A MODEL FOR REGGE CUTS WITH POLE ENHANCEMENT

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It is argued that, since hadronic processes show Regge shrinkage even at large $|t|$, Regge-cut discontinuities must be enhanced in the region of J near the Regge poles (as suggested by some recent theoretical work on the weak-coupling solution to the "infrared problem" in the reggeon calculus) so that the dominant Regge-cut contribution (at current energies) does not come from the region of the discontinuity near the branch point (as it does in eikonal and absorption models). In particular we show that this hypothesis works well for $\pi^- p \rightarrow \pi^0 n$. However, there is less shrinkage in photoproduction processes at large $|t|$, and we find that in $\gamma p \rightarrow \pi^0 p$ (and related processes) there is no pole enhancement of the cuts. We connect this fact with the absence of t-channel unitarity constraints for electromagnetic processes and more speculatively with the possibility that a scaling behaviour due to Regge cuts develops at large $|t|$ as the mass of the external particle is decreased.

1. Introduction

Although Regge poles give quite a good qualitative description of high-energy, two-body scattering processes, predicting fairly successfully the energy dependence, phase, and relative magnitudes of the scattering amplitudes (via SU(3) for the couplings), it has been clear for many years that there are also important qualitative features of the data which poles alone can not explain [1–3]. These include the cross-over zero at $|t| \simeq 0.15$ GeV^2 in meson-baryon elastic scattering, the forward peak in π-exchange processes, and the failure of nonsense-factor dips in $d\sigma/dt$ to occur in all the various processes connected by factorization (e.g. the dip in $d\sigma(\pi^- p \rightarrow \pi^0 n)/dt$ at $|t| \simeq 0.6$ GeV^2 where $\alpha_\rho(t) = 0$ is not seen in $\pi^- p \rightarrow \omega n$ or $\gamma p \rightarrow \eta p$ which are also dominated by ρ exchange).

Various Regge cut models based on the idea of absorption through pomeron (P) exchange in the initial and final states (fig. 1) have been proposed, but again, though they can explain many important features of the data, with each model there are some serious discrepancies [3]. Thus, for example, the weak cut (or Argonne) model [4] which has nonsense zeroes in the Regge pole amplitudes does not get the cross-over zero sufficiently close to $t = 0$, and is incompatible with the polarization in $\pi^- p \rightarrow \pi^0 n$, because the real part of the non-flip amplitude is absorbed much too
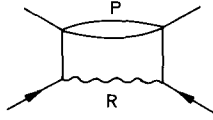
Fig. 1. The absorptive prescription for an $R \otimes P$ cut.

strongly. Also, the cuts are not strong enough to fill in the unwanted nonsense dips (such as those in $\pi^- p \to \omega n$ and $\gamma p \to \eta p$ mentioned above). On the other hand the old strong-cut model (SCRAM) [5] had difficulties with regard to tensor meson exchange processes (e.g. it predicted an unobserved A_2 exchange diffraction minimum in $\pi^- p \to \eta^0 n$ at $|t| \simeq 0.6$ GeV^2) and also had problems with the polarization in $\pi^- p \to \pi^0 n$ inter alia. The dual absorption model (DAM) [6] was constructed to get the zeros of Im A in the correct places, at least in charge-exchange processes, but again gave incorrect phases for $\pi^- p \to \pi^0 n$, as well as being hard to apply to, for example, hypercharge (K^*, K^{**}) exchange processes [7] where the nonsense zeros of the reggeons do not coincide with the diffraction zeroes. In any case this model is theoretically unsatisfactory because it does not attempt to distinguish cuts from poles.

Recently a phenomenologically much more satisfactory model has been suggested by Hartley and Kane [8]. They propose an effective absorbing amplitude (of some complexity) which, when combined with the usual Regge poles, is designed to reproduce the correct amplitude structure. For example the absorption amplitude has a real part to ensure the correct phases for $\pi^- p \to \pi^0 n$.

Though a very sizable body of data has been fitted with this model there remain some difficulties. Firstly, it is not certain that the problem of the absence of dips in A_2 exchange processes has been resolved. They give the A_2 contribution a shorter range than usual (see also Martin and Stevens [9]) thus pushing the absorption dip out to larger $|t|$. Though the data on $\pi^- p \to \eta n$ are not very accurate at large $|t|$ there is no sign of such a dip in recent experiments [10], which is worrying because the dip systematics are an important feature of the model. Of course it is always possible that a more complicated parameterization of the amplitude could remove the problem.

Secondly the rather arbitrary nature of their absorption amplitude is theoretically unsatisfactory. In some recent work [11, 12] it has been shown that absorption with the actual $I_t = 0$ elastic meson-baryon scattering amplitude (effectively $P + P'$) is able to overcome many of the difficulties of the older absorption models. This accords with the observation of Worden [13] that the $\pi^- p \to \pi^0 n$ amplitude phases require a lower-lying J-plane singularity, in addition to the ρ pole and $\rho \otimes P$ cut.

But perhaps the most important problem is that all these cut models seem to get the energy dependence of the scattering amplitude at large $|t|$ wrong. Thus, if the reggeon (R) has the observed linear trajectory

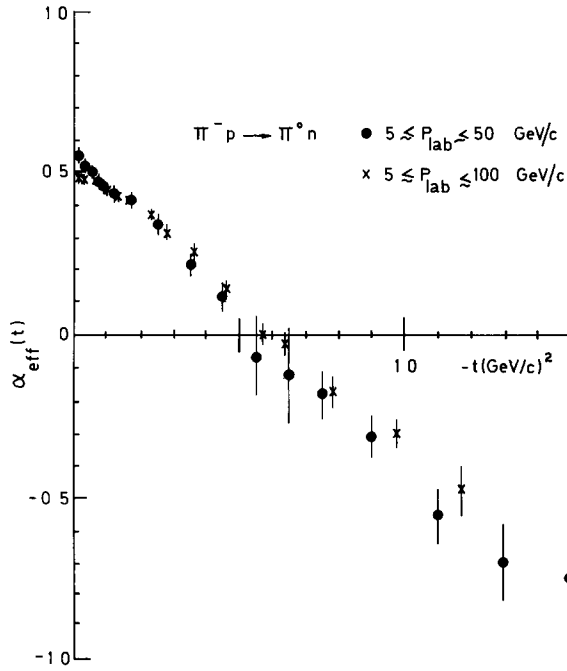$$\alpha_R(t) = \alpha_0 + \alpha' t , \qquad (1.1)$$

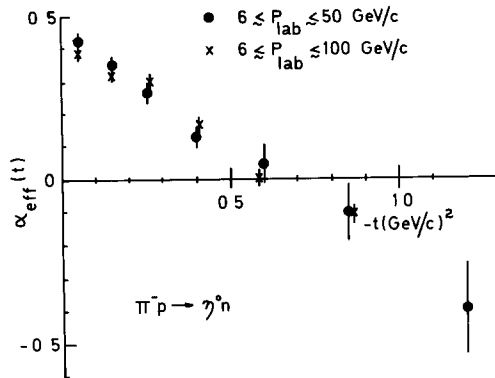Fig. 2. Effective trajectories for various processes (a) $\pi^-p \to \pi^0 n$ from ref. [14,41], ρ exchange.
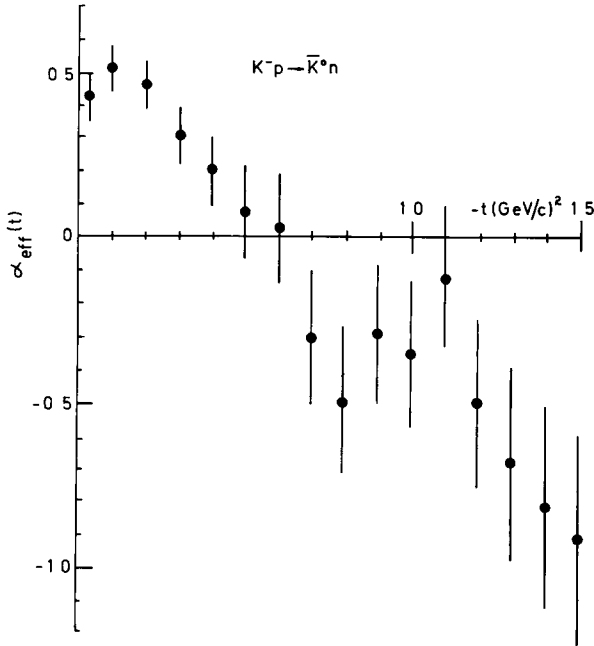

Fig. 2(b). $\pi^-p \to \eta n$ from ref. [10], A_2 exchange.

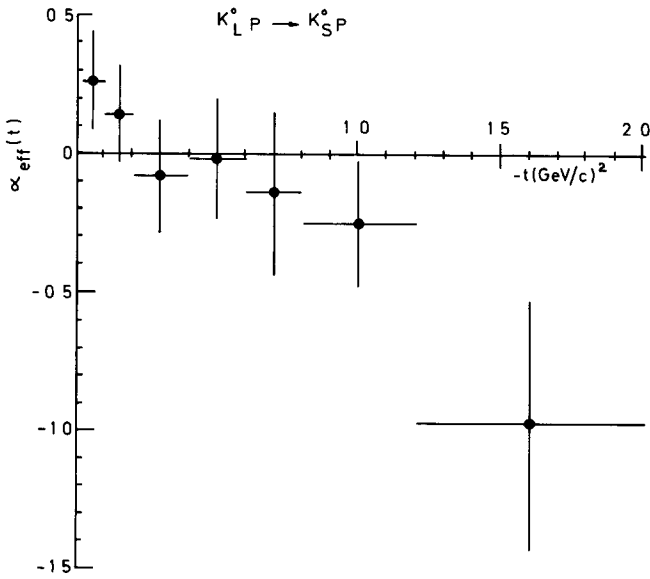Fig. 2(c). $K^- p \to \bar{K}^0 n$ from ref. [15], $\rho + A_2$ exchange.

Fig. 2(d) $K_2^0 p \to K_S^0 p$ from ref. [16], ω exchange.

with $\alpha' \simeq 0.9$ GeV^{-2}, the corresponding $R \otimes P$ cut branch point is at

$$\alpha_{RP}(t) \simeq \alpha_0 + \frac{\alpha'_R \alpha'_P}{\alpha'_R + \alpha'_P} \, t \, , \tag{1.2}$$

which with $\alpha'_P \simeq 0.25$ GeV^2 gives

$$\alpha_{RP}(t) \simeq \alpha_0 + 0.2t \, , \tag{1.3}$$

so one would expect rather a small amount of shrinkage at larger $|t|$, i.e. beyond the diffraction minimum, and of course higher cuts will be flatter still.

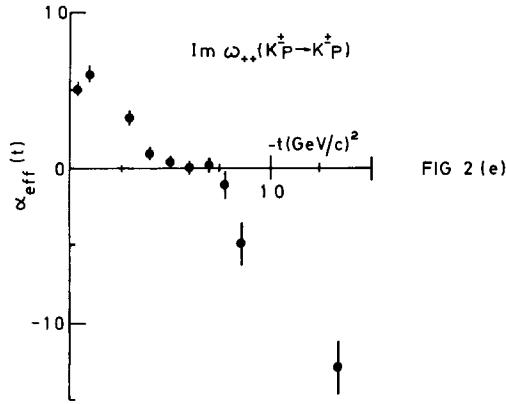In figs. 2−4 we give the effective traje tories obtained from the energy depen-

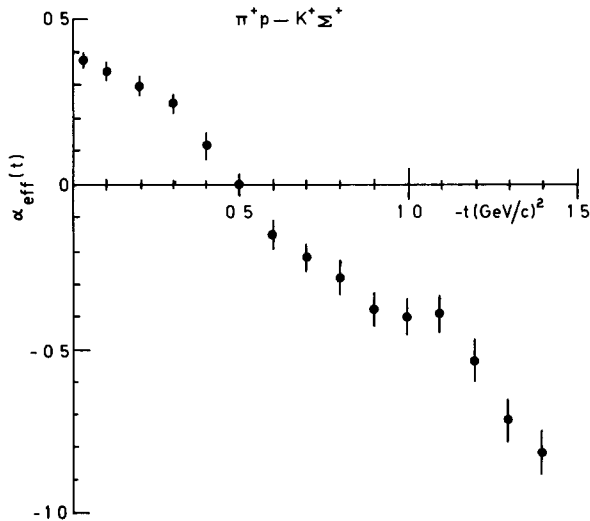Fig. 2(e) $K^{\pm}p \to K^{\pm}p$ from ref, [17], Im ω_{++} exchange amplitude.

Fig. 2(f) $\pi^+p \to K^+\Sigma^+$ from ref. [18], K^*, K^{**} exchange.

dence at fixed t in a variety of processes for which there is sufficient large $|t|$, high-energy data [14–22], using the definition

$$\log\left(\frac{d\sigma}{dt}\right) = (2\alpha_{eff}(t) - 2)\log s + F(t) . \qquad (1.4)$$

(In some cases combinations of processes have been used to isolate a given trajectory.) It is very evident that ρ, ω, A_2, K^*, K^{**}, π and (perhaps) B exchange amplitudes are all compatible with (1.1) rather than (1.3) over the whole observed t range, out to $|t| = 1.5$ GeV^2 in some cases. Indeed, Barger and Phillips [23] have shown that this behaviour persists at low energies right out to $|t| = 5$ GeV^2 in $\pi^- p \to \pi^0 n$. Even more remarkable is fig. 3c from ref. [20] which shows that the effective trajectory of the positive parity amplitude π_c which conspires with the π in $\pi N \to \rho N$, presumably the even parity part of the $\pi \otimes P$ cut, looks very similar to the π trajectory.

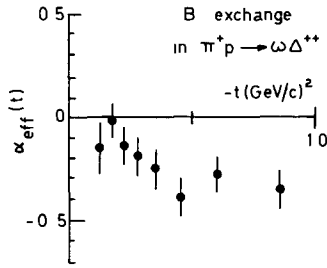These effective trajectories strongly support the view that if there really are im-

Fig. 3. Effective trajectories for unnatural parity exchanges. (a) $\pi^+ p \to \omega\Delta^{++}$ from ref. [19], B exchange.
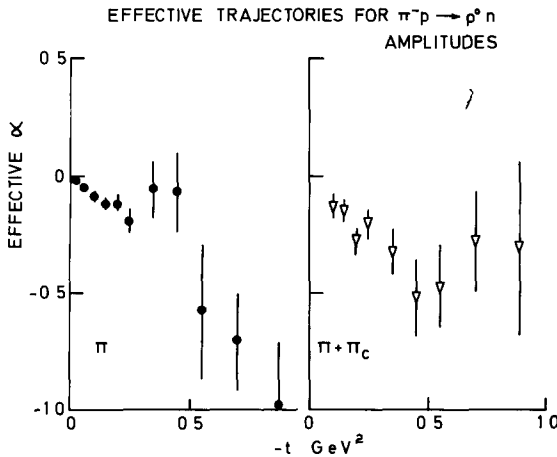

Fig. 3(b). $\pi^- p \to \rho^0 n$ from ref. [20], π exchange.

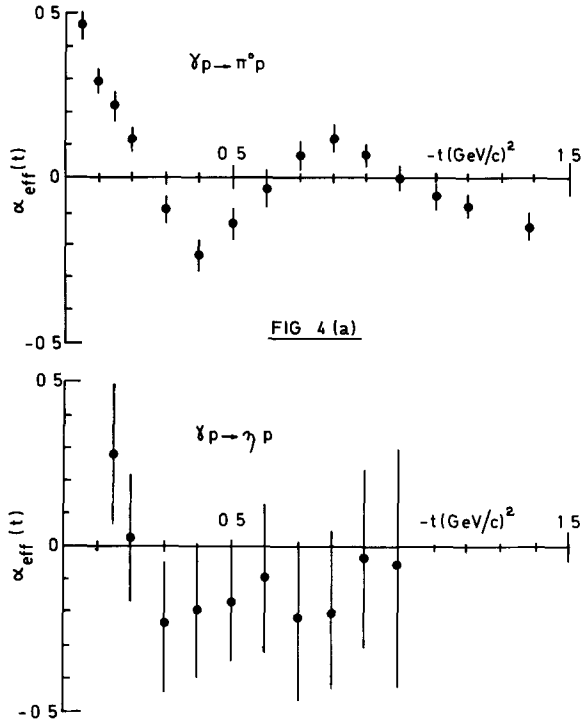Fig. 3(c). $\pi^- p \to \rho^0 n$ from ref. [20], $\pi + \pi_c$ exchange.

Fig. 4. Effective trajectories for photoproduction processes: (a) $\gamma p \to \pi^0 p$ from ref. [21], $\rho + \omega$ exchange and (b) $\gamma p \to \eta p$ from ref. [22], $\rho + \omega$ exchange.

portant contributions from Regge cuts then the energy dependence of cuts is not that of the absorption model (1.3). On the other hand the effective trajectories in photoproduction processes e.g. fig. 4, are much flatter at large $|t|$, and completely in agreement with absorption model fits.

Our purpose in this paper is to examine some of the theoretical developments in our understanding of Regge cuts which suggest that the Regge cut discontinuities may obtain their dominant contribution from the J-plane region near the pole itself, rather than from the branch point, so that at finite energies the effective power behaviour of the cut contribution will be closer to (1.1) than (1.4). We show that this hypothesis gives an excellent fit to the $\pi^- p \to \pi^0 n$ data, and argue that since SU(3) symmetry appears to hold well for Regge residues [2], it will probably (given the freedom available in the parameterisation) be fairly easy to fit most meson-baryon, etc. scattering processes with this model. However, we also find that photoproduction processes require that the cut discontinuity should be dominant near the branch point, not at the pole, a fact which we try to connect with some of the other peculiarities of reggeon coupling to photons.

Some conclusions are presented in sect. 5.

2. Pole-enhanced cuts

The absorption prescription can most plausibly be regarded as an approximation to the eikonal expansion for Regge cuts [1,2,4]. It has been shown in various field theories that if the reggeon is regarded as a sum of ladders like fig. 5a, then the leading diagrams at high energy are those in which the couplings are "nested" as in figs. 5b, at least in the approximation where momentum transfers across the ladders are small. And if the eikonal function is given by the Fourier-Bessel decomposition of the Regge pole amplitude (see e.g. ref. [1])

$$\chi^R(s, b) = \frac{1}{8\pi s} \int\limits_{-\infty}^{0} dt \, J_n(b \sqrt{-t}) \, A(s, t) \,, \tag{2.1}$$

(n = helicity flip, see (3.4)) then the sum of leading terms is given by the eikonal expansion

$$A(s, t) = 4\pi s \int\limits_{0}^{\infty} b\,db \left[\chi^R + \frac{\iota(\chi^R)^2}{2!} - \frac{(\chi^R)^3}{3!} \cdots \right] J_n(b \sqrt{-t}) \,, \tag{2.2}$$

the first term being $A^R(s, t)$, the second the $R \otimes R$ cut, etc. A simple generalization gives the appropriate cuts when more than one type of trajectory is involved (such as $R + R \otimes P + R \otimes P^2 \ldots$), and enhancement of the intermediate states may also be incorporated. Looked at in this way the absorption model (essentially just $R + R \otimes P$) acquires the correct non-planar reggeon-particle couplings required for physical-sheet Regge cuts, and accords with the reggeon calculus [25].

Theoretically this prescription has the rather serious defect that the cuts are "hard", i.e. have a finite discontinuity at the branch-point, and so seem to conflict with t-channel unitarity [26]. The asymptotic behaviour (log $s \to \infty$) of the $R \otimes P$ absorptive cut is
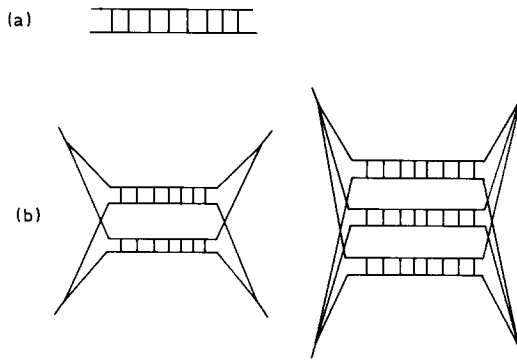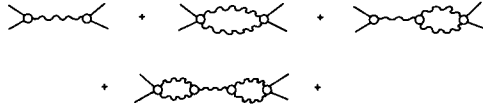

Fig. 5. (a) A Regge pole ladder and (b) two- and three-reggeon cuts with "nested" couplings.

Fig. 6. Two-reggeon unitarity in the t-channel.

$$A^{RP}(s, t) \sim -\beta_{RP}(t) \frac{s^{\alpha_{RP}(t)}}{\log s} , \tag{2.3}$$

where $\alpha_{RP}(t)$ is given by (1.3), and if we define the t-channel partial-wave amplitude by the Mellin transform

$$A_J(t) = \int_0^\infty ds \, s^{-J-1} A(s, t) , \tag{2.4}$$

we get

$$A_J^{RP}(t) \approx \beta_{RP}(t) \log (J - \alpha_{RP}(t)) , \tag{2.5}$$

so the discontinuity is finite at the branch-point $J = \alpha_{RP}(t)$. It was shown by Bronzan and Jones [26] that t-channel unitarity requires the vanishing of the discontinuity at the branch point, a result achieved in the reggeon calculus, and other perturbative approaches, by incorporating all the t-channel iterations (fig. 6) which softens the branch point [27].

This argument is not completely compelling because it has been pointed out by Branson [28] and Paige and Wang [29] that it is possible for the character of the branch point to change from hard to soft as t is increased from 0 to the t-channel threshold. In any case the softening may [27] only modify (2.3) by factors of the form $(1 + a \log \log s)^{-n}$ which would not make much difference at present energies.

However, there is a further difficulty with the reggeon calculus when pomerons with $\alpha_P(0) = 1$ are included because of the accumulation of branch points ($R \otimes P$, $R \otimes P^2, .. R \otimes P^n ..$) at the point $t = 0, J = \alpha_R(0)$, the so-called "infra-red problem". Most of the literature [30–33] has been devoted to $P \otimes P^n$ cuts at $t = 0$, $J = 1$, but, apart from the special constraints of the s-channel Froissart bound, the same self-consistency problems also arise for $R \otimes P$ cuts (see e.g. Cardy, ref. [31]).

Two solutions have been presented, (a) the weak coupling solution [32] in which asymptotically the pole R dominates over the cuts and the R–RP triple coupling vanishes at $t = 0$, and (b) the strong coupling model [33] in which the infinite accumulation of cuts dominates asymptotically. It is hard to decide between these approaches conclusively because all the available data is in the region where $\log s$ is comparatively small (i.e. $\alpha' \log s < a$, where a is a typical value for the logarithmic width of the forward peak of $d\sigma/dt$). But the success of Regge pole phenomenology perhaps argues in favour of the weak coupling solution.

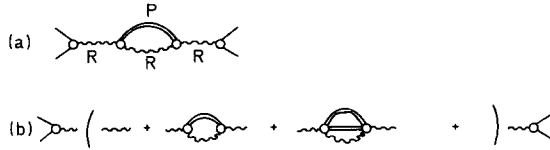The weak coupling model has been extensively explored by Cardy and White

Fig. 7. (a) An $R \otimes P$ cut with R enhancement of the reggeon-particle scattering amplitudes and (b) the sum of enhanced $R \otimes P^n$ cuts.

[30,31] who show that the vanishing of the R—RP coupling at $t = 0$ has the consequence that the leading contribution to the $R \otimes P$ cut stems from the R contribution to the particle + R → particle + P scattering amplitude (see fig. 7a). They thus find [31] that the cut contribution to the partial-wave amplitude is (for $t \approx 0$)

$$A_J(t) \sim \frac{[J - \alpha_{RP}(t)]^2}{[J - \alpha_R(t)]^2} \log (J - \alpha_{RP}(t)) , \tag{2.6}$$

where $[J - \alpha_R(t)]^{-1}$ stems from the reggeon propogator (one on each side of the diagram), the $[J - \alpha_{RP}(t)]$ comes from the vanishing of the R—RP coupling at $t = 0$, $J = \alpha_R(0)$, and the $\log (J - \alpha_{RP}(t))$ gives the branch point due to the $R \otimes P$ loop. So the cut discontinuity is

$$\Delta(J, t) \sim \frac{[J - \alpha_{RP}(t)]^2}{[J - \alpha_R(t)]^2} . \tag{2.7}$$

Their derivation depends on the vanishing of the triple reggeon coupling at $t = 0$, which does not seem to be true phenomenologically at least at present energies, a fact which must throw some doubt on the validity of the weak-coupling solution, though of course the separation between pole and cut contributions becomes more difficult the closer the cuts resemble poles. Also the effective P intercept in elastic scattering is somewhat above 1 ($\alpha_P(0) \approx 1.07$, see ref. [34]) and it is not clear how this effect should be incorporated. One might expect that the poles would become complex for $t < 0$, but phenomenologically this seems undesirable.

Another feature of the model is that because the cuts appear to couple predominantly through the poles the sum $R + R \otimes P + ... + R \otimes P^n + ...$ will approximately factorize, as fig. 7b shows, at least for small $|t|$, and log s. This could account for the frequent success of factorization tests, but it leaves unexplained why factorization sometimes fails badly (e.g. with ρ exchange in $\pi^- p \to \pi^0 n$ and $\pi^- p \to \omega n$ mentioned in sect. 1). Even worse fig. 7 suggests that the parity of the $R \otimes P$ cut should be the same as that of the R pole, whereas the conspiracy problem [1] requires that the cuts contribute to t-channel amplitudes of both parities. For example π_c in fig. 3a (from $\pi N \to \rho N$), though it has a similar trajectory to the pion has opposite parity, and clearly can not couple through the physical pion pole.

Thus although the work of refs. [30,31] gives (2.6) some plausibility we feel that

Fig. 8. Triple-regge representation of the box diagram at high energy.

it is necessary to regard the pole enhancement result as more general than this derivation, and in particular not to take fig. 7 too literally.

Similar results to refs. [30,31] have been obtained by Veneziano [35] from his dual bootstrap model, and closely related conclusions have also been derived by Desai [36] based on rather different arguments.

He notes that if the box diagram of the absorption model (fig. 1) is generalized to include diffractive intermediate states, and the high-energy behaviour of the reggeon-particle scattering amplitude is represented by a Regge pole (see fig. 8) then the resulting contributions are

$$A_J(t) = \beta \log (J - \alpha_c) + \beta_a \frac{\log (J - \alpha_c)}{J - \alpha_R} + \beta_b \frac{\log (J - \alpha_c)}{J - \alpha_R} + \frac{\beta_a \beta_b \log (J - \alpha_c)}{(J - \alpha_R)^2} .$$

(2.8)

and again we must expect the last term to dominate fro small $|t|$ when $J \simeq \alpha_R \simeq \alpha_c$. This differs from ref. [31] in that the generalized box diagram is used which will generate an AFS type of cut rather than the Mandelstam cuts which have non-planar reggeon-particle couplings. It is well known that AFS cuts do not lie on the physical sheet and have the same magnitude but opposite sign to Mandelstam cuts (see refs. [37, 38]) which have a destructive phase relative to the pole. Also Desai does not assume that the triple Reggeon coupling vanishes at $t = 0$. However, the basic idea that the cut discontinuity is enhanced at the position of the pole is a consequence of this model too.

The eikonal model lacks this feature because it includes only the leading term of each diagram which of course is given by the branch point. Clearly, the power behaviour stemming from the cut (2.6) will be $\sim s^{\alpha_{RP}(t)}$ but only as $\log s \to \infty$. At lower energies the power behaviour will be more like $s^{\alpha_R (t)}$ because the important part of the cut discontinuity is near the pole.

In the next sections we examine some of the consequences of this hypothesis.

3. Parameterization of R $\otimes$ P cuts

We write the Regge pole amplitude for an odd-signature reggeon in the form

$$A^R(s, t) = i(s\, e^{-\frac{1}{2}i\pi})^{\alpha_R(0)} (-t)^{\frac{1}{2}(n+x)} G_n^R\, e^{c_n^R t} ,$$

(3.1)

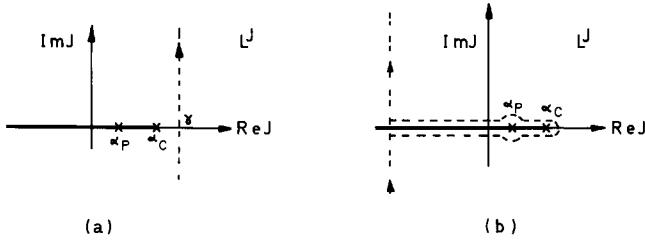$$c_n^R \equiv a_n^R + \alpha_R'(\log s - \tfrac{1}{2}i\pi) ,$$

(3.2)

Fig. 9. (a) Path of contour integration in (3.5) along Re $J = \gamma$. For $t < 0$ we assume that the singularities lie on the real axis. (b) Contour displaced to left embracing the branchpoint at α_c and dipole at α_p.

$$\alpha_R(t) = \alpha_R(0) + \alpha_R{}'t , \tag{3.3}$$

where n is the usual helicity-flip (see ref. [1])

$$n \equiv |(\mu_1 - \mu_2) - (\mu_3 - \mu_4)| , \qquad x \equiv |\mu_1 - \mu_3| + |\mu_2 - \mu_4| - n . \tag{3.4}$$

G_n^R is the coupling, and a_n^R is the slope parameter of the residue in the given amplitude. There are no nonsense factors in the Regge residue.

For the cut contribution we use the inverse Mellin transform to (2.4)

$$A(s, t) = \frac{1}{2\pi i} \int_{-i\infty+\gamma}^{i\infty+\gamma} dJ \, s^J \, A_J(t) , \tag{3.5}$$

where γ is to the right of all the singularities of $A_J(t)$, see fig. 9a with a form like (2.6) for $A_J(t)$ we get

$$A^{RP}(s, t) = \frac{1}{2\pi i} \int_{-i\infty+\gamma}^{i\infty+\gamma} dJ \, G(t) \, (i \, e^{-\frac{1}{2}i\pi J}) \, e^{aJ} s^J$$

$$\times \left[\frac{J - \alpha_{RP}(t)}{J - \alpha_R(t)} \right]^2 \log (J - \alpha_{RP}(t)) \tag{3.6}$$

$$\equiv iG(t) \, F(s, t) . \tag{3.7}$$

Here we have given $A_J(t)$ the arbitrary t dependence, $G(t)$, and an arbitrary J dependence e^{aJ} to provide damping as $J \to -\infty$, and have included the signature factor.

Then displacing the contour to the left (fig. 9b) we can write

$$F(s, t) = D(s, t) - P(s, t) , \tag{3.8}$$

where $D(s, t)$ is the dipole contribution·

$$D(s, t) = \frac{d}{dJ} \left[e^{cJ}(J - \alpha_{RP})^2 \log (J - \alpha_{RP}) \right]_{J=\alpha_R}$$

$$= e^{c\alpha_R}(\alpha_R - \alpha_c) \left[1 + \log |\alpha_R - \alpha_{RP}| (c(\alpha_R - \alpha_{RP}) + 2] \right], \tag{3.9}$$

$$c \equiv a + \log s - \tfrac{1}{2}i\pi$$

and $P(s, t)$ is the principal value integral over the cut discontinuity

$$P(s, t) = \int_{-\infty}^{\alpha_{RP}} e^{cJ}(J - \alpha_{RP})^2 (J - \alpha_R)^{-2} \, dJ. \tag{3.10}$$

By making the substitution $x = (\alpha_{RP} - J) \log s$ the integration may be performed to give

$$P(s, t) = -\frac{ce^{c\alpha_{RP}}}{(\log s)^2} \left[-\beta^2 \, e^{c\beta \log s} \, Ei \left(-\frac{c\beta}{\log s} \right) + \left(\frac{\log s}{c} \right)^2 \right.$$

$$\left. -\frac{\beta \log s}{c} \right] + \frac{2 \, e^{c\alpha_{RP}}}{\log s} \left[\beta \, e^{c\beta/\log s} \, Ei \left(-\frac{c\beta}{\log s} \right) + \frac{\log s}{c} + \frac{\log s}{c} \right], \tag{3.11}$$

$$\beta \equiv (\alpha_R - \alpha_{RP}) \log s.$$

Then using the expansion [39]

$$Ei(x) = \gamma + \log |x| + \sum_{n=1}^{\infty} \frac{x^n}{nn!} , \qquad \gamma \equiv 0.5772 , \tag{3.12}$$

eq. (3.11) can be simplified, and when it is combined with (3.9), we get

$$F(s, t) = e^{c\alpha_R}(\alpha_R - \alpha_{RP}) - e^{c\alpha_R}(\alpha_R - \alpha_{RP}) (2 + (\alpha_R - \alpha_{RP})c) (\gamma + \log c)$$

$$- e^{c\alpha_R}(\alpha_R - \alpha_{RP}) (2 + (\alpha_R - \alpha_{RP})c) \left(\sum_{n=1}^{\infty} \frac{[(\alpha_{RP} - \alpha_R)c]^n}{nn!} \right.$$

$$\left. - e^{c\alpha_{RP}} \frac{1}{c} + (\alpha_R - \alpha_{RP}) \right) \quad . \tag{3.13}$$

When substituted in (3.7) this result gives us our parameterization of the cut contribution. In sect. 4 we compare it with the data.

4. Fits to the data

4.1. $\pi^- p \to \pi^0 n$

For this process there are of course just two independent s-channel helicity amplitudes, which we label by the nucleon helicities as A_{++} and A_{+-}, having $n = 0, 1$

respectively. In terms of these amplitudes the experimental observables are [11]:
the differential cross section

$$\frac{d\sigma}{dt} = \frac{0.3893}{64\pi q_s^2 s} \, [|A_{++}|^2 + |A_{+-}|^2] \, (\text{mb} \cdot \text{GeV}^{-2}) \, ; \tag{4.1}$$

the polarization

$$P = \frac{2 \, \text{Im}(A_{++} A_{+-}^*)}{|A_{++}|^2 + |A_{+-}|^2} \, , \tag{4.2}$$

and the difference between the $\pi^{\pm}$p total cross sections

$$\Delta\sigma_{\text{tot}} = \frac{0.3893}{2q_s\sqrt{s}} \, \text{Im} \, A_{++}(s, 0) \, . \tag{4.3}$$

We require that the model fit the 6 GeV amplitude analysis [40] and the experimental data for (4.1) [41], (4.2) [42] and (4.3) [43] and that the resultant effective trajectory accord with fig. 2a.

For the ρ pole we used (3.1) (with $x = 0$) while for the $\rho \otimes$ P cut we have

$$A_n^{\rho\text{P}}(s, t) = G_n^\rho G_n^{\text{P}}(-t)^{\frac{1}{2}n} \, F_n(s, t) \, , \tag{4.4}$$

where G_n is the effective P absorption coupling in the given amplitude, and $F_n(s, t)$ is given by (3.13) with

$$c \to c_n \equiv a_n^{\rho\text{P}} + \log s - \tfrac{1}{2}i\pi \, , \qquad a_n^{\rho\text{P}} \equiv \frac{a_n^\rho a_n^{\text{P}}}{a_n^\rho + a_n^{\text{P}}} \, . \tag{4.5}$$

Initially we tried to fit with just $\rho + \rho \otimes$ P given by (3.1) + (4.4). However, we encountered the same problems as with the simple absorption model, namely that with the cut adjusted to give a cross-over zero in Im A_{++} at $|t| \approx 0.15$ GeV^2 there was a nearby unwanted zero in Re A_{++} because the pole and cut have a similar phase for small $|t|$. Changing the phase of the cut to correct this defect at 6 GeV required that c (defined in (4.5)) be $\approx -\tfrac{1}{2}i\pi$, but then the energy dependence of the fit is completely incompatible with higher energy data.

So again we were compelled to follow ref. [11] and include a $\rho \otimes$ f cut as well. The latter branch point occurs at $J \approx 0$ for $t = 0$, well below the ρ pole, so we do not expect the cut to be enhanced by the pole, and instead used an absorptive type of cut parameterization

$$A_n^{\rho\text{f}}(s, t) = -i(s \, e^{-\frac{1}{2}i\pi})^{\alpha_\rho\text{f}(0)} \, G_n G_n^{\text{f}}(-t)^{\frac{1}{2}n} \, \frac{e^{c_n^{\rho\text{f}}\alpha'_{\rho\text{f}}t}}{c_n^{\text{f}}} \, (1 + bt) \, , \tag{4.6}$$

$$c_n^{\rho\text{f}} \equiv a_n^{\rho\text{f}} + \log s - \tfrac{1}{2}i\pi, \qquad a_n^{\rho\text{f}} \equiv \frac{a_n^\rho a_n^{\text{f}}}{a_n^\rho + a_n^{\text{f}}},$$
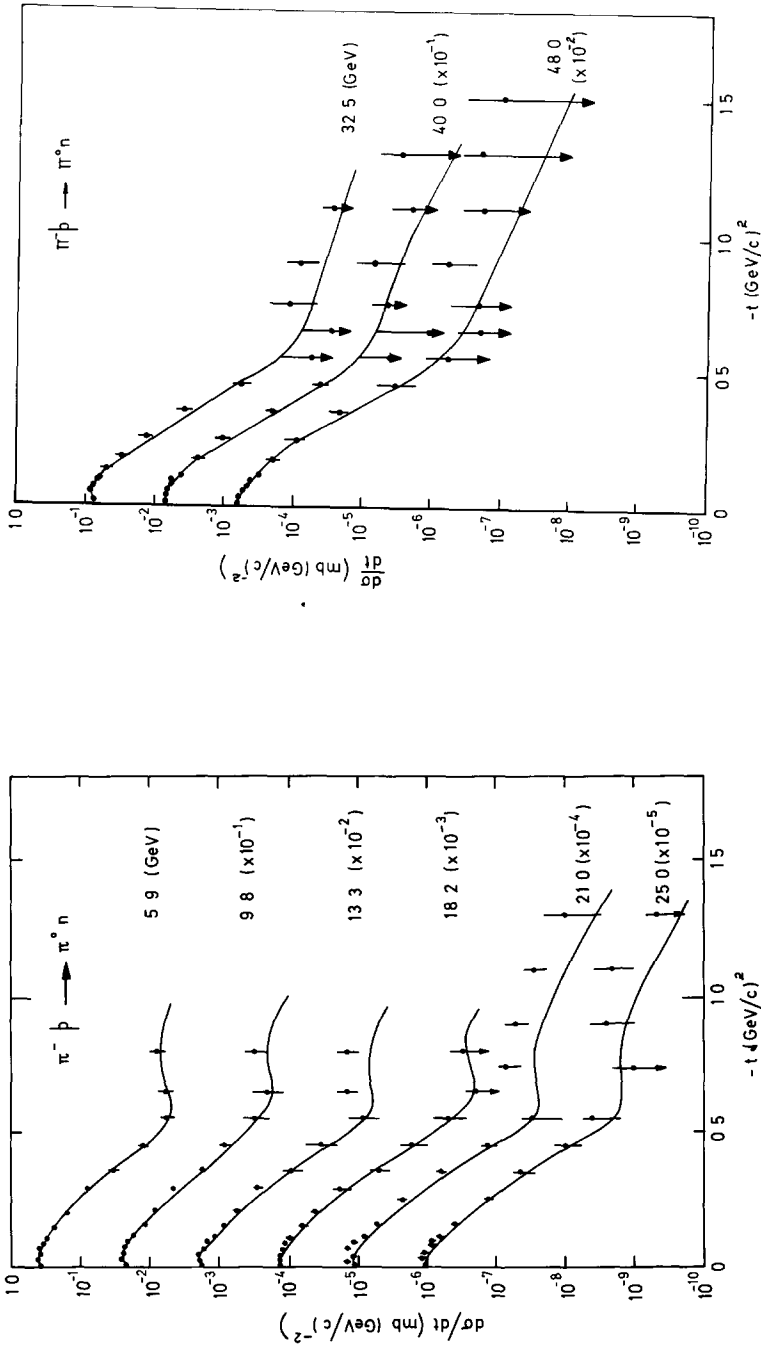
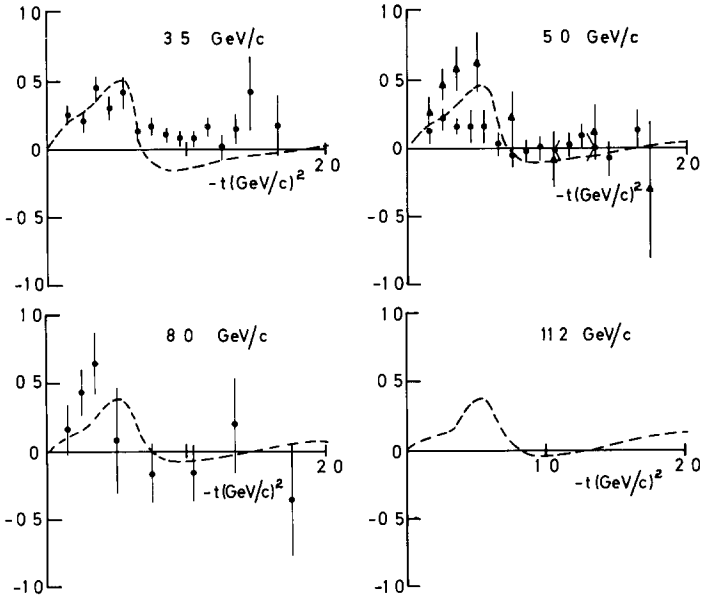Fig. 10. Fits to $\pi^- p \to \pi^0 n$ data. (a) $d\sigma/dt$ ref. [41].

Fig. 10(b). *P* ref. [42].

$$\alpha_{\rho f}(0) \equiv \alpha_\rho(0) + \alpha_f(0) - 1 , \qquad \alpha'_{\rho f} \equiv \frac{\alpha'_\rho \alpha'_f}{\alpha'_\rho + \alpha'_f} .$$

The factor b allowed some additional t dependence (as expectex if the f chooses nonsense) and was taken to be the same in both amplitudes.

The results are shown in fig. 10 and the parameters of our fit are given in table 1. Evidently the agreement with the data, amplitude analysis, and α_{eff} is excellent (except perhaps for the CERN polarization data which disagrees with other measurements). However, it should be noted that we predict a negative polarization in the range $1.0 < |t| < 2.0$ GeV^2 at higher energies, as the $\rho \otimes f$ contribution dies away.

We did not include the low energy (<5 GeV) large angle ($|t| > 2$ GeV^2) data from which Barger and Phillips extracted a linear α_{eff} in ref. [23]. However, in fig. 10e we plot α_{eff} of the fit for various energy ranges and evidently the almost linear behaviour persists at low s even for large $|t|$, but not at higher energies when the branch point becomes more important. Though obviously our parameterization may need some modification to fit large $|t|$ data (all our residues are just simple exponentials) this change of α_{eff} is a qualitative feature of our model which should be tested when higher energy large $|t|$ data becomes available. The model is generally satisfactory down to quite low energies ($P_{lab} \approx 2$ GeV) with only a small movement of the cross-over zero between 2 and 6 GeV. This is in contrast with the eikonal fits
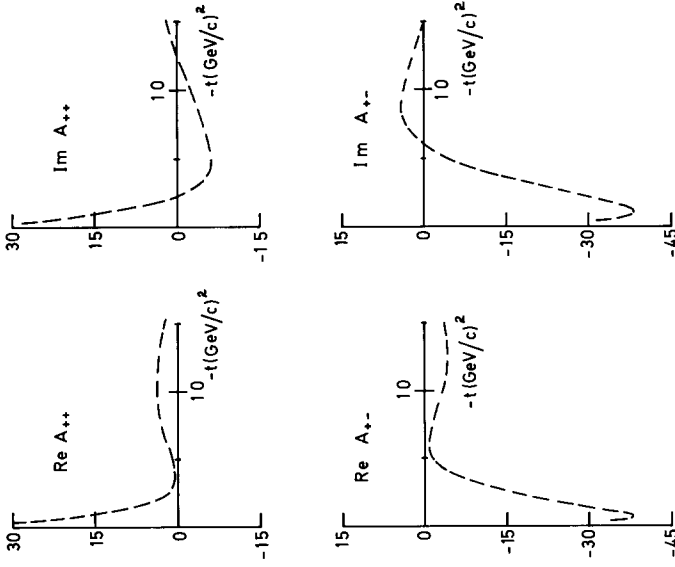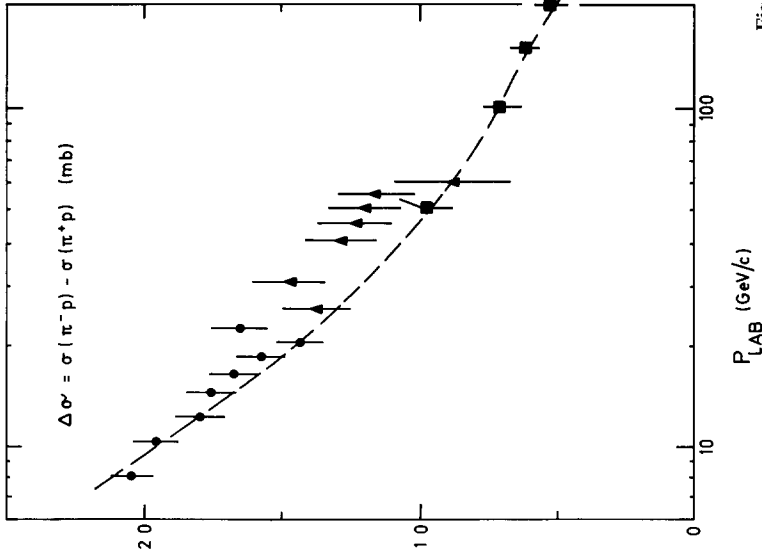
Fig. 10(c). $\Delta\sigma(\pi^\pm p)$ ref. [43].

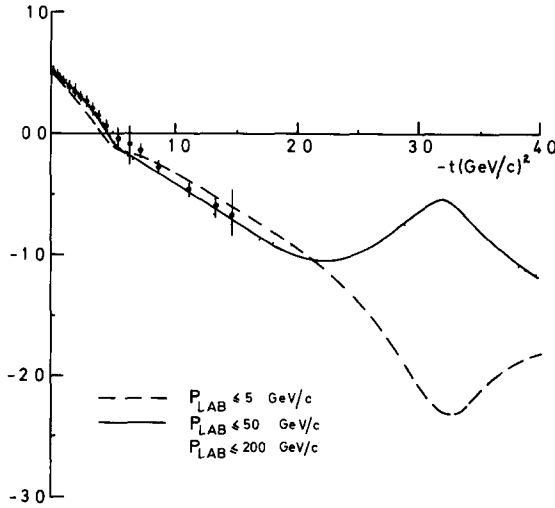Fig. 10(d). Helicity amplitudes at 6.0 GeV.

Fig. 10(e). Effective trajectory data from fig. 2a, and α_{eff} of our fit for various energy ranges.

[11] in which the logarithm factors in the cuts "blow up" at low s, and the cross-over moves rapidly. Hence this model is much more nearly "dual" than the usual eikonal/absorption fits (see ref. [13]).

It has been noticed by many authors that it is not really necessary to include absorption in A_{+-} because the amplitude structure is consistent with that predicted by a nonsense-choosing ρ pole [40]. We therefore also tried a fit with

$$A_{++} = A_0^\rho + A_0^{\rho P} + A_0^{\rho f}, \qquad A_{+-} = A_1^\rho,$$

with A_n^ρ given by (3.1) ($x = 0$) multiplied by $\alpha_\rho(t)$. The fit was almost identical to that reported above. In fact there is really nothing in πN scattering to enable one to decide between a nonsense-choosing ρ and a "fixed pole" coupling ρ which has no

Table 1
Parameters of the fit to $\pi^- p \to \pi^0 n$ using eqs. (3.1), (4.4), (4.6)

	ρ	P	f
a_0	7.8	0.28	0.90
a_1	3.66	0.22	2.06
G_0	14.45	0.22	1.25
G_1	78.39	0.06	0.43
b			−0.90
$\alpha(0)$	0.55	1.0	0.45
α'	0.93	0.28	1.08

The G's are in mb and the other parameters are in GeV units. $\alpha^P(0)$ was fixed at 1.0.

nonsense factors. However, the factorization arguments mentioned in sect. 1, in particular the absence of a $|t| \simeq 0.5$ GeV^2 dip in ρ exchange in $\gamma p \to \eta p$, leads us to prefer the featureless "fixed pole" coupling.

It is fairly obvious that in view of the freedom which we have allowed ourselves in parameterizing the strength of the cuts, and from the success of other pheno-menological work using the absorption approach, that there is no difficulty in fitting the other $0^- \frac{1}{2}^+ \to 0^- \frac{1}{2}^+$ scattering processes related to the above by SU(3). The only problem is to understand why cuts are weaker in tensor meson exchange amplitudes, which is presumably connected with the shorter range of the exchange (see sect. 1).

4.2 $\gamma p \to \pi^0 p$, $\gamma p \to \eta p$

A more interesting challenge is to look again at photoproduction because, as we noted in sect. 1, these processes do not exhibit Regge pole shrinkage at large $|t|$. On the other hand, they have a richer amplitude structure, and ω and ρ exchanges, so it is not apparent at the outset whether or not the cut discontinuities are similarly pole dominated.

We use the notation of refs. [12,44] labelling the helicities by

$$\gamma_\lambda + N_\mu \to 0^- + N_{\mu'} \, ,$$

but as it is only necessary to consider $\lambda = 1$ we again label the amplitudes by the nucleon helicities $A_{\mu'\mu}$, where A_{-+} is the non-flip amplitude ($n = 0$, $x = 2$), A_{++} and A_{--} are single-flip ($n = 1, x = 0$) and A_{+-} is double flip ($n = 2, x = 0$). In terms of these amplitudes to experimental observables are [12].

the differential cross section (m is the nucleon mass)

$$\frac{d\sigma}{dt} = \frac{0.3893}{128\pi(s - m^2)^2} \sum_{\mu'\mu} |A_{\mu'\mu}|^2 \text{ (mb} \cdot \text{GeV}^{-2}) \, , \tag{4.7}$$

the photon asymmetry

$$\Sigma = 2 \text{ Re } [A_{++}A_{--}^* - A_{-+}A_{+-}^*] \left[\sum_{\mu'\mu} |A_{\mu'\mu}|^2 \right]^{-1} , \tag{4.8}$$

the target asymmetry

$$A = 2 \text{ Im } [A_{-+}^*A_{--} - A_{+-}^*A_{++}] \left[\sum_{\mu'\mu} |A_{\mu'\mu}|^2 \right]^{-1} , \tag{4.9}$$

the ratio of π^0 photoproduction on neutrons and protons

$$R\left(\frac{\gamma n \to \pi^0 n}{\gamma p \to \pi^0 p}\right) = \frac{\displaystyle\sum_{\mu'\mu} |A_{\mu'\mu}^s - A_{\mu'\mu}^v|^2}{\displaystyle\sum_{\mu'\mu} |A_{\mu'\mu}^s + A_{\mu'\mu}^v|^2} \, , \tag{4.10}$$

where s and v refer to the isoscalar (ω) and isovector (ρ) exchanges respectively.

The couplings for η production are obtained from those for π production assuming SU(3), see refs. [12,44]. We use the data of refs. [21,22,45].

The poles R = ρ, ω are parameterized as in (3.1) and the $R \otimes P$, $R \otimes f$ like (4.4), (4.6) respectively except that for $A_{-+}(n = 0, x = 2)$ we have multiplied the cut by $(b_1 + b_2 t)$ to give the extra t dependence expected in an $x = 2$ amplitude.

Using the discontinuity (2.7) we found it quite impossible to get the requires structure in A_{-+} and A_{+-}. In fact the fitting program tried to switch off the cuts in these amplitudes, but was then unable to reproduce the polarized photon asymmetry, which essentially measures the strength of the cuts in these amplitudes (see (4.8)). Also since the cut corrections are small we obtain Regge pole shrinkage at large $|t|$ which the data does not possess (fig. 4).

Supposing that the $\gamma P \to \pi R$ amplitude might not be enhanced like the corresponding hadronic amplitude (see fig. 7), we tried replacing (2.7) by

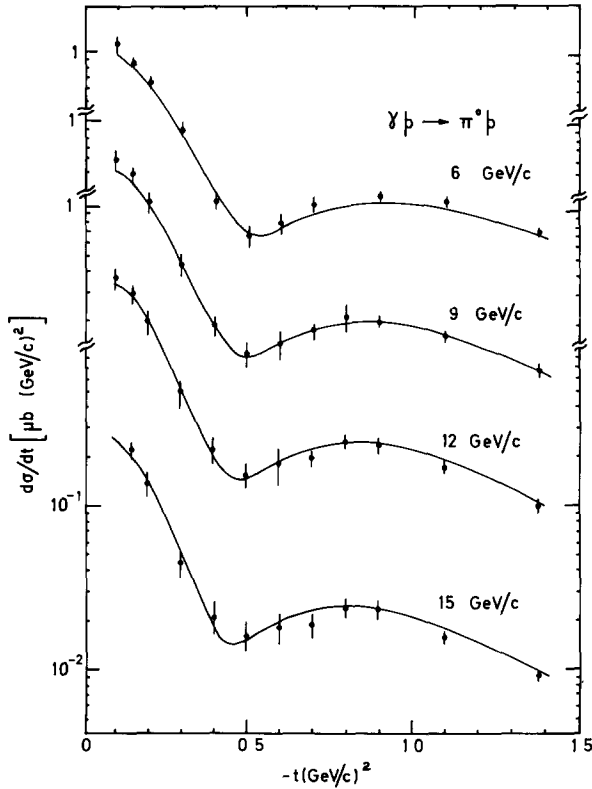$$\Delta(J,t) \sim \frac{[J - \alpha_{RP}(t)]}{[J - \alpha_R(t)]} \ , \tag{4.11}$$

Fig. 11. Fit to $\gamma p \to \pi^0 p$, ηp. (a) $d\sigma(\gamma p \to \pi^0 p) dt$, ref. [21].

i.e. with enhancement at the nucleon end only, but this was not satisfactory either. So instead we tried

$$\Delta(J, t) \sim \text{constant} ,$$ (4.12)

giving

$$F(s, t) = -\frac{e^{c\alpha_{RP}}}{c} ,$$ (4.13)
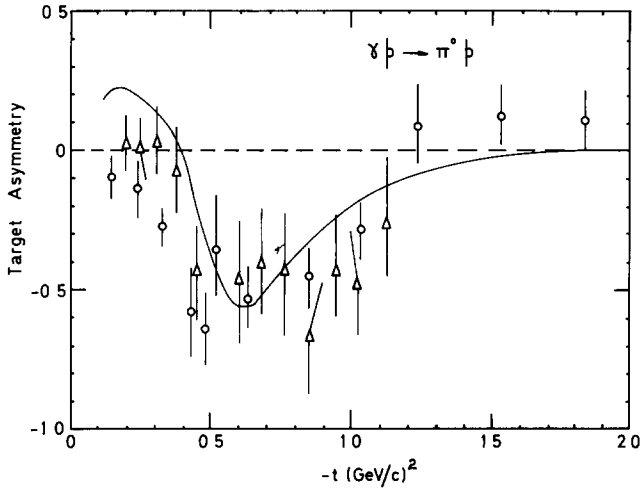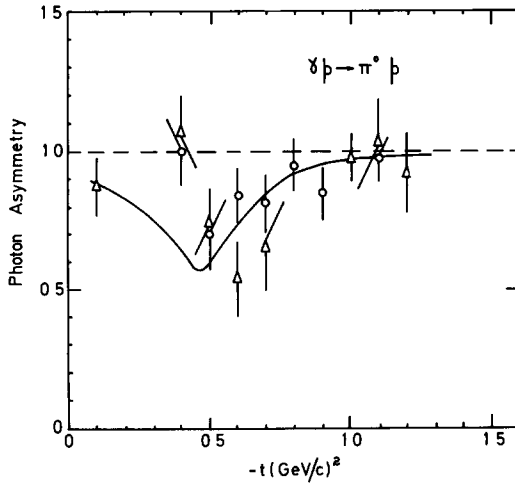

Fig. 11(b). $A(\gamma p \to \pi^0 p)$, ref. [45].
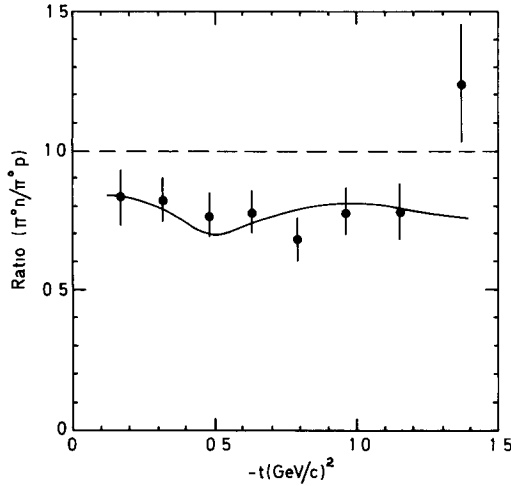

Fig. 11(c). $\Delta\sigma(\pi^{\pm}p)$ ref. [43].

Fig. 11(d). $R(\pi^0 n/\pi^0 p)$, ref. [45].

rather than (3.13). This is of course the same as the usual eikonal/absorption model result, and we parameterized the $R \otimes P$ cut as

$$A_{\mu'\mu}^{RP}(s, t) = -i(s\, e^{-\frac{1}{2}i\pi})^{\alpha_{RP}(t)}\, G_{\mu'\mu}^R\, G_{\mu'\mu}^P\, (-t)^{\frac{1}{2}n}\, \frac{e^{a_n^{RP}t}}{c_n^{RP}}, \qquad (4.14)$$

$$c_n^{RP} \equiv a_n^{RP} + \log s - \tfrac{1}{2}i\pi, \qquad a_n^{RP} \equiv \frac{a_n^R\, a_n^P}{a_n^R + a_n^P}.$$

and similarly for $R \otimes f$. Again the factor $(b_1 + b_2 t)$ was inserted for A_{-+}. We were able to get the excellent fit to the data given in figs. 11 with the parameters of table 2. This is almost good as the full eikonal fit of ref. [12] with fewer parameters, and shows that to fit the data it is necessary that the cut discontinuity be dominated by the branch point region, not near the pole. This is of course just what one would have expected from the behaviour of α_{eff} (fig. 4), but we have now confirmed that the behaviour of the individual amplitudes (and their phases) also requires that this be so.

Another possibility, which might be regarded as somewhat more consistent with factorization, would be to allow enhancement at the nucleon end in A_{++} and A_{--} (which have non-flip couplings to the nucleon, for which we must have strong cuts in $\pi^- p \to \pi^0 n$) but not in A_{+-} and A_{-+} (which have flip couplings to the nucleon, and need have no cuts in $\pi^- p \to \pi^0 n$ if the ρ has a nonsense factor). This gives $\Delta(J, t)$ like (4.11) for A_{++} and A_{--}, but like (4.12) for A_{+-} and A_{-+}. However, we found that in this case the dip in $d\sigma/dt$ at $|t| \approx 0.5$ GeV^2 deepens with energy, contrary to the data.

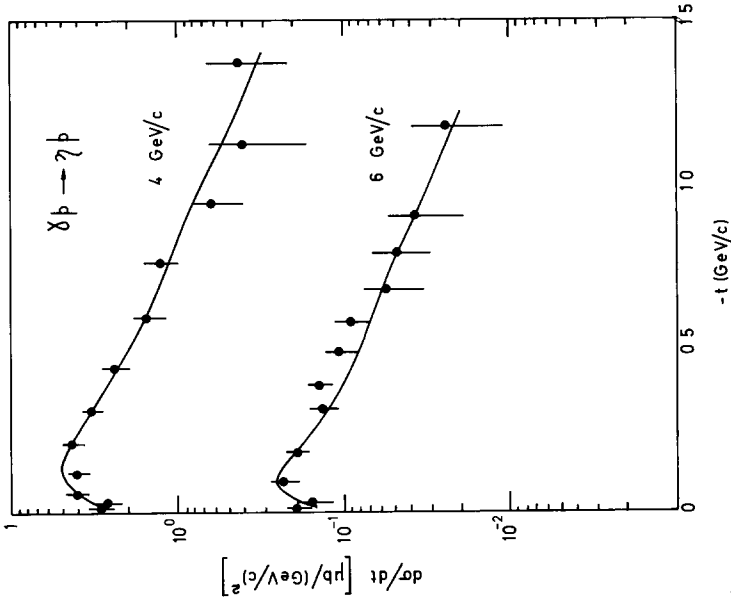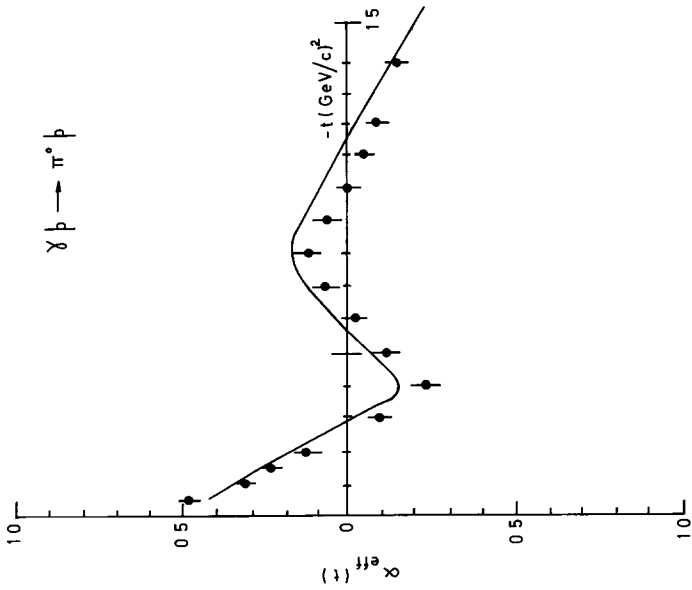Fig. 11(e). $\alpha_{eff}(\gamma p \to \pi^0 p)$.

Fig. 11(f). $d\sigma(\gamma p \to \eta p)/dt$ ref. [22].

Table 2
Parameters of the fit to $\gamma p \to \pi^0 p$ and related processes using (3.1) for the ρ and ω poles, (4.14) for the $R \otimes P$ cuts and (4.6) for the $R \otimes f$ cuts

	ρ	ω	P	f
a_0	3.36	9.48	4.94	9.90
a_1	0.15	1.03	5.10	6.35
G_0	44.13	19.58	0.40	3.61
G_1	1.15	13.61	2.13	1.89
b_1			−0.53	−0.53
b_2			5.96	5.96
$\alpha(0)$	0.55	0.55	1.0	0.45
α'	0.93	0.93	0.25	1.08

The parameters for the two single flip ($n = 1$) amplitudes are the same, and those for non-flip ($n = 0$) are the same as those for double flip ($n = 2$) except that for $n = 0$ we have multiplied the $R \otimes P$ and $R \otimes f$ cuts by $(b_1 + b_2 t)$, b_1 and b_2 being the same for R = ρ and ω

We thus conclude that in πN charge exchange scattering, and other hadronic processes which show shrinkage at large $|t|$, the cut discontinuity obtains its most important contribution in the region of the pole. But in photoproduction processes, which shrink little at large $|t|$, the cuts are dominated by the branch-point region, as in the eikonal/absorption model. We shall attempt to draw some more general conclusions in sect. 5.

5. Conclusions

A fairly consistent picture of hadronic processes seems to be emerging in which the zeros of scattering amplitudes are due to pole-cut interference of the absorptive type, rather than nonsense factors, and the shrinkage, approximate factorization, SU(3) relations and other pole-like features of the amplitudes stem from the enhancement of the cut discontibuities in the region of the Regge poles. This is because t-channel unitarity requires that the cuts couple through the poles (fig. 7a).

This is in constrast with the old eikonal/absorption model for cuts in which the dominant part of the cut discontinuity is in the region of the branch point, so that it is predicts a decrease in the amount of shrinkage, a "harder" interaction, as $|t|$ increases, whereas the data show strong Regge shrinkage in all hadronic processes (except elastic scattering) even at fairly high s and $|t|$ values. The low-energy phase problems are corrected by the inclusion of $R \otimes P'$ cuts. This seems to be better motivated physically than the rather ad hoc modification of the absorption in ref. [8]. It will of course be necessary to test this conclusion more thoroughly over a wider range of high-energy data at large $|t|$ (>1 GeV^2) when this becomes available, but it is very gratifying that the pole-enhancement of cuts which we need to explain the data phenomenologically is supported by the t-channel unitarity arguments of ref. [31].

We have found that photoproduction processes do not enjoy pole enhancement, but of course the restrictions of t-channel unitarity do not apply here (at least directly) because we work only to first order in the electromagnetic coupling.

Several problems connected with the coupling of Reggeons to photons have been noted previously [46], in particular that π-exchange does not decouple from $\gamma p \to \pi^+ n$ at $t = m_\pi^2$ despite the fact that $\alpha_\pi = 0$ is a right-signature nonsense point, and that the pomeron does not decouple from Compton scattering ($\gamma N \to \gamma N$) at $t = 0$ despite the fact that $\alpha_P = 1$ is a nonsense point (admittedly of wrong signature in this case so that a fixed pole can be blamed). It has been argued [46] that both these problems can be solved if one supposes that with (zero mass) photons the t-channel threshold behaviour does not impose a separate constraint (this is discussed in detail in ref. [46], see also ref. [47]).

It is clear from our fits that the absence of t-channel unitarity constraints also affects the cut contributions to photo-production processes, and this is presumably why the eikonal/absorption model with its hard cuts is so much more successful for photoproduction than for hadronic processes. This is true not only in the forward direction as we have shown, but also in the backward direction where $\gamma p \to p\pi^0$, $n\pi^+$ exhibit a quite different behaviour from $\pi N \to N\pi$ (see e.g. ref. [48]) in which the same exchange occur.

An interesting generalization of these conclusions is suggested by recent work of Irving [49]. He compares the magnitude and t-dependence of the cut, C (presumably mainly a $\pi \otimes P$ cut), exchanged together with the π and A_2 poles in the processes $\pi N \to \rho N$ ($q^2 = m_\rho^2$), $\gamma p \to \pi^+ n$ ($q^2 = 0$) and pion electroproduction $\gamma_v p \to \pi^+ n$ ($q^2 < 0$). He finds that the magnitude of $C(q^2)$ increases markedly with $-q^2$, and the interaction becomes more peripheral, an effect which has also been noted in the dependence of cut strength on the dipion mass in $\pi N \to (\pi\pi)N$ [50]. Similarly, it has been found that high mass resonances suffer little absorption in nuclei. Our analysis suggests that the cuts also become harder (i.e. more branch point dominated) and so flatter in t as $-q^2$ increases.

Irving speculates that this transition is connected with the fact that as $-q^2$ increases we get into the scaling region of electroproduction where the hadrons become pointlike, and less structured. This connection between the dominance of Regge cuts and the onset of scaling behaviour has also been noted in high-energy elastic scattering, and has been predicted in asymptotically free theories by Lovelace [52]. It is rather intriguing, therefore, that we have found a transsition from Regge-pole-like cuts suggestive of structured hadrons, to hard cuts suggestive of more pointlike interactions as the mass of the external particles decreases. It remains to be seen whether this is a purely photonic effect, depending on the weak coupling, or whether in hadronic processes the degree of pole enhancement of the cuts increases as the mass of the external hadrons, and hence the distance of the t-channel threshold increases.

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