

Composite Theory of Inclusive Scattering at Large Transverse Momenta*

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The bound-state model which was previously used to describe elastic hadronic scattering is applied to the inclusive reactions $A + B \rightarrow C + X$. The force due to the interchange of common constituents is predicted to be the dominant interaction in inclusive reactions at large transverse momenta, just as it is in large-angle exclusive scattering. In the simplest cases the power-law falloff of the hadronic wave function allows us to introduce effective current operators which are the analogs of the lepton-photon current in deep-inelastic electron-proton scattering. Our predictions for the inclusive cross section take the form of a definite power of s times a scale-invariant function of the remaining invariants which can be simply related to the structure function $\nu W_2(x)$ of the target. It is shown that in most cases our model predicts the same s dependence at fixed missing mass for the incoherent background as for the exclusive resonance contribution to the inclusive cross section. Results for several specific hadron- and photon-induced inclusive reactions are given.

INTRODUCTION

The SLAC-MIT deep-inelastic electron scattering experiments have led to renewed emphasis upon composite theories of the hadrons. In a recent paper¹ we investigated the "constituent interchange" interaction between hadrons [illustrated diagrammatically in Fig. 1(a)] which is naturally present in such theories. The essential power-law dependence of these processes, which derives from the power-law behavior of the form factors, allows the interchange force to dominate in the large-energy, large-angle region over coherent, exponentially falling, multiparticle exchange processes. The resulting description of the elastic scattering processes $p + p \rightarrow p + p$ and $\pi + p \rightarrow \pi + p$ in this region is remarkably good. The predicted energy dependence of the differential cross section [see Eq. (3)] and the predicted energy-independent form of the fixed-angle ratio $(d\sigma/dt)/(d\sigma/dt)_{90^\circ}$ are determined by the asymptotic behavior of the composite wave functions. This asymptotic behavior and the spin structure of the important parton constituents are determined in turn by the electromagnetic form factors of the hadrons.

In this note we explore the consequences of the interchange mechanism for inclusive processes $A + B \rightarrow C + X$ where particle C is detected at large transverse momentum ($P_T^2 \equiv tu/s \gg m^2$). We shall show that the same features which were characteristic of exclusive processes are present and relevant as well in the inclusive case. The basic interchange mechanism is illustrated in Fig. 1(b), for the absorptive amplitude.^{2,3} We shall show that the corresponding inclusive cross section can be calculated in terms of deep-inelastic structure functions

of the target, but with the electromagnetic interaction replaced by an effective local operator determined by the form factors of particles A and C . These effective hadronic operators could be used to form the basis of a current algebra (or light-cone algebra) in order to study a new set of generalized structure functions.

In I we introduced a new formalism for calculating the collisions of hadrons considered as bound states.⁴ The use of an infinite-momentum reference frame and old fashioned perturbation theory tremendously simplifies the calculation, although the results are covariant.

The contribution of the interchange topology of Fig. 1(b) to the single-particle inclusive distribution with missing mass \mathfrak{M}^2 ,

$$R(s, t, \mathfrak{M}^2) \equiv s \frac{d\sigma}{d^3P_C/E_C} = \frac{s^2}{\pi} \frac{d\sigma}{dt d\mathfrak{M}^2},$$

can be written as⁵

$$(2\pi)^3 R = \frac{1}{16\pi^2} \int \int_0^1 \frac{d^2k_\perp dx}{x^3(1-x)^3} \psi_A^2(\vec{k}_\perp + (1-x)\vec{q}_\perp - x\vec{r}_\perp) \times \psi_B^2(\vec{k}_\perp) \psi_C^2(\vec{k}_\perp - x\vec{r}_\perp) \Delta^2 \delta(E_i - E_f), \quad (1)$$

where

$$\begin{aligned} \Delta &= E_p + E_{p+a+r} - E_a - E_c - E_b - E_d \\ &= E_{p+r} - E_a - E_d \\ &= m_c^2 - \frac{[(\vec{k}_\perp - x\vec{r}_\perp)^2 + (1-x)m_a^2 + xm_d^2]}{x(1-x)} \end{aligned}$$

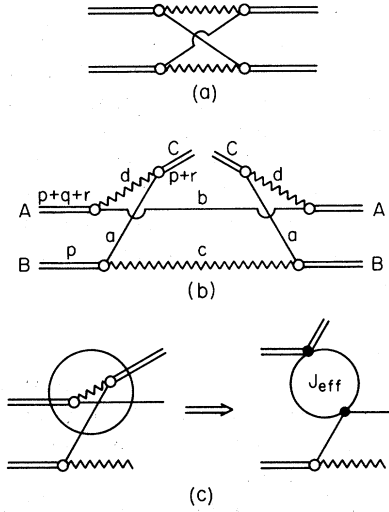


FIG. 1. (a) The interchange topology for exclusive scattering. In the case of proton-proton scattering the solid lines indicate the exchange of common parton constituents (quarks in the quark valence model). The wavy lines represent the remaining cores of the proton (see footnote 2). (b) The interchange topology for inclusive scattering, $A + B \rightarrow C + X$. (c) The reduction of the interchange contribution to an effective "local" current operator J_{eff} which probes a structure function of the target.

and

$$E_i - E_f = \mathfrak{M}^2 + \vec{q}_\perp^2 - \frac{m_b^2 + (\vec{k}_\perp + \vec{q}_\perp)^2}{x} - \frac{m_c^2 + \vec{k}_\perp^2}{1-x}$$

The transverse vectors \vec{r}_\perp and \vec{q}_\perp satisfy $u = -\vec{r}_\perp^2$, $t = -\vec{q}_\perp^2$, and $\mathfrak{M}^2 = m_A^2 + m_B^2 - m_C^2 + 2\vec{r}_\perp \cdot \vec{q}_\perp$. For large t , u , and \mathfrak{M}^2 (with $x_1 = -u/s$, $x_2 = -t/s$, and $\epsilon = \mathfrak{M}^2/s$) there are two regions in the \vec{k}_\perp integral which contribute to the leading asymptotic behavior: $\vec{k}_\perp \sim 0$ and $\vec{k}_\perp + (1-x)\vec{q}_\perp - x\vec{r}_\perp \sim 0$. The latter contribution is obtained from the results we shall give by interchanging \vec{r}_\perp and \vec{q}_\perp and ψ_B and ψ_A . [The interference diagram which corresponds to Fig. 1(b) with p_A and p_C interchanged on one side of the diagram is negligible since only one wave function is evaluated in the nonasymptotic region rather than two for the noninterference terms.]

In general, $\psi_B(\vec{k}_\perp)$ depends on $2p_a \cdot p_B$ and $2p_c \cdot p_B$, which in turn depend on the component \vec{k}_\perp of parton a 's momentum transverse to the momentum of the bound state. In our model ψ_B is taken as a function of the single variable

$$S_B \equiv 2p_B \cdot (p_a + p_c) - m_B^2 \\ = [\vec{k}_\perp^2 + (1-x)m_a^2 + xm_c^2]/x(1-x) \quad (2)$$

for large S_B . In that region $\psi_B = (S_B - M_B^2)^{-1} \phi_B(S_B) \sim S_B^{-1/2}$ for scalar particles. This dependence upon

S_B guarantees the validity of the Drell-Yan relation⁶ between the threshold dependence of the structure function $F_{2B}(x) \sim (1-x)^{2/3B-1}$ and the power falloff of particle B 's electromagnetic form factor $F_B(t) \sim t^{-1/3B}$. This form for ψ then leads to unique predictions for high-energy large-angle cross sections in exclusive processes (see I). For instance, we predicted for $s > 20 \text{ GeV}^2$, $|t|, |u| > 4 \text{ GeV}^2$ that

$$(d\sigma/dt)_{pp} \propto (s \sin \theta_{\text{c.m.}})^{-12} \quad (3)$$

in good agreement with experiment.

Here we shall see that similar results arise in the case of inclusive processes. The asymptotic limit of Eq. (1) is

$$(2\pi)^3 R = \frac{\pi}{s} \int_0^1 \frac{dx}{x^2(1-x)^2} \psi_A^2(x\vec{r}_\perp) \psi_C^2(x\vec{r}_\perp) \left[\frac{x^2 \vec{r}_\perp^2}{x(1-x)} \right]^2 \\ \times \frac{F_{2B}(x)}{\lambda^2(1-x_1)} \delta\left(x - \frac{x_2}{1-x_1}\right). \quad (4)$$

We have identified the target's structure function by making the replacement

$$\frac{1}{(2\pi)^3} \lambda^2 \int \frac{d^2\vec{k}_\perp}{2(1-x)} \psi_B^2(\vec{k}_\perp) \rightarrow F_{2B}(x) = \nu W_{2B} \\ (x = 1/\omega), \quad (5)$$

where λ is the charge of parton a . As in deep-inelastic scattering, one must sum over the contributing partons in this and in all our formulas.

The occurrence of the full structure function in Eq. (5) can be seen to be correct by comparing the corresponding calculation of inclusive electron-proton scattering. The spectrum of masses in the lower core c of Fig. 2(a) allows Regge behavior in the forward antiparton \bar{a} - B amplitude⁷ and this in turn leads to Regge behavior in F_{2B} at small x , $F_{2B} \sim x^{1-\alpha}$. However, for $x \sim 1$ (which requires large S_B), only the simple wave-function expression in Eq. (5) is required. It must be noted that in some processes there is also a contribution to R from the alternate core routing of Fig. 2(b). This contribution may be computed from the asymptotic wave-function dependence, and it is not related to any structure function.⁸

Equation (4) represents a new type of scaling result valid outside the Regge region and where P_T^2 satisfies the following inequalities:

$$P_T^2 \gg \mu^2 \mathfrak{M}^2/s, \mu^2(s+u)^2/s|u|, \mu^2(s+t)^2/s|t|, \quad (6)$$

which we shall call the "full scaling" region to differentiate it from Feynman scaling. μ is a mass characteristic of the bound state which controls the onset of scaling in νW_2 . The asymptotic result can be written in the very simple fashion

$$R \propto \mathcal{O}_{\text{eff}} F_{2B}(x_2/1-x_1)/\lambda^2 + (A \leftrightarrow B, x_1 \leftrightarrow x_2), \quad (7)$$

where

$$\mathcal{O}_{\text{eff}} = s(1-x_1)^3 x_1^2 \epsilon^{-4} F_A^2(sx_1x_2/\epsilon) F_C^2(sx_1x_2/\epsilon) T, \quad (8)$$

with

$$F_A^2 F_C^2 = [\epsilon/x_1x_2s]^{2(l_A+l_C)}$$

and

$$\epsilon = 1 - x_1 - x_2.$$

The effective operator \mathcal{O}_{eff} plays the role of the current commutator in deep-inelastic scattering. It can be considered as arising from the two currents J_{eff} which bring in a large momentum \vec{q}_\perp as shown in Fig. 1(c). This makes it clear that our results can be rephrased in the language of light-cone algebra. (The factor T , which is unity in the present spinless case, contains the net effects of spin and will be given for specific processes later.) The operator \mathcal{O}_{eff} , however, depends on s , x_1 , x_2 , and ϵ in a fashion dictated by the electromagnetic form factors of particles A and C .

Two regions are of particular interest when examining Eq. (7):

(i) The "scaling" Regge region which here corresponds to $x_2/(1-x_1) \ll 1$ or $x_1/(1-x_2) \ll 1$. This is analogous to the $\omega \rightarrow \infty$, $x \rightarrow 0$ region of deep-inelastic scattering where $F_{2B}(x) \sim x^{1-\alpha}$. As in the case of νW_2 this region reflects the Regge behavior of the underlying parton-proton amplitude.

(ii) The threshold region, $\epsilon = \mathfrak{M}^2/s \rightarrow 0$, where

$$F_{2B}\left(\frac{x_2}{1-x_1}\right) \rightarrow \left(\frac{\epsilon}{1-x_1}\right)^{2l_B-1}.$$

There are two ways of reaching a given missing mass \mathfrak{M}^2 , for large s . The first way, which is unreliable, is to take $\epsilon = \mathfrak{M}^2/s$ in the full scaling result Eq. (7). The alternative, and always correct, method is to perform an exact evaluation of Eq. (1). It is important to ask, then, how the power of s at fixed \mathfrak{M}^2 arising from an exact calculation of the above incoherent continuum contribution compares with that resulting from the corresponding exclusive, or resonance, contribution. In nearly all cases the exclusive and incoherent contributions will behave in the same fashion: The magnitude of the resonances will fall at the same rate as that of the background. This is because the δ function of Eq. (1) for fixed \mathfrak{M}^2 requires $\vec{k}_\perp \sim \vec{q}_\perp \sim \sqrt{s}$. Since the falloff in \vec{k}_\perp^2 and the dependence for $x \sim 1$ of ψ (which controls the ϵ power in the scaling region) are correlated, the results are the same. In particular, in the case of inelastic electron scattering, the results of the parton model are completely consistent with the type of duality noticed by

Bloom and Gilman in the resonance region.⁹ That is, $\nu W_2 \sim (1/Q^2)^3$ for both resonance and incoherent contributions. In addition, in the νW_2 case it happens that taking $\epsilon \sim \mathfrak{M}^2/s$ in the scaling answer also gives the correct s dependence for fixed \mathfrak{M}^2 . This is true for νW_2 only if the dependence of the wave function on $1-x$ and \vec{k}_\perp^2 is correlated for large $\vec{k}_\perp^2/(1-x)$. The correct correlation between these variables is present when the wave function depends asymptotically on S_B .

SPECIFIC PREDICTIONS

In I we noted that the spins of the constituents have an important effect upon the angular behavior of the exclusive amplitudes. The same will be true of the inclusive processes.

(a) $p + p \rightarrow p + X$. In paper I we showed that a spin-1 core and a spin- $\frac{1}{2}$ charged parton were required in order to obtain the scaling law for the electromagnetic form factors, G_E and G_M of the proton. Dipole form factors result if $\psi(S) \sim S^{-3}$.¹⁰ This choice leads to the observed angular dependence of elastic pp scattering. Using the same spin assignments here we obtain a contribution of the form given in Eqs. (7) and (8) with

$$T = x_2^2(1-x_1)^{-2}, \quad l_A = l_C \cong 2. \quad (9)$$

In addition we must include the contribution of Fig. 2(b) in which the roles of the partons and cores are interchanged. Also, for both Figs. 2(a) and 2(b), the alternate region of \vec{k}_\perp [$\vec{k}_\perp \sim x\vec{r}_\perp - (1-x)\vec{q}_\perp$] gives contributions which may be obtained by interchanging $t \leftrightarrow u$ in the expressions derived for the first region of \vec{k}_\perp . The net result is

$$R \propto s^{-7} x_1^{-6} x_2^{-6} (1-x_1) \epsilon^4 F_{2p}(x_2/1-x_1)/\lambda^2 + (x_1 \leftrightarrow x_2) \\ + s^{-7} x_1^{-6} x_2^{-4} (1-x_1)^{-7} \epsilon^{10} N_{\psi^2}(x_2/1-x_1) + (x_1 \leftrightarrow x_2), \quad (10)$$

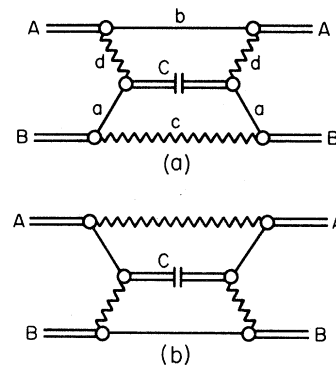


FIG. 2. (a) Normal core routing of the interchange theory. Wavy lines denote the cores. (b) Alternate core routing.

where

$$N_{\psi 2}(x) = \int \frac{d^2 k_{\perp} |\psi(\vec{k}_{\perp})|^2}{x^6 (1-x)^6} \times \left[(1-x)^2 - \frac{\vec{k}_{\perp}^2 x(1-x)}{\Lambda^2} + \frac{\vec{k}_{\perp}^4 - x^2 \Lambda^2 \vec{k}_{\perp}^2}{4\Lambda^4} \right] \quad (11)$$

is a slowly varying function of x , and Λ is the effective mass of the spin-1 core.

The above formulas, which are valid in the full scaling region, should not be used for small \mathfrak{M}^2/s . For fixed \mathfrak{M}^2 the exact evaluation of the incoherent inclusive contribution leads to $R \sim s^{-10}$, which, in fact, has the same s dependence as that predicted for individual exclusive resonance contributions such as $p+p \rightarrow p+N^*$. Note that the reactions $p+p \rightarrow \gamma+X$, $p+p \rightarrow \pi+X$, and $p+p \rightarrow \bar{p}+X$ require anti-quark constituents in the proton and hence would be suppressed in the simplest quark model. Such reactions require higher exotic core states which may have stronger wave-function damping in S and will certainly have a smaller overlap with the physical proton. Thus it is of considerable interest to study such "first-order forbidden" processes and determine this sector of the hadronic wave function.

The above predictions for $p+p \rightarrow p+X$ may be compared with those obtained from the application of Wu-Yang or vector-gluon ideas.^{11,3} If an elementary vector-gluon exchange is present, then the graph topology enables one to avoid any wave-function suppression.² As a result, the final answer can be written in terms of the structure functions of particles A , B , and C . One obtains the scale-invariant answer for all hadrons

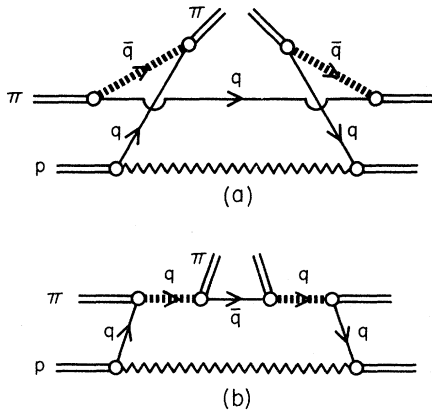


FIG. 3. The ut topology diagram (planar when drawn in the ut channel); the pion releases a core with anti-quark quantum numbers. (b) The st topology diagram; the pion releases a core with quark quantum numbers.

$$R \sim s^{-1} \text{ (elementary vector exchange),} \quad (12)$$

whereas we find

$$R \sim \begin{cases} s^{-7}, & pp \rightarrow pX \\ s^{-3}, & \pi p \rightarrow \pi X \end{cases} \text{ (parton interchange)} \quad (13)$$

in the full scaling region. However, if vector-gluon exchange were present, it would have been expected to dominate elastic pp scattering. The success of the interchange description of elastic scattering argues against a strong coupling for such a particle. There will be, of course, such corrections of order α^2 from photon exchange.

(b) $\pi+p \rightarrow \pi+X$. The diagrams corresponding to parton interchange for this process are illustrated in Fig. 3. In the case of the π we take both the parton and core to have spin $\frac{1}{2}$. The presence of anti-quark quantum numbers in the pion allows the st topology of Fig. 3(b), while preventing any alternate nucleon core routing. (Interference diagrams can be neglected.) The $x_1 \rightarrow x_2$, $p \rightarrow \pi$ contribution is suppressed due to the stronger falloff of the proton wave function relative to that of the pion. The amplitude for Fig. 3(b) is related to that of 3(a) by

$$s \rightarrow u \text{ (or } x_1 \rightarrow 1/x_1, x_2 \rightarrow -x_2/x_1, \epsilon \rightarrow -\epsilon/x_1).$$

The result including spin effects for the ut topology of Fig. 3(a) is

$$R_{ut} \propto s^{-3} x_1^{-2} x_2^{-4} (1-x_1)^3 F_{2p}(x_2/1-x_1)/\lambda^2. \quad (14)$$

Using the simplest quark counting we find the following results:

$$R_+ \equiv \frac{1}{\pi} \frac{d\sigma(\pi^+ p \rightarrow \pi^+ X)}{dx_1 dx_2} = 2R_{ut} + R_{st}, \quad (15)$$

$$R_- \equiv \frac{1}{\pi} \frac{d\sigma(\pi^- p \rightarrow \pi^- X)}{dx_1 dx_2} = R_{ut} + 2R_{st}$$

and that $\pi^+ + p \rightarrow \pi^+ + X$ is suppressed.

Because of the difference in quantum numbers of the proton core and pion core, only the basic routing of Fig. 2(a) is present. Thus the full result is given directly in terms of the structure function of the proton. Because of this, and because of the predicted slow dependence on s , the pion-induced inclusive reactions should be particularly clean tests of this theory.

As in the case of deep-inelastic proton scattering, taking fixed \mathfrak{M}^2 in the full scaling result for this process gives the same $R \sim s^{-6}$ behavior as the exact calculation. This is the same s dependence predicted for exclusive $\pi+p \rightarrow \pi+p$ and $\pi+p \rightarrow \pi+N^*$ processes.

(c) $\pi+p \rightarrow p+X$. Predictions for this inclusive reaction, where the proton is detected at large transverse momentum, are obtained in a similar manner. In general two topologies, us and ut con-

tribute. Because of the core routing, the results depend only on the composite wave function and are not related to the structure function of the proton. We obtain

$$R_{ut}(\pi p \rightarrow pX) \propto s^{-5} x_1^{-4} x_2^{-2} (1-x_1)^{-6} \epsilon^7 \tilde{N}_{\psi^2}(x_2/1-x_1), \quad (16)$$

where $\tilde{N}_{\psi^2}(x)$ is a smooth, slowly-varying function of x , and

$$R_{us}(\pi p \rightarrow pX) = -R_{ut}(x_1 \rightarrow -x_1/x_2, x_2 \rightarrow 1/x_2, \epsilon \rightarrow -\epsilon/x_2). \quad (17)$$

Using simple quark counting, one finds

$$\begin{aligned} R(\pi^+ p \rightarrow pX) &= 2R_{ut} + R_{us}, \\ R(\pi^- p \rightarrow pX) &= R_{ut} + 2R_{us}. \end{aligned} \quad (18)$$

There is one further diagram with the same s dependence in which the pion is coupled to a bare-parton pair and the cores of two protons are present in the final state X . We expect the overlap of the pion with the bare-parton pair to be relatively small, and that this contribution will be suppressed relative to the other contributing diagrams.

It is noteworthy that the incoherent inclusive contribution to the $\pi + p \rightarrow p + X$ cross section at fixed \mathfrak{M}^2 is predicted to decrease faster than the exclusive contribution due to meson resonance production assuming that the mesons have pionlike t^{-1} form factors. This would be a very striking effect in the data if it is not masked by the coherent background.

We turn next to inclusive processes involving photons.

(d) $\gamma + p \rightarrow \pi + X$. The pointlike coupling of the photon to the partons leads to operators Θ_{eff} which fall off quite slowly in s . For this process we define, for the contribution of the parton a ,

$$R_{ut}^{(a)} \propto s^{-2} x_1^{-1} x_2^{-3} (1-x_1)^3 \epsilon^{-1} F_{2p}^{(a)}(x_2/1-x_1) / \lambda_{(a)}^2, \quad (19)$$

$$R_{st}^{(a)} = -R_{ut}^{(a)}(x_1 \rightarrow 1/x_1, x_2 \rightarrow -x_2/x_1, \epsilon \rightarrow -\epsilon/x_1), \quad (20)$$

where

$$\sum_a F_{2p}^{(a)}(x) = \nu W_{2p}(x) = F_{2p}(x) \quad (21)$$

is the inelastic proton structure function. Using quark assignments (p), (pn), etc. for the partons and core we obtain

$$\begin{aligned} R(\gamma p \rightarrow \pi^+ X) &= \frac{1}{9} R_{ut}^{(p)} + \frac{4}{9} R_{st}^{(p)}, \\ R(\gamma p \rightarrow \pi^- X) &= \frac{4}{9} R_{ut}^{(n)} + \frac{1}{9} R_{st}^{(n)}, \\ R(\gamma p \rightarrow \pi^0 X) &= \frac{4}{9} (R_{ut}^{(p)} + R_{st}^{(p)}) + \frac{1}{9} (R_{ut}^{(n)} + R_{st}^{(n)}). \end{aligned} \quad (22)$$

Thus the result for neutral-pion photoproduction is directly proportional to νW_{2p} since the charge factors enter in the same way. The process $\gamma + p$

$\rightarrow p + X$ may be calculated in a fashion analogous to that used for $\pi + p \rightarrow p + X$. Our prediction is that $R \sim s^{-4}$.

(e) $\gamma + p \rightarrow \gamma + X$. In the case of deep-inelastic Compton scattering, the parton interchange model is identical to the parton model of Bjorken and Paschos,¹² but with the predicted region of validity given by Eq. (6). The result in our notation is

$$\begin{aligned} R(\gamma p \rightarrow \gamma X) &= 2\alpha^2 s^{-1} x_1^{-1} x_2^{-2} (1-x_1)(1+x_1^2) \\ &\quad \times \sum_a \lambda_a^2 F_{2p}^{(a)}(x_2/1-x_1). \end{aligned} \quad (23)$$

The Bjorken-Paschos process, which involves simple Compton scattering on the bare-parton constituents of the proton, dominates over processes involving more complicated constituent states of the photon.¹³ The latter processes are suppressed in the full scaling region due to the additional wave-function falloff which is present in our theory for multiparticle emission from the photon. Note that the decay background due to π^0 production in the deep-inelastic Compton scattering is predicted to fall faster by a factor of s in the scaling region. [See Eqs. (22) and (19).]

CONCLUSIONS

The essential element of our theory is that large transverse momentum processes occur at such small distances that only the free-propagation properties of the simplest constituents are involved. The dominant interaction is thus the interchange of the common constituents of the hadrons. During the interchange process, the binding interactions may be neglected, so that the scattering amplitude is determined purely in terms of the wave functions describing the breakup of hadrons into their constituents. The asymptotic structure of the wave function is determined by the electromagnetic form factors, and the resulting predictions appear to be in good agreement with the exclusive scattering data. The power-law nature of this behavior allows us to define effective current operators which, like the lepton-photon current in deep-inelastic electron-proton scattering, bring in a large transverse momentum $q = p_A - p_C$ [see Fig. 1(c)]. This allows us to write inclusive cross sections in the full scaling limit in terms of generalized proton structure functions. Inevitably hadronic radiation processes will smear these predictions just as radiative corrections smear deep-inelastic $e-p$ scattering. The photon-induced inclusive reactions should be least affected by these soft-bremsstrahlung processes (see Ref. 2).

Our result for a given inclusive cross section takes the form of a definite power of s times a scale-invariant function of the remaining kinematic invariants. One particular feature of the predic-

tions is the strong correlation between the power of the s falloff and the angular behavior. The expected power decrease associated with photon processes relative to their vector-meson analogs is in sharp contrast with theories of the electromagnetic interaction based on vector dominance. The interchange theory also justifies the Bjorken-Paschos prediction for deep-inelastic Compton scattering in the full scaling region, Eq. (6).

In a sense, the new operators J_{eff} which are defined for inclusive scattering processes at high transverse momentum may be added to the electromagnetic and weak currents as probes of hadronic structure. The various inclusive reactions such as $\pi + p \rightarrow \pi + X$ probe different contributions to the instantaneous structure of the nucleon. If the theory is correct, it thus becomes possible to determine the composite wave functions of the hadron, and to determine the quantum numbers and spins of their constituents. A study of processes which are forbidden for a particular choice of the parton quantum numbers is particularly important.

We have also shown that, in general, our parton bound-state model predicts the same s dependence

at fixed missing mass for the incoherent background as for the exclusive-resonance contribution to the inclusive cross section. Thus there is no conflict with Bloom-Gilman duality in which resonance and background contributions decrease at the same rate. An exceptional case, which should be studied experimentally, is $\pi + p \rightarrow p + X$, for which the resonances are expected to fall at a slower rate than the continuum contribution.

The interchange theory for reactions at large transverse momenta is perhaps the simplest imaginable since the binding forces need never be specified. It is very important to check this theory for inclusive reactions in the full scaling region of large transverse momentum. Since the common expectation has been that hadronic reactions fall exponentially in p_{\perp} , it will be very exciting to see the emergence of the underlying interchange contributions in inelastic processes.

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²As in paper I we treat the proton as being composed of a parton and a core (representing the combined effects of all the other constituents). This is justified for large-angle exclusive processes if the parton interaction vertices have power-law falloff. In this case the most favorable configuration involves a single hard vertex. The situation is somewhat different in the case of inclusive scattering where there is the additional possibility of soft bremsstrahlung emission from both the virtual cores and the external hadrons. (These contributions are suppressed in the exclusive case as the soft emission of an initial hadron must be reabsorbed by one of the final particles.) Inclusive processes of the form $A + B \rightarrow C^* + X$, where $C^* \rightarrow C + X$, C being the observed particle, have also been considered. Neither of these processes change the s dependence predicted for an inclusive process though they will smear the simple results given in the text. Just as unfolding of the radiative corrections to deep-inelastic $e-p$ scattering is necessary to uncover the fundamental electromagnetic currents, it would be necessary to follow a similar procedure here. In both cases, however, the *target* structure function contains, by definition, the effects of its own radiation. In the case of photon-induced reactions, for which only the bare parton-photon coupling is important at large P_T , bremsstrahlung problems for the projectile and its core are eliminated. In addition, the requirement that the final hadron be unaccompanied suppresses its bremsstrahlung and provides the cleanest test of the theory.

In all cases, this smearing effect is eliminated at small ϵ . In the case of deep-inelastic $e-p$ scattering, one knows that near threshold, $x \sim 1$, bremsstrahlung emission from the target B and its core is suppressed. The leading threshold dependence results from the simplest two-particle wave function. Similarly, in the inclusive case, one can suppress soft radiation from both A and C by requiring that ϵ be small. This is the threshold region for A and C .

The mechanism described here for the production of C should be contrasted with that for the production of hadrons in the inclusive state X , in which the hadronic distribution is not specified in any way, and thus has no form factor suppression. The conversion process of the state X into physical hadrons is the same as that in deep-inelastic scattering of electrons to which we relate all our results.

The role of the topology of the interchange diagrams is crucial for the above contrast, since it is not possible to create a triple inclusive process for A , B , and C without having a disconnected graph. This can be contrasted with the situation when photon or vector-gluon exchange is present. The topology of such exchange graphs allows sufficient freedom in the loop momenta that the arguments of none of the wave functions need be asymptotic. The result can be expressed as a triple inclusive expression in terms of F_{2A} , F_{2B} , F_{2C} such as that given by Berman, Bjorken, and Kogut, Ref. 3 (Eq. 3.56).

³S. M. Berman, J. D. Bjorken, and J. B. Kogut, Phys. Rev. D **4**, 3388 (1971). See also S. M. Berman and M. Jacob, Phys. Rev. Letters **25**, 1683 (1970).

⁴In a separate paper, we shall give a derivation of this formalism based on a multichannel Hamiltonian

approach to the problem of computing the S matrix in the presence of bound states. The types of wave functions used in this and the previous letter arise naturally in this formalism.

⁵The infinite-momentum frame used is defined by

$$p_B = p = \left(P + \frac{m_B^2}{2P}, \vec{0}_\perp, P \right),$$

$$q = \left(\frac{p \cdot q}{P}, \vec{q}_\perp, 0 \right),$$

$$r = \left(\frac{p \cdot r}{P}, \vec{r}_\perp, 0 \right),$$

$$p_a = \left(xP + \frac{\vec{k}_\perp^2 + m_a^2}{2xP}, \vec{k}_\perp, xP \right).$$

The other vectors follow from 3-momentum conservation.

⁶S. D. Drell and T.-M. Yan, *Phys. Rev. Letters* **24**, 181 (1970).

⁷P. Landshoff, J. C. Polkinghorne, and R. Short, *Nucl. Phys.* **B28**, 225 (1971). See also S. J. Brodsky, F. E. Close, and J. F. Gunion, *Phys. Rev. D* **5**, 1384 (1972).

⁸We assume that the parton- B amplitude at large momentum transfer reduces to a product of the two wave functions describing the breakup of the particle B into its parton and core constituents, just as in the form-

factor calculations. We also make this assumption in the case of the forward amplitude when the relative momentum of the parton and core is large — this corresponds to the assumption of a strong off-shell damping of the coherent Regge terms. (See Ref. 7). It is due to this last assumption that the contribution of the alternate core routing of Fig. 2(b) is not given in terms of a structure function, but rather is related only to the simple breakup wave functions.

⁹E. D. Bloom and F. J. Gilman, *Phys. Rev. Letters* **25**, 1140 (1970).

¹⁰As in the exclusive case, one should use $l_p = 2$ only when the wave-function arguments are $\gtrsim 10 \text{ GeV}^2$, as it is only there that the form factor falls as t^{-2} . Between 5 and 10 GeV^2 , $t^{-1.7}$ is a better description of the form-factor data, and the results given should be modified accordingly.

¹¹T. T. Wu and C. N. Yang, *Phys. Rev.* **137**, B708 (1965); H. D. I. Abarbanel, S. D. Drell, and F. J. Gilman, *ibid.* **177**, 2458 (1969); J. V. Allaby *et al.*, CERN Report No. 70-12 (unpublished). J. Kogut and L. Susskind, IAS report, 1972 (unpublished).

¹²J. D. Bjorken and E. A. Paschos, *Phys. Rev.* **185**, 1975 (1969).

¹³S. J. Brodsky and D. P. Roy, *Phys. Rev. D* **3**, 2914 (1971); M. Bander, University of California, Irvine report (unpublished).

Multiparticle Spectra in Simple Theoretical Models: A Cluster-Decomposition Analysis*

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We present a detailed investigation of inclusive and exclusive multiparticle spectra in several simple models of high-energy hadronic interactions. To clarify the origins of particular structure in these spectra we divide the study into two sections. First, we discuss models designed to isolate the features arising from kinematic constraints. Specifically, we treat both pure longitudinal phase space and a longitudinal phase space modified to produce a "leading-particle" effect. In the second section we consider examples of specific dynamical models. Here we investigate the simplified Chew-Pignotti model, in both the strong-ordered and general versions, and a " φ^3 -type" multiperipheral model. Our analysis is based on a cluster-decomposition approach analogous to that used in statistical mechanics. In particular, we apply a set of "cluster-function sum rules" to relate the correlations found in inclusive spectra to those observed in exclusive processes. Within the context of the models, we establish that these sum rules provide both an attractive qualitative picture of correlations in multiparticle spectra and a useful quantitative framework in which to calculate these correlations. We conclude with a discussion of the possible extensions of these cluster techniques to phenomenological analyses of high-energy interactions.

I. INTRODUCTION

Attempts to describe the behavior of strong interactions at very high energy have recently stimulated considerable interest in multiparticle produc-

tion reactions. The obvious complexities involved in analyzing the many-body final states, however, have led theorists to seek first a general conceptual framework in which to discuss the gross features of these reactions and experimentalists to