\mu u$, $\bar{u}' \gamma_5 u$, and $\bar{u}' \sigma_{\mu\nu} u$ all have $G' = 1$, whilst $\bar{u}' \gamma_5 \gamma_\mu u$ corresponds to $G' = -1$. (For instance, $G' = +1$ for a pion which, as is well known, couples to $\bar{u}' \gamma_5 u$.)

We now assume that Regge poles of definite G' and P' describe the $\gamma + p \rightarrow \pi^+ + n$ amplitude and that they build up the tensor coupling (71). We classify their contributions according to P' and G' .

a) $P' = G' = +1$ (Regge trajectories of ρ , A_2 mesons, for instance)

$$\left\{ \bar{u}' \gamma_\mu u \beta_1^+(t) + i \bar{u}' u (p_\mu^2 + p_\mu^4) \beta_2^+(t) \right\} \epsilon_{\mu\nu\rho\sigma} e_\nu p_\rho^1 p_\sigma^3 \times s^{\alpha_+(t)-1} \frac{1 + \xi_+ e^{-i\pi\alpha_+(t)}}{\sin \pi\alpha_+(t)} . \quad (76)$$

b) $P' = -G' = -1$ (π , B mesons, for instance)

$$\left\{ i\bar{u}'\gamma_5 u(p_\mu^2 + p_\mu^4) [e_\mu(p^1 \cdot p^3) - p_\mu^1(e \cdot p^3)] \right\} \beta^-(t) \times s^{\alpha_-(t)-1} \frac{1 + \xi_- e^{-i\pi\alpha_-(t)}}{\sin \pi\alpha_-(t)} .$$

c) $P' = G' = -1$: no contribution to $\sigma_{\mu\nu}$ terms.

In these expressions the β 's denote the Regge residues, the α 's the trajectories, and the ξ 's the signatures; the power of s is $\alpha - 1$ because the kinematic factors behave as s at high energy. The $t^{-1} = -q^{-2}$ singularity in Eq. (75) must now appear in the residues in such a way that

$$s^{+\alpha_+(t)-1} \beta_1^+(t) \propto -\frac{2bm_2}{t} + 0(t^0)$$

$$s^{+\alpha_+(t)-1} \beta_2^+(t) \propto \frac{b}{t} + 0(t^0)$$

$$s^{-\alpha_-(t)-1} \beta^-(t) \propto \frac{2b}{t} + 0(t^0) ,$$

where $b = b(s) = A(s, t = 0)$. The cancellation mechanism is provided by the fact that near $t = 0$ the three contributions depend on the same coefficient $b(s)$. Since this cancellation must hold independently of the energy, the energy dependences of the three terms must be the same at $t = 0$, which implies

$$\alpha_+(0) = \alpha_-(0) .$$

One can thus write the residues in the following way:

$$\beta_1^+(t) = \frac{2cm_2}{t} + \text{regular terms}$$

$$\beta_2^+(t) = \frac{c}{t} + \text{regular terms}$$

$$\beta^-(t) = \frac{2c}{t} + \text{regular terms}$$

where c now is a constant independent of s and t . One sees, then, that in order to give a total amplitude which is regular, the two trajectories have to conspire, firstly through their intercepts $\alpha(0)$ and secondly through singular residues at $t = 0$.

We conclude these considerations about the conspiracy in π^+ photo-production by reviewing the actual experimental situation. In Fig. 16

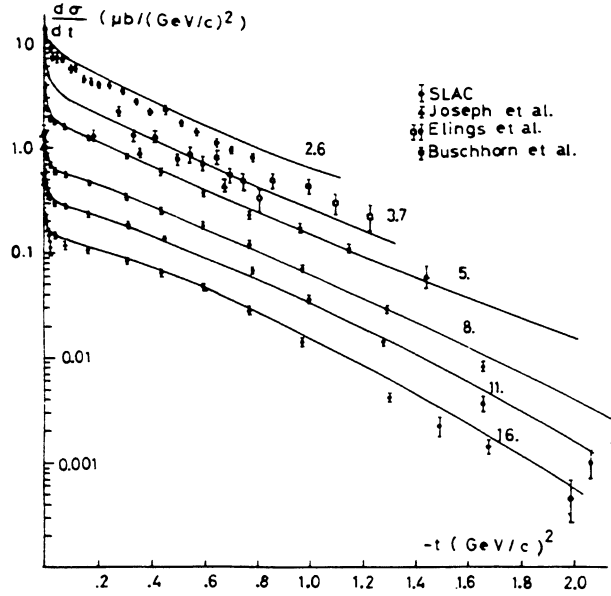


Fig. 16

the experimental points for $d\sigma/dt$ show clearly the existence of a narrow forward peak in the differential cross-section, which certainly rules out a simple-minded, non-conspiratorial Regge-pole model. Figure 17, which is a plot of $s^2(d\sigma/dt)$ versus $\sqrt{-t}$, indicates that the cross-section behaves roughly like s^{-2} , and that the width of the forward peak is one pion mass. The data in both figures are from Colley et al.³³⁾, and the fits from Amati et al.³⁴⁾. Both s - and t -dependences of $d\sigma/dt$ strongly suggest that the conspiracy has something to do with the pion exchange. Indeed, $\alpha_\pi(t)$ is ≈ 0 for small values of t , since $\alpha_\pi(m_\pi^2) = 0$ and $m_\pi^2 = 0.02 (\text{GeV}/c^2)^2$. According to the $s^{2\alpha-2}$ energy dependence of the differential cross-section, one then expects $s^2(d\sigma/dt)$ to be almost energy independent. On the other hand, since

$$\frac{1}{\sin \pi \alpha_\pi(t)} \approx \frac{1}{\pi \alpha'_\pi(t - m_\pi^2)} \quad \text{for } t \text{ small,}$$

one also expects a width for the forward peak of the order of m_π^2 .

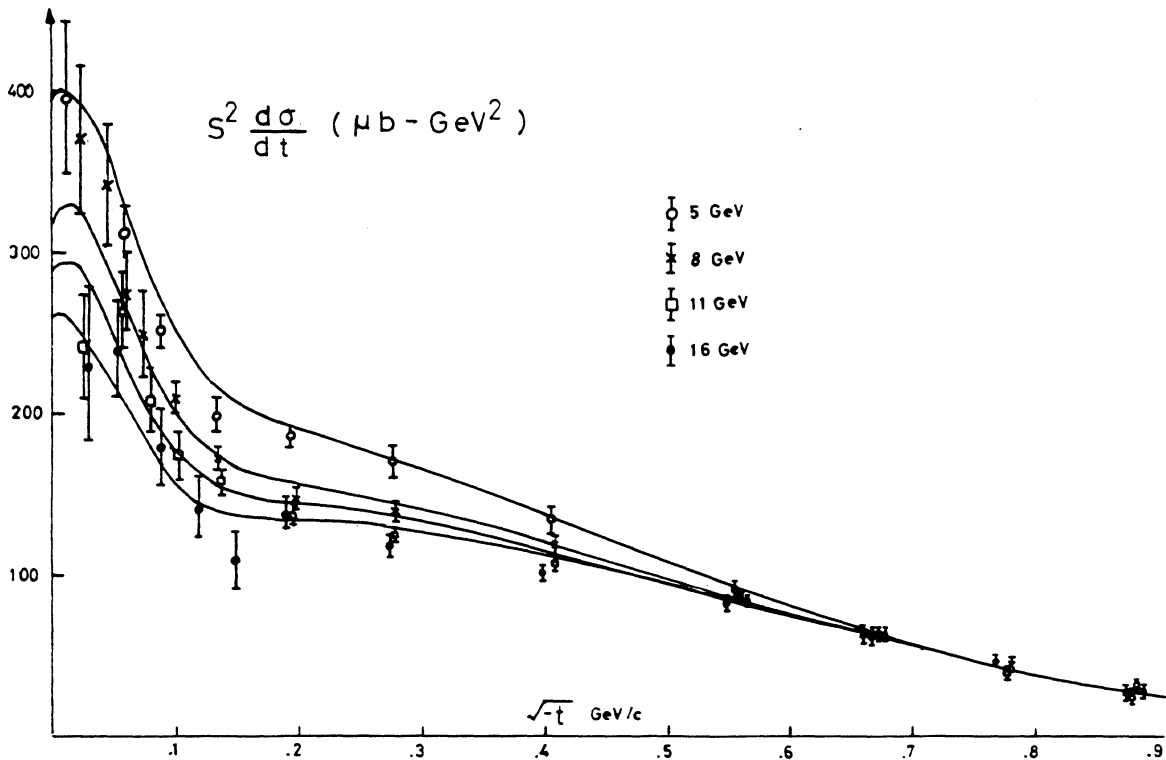


Fig. 17

Despite these indications for π exchange, the situation is very curious: when one tries a pion conspiracy in Eq. (75) it appears that the pion term which comes from $\bar{u}'\gamma_5 u$ has only a $1/\sqrt{-t}$ singularity because $\bar{u}'\gamma_5 u \propto \sqrt{-t}$. The conspirator, on the contrary [$P' = +1$ term in (75)], has the full $1/t$ singularity in each of its two terms (in $\bar{u}'\gamma_\mu u$ and in $\bar{u}'u$) but they cancel with each other in $1/t$ leaving only $1/\sqrt{-t}$ to cancel with the π term. No particle is known which would belong naturally to the conspirator trajectory.

In order to clarify further this rather puzzling situation, a careful experimental study of the energy dependence of the forward peak will be very important. Suppose, for instance, that the peak shows a tendency to disappear at very high energy; this would imply that the conspiracy mechanism is imperfect. Absorptive corrections to π exchange would give an imperfect conspiracy of this type because α_+ runs over a cut, whilst α_- has for each t a single value corresponding to a pole.

We end our discussion of photoproduction by mentioning that the behaviour of $\gamma + p \rightarrow \pi^0 + p$, i.e. π^0 photoproduction, is found experimentally to be very different from $\gamma + p \rightarrow \pi^+ + n$. For $\gamma + p \rightarrow \pi^0 + n$, experiment reveals a dip in $d\sigma/dt$ in the forward direction, and the data are easily explained by assuming that Reggeized ω exchange dominates (ω exchange has natural parity, $P' = +1$). The corresponding vector meson exchange for $\gamma + p \rightarrow \pi^+ + n$, which is ρ exchange, is apparently very strongly suppressed, a fact that can be understood from the circumstance that the $\rho \rightarrow \pi + \gamma$ coupling is much weaker than the $\omega \rightarrow \pi + \gamma$ one.

12. VECTOR MESON PRODUCTION

We continue our investigation of the conspiracy mechanism by considering a process which shows strong analogy with pion photoproduction, namely vector meson production. Consider, for example, the reaction

$$\pi^- + p \rightarrow \rho^0 + n,$$

the time reversed of which is very similar to $\gamma + p \rightarrow \pi^+ + n$. For $\pi^- + p \rightarrow \rho^0 + n$ the number of independent amplitudes is $\frac{3}{2}$ times the one for photoproduction since the ρ meson has three spin states (instead of two for the photons). Under P invariance there are six independent amplitudes, of which two survive in the forward direction. The analysis in terms of invariant amplitudes can be performed as for photoproduction. We study separately $P' = +1$ and -1 exchanges. The notation is defined in Fig. 18. The polarization vector of ρ_0 verifies $e \cdot p^3 = 0$

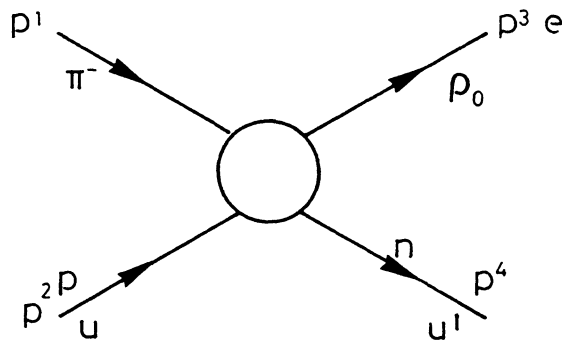


FIG.18

Case a: exchange of $P' = +1$

The vertex functions V^{13} and V^{24} take exactly the same forms as before, see Eqs. (63) and (64). There are again two amplitudes of this type, both giving at high energy a dip in $d\sigma/dt$ in the forward direction. Both have $G' = +1$. I and G conservation now imply $G' = G(-1)^{I+j} = (-1)^{I+j+1} = (-1)^j$ because $G = -1$, $I = 1$. Hence we have j even.

Case b: exchange of $P' = -1$

There are two possible values of V^{13} instead of the single one in Eq. (68). They are

$$V_{\mu_1 \dots \mu_j}^{13} = \begin{cases} \left\{ e_{\mu_1} (p_{\mu_2}^1 + p_{\mu_2}^3) \dots (p_{\mu_j}^1 + p_{\mu_j}^3) \right\}_{\text{sym. in } \mu_1, \dots, \mu_j} \\ (e \cdot p^1) (p_{\mu_1}^1 + p_{\mu_1}^3) \dots (p_{\mu_j}^1 + p_{\mu_j}^3) \end{cases} \quad (77)$$

V^{24} takes again the two values given in Eq. (69). Combining the two choices (77) with the two choices (69) one gets four amplitudes. The two amplitudes containing $\bar{u}' \gamma_5 u$ have $G' = +1$; since G' is here $(-1)^j$, this gives j even. The two amplitudes containing $\bar{u}' \gamma_5 \gamma_\mu u$ have $G' = -1$, i.e. odd j . The first vertex function of Eq. (77) combined with the V^{24} containing $\bar{u}' \gamma_5 \gamma_\mu u$ gives an amplitude which does not have a dip in the forward direction. The three other combinations develop such a dip at high energy, either because $\bar{u}' \gamma_5 u \propto (-t)^{1/2}$ or because $e \cdot p^1 = e \cdot q$ with $q = p^1 - p^3$.

The two non-vanishing forward amplitudes are

$$\bar{u}' \sigma_{\mu\nu} u \epsilon_{\mu\nu\rho\sigma} e_\rho p_\sigma^3 A_1(s, t), \quad (78)$$

$$i \bar{u}' \gamma_5 \gamma_\mu u e_\mu A_2(s, t). \quad (79)$$

The latter one is the $P' = -1$ amplitude without the forward dip already mentioned before. Amplitude (78) is essentially the same as in photoproduction; it mixes $P' = \pm 1$ in the way discussed in detail for $\gamma + p \rightarrow \pi^+ + n$.

It is interesting to interpret formulae (78) and (79) for $s \rightarrow \infty$ by taking $\bar{u}'\sigma_{\mu\nu}u$ and $i\bar{u}'\gamma_5\gamma_\mu u$ in the lab. system, where both nucleons are at rest (the proton exactly and the neutron approximately). One then has

$$\bar{u}'\sigma_{12,23,31}u = \chi'^*\sigma_{1,2,3}\chi, \quad \bar{u}'\sigma_{14,24,34}u = 0 \quad (80)$$

$$i\bar{u}'\gamma_5\gamma_{1,2,3}u = \chi'^*\sigma_{1,2,3}\chi, \quad i\bar{u}'\gamma_5\gamma_4u = 0, \quad (81)$$

where χ , χ' , and $\vec{\sigma}$ are Pauli spinors and spin matrices in the lab. system. As to the polarization e and momentum p^4 of the ρ meson, we write them:

$$e_\rho = L_{\rho\rho'}e_\rho^r = iL_{\rho 4}m_3, \quad (82)$$

where e_ρ^r is the polarization vector of the ρ in the latter's rest system ($e_4^r = 0$), and $L_{\mu\nu}$ the Lorentz transformation between the ρ rest system and the lab. system. The crucial point is that

$$L_{11} = L_{22} = 1, \quad L_{33} = L_{44} \simeq iL_{43} \simeq -iL_{34} \simeq s/2m_2m_3, \quad (83)$$

where direction 3 is the incident, i.e. longitudinal direction, and 1, 2 are the transverse directions.

We first consider the amplitude (78). Insert Eqs. (80), (82), and (83) into (78), and keep the leading terms which are of order sA_1 ; one sees that these terms turn out to be

$$\chi'^*(\sigma_1 e_1^r + \sigma_2 e_2^r)\chi \frac{isA_1}{2m_2}.$$

They involve only transversally polarized ρ mesons.

The amplitude (79), through the same substitutions and to order sA_2 , reduces to

$$\chi'^*\sigma_3\chi e_3 \frac{sA_2}{2m_2m_3}$$

and involves only longitudinally polarized ρ mesons. Clearly, separation of the various polarization states of the ρ will allow one to study the two couplings (78) and (79) near the forward direction.

Experimentally, the $d\sigma/dt$ of $\pi^- + p \rightarrow \rho^0 + n$ decreases roughly as s^{-2} , which is compatible with dominance of π exchange. No evidence

for a forward dip exists. The ρ^0 is polarized mostly in the longitudinal direction near $t = 0$ ³⁴). This suggests dominance of coupling (79).

It is clear, however, that available data are not yet sufficient to draw definite conclusions, and much higher statistics experiments are called for. Also production of other vector meson states is of the greatest interest.

13. NUCLEON-NUCLEON SCATTERING

We conclude our discussion about conspiracy by considering the nucleon-nucleon elastic scattering problem. For this process, parity conservation, time reversal invariance, and the fact that the final particles are the same as the initial ones reduce the number of independent amplitudes to five. The decomposition into kinematically regular, invariant amplitudes is the following ³⁵) (the notations are indicated in Fig. 19):

$$T = \bar{u}'u\bar{v}'vA_1(s,t) + \bar{u}'\gamma_\mu u\bar{v}'\gamma_\mu vA_2(s,t) + \frac{1}{2}\bar{u}'\sigma_{\mu\nu}u\bar{v}'\sigma_{\mu\nu}vA_3(s,t) + i\bar{u}'\gamma_5\gamma_\mu u\bar{v}'\gamma_5\gamma_\mu vA_4(s,t) + \bar{u}'\gamma_5u\bar{v}'\gamma_5vA_5(s,t) . \quad (84)$$

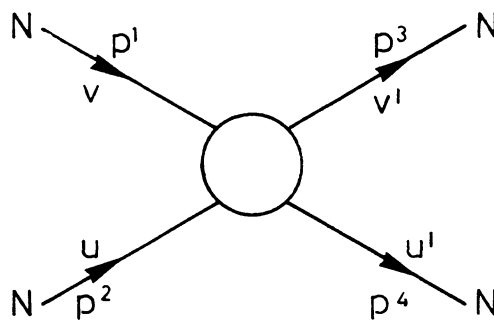


FIG.19

In the forward direction where $p^1 = p^3$ and $p^2 = p^4$, three of the five amplitudes give non-vanishing contributions. They are A_3 , A_4 , and a combination A'_1 of A_1 and A_2 .

Using the Lorentz transformation (83) from the rest system of one nucleon to the rest system of the other, we write them in terms of

rest-system spinors of each nucleon (χ corresponding to u and ξ to v). One finds to leading order in s (A_1 reduces to A_2 in this order):

$$\bar{\chi}' \chi \bar{\xi}' \xi s A_2(s, t), \bar{\chi}' \sigma_{\parallel} \chi \bar{\xi}' \sigma_{\parallel} \xi s A_4(s, t), \bar{\chi}' \vec{\sigma}_{\perp} \chi \bar{\xi}' \vec{\sigma}_{\perp} \xi s A_3(s, t) .$$

σ_{\parallel} refers to the longitudinal spin component, $\vec{\sigma}_{\perp}$ to the transverse components. As discussed in connection with Eqs. (64) and (69), the amplitudes A_1 and A_2 of Eq. (84) correspond to pure $P' = +1$ exchange, whereas A_4 and A_5 correspond to pure $P' = -1$ exchange. The A_3 amplitude corresponds to mixed $P' = +1$ and $P' = -1$ exchange, so that $A_3 \neq 0$ at $t = 0$ (forward direction) leads to the same conspiracy situation as in the π^+ photoproduction case.

Experimentally, it appears that the forward charge exchange process $np \rightarrow pn$ has a differential cross-section $d\sigma/dt$ which shows a narrow peak³⁶⁾ in striking analogy with the $\gamma p \rightarrow \pi^+ n$ one. The width of the peak suggests π exchange, and so does the energy variation of $d\sigma/dt$. (In nucleon-nucleon elastic scattering the neutron-proton charge exchange is the only small-angle collision in which one can expect pion exchange to dominate, since all others are diffractive-like.)

14. REMARK ON FERMION TRAJECTORIES

Amongst the many aspects of Regge-pole theory not treated in these lectures, there is one that concerns a parity-mixing phenomenon somewhat similar to the conspiracy discussed above, but more natural because it is theoretically bound to occur. It concerns the Regge trajectories for baryons (nucleons, hyperons, and their resonances). Because of their fermion character, such trajectories carry both relative parities. Indeed, the partial wave amplitudes for scattering of a spin 0 with a spin $1/2$ particle obey the following equation (MacDowell symmetry)

$$f_{+}^J(W) = f_{-}^J(-W) ,$$

$W =$ c.m. energy or its analytic continuation, $\pm =$ parity, $J =$ angular momentum. Taking this property in the t-channel, one finds that baryon Regge trajectories are analytic functions $\alpha(\sqrt{t})$ of \sqrt{t} rather than of t . For $t = M^2 > 0$, $\alpha(M) = J$ and $\alpha(-M) = J$ correspond to baryon states of

mass M , spin J , and opposite parities. For $t < 0$, the baryon trajectory contributes twice, once through $\alpha(i\sqrt{-t})$ and once through $\alpha(-i\sqrt{-t})$, with degeneracy between the two contributions at $t = 0$. This form of "conspiracy" was first recognized by Gribov³⁷⁾.

* * *

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