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Bootstrap approach to scaling and fixed poles in virtual Compton amplitudes*

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It is shown that scaling and the $l=0$ fixed poles can be explained in terms of bootstrap elements. This approach is strongly supported by recent investigations of the nature of the fixed poles, while, in the scaling region, it turns out to be equivalent to the parton model.

I. INTRODUCTION

The scaling behavior observed in deep-inelastic electron-proton scattering has motivated a series of parton models of photon-hadron interactions.[1] All of these models postulate that the hadrons are built up of certain pointlike constituents (partons) which carry the electromagnetic current. The partons are understood to be underlying fields of some future field theory, but need not have a particle interpretation.[2] Some authors[3] identify the partons with quarks which throws a bridge to strong-interaction dynamics.[4] But this conjecture still awaits an experimental test.

Recently a further attribute of photon-hadron amplitudes has been established. The high-energy data of forward nucleon Compton scattering require, in addition to the standard Regge terms, the presence of $l=0$ fixed poles, whose residues agree with the Thomson limit[5]

$$T^{p}_{1\,\mathrm{FP}} \simeq -2\,, \quad T^{n}_{1\,\mathrm{FP}} \simeq 0\,. \tag{1.1}$$

This provides a sensitive test of existing parton models and the quark-parton picture as well. The simple three-quark model of the nucleon would give

$$T^{p}_{1\,\mathrm{FP}} = \tfrac{3}{2} T^{n}_{1\,\mathrm{FP}}\,, \quad T^{p}_{1\,\mathrm{FP}} \simeq -6\,, \tag{1.2}$$

which obviously contradicts Eq. (1.1). In the more general case, the parton model is much less specific, but it always predicts a nonzero fixed-pole contribution of the neutron.[6] This points out that the parton model in its present form fails to explain the fixed poles, which has interesting theoretical consequences.

The fact that the fixed poles seem to be sensitive only to the total charge indicates that some kind of self-consistency condition between the constituent currents and the total electromagnetic current might be involved. On the one hand, the scaling behavior requires a composite theory of the hadrons, while, on the other hand, the fixed poles are closely associated with the (noncomposite) Born terms.[7] This "duality" comes as no surprise to us. There is strong evidence that at least the low-lying hadron states are governed by bootstrap principles,[8] which has to be respected by any future field theory. In the bootstrap picture the "Born singularities" and multiline connected parts are intimately related, allowing for both scaling graphs *and* (fixed-pole) Born terms.

In view of this it seems to be attractive to consider the alternative that partons are *ordinary* (bootstrapping) hadrons. It has been argued recently by Zachariasen[9] that the scaling behavior does not necessarily lead to pointlike constituents. This can best be understood from some model calculations[10] which indicate that the constituent form factors need no longer vanish as $q^2 \to -\infty$ when both legs are off-shell. So, even in the bootstrap picture some internal lines represent more elementary entities.

In this paper we shall demonstrate that scaling and the "right" fixed-pole behavior can be explained in terms of bootstrap elements. We will base our discussion on S-matrix theory although we have something like a Bethe-Salpeter equation in mind. For our aims we need not go into dynamical details of the bootstrap program. Dashen and Frautschi[11] have shown that the bootstrap currents fulfill a current algebra. This provides a q^2-normalization condition of the current-current amplitudes which is all we need.

In order to avoid spin complications we shall restrict ourselves to a pion target. But we believe that our conclusions can be carried over to the spin-$\frac{1}{2}$ case.

The paper is organized as follows. In Sec. II we define invariant amplitudes free of kinematical singularities and zeros that describe pion Compton scattering. For these amplitudes a Mandelstam representation is assumed, on the basis of which we derive t-channel partial-wave dispersion relations for the double-helicity-flip amplitude (Sec. III). The partial-wave series is continued to the physical region of the s channel by means of a Sommerfeld-Watson transform. In Sec. IV we discuss the current-algebra constraint. It gives rise to an $l=1$ fixed pole in the odd amplitude which imposes decisive restrictions on the dynamical input. Then, by incorporating this information, we calculate the $l=0$ fixed pole and show (in Sec. V) that our model gives rise to nontrivial scaling. In these cases the partial-wave dispersion relation can be solved explicitly. Finally, in Sec. VI we add some concluding remarks.

II. COMPTON AMPLITUDES

We consider the process

$$\gamma_\mu^\alpha(q)+\pi^\rho(p)\to\gamma_\nu^\beta(q')+\pi^\rho(p'), \tag{2.1}$$

where $\gamma_\mu^\alpha, \gamma_\nu^\beta$ may be neutral or charged isovector photons. Isoscalar photons are not taken into account since they do not contribute to the fixed poles and in the deep-inelastic limit as will be shown later. This, however, is not true in the case of nucleon Compton scattering. Our notation will be[12] $s=(q+p)^2$, $u=(q-p')^2$, and $t=(q-q')^2$. The scattering amplitude is given by

$$T_{\mu\nu}^{\alpha\beta,\rho}=i\int d^4x\, e^{iq'\cdot x}\theta(x)\langle\pi^\rho(p')|\,[j_\nu^\beta(x),j_\mu^\alpha(0)]\,|\pi^\rho(p)\rangle. \tag{2.2}$$

Due to crossing and isospin invariance it can be reduced to three independent t-channel isospin amplitudes (the upper index labeling the isospin)

$$\begin{aligned}T_{\mu\nu}^{(0)}&=2[C_{\mu\nu}(s,t)+C_{\mu\nu}(u,t)]-N_{\mu\nu},\\ T_{\mu\nu}^{(1)}&=C_{\mu\nu}(s,t)-C_{\mu\nu}(u,t),\\ T_{\mu\nu}^{(2)}&=-[C_{\mu\nu}(s,t)+C_{\mu\nu}(u,t)]+2N_{\mu\nu},\end{aligned} \tag{2.3}$$

where $C_{\mu\nu}=T_{\mu\nu}^{-+,+}$, i.e., the tensor associated with "charged" Compton scattering off π^+, and $N_{\mu\nu}=T_{\mu\nu}^{00,0}$ corresponding to "neutral" Compton scattering off π^0. This representation will prove to be very useful. It is evident that $N_{\mu\nu}$ gets *no* contribution from the pion pole terms and related diagrams. The currents are assumed to be conserved and to satisfy the current algebra which gives

$$q^\mu T_{\mu\nu}^{(0,2)}=T_{\mu\nu}^{(0,2)}q'^\nu=0, \tag{2.4}$$

$$q^\mu T_{\mu\nu}^{(1)}=T_{\nu\mu}^{(1)}q'^\mu=2\Delta_\nu F(t), \tag{2.5}$$

where $\Delta=p+p'$ and $F(t)$ is the pion electromagnetic form factor.

Next we express the tensors $T_{\mu\nu}^{(i)}$ in terms of (independent) invariant amplitudes free of kinematical singularities and zeros.[13] These amplitudes are the most suitable ones having a Mandelstam representation without *ad hoc* subtractions. Let us first consider the isospin-0 and isospin-2 cases. Here the method of Bardeen and Tung[14] can be applied. We obtain

$$T_{\mu\nu}^{(0,2)}=\sum_{n=1}^{5}I_{\mu\nu}^{n}A_n^{(0,2)}, \tag{2.6}$$

where

$$\begin{aligned}I_{\mu\nu}^1&=q\cdot q'g_{\mu\nu}-q'_\mu q_\nu,\\ I_{\mu\nu}^2&=q\cdot q'\Delta_\mu\Delta_\nu-q\cdot\Delta q'_\mu\Delta_\nu-q'\cdot\Delta\Delta_\mu q_\nu\\ &\quad+q\cdot\Delta q'\cdot\Delta g_{\mu\nu},\\ I_{\mu\nu}^3&=q\cdot q'q_\mu\Delta_\nu-q^2q'_\mu\Delta_\nu-q'\cdot\Delta q_\mu q_\nu\\ &\quad+q^2q'\cdot\Delta g_{\mu\nu},\\ I_{\mu\nu}^4&=q\cdot q'\Delta_\mu q'_\nu-q\cdot\Delta q'_\mu q'_\nu-q'^2\Delta_\mu q_\nu+q'^2q\cdot\Delta g_{\mu\nu},\\ I_{\mu\nu}^5&=q\cdot q'q_\mu q'_\nu-q^2q'_\mu q'_\nu-q'^2q_\mu q_\nu+q^2q'^2g_{\mu\nu}.\end{aligned} \tag{2.7}$$

In forward direction and for equal photon mass the A_n are related to the usual invariant amplitudes T_1, T_2 by (omitting the isospin)

$$\begin{aligned}T_1+\frac{1}{4}\frac{(q\cdot\Delta)^2}{q^2}T_2&=-A_1-q^2A_5,\\ T_2&=4q^2A_2.\end{aligned} \tag{2.8}$$

In the isospin-1 case we can proceed similarly. Instead of the gauge condition (2.4) we have to incorporate the current-algebra constraint (2.5). We find

$$T_{\mu\nu}^{(1)}=\sum_{n=1}^{6}L_{\mu\nu}^{n}A_n^{(1)}, \tag{2.9}$$

where

$$\begin{aligned}L_{\mu\nu}^m&=I_{\mu\nu}^m,\quad m=1,3,4,5\\ L_{\mu\nu}^2&=\Delta_\mu\Delta_\nu,\\ L_{\mu\nu}^6&=q'_\mu\Delta_\nu+\Delta_\mu q_\nu-q\cdot\Delta g_{\mu\nu}.\end{aligned} \tag{2.10}$$

Here only five of the invariant amplitudes $A_n^{(1)}$ are independent. We have

$$A_6^{(1)}=\frac{1}{q\cdot q'}[2F(t)-q\cdot\Delta A_2^{(1)}]. \tag{2.11}$$

III. DISPERSION RELATIONS

Our *basic assumption* will be that the invariant amplitudes $A_n^{(i)}$ satisfy an *unsubtracted* Mandelstam representation for $q^2, q'^2<4$. This is quite a natural assumption from the bootstrap point of view which we will hold throughout this work. On the other hand, there are no subtractions required on the basis of low-energy theorems.

Since our further discussion will mainly be concerned with the invariant amplitudes $A_2^{(i)}$, we shall restrict our attention to these amplitudes only. The Mandelstam representation for the $A_2^{(i)}$ can be written

$$A_2^{(0,2)}=\frac{1}{\pi}\int ds'\frac{\rho_1^{(0,2)}(s')}{(s'-s)(s'-u)}+\frac{1}{\pi^2}\int ds'\int dt'\frac{\rho_2^{(0,2)}(s',t')}{t'-t}\left(\frac{1}{s'-s}+\frac{1}{s'-u}\right)+\frac{1}{\pi^2}\int ds'\int du'\frac{\rho_3^{(0,2)}(s',u')}{(s'-s)(u'-u)}, \tag{3.1}$$

$$A_2^{(1)}=\frac{1}{\pi}\int ds'\rho_1^{(1)}(s')\left(\frac{1}{s'-s}-\frac{1}{s'-u}\right)+\frac{1}{\pi^2}\int ds'\int dt'\frac{\rho_2^{(1)}(s',t')}{t'-t}\left(\frac{1}{s'-s}-\frac{1}{s'-u}\right)$$
$$+\frac{1}{\pi^2}\int ds'\int du'\frac{\rho_3^{(1)}(s',u')}{(s'-s)(u'-u)}. \tag{3.2}$$

The crossing conditions as expressed by Eq. (2.3) are explicitly satisfied for $\rho_3^{(0,2)}(s',u')=\rho_3^{(0,2)}(u',s')$ and $\rho_3^{(1)}(s',u')=-\rho_3^{(1)}(u',s')$. Single dispersion integrals are not allowed for $A_2^{(0,2)}$ because they would exceed the unitarity bound.[15] The spectral functions apparently depend on q^2, q'^2 and so do the boundary curves.

The single-spectral-function integrals in Eqs. (3.1) and (3.2) represent the pion pole terms and the whole set of associated bootstrap diagrams as shown in Fig. 1. The pole terms which are responsible for the low-energy theorems are given by

$$A_{2P}^{(0)}=\frac{2}{(s-1)(u-1)}F(q^2)F(q'^2),$$
$$A_{2P}^{(2)}=-\tfrac{1}{2}A_{2P}^{(0)}, \tag{3.3}$$
$$A_{2P}^{(1)}=\left(\frac{1}{s-1}-\frac{1}{u-1}\right)F(q^2)F(q'^2)$$

corresponding to the δ-function part of $\rho_1^{(i)}$. It has already been pointed out that the contributions connected with the single-spectral-function integrals come entirely from $C_{\mu\nu}$. So, we expect that the $\rho_1^{(i)}$ are related to each other similar to the pole terms (3.3). This is indeed the case. We have

$$\tfrac{1}{2}\rho_1^{(0)}=\rho_1^{(1)}=-\rho_1^{(2)}, \tag{3.4}$$

which can be verified by evaluating $T_{\mu\nu}^{(2)}$ in the infinite-momentum frame ($|\vec{\Delta}|\to\infty$; s,t,u fixed). Here the leading contribution is proportional to $q\cdot q'A_2^{(0,2)}$ and $A_2^{(1)}$, respectively. The single-spectral-function part of $q\cdot q'A_2^{(0,2)}$ may be written

$$q\cdot q'\int_1^\infty ds'\frac{\rho_1^{(0,2)}(s')}{(s'-s)(s'-u)}=-\tfrac{1}{2}\int_1^\infty ds'\rho_1^{(0,2)}(s')\left(\frac{1}{s'-s}+\frac{1}{s'-u}\right)+\int_9^\infty ds'\int_9^\infty du'\frac{\tilde{\rho}_1^{(0,2)}(s',u')}{(s'-s)(u'-u)}, \tag{3.5}$$

where $\tilde{\rho}_1^{(0,2)}(s',u')=\delta(s'-u')(s'-1)\rho_1^{(0,2)}(s')$. By comparison with Eq. (2.3) this gives the desired result.[16] Equation (3.4) will become important in the next section where we shall discuss the restrictions imposed on $\rho_1^{(i)}$ by self-consistency requirements.

It will prove to be useful to analyze $A_2^{(i)}$ in terms

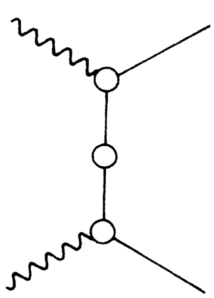

FIG. 1. Single-spectral-function contribution to the Mandelstam representation.

of t-channel partial-wave amplitudes. The $A_2^{(i)}$ correspond to the t-channel helicity-flip amplitudes:

$$T_{+-}^{(0,2)}=\tfrac{1}{4}(t-q^2-q'^2)(t-4)\sin^2\theta A_2^{(0,2)},$$
$$T_{+-}^{(1)}=\tfrac{1}{2}(t-4)\sin^2\theta A_2^{(1)}. \tag{3.6}$$

This naturally leads to the definition of helicity amplitudes free of kinematical singularities and zeros:

$$M_{+-}^{(i)}=A_2^{(i)}, \tag{3.7}$$

which have the partial-wave expansion

$$M_{+-}^{(i)}=\sum_{l=2}^\infty(2l+1)f_l^{(i)}(t)(1+e^{i\pi(l+i)})\frac{d_{20}^l(\cos\theta)}{\sin^2\theta}, \tag{3.8}$$

where

$$d^l_{20}(z)=\frac{[(l-1)l(l+1)(l+2)]^{1/2}}{(2l-1)(2l+1)(2l+3)}[(2l-1)P_{l+2}(z)+(2l+3)P_{l-2}(z)-2(2l+1)P_l(z)]\frac{1}{1-z^2}. \tag{3.9}$$

The singularities of the partial-wave amplitudes $f_l^{(i)}$ are then completely determined by the Mandelstam representation.

For $f_l^{(i)}$ we can write down partial-wave dispersion relations. However, we must be careful about the branch cuts due to unequal-mass kinematics. In the following we set $q^2=q'^2$. Then we have $f_l^{(i)} \sim[(t-4)(t-4q^2)]^{(l-2)/2}$ as $t\to 4$ or $4q^2$, which gives rise to a branch cut from $t=4$ to $t=4q^2$. We see that for physical l (i.e., l even for $i=0,2$ and odd for $i=1$) only $f_l^{(1)}$ has an extra branch cut. In this case and for general l we draw the branch cuts along the positive real axis. So we can write generally

$$f_l^{(i)}(t)=f_{Bl}^{(i)}(t)+\frac{1}{\pi}\int_{-\infty}^{t_L}dt'\frac{\mathrm{Disc}f_l^{(i)}(t')}{t'-t}+\frac{1}{\pi}\int_4^{4q^2}dt'\frac{\mathrm{Disc}f_l^{(i)}(t')}{t'-t}+\frac{1}{\pi}\int_4^{\infty}dt'\frac{\mathrm{Im}f_l^{(i)}(t')}{t'-t}, \tag{3.10}$$

where we have projected out the Born term:

$$\begin{aligned} f_{Bl}^{(0,2)}(t)&=-\frac{2}{[(t-4)(t-4q^2)]^{1/2}}\frac{[(l-1)l(l+1)(l+2)]^{1/2}}{(2l-1)(2l+1)(2l+3)}\frac{1}{\pi}\int_1^{\infty}ds'\frac{\rho_1^{(0,2)}(s')}{2s'-2+t-2q^2}\mathcal{Q}_l(z'),\\ f_{Bl}^{(1)}(t)&=-\frac{2}{[(t-4)(t-4q^2)]^{1/2}}\frac{[(l-1)l(l+1)(l+2)]^{1/2}}{(2l-1)(2l+1)(2l+3)}\frac{1}{\pi}\int_1^{\infty}ds'\rho_1^{(1)}(s')\mathcal{Q}_l(z'). \end{aligned} \tag{3.11}$$

Here

$$\mathcal{Q}_l(z')=(2l-1)Q_{l+2}(z')+(2l+3)Q_{l-2}(z')-2(2l+1)Q_l(z')$$

and

$$z'=\frac{2s'-2+t-2q^2}{[(t-4)(t-4q^2)]^{1/2}}.$$

In the physical region unitarity tells us

$$f_l^{(0,2)}(t)\lesssim\frac{1}{t^2},\qquad f_l^{(1)}(t)\lesssim\frac{1}{t}, \tag{3.12}$$

so that no subtractions are needed.

The left-hand cut starts at

$$t_L=\tfrac{1}{16}[40q^2-4(q^2)^2-36]. \tag{3.13}$$

Its discontinuity has the form

$$\begin{aligned}\mathrm{Disc}f_l^{(i)}(t)=\frac{1}{[(t-4)(t-4q^2)]^{1/2}}\frac{[(l-1)l(l+1)(l+2)]^{1/2}}{(2l-1)(2l+1)(2l+3)}\frac{1}{\pi}\Big[\int ds'\Big(\int dt'\frac{\rho_2^{(i)}(s',t')}{t'-t}+\int du'\frac{\rho_3^{(i)}(s',u')}{s'+u'-2+t-2q^2}\Big)\mathcal{P}_l(z')\\ -2\int ds'\rho_3^{(i)}(s',2-s'-t+2q^2)\mathcal{Q}_l(z')\Big],\end{aligned} \tag{3.14}$$

where

$$\mathcal{P}_l(z')=[(2l-1)P_{l+2}(z')+(2l+3)P_{l-2}(z')-2(2l+1)P_l(z')]\,\theta(1+z')\theta(1-z').$$

On the right-hand cut we shall use elastic unitarity[17] ($\rho(t)=[(t-4)/t]^{1/2}$):

$$\mathrm{Im}f_l^{(i)}(t)=\rho(t)f_l^{(i)}(t)t_l^{(i)*}(t), \tag{3.15}$$

where $t_l^{(i)}$ is the elastic isospin-i $\pi\pi$ amplitude. We believe that this approximation is justified for our aims. Later on we are only interested in $f_l^{(i)}$ at $t=0$. For $t_l^{(i)}$ we make the ansatz

$$t_l^{(i)}(t)=\frac{\beta_l^{(i)}(t)}{\alpha^{(i)}(t)-l}, \tag{3.16}$$

where $\beta_l^{(i)}(t)$ (generally complex) is only restricted by unitarity. In the case when there is no extra branch cut and $f_{Bl}^{(i)}$ is continuous at $t=4$, the dispersion relation (3.10) can be solved by standard methods[18] (provided that the left-hand-cut integral, for short $f_{Ll}^{(i)}$, is known). The solution is

$$f_l^{(i)}(t)=f_{Bl}^{(i)}(t)+f_{Ll}^{(i)}(t)-\frac{1}{D_l^{(i)}(t)}\frac{1}{\pi}\int_4^{\infty}dt'\frac{\mathrm{Im}D_l^{(i)}(t')}{t'-t}[f_{Bl}^{(i)}(t')+f_{Ll}^{(i)}(t')]+\frac{P(t)}{D_l^{(i)}(t)}, \tag{3.17}$$

where (formally)

$$D_l^{(i)}(t)=\exp\left\{-\frac{i}{2\pi}\int_4^{\infty}dt'\,\frac{\ln\left[\dfrac{\alpha^{(i)}(t')-l-2i\rho(t')\beta_l^{(i)}(t')}{\alpha^{(i)}(t')-l}\right]}{t'-t}\right\}. \tag{3.18}$$

The polynomial $P(t)$ must be zero if $D_l^{(0,2)}(t) \lesssim t^{2-\epsilon}$ and $D_l^{(1)}(t) \lesssim t^{1-\epsilon}$; otherwise this would conflict with Eq. (3.12). In the case of unequal-mass branch cuts (which will concern us later) one has to take care of the discontinuities of $f_l^{(i)}$ across this cut, which gives a slightly different solution.[19] Equations (3.16)–(3.18) can easily be generalized to include several Regge poles.

Equation (3.8) will be continued to the physical region of the s-channel by means of the Sommerfeld-Watson transform assuming analyticity in l:

$$\tilde{M}_{+-}^{(i)} = -\frac{1}{2i}\int_C \frac{dl}{\sin\pi l}(2l+1)f_l^{(i)}\frac{d_{20}^l(-\cos\theta)}{\sin^2\theta}. \quad (3.19)$$

Here $\tilde{M}_{+-}^{(i)}$ are amplitudes of definite signature [signature $(-1)^i$]. The contour C of the integration is shown in Fig. 2. The leading l-plane singularities are given by the zeros of $D_l^{(i)}$ [see Eq. (3.17)] and the finite parts of $f_l^{(i)}$ at $l=0, 1$. The former lead to moving poles while the latter correspond to (right- or wrong-signature nonsense) fixed poles.

IV. FIXED POLES

The current-algebra constraint (2.5) requires the existence of an $l=1$ fixed pole in the isospin-1 amplitude[20]

$$A_{2\,\mathrm{FP}}^{(1)} \simeq 2\frac{F(t)}{s}. \quad (4.1)$$

This can be read off (alternatively to the original derivation) from Eq. (2.11). The highest fixed pole that may occur in $A_6^{(1)}$ behaves like $1/s$ corresponding to the largest possible right-signature nonsense point. So the form factor must asymptotically be canceled by $A_2^{(1)}$.

We shall now evaluate the $l=1$ fixed-pole contribution in terms of Eqs. (3.2) and (3.10) and discuss what kind of restrictions Eq. (4.1) imposes on the spectral functions. This forces us to solve Eq. (3.10) for i, $l=1$. From Eq. (3.14) we deduce that there is no left-hand-cut contribution in this case.[21] The first term of the discontinuity across the left-hand cut vanishes because of the factor $(l-1)^{1/2}$ [note that there is an additional factor $(l-1)^{1/2}$ coming from $d_{20}^l(\cos\theta)$; see Eq. (3.8)]. The second term, corresponding to the su-double-spectral function, does quite generally not contribute to the right-signature points, which results from the following symmetry properties under the substitution $s' \to 2-s'-t+2q^2$ (of course, this term gives rise to wrong-signature fixed poles):

$$\rho_3^{(1)}(s', 2-s'-t+2q^2) \to -\rho_3^{(1)}(s', 2-s'-t+2q^2),$$
$$z' \to -z', \quad \mathcal{Q}_l(z') \to -e^{i\pi l}\mathcal{Q}_l(z'). \quad (4.2)$$

Here the factor $l-1$ (as well as the factor l which becomes significant as $l\to 0$) is canceled by the pole of Q_{l-2}. The same holds also for the Born term. So, if we absorb the $(l-1)^{1/2}$ of Eq. (3.9) into $f_l^{(1)}$ by defining $\bar{f}_l^{(1)} = (l-1)^{1/2}f_l^{(1)}$, we obtain the very simple integral equation

$$\bar{f}_1^{(1)}(t) = \frac{2(\frac{2}{3})^{1/2}}{[(t-4)(t-4q^2)]^{1/2}}\frac{1}{\pi}\int_1^\infty ds'\rho_1^{(1)}(s') + \frac{1}{\pi}\int_4^{4q^2} dt'\frac{\mathrm{Disc}\bar{f}_1^{(1)}(t')}{t'-t} + \frac{1}{\pi}\int_4^\infty dt'\frac{\bar{f}_1^{(1)}(t')}{t'-t}\frac{\rho(t')\beta_1^{(1)}(t')^*}{\alpha^{(1)}(t')^*-1}. \quad (4.3)$$

The only unknown parameter (besides the right-hand-cut parametrization) is $R \equiv (1/\pi)\int_1^\infty ds'\rho_1^{(1)}(s')$. Since the fixed pole is the leading term in the high-energy limit, we obtain from Eqs. (3.2) and (4.1)

$$\int ds'\rho_1^{(1)}(s') + \int ds'\int dt'\frac{\rho_2^{(1)}(s',t')}{t'-t} = F(t), \quad (4.4)$$

which tells us that R has to be independent of q^2.

We are now looking for a solution of Eq. (4.3) which meets the requirement (4.1). That means

$$\bar{f}_1^{(1)}(t) = \frac{2(\frac{2}{3})^{1/2}}{[(t-4)(t-4q^2)]^{1/2}}F(t). \quad (4.5)$$

Therefore it is convenient to deal with the once-subtracted dispersion relation. We make a subtraction at $t=0$ and fix the subtraction constant by the constraint (4.5). The solution of this equation has the form[22] (taking proper care of the right-hand-cut singularities)

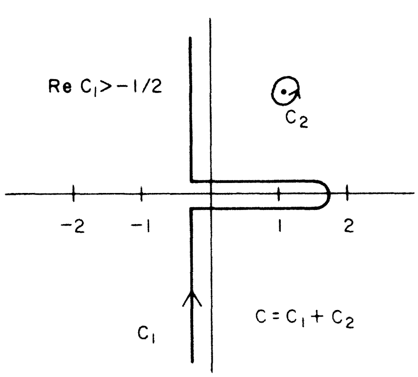

FIG. 2. The contour of integration of the Sommerfeld-Watson transform showing the background and fixed-pole contribution (C_1) and one Regge pole (C_2).

$$\bar{f}_1^{(1)}(t) = +\frac{2(\frac{2}{3})^{1/2}}{(16q^2)^{1/2}}(1-R) + \frac{2(\frac{2}{3})^{1/2}}{[(t-4)(t-4q^2)]^{1/2}}R - \frac{2(\frac{2}{3})^{1/2}}{[(t-4)(t-4q^2)]^{1/2}D_1^{(1)}(t)}\frac{t}{\pi}\int_4^\infty dt'\frac{\mathrm{Im}D_1^{(1)}(t')}{(t'-t)t'}\left(R+\frac{[(t'-4)(t'-4q^2)]^{1/2}}{(16q^2)^{1/2}}(1-R)\right) + \frac{tP(t)}{[(t-4)(t-4q^2)]^{1/2}D_1^{(1)}(t)}. \tag{4.6}$$

Let us first assume $D_1^{(1)}(t) \lesssim \text{const}$. Then we observe that Eq. (4.6) is consistent with Eq. (4.5) if $R=1$ and $P(t)\equiv 0$. In this case the solution can be rewritten[23]

$$\bar{f}_1^{(1)}(t) = \frac{2(\frac{2}{3})^{1/2}}{[(t-4)(t-4q^2)]^{1/2}}\frac{D_1^{(1)}(0)}{D_1^{(1)}(t)}, \tag{4.7}$$

where (in our approximation) $D_1^{(1)}(0)/D_1^{(1)}(t)$ corresponds to the pion electromagnetic form factor. If $D_1^{(1)}(t) \simeq b_1^1 t$ we notice that $R=1$ and $P(t)=2(\frac{2}{3})^{1/2}b_1^1$ is a solution which fulfills Eq. (4.5) (and similarly for any higher power). Here, Eq. (4.6) can again be written in the form[24] (4.7). Now we show that $R=1$ is also a necessary condition. Therefore we consider the limit $q^2\to 0$, $t\neq 0$. Since $\bar{f}_1^{(1)}(t)$ has to be finite at this point, we must have

$$(1-R)\left(1-\frac{1}{[(t-4)t]^{1/2}D_1^{(1)}(t)}\frac{t}{\pi}\int_4^\infty dt'\frac{\mathrm{Im}D_1^{(1)}(t')}{(t'-t)t'}[(t'-4)t']^{1/2}\right)+\frac{t}{[(t-4)t]^{1/2}}\frac{\tilde{P}(t)}{D_1^{(1)}(t)}=0, \tag{4.8}$$

which can only be fulfilled if $R=1$ and $\tilde{P}(t)\equiv 0$. Thus the current-algebra constraint normalizes the single-spectral functions to a fixed value. This looks quite similar to the usual parton model but the parton lines are replaced by the diagram shown in Fig. 1.

In the second part of this section we shall calculate the $l=0$ fixed-pole contribution to the isospin-even amplitudes using the normalization condition of the single-spectral functions as input. That is [by means of Eq. (3.4)]

$$\int ds'\rho_1^{(0)}(s') = -2\int ds'\rho_1^{(2)}(s') = 2. \tag{4.9}$$

The formal manipulations will be much the same as in the $l=1$ case. The dispersion relation for $f_0^{(0,2)}$ again has no left-hand-cut contribution for the same reasons as before (we only have to replace $l-1$ by l in our arguments). If we define $\bar{f}_l^{(0,2)} = i\sqrt{l}f_l^{(0,2)}$ we obtain the integral equation (not allowing for "Kronecker δ" terms)

$$\bar{f}_0^{(0,2)}(t) = -\frac{2\sqrt{2}\,\delta^{(0,2)}}{(t-4)(t-4q^2)} + \frac{1}{\pi}\int_4^\infty dt'\frac{\bar{f}_0^{(0,2)}(t')}{t'-t}\frac{\rho(t')\beta_0^{(0,2)}(t')^*}{\alpha^{(0,2)}(t')^*}, \tag{4.10}$$

where $\delta^{(0)}=2$ and $\delta^{(2)}=-1$. Here are no extra branch cuts involved. We solve Eq. (4.10) for $q^2<1$, which gives[19]

$$\bar{f}_0^{(0,2)}(t) = -\frac{2\sqrt{2}\,\delta^{(0,2)}}{(t-4)(t-4q^2)} + \frac{2\sqrt{2}\,\delta^{(0,2)}}{(t-4)D_0^{(0,2)}(t)}\frac{1}{\pi}\int_4^\infty dt'\frac{\mathrm{Im}D_0^{(0,2)}(t')}{(t'-t)(t'-4q^2)}. \tag{4.11}$$

The polynomial term (i.e., the solution of the homogeneous equation) has been omitted, which corresponds to the view that the photons couple to the pions only as illustrated in Figs. 1 and 3. The polynomial term must be zero if $D_0^{(0,2)} \lesssim \text{const}$, which is the most likely behavior [i.e., no $l=0$, isospin-0, 2 resonance; see Eq. (3.12)], and, in general, it does not affect the fixed-pole residues at $t=q^2=0$ because $d_{20}^0(\cos\theta)/\sin^2\theta \sim t-4q^2$.

For $t=0$ the solution (4.11) can be written in a similar manner as in the $l=1$ case[23,24] giving

$$\bar{f}_0^{(0,2)}(0) = -\frac{\sqrt{2}}{8q^2}\delta^{(0,2)}\frac{\bar{D}_0^{(0,2)}(4q^2)}{\bar{D}_0^{(0,2)}(0)}, \tag{4.12}$$

where (higher powers in t will not be considered in the following)

$$\bar{D}_0^{(0,2)}(t) = \begin{cases} D_0^{(0,2)}(t) \text{ if } D_0^{(0,2)}(t) \lesssim \text{const}, \\ D_0^{(0,2)}(t) - b_0^{0,2}t \text{ if } D_0^{(0,2)}(t) \simeq b_0^{0,2}t. \end{cases}$$

Equation (4.12) now predicts the following fixed poles ($t=0$):

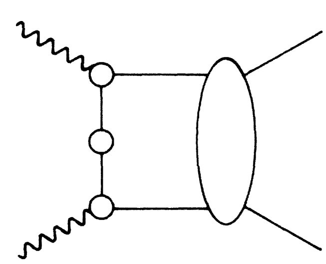

FIG. 3. Diagram contributing to the fixed poles and to the Bjorken limit.

$$A_{2\,\mathrm{FP}}^{(0,2)} \simeq -\frac{\delta^{(0,2)}}{s^2}\frac{\overline{D}_0^{(0,2)}(4q^2)}{D_0^{(0,2)}(0)} \tag{4.13}$$

and [by means of Eq. (2.8)]

$$T_{2\,\mathrm{FP}}^{\pi^\pm} \simeq \frac{q^2}{\nu^2}\frac{4}{3}\left(\frac{\overline{D}_0^{(0)}(4q^2)}{D_0^{(0)}(0)} + \frac{1}{2}\frac{\overline{D}_0^{(2)}(4q^2)}{D_0^{(2)}(0)}\right),$$
$$T_{2\,\mathrm{FP}}^{\pi^0} \simeq \frac{q^2}{\nu^2}\frac{4}{3}\left(\frac{\overline{D}_0^{(0)}(4q^2)}{D_0^{(0)}(0)} - \frac{\overline{D}_0^{(2)}(4q^2)}{D_0^{(2)}(0)}\right). \tag{4.14}$$

The fixed-pole contribution to the amplitude T_1 can be calculated from Eqs. (2.8) and (4.14). The highest right-signature nonsense point of $A_1+q^2A_5$ is $l=-2$ so that T_1 has to cancel the fixed-pole contribution of A_2 [a similar situation has led to Eq. (4.1)]. Thus we have

$$T_{1\,\mathrm{FP}} = -\frac{\nu^2}{q^2}T_{2\,\mathrm{FP}}\,. \tag{4.15}$$

First of all we notice that the fixed poles coincide with the Thomson limit at $q^2=0$ $[\overline{D}_0^{(0,2)}(0)=D_0^{(0,2)}(0)]$, i.e., $T_{1\,\mathrm{FP}}^{\pi^\pm}\simeq -2$ and $T_{1\,\mathrm{FP}}^{\pi^0}\simeq 0$, which agrees with experiment[5] (we do not believe that the inclusion of spin and minor changes in the isospin description will make any difference). Second, we obtain that the residue of the fixed poles is not a simple polynomial in q^2 although it goes to a constant at large negative q^2 as has been argued by several authors.[7,25] The residue has a cut along the positive real axis starting at $q^2=1(=m_\pi{}^2)$, so that the polynomial behavior very likely breaks down near this cut.

We shall now discuss what else our model predicts for small q^2. Therefore we go to the realistic case where $D_l^{(0)}$ contains several Regge poles and where $D_l^{(2)}$ has no resonances (because of the absence of exotics). We furthermore assume $\overline{D}_0^{(0,2)}=D_0^{(0,2)}$, i.e., $D_0^{(0,2)}\lesssim$ const. Then $D_0^{(0)}(4q^2)$ will have a zero where (say) the f trajectory $\alpha_f(4q^2)$ passes through zero (for our following arguments it can be any other trajectory with intercept >0). This is expected to be somewhere between -1 $(\mathrm{GeV}/c)^2 \leq 4q^2 \leq 0$. On the other hand, $D_0^{(2)}(4q^2)$ is very likely a slowly varying function without any zeros in this region. So the residues of $T_{1\,\mathrm{FP}}^{\pi^\pm}$ *and* $T_{2\,\mathrm{FP}}^{\pi^\pm}$ *have a zero for small negative* q^2 if $D_0^{(0)}(4q^2)$ reaches at least half its value at $q^2=0$ after it changes sign.[26] This is exactly what comes out in recent finite-energy sum-rule calculations[27] (again for protons). The residues of $T_{1\,\mathrm{FP}}^{\pi^0}$ and $T_{2\,\mathrm{FP}}^{\pi^0}$ have no further zeros at small q^2 (besides that at $q^2=0$).

The arguments of Cheng and Tung[7] and Cornwall, Corrigan, and Norton[25] leading to polynomial fixed-pole residues fail for various reasons. The conclusion of Cheng and Tung is only valid if, e.g., there are no fixed poles in photoproduction amplitudes, which is a rather doubtful assumption (this will be discussed elsewhere). Cornwall, Corrigan, and Norton, on the other side, make the basic assumption that ImT has no fixed poles. This already anticipates their result and is not justified on the more general grounds of the Deser-Gilbert-Sudarshan representation.[28]

We still have to show that isoscalar currents do not contribute to the $l=0$ fixed poles. It is obvious that the isoscalar currents only enter into the double-spectral functions. This means that the integral Eq. (4.10) will not be altered by the inclusion of these terms. So we can conclude that they do not contribute at all.

V. DEEP-INELASTIC SCATTERING

We shall now discuss the scaling limit. In this limit $q^2\to-\infty$ and $(t=0)$

$$\cos\theta \simeq -\tfrac{1}{2}\omega(q^2)^{1/2}\,. \tag{5.1}$$

Here we consider only the scaling function νW_2 given by (omitting the isospin)

$$\nu W_2 = -\frac{2\nu q^2}{\pi}\,\mathrm{Im}\,A_2\,. \tag{5.2}$$

In order to calculate W_1 and W_L we had to go through the same kind of calculations for A_1 and A_5 as for A_2.

The scaling function νW_2 is made up of the Regge-pole terms and the background integral [Eqs. (3.17)–(3.19)]. We solve the integral equation (3.10) (in this case for general l) as before and then continue the solution to $q^2\to-\infty$. In this limit the left-hand-cut integral vanishes relative to the Born term for a large range of l, which can be verified as follows. We assume that the integrals $\int ds'\int dt'\rho_2^{(0,2)}(s',t')$ and $\int ds'\int du'\rho_3^{(0,2)}(s',u')$ are finite for all $q^2<4$. This should be true in order to fulfill the unitarity bound (3.12). Then, for fixed t we get the asymptotic behavior [Eqs. (3.11), (3.13), and (3.14)]

$$f_{Bl}^{(0,2)} \lesssim \frac{1}{(q^2)^{(l+2)/2}},$$
$$f_{Ll}^{(0,2)} \lesssim \frac{1}{(q^2)^4}\ \text{(within logarithms of } q^2). \tag{5.3}$$

For those values of l which are involved in the Sommerfeld-Watson transform (3.19) (i.e., Rel ≤ 1) $f_{Ll}^{(0,2)}$ decreases at least by a factor of $(q^2)^{-5/2}$

faster than $f_{Bl}^{(0,2)}$. The main reason behind this is that the left-hand branch point moves to $-\infty$ as $q^2 \to -\infty$ $[t_L \simeq -\frac{1}{4}(q^2)^2]$. So we have the same situation as in the parton model. That is, only the "handbag" diagram as shown in Fig. 3 contributes in the scaling limit.[1] The solution has the asymptotic form (again neglecting the possible polynomial term)

$$f_l^{(0,2)}(0) \simeq -(-i)^{-l}\frac{\sqrt{\pi}}{4}\frac{[(l-1)l(l+1)(l+2)]^{1/2}}{(2l-1)(2l+1)}\frac{\Gamma(l-1)}{\Gamma(l-\frac{1}{2})}\frac{\delta^{(0,2)}}{(\sqrt{-q^2})^{l+2}}\left(1-\frac{1}{D_l^{(0,2)}(0)}\frac{1}{\pi}\int dt'\frac{\operatorname{Im}D_l^{(0,2)}(t')}{t'}\right), \tag{5.4}$$

where we have taken the q^2 limit under the integrals. Equation (5.4) reduces to the simple expression

$$f_l^{(0,2)}(0) \simeq -(-i)^{-l}\frac{\sqrt{\pi}}{4}\frac{[(l-1)l(l+1)(l+2)]^{1/2}}{(2l-1)(2l+1)}\frac{\Gamma(l-1)}{\Gamma(l-\frac{1}{2})}\frac{\delta^{(0,2)}}{(\sqrt{-q^2})^{l+2}}\frac{a_l^{0,2}}{D_l^{(0,2)}(0)} \tag{5.5}$$

if we write

$$D_l^{(0,2)}(t) = a_l^{0,2} + b_l^{0,2}t + \frac{1}{\pi}\int dt'\frac{\operatorname{Im}D_l^{(0,2)}(t')}{t'-t}. \tag{5.6}$$

In the following we shall set $a_l^{0,2}=1$ (provided $a_l^{0,2}\neq 0$). This can always be achieved since the normalization of $D_l^{(0,2)}(t)$ is not determined by the integral equation.

The scaling function νW_2 can now be written down explicitly. By using Eq. (5.1) and the asymptotic expansion of Eq. (3.9) we obtain from Eq. (3.19)

$$\nu W_2^{(0,2)} = \tfrac{1}{2}\delta^{(0,2)}\left[\gamma^{(0,2)}\omega^{\alpha^{(0,2)}(0)-1} + \operatorname{Im}\left(\frac{1}{2\pi i}\int_C dl\frac{\omega^{l-1}}{D_l^{(0,2)}(0)\sin\pi l} - (\omega \longleftrightarrow -\omega)\right)\right], \tag{5.7}$$

where

$$\gamma^{(0,2)} = \left[\frac{\partial D_l^{(0,2)}(0)}{\partial l}\right]^{-1}_{l=\alpha^{(0,2)}(0)}. \tag{5.8}$$

The contour of integration is parallel to the imaginary axis with $0 > \operatorname{Re} l > -\frac{1}{2}$. If we assume that there are no Regge poles in $D_l^{(2)}$ this gives for scattering off charged and neutral pions (including several Regge poles in $D_l^{(0)}$)

$$\nu W_2^{\pi^\pm} = -\frac{2}{3}\left\{\sum_i \gamma_i^{(0)}\omega^{\alpha_i^{(0)}(0)-1} + \operatorname{Im}\left[\frac{1}{2\pi i}\int_C dl\frac{\omega^{l-1}}{\sin\pi l}\left(\frac{1}{D_l^{(0)}(0)}+\frac{1}{2D_l^{(2)}(0)}\right) - (\omega \longleftrightarrow -\omega)\right]\right\}, \tag{5.9}$$

$$\nu W_2^{\pi^0} = -\frac{2}{3}\left\{\sum_i \gamma_i^{(0)}\omega^{\alpha_i^{(0)}(0)-1} + \operatorname{Im}\left[\frac{1}{2\pi i}\int_C dl\frac{\omega^{l-1}}{\sin\pi l}\left(\frac{1}{D_l^{(0)}(0)}-\frac{1}{D_l^{(2)}(0)}\right) - (\omega \longleftrightarrow -\omega)\right]\right\}. \tag{5.10}$$

For large ω the background certainly can be neglected, leaving only the familiar Regge terms. However, near threshold, i.e., $\omega \to 1$, the background integral becomes important. It is responsible for the threshold behavior and suggests a new type of Drell-Yan relation.[29] For $\omega = 1$ the contour can be shifted back to the original contour around the positive real axis provided that $[D_l^{(0,2)}(0)]^{-1}$ vanishes at infinity. This cancels the Regge terms and, because the integral along the positive axis is real, gives $\nu W_2(\omega=1)=0$. The same can be done for the derivatives

$$\left(\frac{\partial^n \nu W_2}{\partial\omega^n}\right)_{\omega=1}$$

whenever the integral over the semicircle vanishes. This would give

$$\nu W_2 \underset{\omega\to 1}{\sim} \text{const}(\omega-1)^{n+1}.$$

Because the derivatives get extra factors of l under the integral the highest n is determined by the asymptotic behavior of $D_l^{(0,2)}(0)$. In the case where $D_l^{(0,2)}$ contains only Regge poles and no cuts there is a simple relation between the asymptotic behavior (in l) and the number of Regge poles. This will be discussed in a forthcoming paper.

We shall now show that νW_2 satisfies the Adler sum rule[30]

$$\int_1^\infty \frac{d\omega}{\omega}\nu W_2^{(0,2)} = \tfrac{1}{2}\delta^{(0,2)}\frac{1}{D_1^{(0,2)}(0)} \tag{5.11}$$

if $\alpha^{(0,2)}(0) < 1$. Therefore we consider the background integral and perform the integration [Eq. (5.11)] over ω. Since $\alpha^{(0,2)}(0)<1$ the integral exists and gives

$$-\tfrac{1}{2}\delta^{(0,2)}\operatorname{Im}\left[\frac{1}{2\pi i}\int_C dl\frac{1}{D_l^{(0,2)}(0)\sin\pi l}\frac{1+e^{i\pi l}}{l-1}\right]. \tag{5.12}$$

Now we displace the contour C to the (original) contour along the positive real axis (here $[D_l^{(0,2)}(0)]^{-1}$ has to vanish at infinity again) as before. Then only the pole at $l=1$ contributes to the imaginary part, giving

$$\tfrac{1}{2}\delta^{(0,2)}\frac{1}{2\pi i}\oint_{C_0} dl\frac{1}{D_l^{(0,2)}(0)(l-1)} = \tfrac{1}{2}\delta^{(0,2)}\frac{1}{D_1^{(0,2)}(0)}, \qquad (5.13)$$

where C_0 is a small circle around $l=1$. This proves the sum rule (5.11).

Finally we derive the fixed-pole sum rule

$$\lim_{q^2\to-\infty} T_{1\,\mathrm{FP}}^{(0,2)} = -\lim_{q^2\to-\infty}\frac{\nu^2}{q^2}T_{2\,\mathrm{FP}}^{(0,2)} = 2\left\{\int_1^\infty d\omega\,\nu\overline{W}_2^{(0,2)} - \sum_i \frac{\frac{1}{2}\delta^{(0,2)}}{\alpha_i^{(0,2)}(0)}\gamma_i^{(0,2)}\right\}, \qquad (5.14)$$

where

$$\nu\overline{W}_2^{(0,2)} = \nu W_2^{(0,2)} - \sum_i \tfrac{1}{2}\delta^{(0,2)}\gamma_i^{(0,2)}\omega^{\alpha_i^{(0,2)}(0)-1}. \qquad (5.15)$$

This sum rule has been derived in a slightly different form by Cornwall, Corrigan, and Norton[25] and Brodsky, Close, and Gunion.[1] The integral in Eq. (5.14) exists since $\mathrm{Re}\,l<0$ in the background integral. The integration over ω gives

$$\int_1^\infty d\omega\,\nu\overline{W}_2^{(0,2)} = -\tfrac{1}{2}\delta^{(0,2)}\,\mathrm{Im}\left[\frac{1}{2\pi i}\int_C dl\frac{1}{D_l^{(0,2)}(0)\sin\pi l}\,\frac{1+e^{i\pi l}}{l}\right]. \qquad (5.16)$$

Now the contour can be displaced as before, which gives back the Regge terms and a contribution from the pole at $l=0$. Then the right-hand side of Eq. (5.16) becomes

$$\tfrac{1}{2}\delta^{(0,2)}\left(\sum_i\frac{1}{\alpha_i^{(0,2)}(0)}\gamma_i^{(0,2)} + \frac{1}{D_0^{(0,2)}(0)}\right). \qquad (5.17)$$

This proves the sum rule if we remember that we have chosen the normalization

$$\lim_{q^2\to-\infty}\overline{D}_0^{(0,2)}(4q^2)=1$$

[Eqs. (4.3) and (5.6)].

In the scaling limit isoscalar currents again do not contribute. This follows from the fact that the left-hand-cut integral vanishes, so that the arguments given at the end of the last section hold.

VI. CONCLUSIONS

We have presented a bootstrap approach to Bjorken scaling and $l=0$ fixed poles in virtual Compton amplitudes. The characteristic feature of this approach is that it only deals with physical particles and provides a connection between the Regge and the scaling region. It is striking that scaling follows directly from current algebra.

In the scaling region this approach turns out to be equivalent to the parton model from the point of view of perturbation diagrams[1] (only the "handbag" diagram contributes). The single dispersion integrals of the Mandelstam representation (3.1) take over the role of the parton propagator, i.e.,

$$\int_1^\infty ds'\rho_1(s') = \text{const (independent of } q^2),$$

and lead to the same pointlike structure as the parton model. The scaling function νW_2 is completely determined by the denominator functions D_l of the $\pi\pi$ amplitude (in our approximation). Near threshold (i.e., $\omega\to1$) $\nu W_2 \lesssim \mathrm{const}(\omega-1)^m$ where m is given by the asymptotic behavior (for integer m)

$$D_l(0) \underset{l\to\infty}{\sim} l^m.$$

At large ω, νW_2 has Regge behavior: $\nu W_2 \simeq \gamma\omega^{\alpha-1}$.

The fixed-pole sum rule (5.14) differs from the Cornwall, Corrigan, and Norton[25] and Brodsky, Close, and Gunion[1] sum rules with respect to the fixed-pole residues which have to be taken here at $q^2=-\infty$. The sum rule (5.14) does *not* require a polynomial residue (in q^2) as in the parton model[6] which is indeed not the case.

We found that the $l=0$ fixed-pole residues have a cut in q^2 starting at $q^2=1$. The residues behave like a polynomial for $q^2\to-\infty$. But near the cut the polynomial behavior breaks down. Such a behavior is strongly supported by the Regge-pole analysis of forward virtual Compton scattering.[27] The specific form of the residues is a direct consequence of t-channel unitarity. This need not apply in the parton model (can partons be produced?). If not, the parton model definitely gives a different behavior (e.g., polynomial residues[6]).

At $q^2=0$ the fixed poles coincide with the Thomson limit. This is in excellent agreement with experiment. Further tests of this model will be discussed in a subsequent paper in which this approach is applied to deep-inelastic e^+e^- annihilation.

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[23]This follows from the dispersion relation

$$D_l^{(i)}(t) = D_l^{(i)}(0) + \frac{t}{\pi} \int dt' \, \frac{\mathrm{Im}\, D_l^{(i)}(t')}{(t'-t)t'} \,.$$

[24]In this case the dispersion relation reads

$$D_l^{(i)}(t) = D_l^{(i)}(0) + b_l^i t + \frac{t}{\pi} \int dt' \, \frac{\mathrm{Im}\, D_l^{(i)}(t')}{(t'-t)t'} \,.$$

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