Dual model with Mandelstam analyticity for deep-inelastic electroproduction and annihilation*

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We present a dual model for virtual Compton amplitudes which satisfies the current-algebra constraints and exhibits Mandelstam analyticity. Mandelstam analyticity is crucial for making any contact to the deep-inelastic annihilation region, i.e., s, q^2, $q'^2 \to +\infty$, and is accomplished by introducing nonlinear trajectories. The scaling properties of this model are mainly shaped by the current-algebra constraints. In terms of dual variables, the scaling limit occurs from the same configuration as the current-algebra fixed pole. We deal with two different choices of the trajectory. One gives back all the nice properties of the original Veneziano model. The other is inspired by the experimental finding of power behavior in large-angle scattering and its theoretical elucidation, the interchange model which requires asymptotically constant trajectories. The first choice leads to a light-cone representation and exhibits a new relation between the large-x behavior of the deep-inelastic annihilation structure function and the asymptotic behavior of the electromagnetic $(2^+) \to (1^-)$ transition form factor.

I. INTRODUCTION

Since Bloom and Gilman[1] observed that the structure function $\nu W_2(\nu, q^2)$ for inelastic electron-proton scattering is dual in the sense that the scaling limit mediates the resonance region, it has been a constant task to construct dual current amplitudes in respect to the deep-inelastic phenomena.[2-6]

The most direct approach to include currents in dual hadronic amplitudes is to employ the minimal electromagnetic coupling principle in the dual operator formalism,[7,8] as in ordinary field theory, which, however, leads to certain nondual features.[9] Another approach in the dual operator formalism stresses the factorization aspect in the construction of off-shell photon amplitudes.[10] In all these models the spectrum is not realistic and up to now current algebra and low-energy theorems are not fulfilled.

A second and more phenomenological approach is based on suitable modifications of the (known) hadronic n-point functions of the generalized Veneziano model.[2] In the most veritable of these models (in our opinion) the (off-mass-shell) currents are constructed from pairs of fictitious particles ("spurions") corresponding, e.g., to a six-point function for Compton scattering. The trajectories in mixed ("spurion"-hadron) channels are set constant in order that the amplitude does not depend on the corresponding channel energy.[11,12] These constant trajectories give rise, of course, to fixed poles in addition to Regge behavior in all channels. But we shall see that fixed poles are indeed required by current algebra and power behavior of the electromagnetic elastic and transition form factors.

This second approach is, however, not without its difficulties. Current algebra is generally not satisfied in these models and any attempts so far made to fulfill the current-algebra constraints seem to have no general validity.[13] Furthermore, these models are far from being convincing for the discussion of deep-inelastic electron scattering and annihilation as they lack Mandelstam analyticity. This is common to all Veneziano-type models with linearly rising trajectories[14] and means that the structure functions can only be explored in a limited kinematical region. There is, e.g., little hope to get sense out of these models in the domain of deep-inelastic electron-positron annihilation, which will become accessible experimentally with the advent of the new generation of electron-positron storage rings, SPEAR and DORIS. But even in the deep-inelastic scattering region some of these models have very peculiar effects. The deep-inelastic scattering structure function derived by Landshoff and Polkinghorne,[3] who managed to fulfill all the current-algebra constraints, does not, e.g., have the proper Regge limit and violates the Drell-Yan relation.[15]

In this paper we shall present a dual model of the second category, but with Mandelstam analyticity built in right from the beginning. This is accomplished by introducing nonlinear trajectories,[14] which is necessary in order to make contact with the deep-inelastic annihilation region, i.e., $s, q^2, q'^2 \to +\infty$. The current-algebra constraints will play an important role in this discussion. As we shall see, current algebra shapes the scaling properties of this model to a large extent.[16]

In order to avoid spin complications we shall restrict our discussion to a pion target. We believe that our conclusions can be carried over to the spin-$\frac{1}{2}$ case without serious difficulties.

The paper is organized as follows. In Sec. II and the Appendix we discuss the basic properties of pion Compton amplitudes. In particular, we give an explicit construction of the invariant amplitudes fulfilling the current-algebra constraints. One of our results is that the divergence condition can be satisfied by any model amplitude in a very simple (and generalizable) way. In Sec. III we give a brief review of dual amplitudes satisfying Mandelstam analyticity and consider two extreme *Ansätze* for the trajectory and the residue function which will lead us to two models for the Compton scattering amplitude in Sec. IV. Both models have nice analytic properties and there is good reason to believe (at least for the first model) that we have obtained a realistic solution to the problem of constructing dual Compton amplitudes. After having discussed the scaling properties of these models, we give a brief discussion of our results and some concluding remarks in Sec. V.

II. COMPTON AMPLITUDES

We consider Compton scattering of neutral or charged isovector photons off pions:

$$\gamma_\mu^\alpha(q)+\pi^\rho(p)\to\gamma_\nu^\beta(q')+\pi^\rho(p')\ . \tag{2.1}$$

Isoscalar photons are not taken into account for the moment but will be discussed later. The scattering amplitude is given by

$$T_{\mu\nu}^{\alpha\beta,\rho}=i\int d^4x\, e^{iq'x}\,\theta(x)\times\langle\pi^\rho(p')\,|\,[\,j_\nu^\beta(x),j_\mu^\alpha(0)]\,|\,\pi^\rho(p)\rangle\ . \tag{2.2}$$

It may be expanded in terms of t-channel isospin amplitudes[17,18] (the upper index labeling the isospin),

$$\begin{aligned}T_{\mu\nu}^{(0)}&=2[C_{\mu\nu}(s,t)+C_{\mu\nu}(u,t)]-N_{\mu\nu}\ ,\\ T_{\mu\nu}^{(1)}&=C_{\mu\nu}(s,t)-C_{\mu\nu}(u,t)\ ,\\ T_{\mu\nu}^{(2)}&=-[C_{\mu\nu}(s,t)+C_{\mu\nu}(u,t)]+2N_{\mu\nu}\ ,\end{aligned} \tag{2.3}$$

where $(\Delta=p+p')$

$$\begin{aligned}C_{\mu\nu}(s,t)&=T_{\mu\nu}^{-+,+}\\ &=\Delta_\mu\Delta_\nu A(s,t)+q'_\mu q_\nu B(s,t)+\cdots\ ,\end{aligned} \tag{2.4}$$

i.e., the amplitude corresponding to charged Compton scattering off π^+, and

$$\begin{aligned}N_{\mu\nu}&=T_{\mu\nu}^{00,0}\\ &=\Delta_\mu\Delta_\nu\bar{A}(s,t)+q'_\mu q_\nu\bar{B}(s,t)+\cdots\ ,\end{aligned} \tag{2.5}$$

representing neutral Compton scattering off π^0.

The hypothesis of conserved vector currents and of current algebra requires $T_{\mu\nu}^{(i)}$ to have the following properties:

$$\begin{aligned}&q^\mu T_{\mu\nu}^{(0,2)}=T_{\mu\nu}^{(0,2)}q'^\nu=0\ ,\\ &q^\mu T_{\mu\nu}^{(1)}=T_{\nu\mu}^{(1)}q'^\mu=2\Delta_\nu F(t)\end{aligned} \tag{2.6}$$

or equivalently

$$q^\mu C_{\mu\nu}(s,t)=C_{\nu\mu}(s,t)\,q'^\mu=\Delta_\nu F(t)\ , \tag{2.7a}$$

$$q^\mu N_{\mu\nu}=N_{\mu\nu}q'^\nu=0\ , \tag{2.7b}$$

where $F(t)$ is the electromagnetic form factor of the pion. In terms of the invariant amplitudes the current-algebra requirement (2.7a) gives rise to the following constraint[19] (the so-called divergence condition):

$$A(s,t)=\frac{F(t)}{s-m_\pi^2}\quad\text{for}\begin{cases}t=q^2,\ q'^2=0\\ t=q'^2,\ q^2=0\ .\end{cases} \tag{2.8}$$

In the Appendix we shall give a construction of the tensor amplitudes $C_{\mu\nu}$ and $N_{\mu\nu}$, assuming the divergence condition (2.8).

The constraint (2.8) can be accomplished for any model amplitude by setting

$$\begin{aligned}A(s,t;q^2,q'^2)\to\frac{F(q^2)F(q'^2)}{s-m_\pi^2}&+[A(s,t;q^2,q'^2)-A(s,q^2;q^2,0)F(q'^2)\\ &-A(s,q'^2;0,q'^2)F(q^2)+A(s,0;0,0)F(q^2)F(q'^2)]\ ,\end{aligned} \tag{2.9}$$

which clearly exposes the exceptional role of the Born term.[20] This is not the only possible representation satisfying the divergence relations, but (in the framework of our construction scheme) it is the only one which is consistent with the current-algebra fixed pole that we shall discuss now.

On the further assumption that the invariant amplitudes $A,B,\ldots$ satisfy unsubtracted dispersion relations in s, it has been shown[19,21] that $A(s,t)$ must have a fixed pole,

$$A(s,t)\simeq\frac{F(t)}{s}\ , \tag{2.10}$$

whose residue is independent of q^2 and q'^2. While the divergence relations can be fulfilled explicitly (see the Appendix), the current-algebra fixed pole (2.10) imposes decisive restrictions on the dynamics of Compton amplitudes.[18] Any model amplitude

A has to definitely include the fixed pole (2.10) irrespective of the divergence conditions. It is evident that the fixed-pole contribution will not be altered by passing to expression (2.9).

After having discussed the current-algebra implications, we shall proceed in our task to construct a dual amplitude for pion Compton scattering. To be more specific, by this we understand to construct invariant amplitudes $A, B, \ldots, \bar{A}$, $\bar{B}, \ldots$, having the following properties:

(i) divergence condition (2.8) (this can, however, be explicitly fulfilled),

(ii) current-algebra fixed pole (2.10),

(iii) vector-meson dominance (at the vector meson pole, i.e., $q^2 = q'^2 = m_V^2$, the amplitudes are reduced to ordinary dual hadronic amplitudes),

(iv) Regge behavior and particle spectrum (the amplitudes contain resonances on Regge trajectories and duality between s and t channels),

(v) factorization (we here require factorization only at the pion pole in order to consistently restore the right current-algebra fixed-pole residue), and

(vi) Mandelstam analyticity (with the advent of dual models with Mandelstam analyticity,[14] we feel that this requirement is not too ambitious).

In the following we shall mainly be concerned with the invariant amplitudes $A(s, t)$ and $\bar{A}(s, t)$. They are related to the familiar forward Compton scattering amplitude $T_2(s, q^2)$ by $(q^2 = q'^2)$

$$\begin{aligned} T_2^{\pi^\pm}(s, q^2) &= [A(s, t) + A(u, t)] - \bar{A}\,, \\ T_2^{\pi^0}(s, q^2) &= \bar{A}\,. \end{aligned} \tag{2.11}$$

As we shall see, $T_2(s, q^2)$ is to a great extent shaped by the current-algebra constraints on $A(s, t)$, whereas $T_1(s, q^2)$ involves all the other invariant amplitudes which lack in similar restrictions. However, without going into detailed calculations, we still have arguments leading to the experimental result

$$T_1 \underset{s, -q^2 \to \infty}{\sim} -\frac{\nu^2}{q^2} T_2\,, \tag{2.12}$$

i.e., $\sigma_L \to 0$ [note that Eq. (2.12) is identically fulfilled at $q^2 = 0$]. In order that Eq. (2.12) hold, $\tilde{A}_1(s, t=0) + q^2 \tilde{A}_5(s, t=0)$ (and similarly the combination of invariant amplitudes belonging to $N_{\mu\nu}$; for the definition of $\tilde{A}_1$, $\tilde{A}_5$ see the Appendix) must vanish in the scaling limit.[22] This combination corresponds to the helicity amplitude C_{00} (for longitudinal polarized photons), i.e.,

$$C_{00} \underset{t=0}{\sim} q^2(\tilde{A}_1 + q^2 \tilde{A}_5)\,. \tag{2.13}$$

If we now assume that C_{00} has no Kronecker δ's, the highest fixed pole that may occur in $\tilde{A}_1 + q^2\tilde{A}_5$ has $J = -1$ corresponding to the largest nonsense point. In the context of our model this means (as will become clear in Sec. IV)

$$\tilde{A}_1 + q^2 \tilde{A}_5 \underset{s, -q^2 \to \infty}{\leqslant} \frac{1}{q^2}\,, \tag{2.14}$$

which proves Eq. (2.12) (under similar considerations for N_{00}). Actually this is a result of the fact that the mixed channel "trajectory" c_2 (to be introduced later), which governs the fixed pole *and* scaling limit, is restricted to $c_2 \leqslant 0$ by the requirement of no Kronecker δ's.

From Eq. (2.3) and the corresponding s-channel isospin amplitudes

$$\begin{aligned} \tilde{T}_{\mu\nu}^{(0)} &= -2C_{\mu\nu}(u, t) + 3N_{\mu\nu}\,, \\ \tilde{T}_{\mu\nu}^{(1)} &= 2C_{\mu\nu}(s, t) + C_{\mu\nu}(u, t) - 2N_{\mu\nu}\,, \\ \tilde{T}_{\mu\nu}^{(2)} &= C_{\mu\nu}(u, t)\,, \end{aligned} \tag{2.15}$$

it is apparent, within the context of the dual resonance model, that once we have constructed $A(s, t)$ then $\bar{A}(s, t)$ is determined, apart from possible su terms. Assuming that there are no exotic resonances, we obtain that $A(s, t)$ is a pure st term and

$$\bar{A}(s, t) = \tfrac{1}{2}[A(s, t) + A(u, t) + A'(s, u)]\,, \tag{2.16}$$

where the su term A' must cancel the pion pole term in $A(s, t)$ and $A(u, t)$ (i.e., must have equal strength but opposite sign). In the following we shall not discuss su terms any further since they do not contribute to the fixed pole[5] and, because of the intimate relation between fixed pole and scaling limit in our model, not to the scaling functions.

So far we only have taken isovector currents into account. By use of crossing, SU(3), nonexoticity, and the (experimentally well-justified) hypothesis of U-spin conservation in electromagnetic interactions (i.e., the photon is a U-spin scalar) we now can calculate the contribution of isoscalar currents.[23] We find a contribution $\tilde{T}_{\mu\nu}^{(1)\,\text{isoscalar}} = \frac{1}{9} N_{\mu\nu}$, which finally gives

$$\begin{aligned} T_2^{\pi^\pm}(s, q^2) &= [A(s, t) + A(u, t)] - \tfrac{8}{9}\bar{A}\,, \\ T_2^{\pi^0}(s, q^2) &= \tfrac{10}{9}\bar{A}\,. \end{aligned} \tag{2.17}$$

So we are left with the construction of only the amplitude A. In Sec. IV we shall associate the scaling functions with this amplitude. The physical scaling functions are then given by Eqs. (2.16) and (2.17).

We shall base our construction of the invariant amplitudes on the dual model proposed by Ademollo and Del Giudice[11] and Ohba[12] except for the fact that Mandelstam analyticity will be built in from the beginning. Before we go into details we would like to discuss certain aspects of dual amplitudes with Mandelstam analyticity.

III. DUAL AMPLITUDES WITH MANDELSTAM ANALYTICITY

As is well known, the Regge trajectories in the Veneziano-type amplitudes must be linear in the Mandelstam variables in order not to introduce ancestors. This results in an infinite set of zero-width resonances which destroy Regge behavior on the positive real axis. The question then is how to shift the resonance poles to the second sheet, keeping as much of the good properties of the Veneziano-type amplitudes as possible.

Several authors proposed the following generalization of the Veneziano amplitude[14] (the extension to n-point functions is straightforward):

$$M(s,t)=\int_0^1 dx\, x^{-\alpha(t,x)-1}(1-x)^{-\alpha(s,1-x)-1} \times f(t,x)f(s,1-x)\,, \tag{3.1}$$

where

$$\alpha(t,0)=\alpha(t),\quad f(t,0)=f(t)\,, \qquad \alpha(t,1)=\alpha(0),\quad f(t,1)=f(0)\,. \tag{3.2}$$

This *Ansatz* allows nonlinear trajectories without having ancestors so that $\alpha(t)$, $\alpha(s)$ and $f(t), f(s)$ can now be considered as general real analytic functions with a cut on the positive real axis starting at $t=t_0$, $s=s_0$. For $\alpha(u,x)\lesssim C\sqrt{u}$ it has been shown[24] that Eq. (3.1) has Mandelstam analyticity and Regge behavior as $|s|\to\infty$ (i.e., in all directions of the s plane). The price one has to pay for this, however, is to definitely go off from an infinite number of observable resonances lying on the same Regge trajectory. Here resonances are characterized as complex poles on the second sheet near the real axis.

Mandelstam analyticity becomes particularly important in dealing with the deep-inelastic structure functions, as we shall see. Before we now adopt these methods to construct dual two-current amplitudes, we shall consider two different parameterizations of the Regge trajectories α and the residue function f.

Model A:

$$\alpha(t,x)=\alpha(0)+t\alpha'(t(1-x)^2),\quad f(t,x)\equiv 1\,, \qquad \alpha'(t)=O(|t|^{-1/2})\,. \tag{3.3}$$

Model B:

$$\alpha(t,x)=\alpha(t(1-x)),\quad \alpha(t)\to -n \text{ for } |t|\to\infty\,, \qquad f(t,x)=f(t(1-x)),\qquad f(t)=O(|t|^{-n}) \text{ for } |t|\to\infty\,. \tag{3.4}$$

Model A is a choice close to the original Veneziano model.[25] The form (3.3) of the trajectory restores factorization and gives back the Veneziano formula in the limit $\alpha'\to 1$. Model B [i.e., the original CTHKZ (Cohen-Tannoudji–Henyey–Kane–Zakrzewski) model[26]], with asymptotically constant trajectories and residue function $f\neq 1$, is inspired by the fact that the high-energy large-angle scattering amplitude has power behavior (note that the original Veneziano model gives an exponential behavior). One of the authors[27] has demonstrated a resemblance between this model (model B) and the interchange model of Gunion, Brodsky, and Blankenbecler.[28] The most unorthodox feature of model B is that it includes some notion of "short range" forces, taken care of by the residue function f, in contrast to the usual dual models basically describing "long range" effects. When extended to n-point functions model B does, however, not factorize anymore beyond the lowest-order pole.

Owing to the asymptotically constant trajectories model B gives rise to fixed poles. The same holds true in model A if $\alpha'(t)\to[-n-\alpha(0)]/t$ for $t\to\infty$. This can, however, be cured by introducing secondary nonleading trajectories as found in the interchange model.[29]

IV. THE MODEL

We now shall extend these ideas to two-current amplitudes. We imagine that the currents are coupled to pairs of outer leptons as shown in Fig. 1, and, corresponding to this picture, we write a dual six-point function *Ansatz* for the invariant amplitudes. In order that this *Ansatz* be consistent with a two-current amplitude we choose constant trajectories in the mixed lepton-hadron channels so as to eliminate the dependence on the corresponding channel energies.[11,12] The technique of how to construct dual six-point functions is standard. In the following we shall present two models according to our different choices of the Regge trajectory and residue function as discussed in Sec. III.

Model A. According to our first choice we are concerned with the *Ansatz* [generally $\alpha(s)$ is taken to be different from the other trajectories, which all are assumed to be equal]

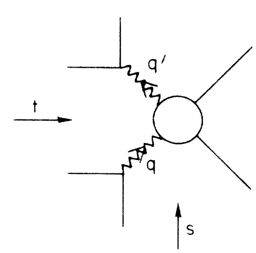

FIG. 1. Typical six-point function *Ansatz*.

$$A(s,t;q^2,q'^2)=N\int_0^1 dx\int_0^1 dy\int_0^1 dz(1-xy)^{-1}(1-zy)^{-1}(1-xyz)^{c_2}\left(\frac{1-x}{1-xy}\right)^{c_1}\left(\frac{1-z}{1-zy}\right)^{c_1}\left(\frac{1-xy}{1-xyz}\right)^{c_1'}\left(\frac{1-zy}{1-xyz}\right)^{c_1'}$$
$$\times x^{-\alpha(q^2,x)}\,z^{-\alpha(q'^2,z)}\,y^{-\alpha(s,y)-1}\left[\frac{(1-y)(1-xyz)}{(1-xy)(1-zy)}\right]^{-\alpha(t,[(1-y)(1-xyz)/(1-xy)(1-zy)])+1}, \tag{4.1}$$

where c_1, c_1', and c_2 correspond to constant mixed-channel trajectories and [still requiring $\alpha'(s)=O(|s|^{-1/2})$], e.g.,

$$\alpha(s,y)=\alpha(0)+s\,\alpha'(s(1-y)^2)\,. \tag{4.2}$$

The dual variables x, y, and z are as described in Fig. 2, and N is a normalization constant to be fixed later. As can easily be verified, our amplitude (4.1) (corresponding to a t-channel double-helicity-flip amplitude) has the correct spin structure in the q^2, q'^2, s, and t channel.

At the pion pole (i.e., $y\simeq 0$), Eq. (4.1) is reduced to [this is consistent with Eqs. (2.11) and (2.17)]

$$\frac{F(q^2)\,F(q'^2)}{s-m_\pi^{\,2}}, \tag{4.3}$$

which gives us the pion electromagnetic form factor

$$F(q^2)=\left(\frac{N}{\alpha'(m_\pi^{\,2})}\right)^{1/2}\int_0^1 dx\,x^{-\alpha(q^2,x)}(1-x)^{c_1}\,. \tag{4.4}$$

For large q^2 it behaves like

$$F(q^2)\simeq\frac{1}{(q^2)^{c_1+1}}\left(\frac{N}{\alpha'(m_\pi^{\,2})}\right)^{1/2}\int_0^\infty d\mu\,e^{-\mu\alpha'(0)}\,\mu^{c_1}\,, \tag{4.5}$$

relating c_1 to the asymptotic behavior of the form factor. The widely accepted monopole behavior of the pion form factor would correspond to $c_1=0$.

We now discuss the large-s behavior of Eq. (4.1). Current algebra requires that this is given by the $J=1$ current-algebra fixed pole (2.10).[19,21] Changing the dual variables x, y, z to $x'=x$, $y'=(1-xy)(1-zy)/(1-x)(1-xyz)$, $z'=(1-y)(1-xyz)/(1-xy)(1-zy)$, and performing a Mellin transform we can easily separate a fixed pole at $J=1-c_2$ (corresponding to $x, y, z\simeq 1$). In order that this be consistent with the current-algebra requirement we must set $c_2=0$. (The same choice of c_2 will also lead to scaling as we shall see.) The fixed-pole residue turns out to be

$$\frac{N}{\alpha'(0)}\int_0^1 dz'\,z'^{-\alpha(t,z')}(1-z')^{c_1}\int_0^1 dy'\,y'^{c_1'-1}(1-z'y')^{-c_1'}=\frac{N}{\alpha'(0)}B_4(c_1',c_1')\int_0^1 dz'\,z'^{-\alpha(t,z')}(1-z')^{c_1}$$
$$\times {}_2F_1(c_1',c_1',2c_1',z')\,, \tag{4.6}$$

which, as a further current-algebra constraint [postulate (ii)], should coincide with the form factor $F(t)$. *A priori* this is not the case although Eq. (4.6) is very similar to the expression (4.4), i.e., has the same large-t behavior as the form factor (the necessity for having the same trajectory in the q^2, q'^2, and t channels is apparent). The discrepancy, due to the extra z' dependence of the second integral in Eq. (4.6), can be removed by adding an infinite number of satellite terms to Eq. (4.1):

$$z'^{-\alpha(t,z')+1}\to\sum_{m=0}^{\infty}c_m\,z'^{-\alpha(t,z')+1+m}\,,$$
$$z'=\frac{(1-y)(1-xyz)}{(1-xy)(1-zy)}\,, \tag{4.7}$$

such that

$$\sum_{m=0}^{\infty}c_m z'^{\,m}=\alpha'(0)\left[B_4(c_1',c_1')\,{}_2F_1(c_1',c_1',2c_1',z')\right]^{-1}\,, \tag{4.8}$$

where N is now determined through the normalization of the residue. The satellite terms affect the normalization of the form factor (4.4) as well, but leave the general structure unchanged. As can be deduced from Eqs. (4.1) and (4.6), the normalization of the form factor and the fixed-pole residue is generally not consistent. This need not be the case because of the exceptional role of the Born term in Eq. (2.9). In the following

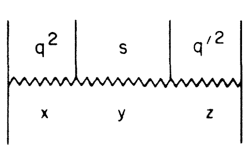

FIG. 2. Choice of the dual variables x, y, and z.

discussion we shall ignore the satellite terms for simplicity and only comment on how they affect our final results.

After subtracting the leading fixed-pole contribution, we obtain the Regge behavior ($t=0$; q^2, q'^2 fixed)

$$A(s, 0; q^2, q'^2) \simeq C s^{\alpha(0)-2} . \tag{4.9}$$

This corresponds to $y \simeq 1$, but (generally) $x, z \not\simeq 1$ so that we have to expect a q^2- and q'^2-dependent residue (the same holds for the nonleading fixed pole, e.g., the $J=0$ fixed pole, which will be discussed elsewhere). The satellite terms ($m>0$) as introduced through Eq. (4.7) are suppressed by at least one power of s. They have the asymptotic behavior (again subtracting the leading fixed-pole contribution) $C' s^{\alpha(0)-2-m}$.

Now we will discuss the scaling limit of the amplitude (4.1). We first concentrate on deep-inelastic scattering, i.e., $s, -q^2, -q'^2 \to +\infty$. For further convenience we keep $q^2 \neq q'^2$. The constant c_2, which was required to be $c_2=0$ by current algebra, will for the moment be left open. Substituting

$$\lambda = 3-x-y-z, \quad \alpha = \frac{1-y}{\lambda}, \quad \beta = \frac{x-z}{\lambda} \tag{4.10}$$

we obtain for large negative s, q^2, and q'^2

$$A(s,0;q^2,q'^2) \simeq N\int_0^3 d\lambda \int_0^1 d\alpha \int_{-(1-\alpha)}^{1-\alpha} d\beta \exp\{\lambda\alpha'(0)[\tfrac{1}{2}(q^2+q'^2)(1-\alpha)+\tfrac{1}{2}(q^2-q'^2)\beta + s\alpha]\}$$
$$\times\left(\frac{\lambda^{c_2}}{2}\right)\left(\frac{1-\alpha+\beta}{1+\alpha+\beta}\right)^{c_1}\left(\frac{1-\alpha-\beta}{1+\alpha-\beta}\right)^{c_1}\left(\frac{1+\alpha+\beta}{2}\right)^{c_1'-1}\left(\frac{1+\alpha-\beta}{2}\right)^{c_1'-1}$$
$$\times\left(\frac{4\alpha}{(1+\alpha+\beta)(1+\alpha-\beta)}\right)^{-\alpha(0)+1} . \tag{4.11}$$

The integration over λ can be done explicitly, which finally gives

$$A(s,0;q^2,q'^2) \simeq \tfrac{1}{2}N\Gamma(c_2+1)[-\alpha'(0)]^{-c_2-1}\int_0^1 d\alpha \int_{-(1-\alpha)}^{1-\alpha} d\beta\, \alpha^{-\alpha(0)+1}\{\tfrac{1}{4}[(1-\alpha)^2-\beta^2]\}^{c_1}\{\tfrac{1}{4}[(1+\alpha)^2-\beta^2]\}^{c_1'-c_1+\alpha(0)-2}$$
$$\times[\tfrac{1}{2}(q^2+q'^2)(1-\alpha)+\tfrac{1}{2}(q^2-q'^2)\beta+s\alpha]^{-c_2-1}. \tag{4.12}$$

This form can be continued analytically to the physical region s, $-q^2$, $-q'^2 \geq 0$.

We now expect that $\nu A(s,0,q^2,q'^2)$ scales nontrivially (i.e., does not vanish in the scaling limit). A necessary and sufficient condition for this to happen is $c_2=0$, i.e., the current-algebra result, as can be verified from Eq. (4.12). This means there is a strong connection between current algebra and scaling. In other words, once we have built in the current-algebra fixed pole, we automatically get scaling. Formally this is established by the fact that both the fixed pole and the scaling limit correspond to the behavior near $x, y, z \simeq 1$. We remark that the same strong connection between the current-algebra fixed pole and scaling is true in parton models[3] and in the bootstrap model proposed by one of the authors.[18]

Equation (4.12) has exactly the form ($c_2=0$ now) as derived by Gatto and Preparata[30] from a light-cone-dominated current commutator, i.e.,

$$A(s,0;q^2,q'^2) = -\int_0^1 d\alpha \int_{-(1-\alpha)}^{1-\alpha} d\beta\, F(\alpha,\beta,0)\,[\tfrac{1}{2}(q^2+q'^2)(1-\alpha)+\tfrac{1}{2}(q^2-q'^2)\beta+s\alpha]^{-1}, \tag{4.13}$$

which essentially is a DGS (Deser-Gilbert-Sudarshan)–Nakanishi representation[31] with all mass terms neglected. This leads to the following *Ansatz* for the light-cone spectral function[32] (the generalization to $t\neq 0$ is straightforward):

$$F(\alpha,\beta,t) = \frac{N}{2\alpha'(0)}\,\alpha^{-\alpha(t,4\alpha/[(1+\alpha)^2-\beta^2])+1}\{\tfrac{1}{4}[(1-\alpha)^2-\beta^2]\}^{c_1}\{\tfrac{1}{4}[(1+\alpha)^2-\beta^2]\}^{-c_1+c_1'+\alpha(t,4\alpha/[(1+\alpha)^2-\beta^2])-2} . \tag{4.14}$$

If we take the satellite terms (4.7) into account, this becomes

$$F(\alpha,\beta,t)\to F(\alpha,\beta,t)\sum_{m=0}^{\infty} c_m\left(\frac{4\alpha}{(1+\alpha)^2-\beta^2}\right)^m . \tag{4.15}$$

From Eq. (4.14) we explicitly see how the light-cone expansion incorporates some notion of compositeness which will become even clearer when we discuss the physical meaning of the constant c_1'.

The deep-inelastic (scattering) structure function

$$F_2=\nu W_2=\frac{2\nu}{\pi}\,\mathrm{Im}A(s,0;q^2,q^2)$$

is now given by

$$F_2(x)=\int_{-(1-x)}^{1-x} d\beta\, F(x,\beta,0)$$
$$=\frac{N}{2\alpha'(0)}\,x^{-\alpha(0)+1}(1-x)^{2c_1+1}\int_{-1}^{+1} d\beta'\,[\tfrac14(1-\beta'^2)]^{c_1}\{\tfrac14[(1+x)^2-(1-x)^2\beta'^2]\}^{-c_1+c_1'+\alpha(0)-2}, \tag{4.16}$$

where $x=1/\omega=-q^2/2\nu$. At threshold, i.e., $x\to1$, $F_2(x)$ behaves like $(1-x)^{2c_1+1}$, which restores the Drell-Yan relation[15] [see Eq. (4.5)] between the threshold behavior of $F_2(x)$ and the large-momentum-transfer behavior of the electromagnetic form factor. For $x\to0$ ($\omega\to\infty$), $F_2(x)$ is Regge-behaved, i.e., $F_2(x)\sim x^{-\alpha(0)+1}$ (satellite terms: $\sim x^{-\alpha(0)+1+m}$) as one expects. The residue of the current-algebra fixed pole at $t=0$ is in terms of the structure function (4.16) given by

$$\int_0^1 \frac{dx}{x}\,F_2(x) \tag{4.17}$$

as can be verified from Eqs. (4.13) and (4.16). This also holds in the presence of satellite terms. Equation (4.17) gives essentially the Adler sum rule[33] since the fixed-pole residue is normalized to unity at $t=0$.

We now discuss our model in the deep-inelastic annihilation region, i.e., $s, q^2, q'^2\to+\infty$. The amplitude (4.1) is well defined in this region in contrast to the usual Veneziano-type amplitudes. The derivation of the light-cone representation (4.12), however, is strictly restricted to negative q^2, q'^2. In order to obtain an appropriate representation for the annihilation structure function well adapted for the discussion of the relation between the scattering and annihilation region, we shall continue Eq. (3.12) analytically to positive q^2, q'^2. One can show from our assumptions that this gives the correct structure function. That means that it is well justified to continue the asymptotic form (3.12), instead of continuing Eq. (4.1) first and then going to the scaling limit.

The continuation of Eq. (4.12) to the deep-inelastic annihilation region will cross the line

$$\tfrac12(q^2+q'^2)(1-\alpha)+\tfrac12(q^2-q'^2)\beta+s\alpha=0$$

in the integration region, so that we must distort the integration path. We end up with the representation

$$A(s,0;q^2,q'^2)=-\int_0^1 d\alpha\int_{C_\beta} d\beta\, F(\alpha,\beta,0)\,[\tfrac12(q^2+q'^2)(1-\alpha)+\tfrac12(q^2-q'^2)\beta+s\alpha]^{-1}, \tag{4.18}$$

where C_β is shown in Fig. 3. Because the integration path C_β goes to infinity, Eq. (4.18) does generally not scale as in the scattering case, but the high-s, $-q^2$, and $-q'^2$ behavior may depend on c_1'. However, the imaginary part always scales and leads to the annihilation structure function $\overline{F}_2=\nu\overline{W}_2=(2\nu/\pi)\mathrm{Im}A(s,0;q^2+i\epsilon,q^2-i\epsilon)$:

$$\overline{F}_2(x)=-\int_{-(1-x)}^{1-x} d\beta\, F(x,\beta,0)$$
$$=\frac{N}{2\alpha'(0)}\,x^{-\alpha(0)+1}(x-1)^{2c_1+1}\int_{-1}^{+1} d\beta'\,[\tfrac14(1-\beta'^2)]^{c_1}\{\tfrac14[(1+x)^2-(1-x)^2\beta'^2]\}^{-c_1+c_1'+\alpha(0)-2}. \tag{4.19}$$

We now ask how $\overline{F}_2(x)$ is related to the analytic continuation of $F_2(x)$ to $x>1$. Generally, we can write

$$\overline{F}_2(x)=-\mathrm{Re}F_2(x)+G(x), \tag{4.20}$$

where

$$G(x)=\frac{N}{\alpha'(0)}\sin^2\pi c_1\,x^{-\alpha(0)+1}(x-1)^{2c_1+1}\int_{-1}^{+1}d\beta'\left[\tfrac{1}{4}(1-\beta'^2)\right]^{c_1}\left\{\tfrac{1}{4}\left[(1+x)^2-(1-x)^2\beta'^2\right]\right\}^{-c_1+c_1'+\alpha(0)-2}, \qquad (4.21)$$

which can easily be deduced from Eqs. (4.16) and (4.19). We see that $F_2(x)$ has for noninteger c_1 (c_1 not half-integer either) a branch cut starting at $x=1$ which has been taken care of by $G(x)$. For integer c_1, $G(x)$ vanishes, giving

$$\overline{F}_2(x)=-F_2(x)\,, \qquad (4.22)$$

where $\overline{F}_2(x)$ is the analytic continuation of $F_2(x)$. For half-integer c_1 we obtain from Eqs. (4.16) and (4.19) [or (4.20) and (4.21)]

$$\overline{F}_2(x)=F_2(x)\,. \qquad (4.23)$$

This corresponds to the case where $F_2(x)$ has no branch cut, but $\overline{F}_2(x)$ is not given by the analytic continuation of $F_2(x)$, i.e., $G(x)\neq 0$. The generalization of the continuation procedure to include satellite terms is straightforward.

From Eqs. (4.16) and (4.19) we see that $F_2(x)$ and $\overline{F}_2(x)$ fulfill the remarkable reciprocity relation ($x>1$)

$$\overline{F}_2(x)=x^{2c_1'-1}F_2(1/x)\,, \qquad (4.24)$$

which relates F_2 and $\overline{F}_2$ in their physical regions. For $c_1'=2$ this is the well-known Gribov-Lipatov relation.[34] Once the constant c_1' is known, Eq. (4.24) now allows us to predict the large-x behavior of $\overline{F}_2(x)$ if $F_2(x)$ is supposed to be known. Then there remains the question about the physical meaning of c_1'.

If we go back to Eq. (4.1) we find that c_1' governs the large-momentum-transfer behavior of the $(2^+)\to(1^-)$ electromagnetic transition form factor (e.g., $A_2\to\rho\gamma$),

$$F_{\text{trans}}(q^2)\sim|q^2|^{-c_1'-1}\,. \qquad (4.25)$$

Equation (4.25) results from going to the vector-meson pole in the q'^2 channel ($z\simeq 0$) and to the tensor-meson pole in the t channel ($z'\simeq 0$), and then taking the limit $q^2\to\infty$ ($x\simeq 1$). Equation (4.25) gives rise to a Drell-Yan type of relation between the large-x behavior of $\overline{F}_2(x)$ [suppose $F_2(x)$ is known] and the decrease of the transition form factor. The Gribov-Lipatov case, i.e., $c_1'=2$, corresponds in our model to $F_{\text{trans}}(q^2)\sim|q^2|^{-3}$. It is obvious that Eqs. (4.24) and (4.25) also hold if we include satellite terms.

Equation (4.25) provides another example of how the light cone carries some information of compositeness which deserves further investigation.

Model B. Now we shall discuss our second model. In this case we write

$$A(s,t;q^2,q'^2)=\int_0^1dx\int_0^1dy\int_0^1dz\,(1-xy)^{-1}(1-zy)^{-1}(1-xyz)^{c_2}\left(\frac{1-x}{1-xy}\right)^{c_1}\left(\frac{1-z}{1-zy}\right)^{c_1}\left(\frac{1-xy}{1-xyz}\right)^{c_1'}\left(\frac{1-zy}{1-xyz}\right)^{c_1'}$$
$$\times x^{-\alpha(q^2(1-x))}\,z^{-\alpha(q'^2(1-z))}\,y^{-\alpha(s(1-y))-1}$$
$$\times\left[\frac{(1-y)(1-xyz)}{(1-xy)(1-zy)}\right]^{-\alpha(t\{1-[(1-y)(1-xyz)/(1-xy)(1-zy)]\})+1}$$
$$\times f_1(q^2(1-x))\,f_1(q'^2(1-z))\,f_2(s(1-y))\,f_3\left(t\left\{1-\left[\frac{(1-y)(1-xyz)}{(1-xy)(1-zy)}\right]\right\}\right), \qquad (4.26)$$

with real analytic functions α, f_i of the type (3.4). The dual structure of this amplitude is very much the same as that of model A except that it does not factorize except for the pion pole and has no Veneziano limit. However, it provides a more realistic description of high-energy large-angle scattering.

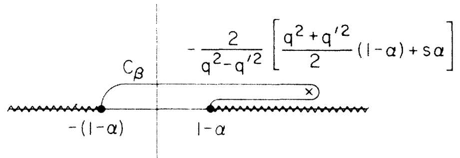

FIG. 3. The integration path C_β. The wavy lines indicate branch cuts of the spectral function.

Our following discussion will be much like in the case of model A. Through factorization at the pion pole we obtain the pion electromagnetic form factor

$$F(q^2)=\left(\frac{f_2(m_\pi^2)f_3(0)}{\alpha'(m_\pi^2)}\right)^{1/2}\int_0^1dx\,x^{-\alpha(q^2(1-x))}(1-x)^{c_1}$$
$$\times f_1(q^2(1-x))\,, \qquad (4.27)$$

which for large q^2 behaves like

$$F(q^2)\simeq\frac{1}{(q^2)^{c_1+1}}\left(\frac{f_2(m_\pi^2)f_3(0)}{\alpha'(m_\pi^2)}\right)^{1/2}$$
$$\times\int_0^\infty d\mu\,\mu^{c_1}f_1(-\mu) \qquad (4.28)$$

if the integral exists, i.e., $f_1 < O(\mu^{-c_1-1})$.

As in model A the amplitude (4.26) is forced to have the current-algebra fixed pole (2.10). This again requires $c_2=0$. Employing the same technique as before, we obtain the residue

$$\left[f_1{}^2(0)\int_0^\infty d\mu f_2(-\mu)\right] B_4(c_1', c_1') \times \int_0^1 dz' z'^{-\alpha(t(1-z'))}(1-z')^{c_1} f_3(t(1-z')) \times {}_2F_1(c_1', c_1', 2c_1', z') . \tag{4.29}$$

In order to bring Eq. (4.29) into accordance with the pion form factor, we take $f_3 \equiv f_1$ and remove the remaining discrepancy as before by adding an infinite set of satellite terms such that

$$\sum_{m=0}^{\infty} c_m z'^m = \left[f_1{}^2(0)\int_0^\infty d\mu f_2(-\mu)\right]^{-1} \times [B_4(c_1', c_1')\,{}_2F_1(c_1', c_1', 2c_1', z')]^{-1}. \tag{4.30}$$

Now we shall investigate the scaling properties of the model amplitude (4.26). Substituting

$$u=q^2(1-x), \quad v=q'^2(1-z), \quad w=s(1-y) , \tag{4.31}$$

we obtain the deep-inelastic (scattering) structure function (again leaving c_2 open for the moment)

$$F_2(x)=\frac{1}{2\pi}(q^2)^{-c_2}\frac{1}{x}\left(\frac{1-x}{x}\right)^{2c_1+1}\int_{-\infty}^0 du\int_{-\infty}^0 dv\int_{s_0}^\infty dw(-u)^{c_1}(-v)^{c_1} w^{-\alpha(0)+1}\left[w-\frac{1-x}{x}(u+v)\right]^{-\alpha(0)+c_2-2c_1'+1} \times\left[\left(w-\frac{1-x}{x}u\right)\left(w-\frac{1-x}{x}v\right)\right]^{\alpha(0)-c_1+c_1'-2} f_1(u)f_1(v)\,\mathrm{Im}f_2(w)f_3(0) . \tag{4.32}$$

First of all, we notice that Eq. (4.32) scales nontrivially if and only if $c_2=0$ as before. Moreover ($c_2=0$ now), $F_2(x)$ is Regge-behaved and obeys the Drell-Yan relation[15] [see Eq. (4.28)] as the integral in Eq. (4.32) stays finite in the limit $x\to 1$, provided that $s_0>0$.

The structure function for deep-inelastic annihilation is similarly given by

$$\overline{F}_2(x)=\frac{1}{2\pi}\frac{1}{x}\left(\frac{x-1}{x}\right)^{2c_1+1}\int_0^\infty du\int_0^\infty dv\int_{s_0}^\infty dw\, u^{c_1} v^{c_1} w^{-\alpha(0)+1}\left[w-\frac{1-x}{x}(u+v)\right]^{-\alpha(0)-2c_1'+1} \times\left[\left(w-\frac{1-x}{x}u\right)\left(w-\frac{1-x}{x}v\right)\right]^{\alpha(0)-c_1+c_1'-2} f_1(u+i\epsilon)f_1(v-i\epsilon)\,\mathrm{Im}f_2(w)f_3(0) . \tag{4.33}$$

The main difference between the structure functions (4.32) and (4.33) is that the u and v integrations in Eq. (4.33) are along the cuts of the functions f_1.

The generalization of Eqs. (4.32) and (4.33) to satellite terms is straightforward. We simply have to replace $\alpha(0)$ by $\alpha(0)-m$, multiply by c_m, and sum over m.

Now we ask how $\overline{F}_2(x)$ is related to the analytically continued (in x) structure function $F_2(x)$. The answer is

$$\overline{F}_2(x)=-\mathrm{Re}F_2(x)+G(x) \tag{4.34}$$

like Eq. (4.20), where

$$G(x)=\frac{1}{2\pi}\frac{1}{x}\left(\frac{x-1}{x}\right)^{2c_1+1}\int_{t_0}^\infty du\int_{t_0}^\infty dv\int_{s_0}^\infty dw\, u^{c_1} v^{c_1} w^{-\alpha(0)+1}\left[w-\frac{1-x}{x}(u+v)\right]^{-\alpha(0)-2c_1'+1} \times\left[\left(w-\frac{1-x}{x}u\right)\left(w-\frac{1-x}{x}v\right)\right]^{\alpha(0)-c_1+c_1'-2}\mathrm{Im}f_1(u)\,\mathrm{Im}f_1(v)\,\mathrm{Im}f_2(w)f_3(0) . \tag{4.35}$$

This result is similar to that of Dahmen and Steiner.[35] Here the extra term $G(x)$ corresponds to the triple discontinuity of the amplitude (4.26). If the function f_1 has zero discontinuity, we have

$$\overline{F}_2(x)=-F_2(x) . \tag{4.36}$$

In order to prove Eq. (4.34), we rotate the u and v integrations in Eq. (4.32) to an integral along the positive real axis and then continue in x. To avoid branch-cut contributions from the exponential factors the rotation has to be over the upper (lower) u *and* v plane if one intends to approach the real-x axis ($1 \leq x < \infty$) from above (below). If the function f_1 is analytic except for the cut (see Sec. III), this rotation is always possible. This way we obtain for the analytic continuation of $F_2(x)$ as (say) $x \to \mathrm{Re}\,x + i\epsilon$ the same expression as Eq. (4.33), but $f_1(v - i\epsilon)$ replaced by $f_1(v + i\epsilon)$, which proves Eq. (4.34).

We notice that $\overline{F}_2(x)$ also fulfills the Drell-Yan relation.[15] A Gribov-Lipatov relation[33] cannot, however, be derived even if $\mathrm{Im} f_1 = 0$.

V. DISCUSSION

Both of our models discussed in Sec. IV fulfill the requirements (i)–(vi) listed in Sec. II. Model A, which stands in close analogy to the generalized Veneziano model, appears to be very appealing since it factorizes and has some very attractive consequences in the scaling region: It is reduced to a light-cone representation and gives rise to a new relation between the large-x behavior of $\overline{F}_2(x)$ and the asymptotic behavior of the electromagnetic $(2^+) \to (1^-)$ transition form factor, together with a Gribov-Lipatov–type of reciprocity relation. Model B does not factorize and corresponds to the original CTHKZ model[26] which was invented to cure the bad analytic properties of the Veneziano model. This model probably would have been abandoned by the purists among the dualists, but (in the four-point function version) it is a first attempt to incorporate the power laws for the energy dependence of hadronic processes at fixed angle[28,36] in a dual model. This is an interesting subject to be investigated in more detail.

We have (again) the appealing relationship between the current-algebra fixed pole and scaling in deep-inelastic electron scattering and annihilation. In both models the fixed pole and scaling arise from the same limit (i.e., $x, y, z \simeq 1$). In this limit the amplitude does not depend on the details of the trajectory, i.e., on the resonance spectrum, anymore as long as the trajectory guarantees Mandelstam analyticity. Hence, scaling is *not* a matter of the particular form of the spectrum of the vector mesons as proposed by the generalized vector-meson-dominance model.[37] The divergence condition can be fulfilled in our model without imposing restrictions on the scaling functions. One can easily check from Eq. (2.9) that in the scaling region only $A(s, t; q^2, q'^2)$ contributes.

In our model we have taken only one (exchange-degenerate) trajectory into account and the Pomeron has been left out so far. Various possibilities of how to include the Pomeron by hand have been reported.[38] In these models scaling of the Pomeron contribution requires $\alpha_P(0) = 1$ similar to scaling in strong interactions and bears no resemblance to the current-algebra construction. Alternatively, one could think of a Pomeron being dual to some background trajectory according to the Harari-Freund hypothesis[39] since, in our approach, we are not restricted to linear trajectories. This again would not fit into the current-algebra construction scheme and hence the Pomeron would couple with arbitrary strength. However, in this case one is not forced to set $\alpha_P(0) = 1$ such that the Pomeron contribution scales. In order to consistently include the Pomeron in our scheme the Pomeron has to be accompanied by an exchange-degenerate $C = -1$ "photon" trajectory,[40] meaning that the Pomeron and its exchanged-degenerate partner are dual to themselves. So, if the reader is willing to accept a "photon" trajectory being exchange-degenerate to the Pomeron, we have a perfect model (the extension of our formalism to two trajectories is straightforward). We feel, however, that this concept is far from being settled but deserves further investigations.

Concluding, we remark that we have worked out a restrictive model for virtual Compton scattering with a large number of good properties, especially current algebra, analyticity, and Bjorken scaling. There remains the problem of constructing general current amplitudes with factorization even on the daughter level. Despite this we feel that our model provides a useful means for understanding deep-inelastic electron scattering. The model can be extended straightforwardly to various other processes as $\gamma\gamma \to \pi\pi$ and $\gamma\gamma \to$ any resonance.

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APPENDIX

The tensor $C_{\mu\nu}(s, t)$ can be rewritten in the form

$$C_{\mu\nu}(s, t) = \tilde{C}_{\mu\nu}(s, t) + (q'_\mu \Delta_\nu + \Delta_\mu q_\nu - q\Delta g_{\mu\nu}) \frac{F(t)}{qq'}, \tag{A1}$$

where

$$q^\mu \tilde{C}_{\mu\nu}(s, t) = \tilde{C}_{\mu\nu}(s, t)\, q'^\nu = 0 \tag{A2}$$

so that the divergence relation (2.7a) is explicitly fulfilled. By construction, $\tilde{C}_{\mu\nu}(s, t)$ may be expanded in terms of a gauge-invariant tensor basis employing the projection formalism of Bardeen and Tung,[41] i.e.,

$$\tilde{C}_{\mu\nu}(s,t)=\sum_{n=1}^{5} I^{n}_{\mu\nu}\tilde{A}_n(s,t)\ , \tag{A3}$$

where[18]

$$\begin{aligned}
I^1_{\mu\nu}&=qq'g_{\mu\nu}-q'_\mu q_\nu\ ,\\
I^2_{\mu\nu}&=qq'\Delta_\mu\Delta_\nu-q\Delta q'_\mu\Delta_\nu-q'\Delta\Delta_\mu q_\nu+q\Delta q'\Delta g_{\mu\nu}\ ,\\
I^3_{\mu\nu}&=qq'q_\mu\Delta_\nu-q^2q_\mu\Delta_\nu+q'\Delta q_\mu q_\nu+q^2q'\Delta g_{\mu\nu}\ ,\\
I^4_{\mu\nu}&=qq'\Delta_\mu q'_\mu-q\Delta q'_\mu q'_\nu-q'^2\Delta_\mu q_\nu+q'^2q\Delta g_{\mu\nu}\ ,\\
I^5_{\mu\nu}&=qq'q_\mu q'_\nu-q^2q'_\mu q'_\nu-q'^2q_\mu q_\nu+q^2q'^2g_{\mu\nu}\ .
\end{aligned}\tag{A4}$$

In order to show that Eq. (A1) has a similar representation we have to make sure that the second term on the right-hand side of Eq. (A1) does not give rise to a kinematical singularity at $qq'=0$.

We notice that the tensors $I^{1,\cdots,5}_{\mu\nu}$ and $I^6_{\mu\nu}=q'_\mu\Delta_\nu+\Delta_\mu q_\nu-q\Delta g_{\mu\nu}$ become linearly dependent at $qq'=0$ $[qq'=\frac{1}{2}(t-q^2-q'^2)]$:

$$q^2q'^2I^2_{\mu\nu}-q\Delta q'^2I^3_{\mu\nu}-q\Delta q^2I^4_{\mu\nu}+(q\Delta)^2I^5_{\mu\nu}=0\ , \tag{A5}$$

$$I^2_{\mu\nu}+q\Delta I^6_{\mu\nu}=0\ . \tag{A6}$$

On the other hand, the divergence condition (2.8) tells us that $\tilde{A}_2(s,t)$ must be of the form

$$\tilde{A}_2(s,t)=\frac{1}{qq'}\frac{F(t)}{s-m_\pi{}^2}+\frac{q^2q'^2}{qq'}R(s,t)$$
$$+\text{terms nonsingular at } qq'=0\ . \tag{A7}$$

This means that the kinematical singularity of the second term of Eq. (A1) is canceled by the counterpart in $\tilde{A}_2(s,t)$ because of the linear dependence (A6). There is still a further possible singularity in $\tilde{A}(s,t)$ [Eq. (A7)]. But this is to be canceled by similar terms in $\tilde{A}_3$, $\tilde{A}_4$, and $\tilde{A}_5$ employing relation (A5).

The tensor $N_{\mu\nu}$ may be expanded in terms of the tensor basis (A4) so that the divergence relation (2.7b) is explicitly fulfilled.

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Single-particle distribution in the hydrodynamic and statistical thermodynamic models of multiparticle production

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We find that the single-particle distribution EdN/d^3p for an expanding relativistic gas described by a distribution function obeying the Boltzmann transport equation is not of the form of an integral over collective motions of a velocity weight function times a "Lorentz-transformed" rest-frame distribution function. This casts doubt on the algorithms of Milekhin and Hagedorn for determining the single-particle distribution function in their models of particle production. For the hydrodynamic model, the correct algorithm is presented.

With the advent of new high-energy accelerators, there has been a revival of interest in many-body approaches to particle production. In particular, the statistical thermodynamic model of Hagedorn[1] and Landau's hydrodynamic model[2] have had considerable success in fitting single-particle inclusive data. Recent review papers have summarized the history and successes of these models.[3-5] In both models, one assumes that the collision process yields a distribution of collective motions. In Hagedorn's approach these collective motions are called fireballs; in Landau's approach the collective motions are that of the hadronic fluid and one has an entropy and energy distribution in terms of the fluid velocity. In both models one assumes that in the local rest frame the distribution of momenta is isotropic and is described by either a Bose or a Fermi distribution of the observed particle.

The question to which we address ourselves is whether the momentum distribution in the center-of-mass frame is given by the probability of finding a particle with collective velocity $\vec{v}$ times the Lorentz-boosted thermal distribution normalized to the total number of particles. The invariant single-particle distribution that follows from this assumption is[1b,6]

$$E\frac{dN}{d^3p} \overset{?}{=} \int \frac{dN}{d^3v}\,\frac{g(\bar{E}, \bar{T}(\vec{v}))}{\bar{n}(\bar{T}(\vec{v}))}\,\bar{E}\,d^3v\,, \qquad (1)$$

where $\bar{E}$ and $\bar{T}$ are, respectively, the energy and temperature in the comoving or local rest frame