Duality and Final-State Interaction in the Description of $\bar{p}n \to \pi^+\pi^-\pi^-$ at Rest (*).

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Summary. — A pole expansion of Veneziano four-point amplitudes which is convergent in the annihilation channel is used *a*) to analyse the resonance content of the previously proposed dual descriptions of the reaction $\bar{p}n \to \pi^+\pi^-\pi^-$ at rest, and *b*) to construct a dual model for this reaction which exhibits a clear connection to the final-state interaction model. The important predictions of duality and crossing symmetry are the existence of the daughter resonances in the $\pi\pi$ system (particularly ε and ε'), and the presence of a nonresonant background in the annihilation amplitude. A reasonable fit to the data is found with this model.

1. – Introduction.

The literature on the reaction $\bar{p}n \to \pi^+\pi^-\pi^-$ at rest is now quite extensive (1). The analysis within the framework of a simple final-state interaction

(*) To speed up publication, the authors of this paper have agreed to not receive the proofs for correction.

(1) See, for example, J. D. JACKSON: in *Proceedings of the Lund International Conference on Elementary Particles* (Lund, 1969), p. 96. Also see the references below.

model (2) (*) was followed by several fits with Veneziano-type amplitudes (4-6). The interest in dual models for the annihilation reaction stems in part from the possibility of giving a simple explanation for the hole (2) near the centre of the Dalitz plot. As proposed by LOVELACE (4), the hole is due to a zero of the Veneziano amplitude. However, this amplitude gives only a qualitative description of the remainder of the Dalitz plot. ALTARELLI and RUBINSTEIN (5) pointed out that an improved description of the hole region could be obtained by using five Veneziano amplitudes instead of the Lovelace amplitude. In their model with finite widths, the hole is due more to a cancellation of several terms than to the zeros of the Veneziano amplitudes (7). Up to now the best fit to the considered reaction obtained with dual amplitudes is presented in ref. (6).

An unattractive feature of Veneziano-type models has been that a simple connection does not exist between them and the physically more comprehensible final-state interaction model. In the final-state interaction model the annihilation amplitude is built up from the resonance poles with the masses and total widths of the resonances being the same as those observed in production channels.

The purpose of this paper is to propose a dual model which has a clear connection with the final-state interaction model. In this way those features of the description which indeed come from dual amplitudes will be more apparent. We shall also discuss the resonant content of previously proposed dual descriptions of the annihilation reaction.

2. – The model.

As emphasized in a recent note (8), the physical content of dual models for the annihilation reaction can be obtained using the expansion

$$\frac{\Gamma(1-\alpha_s)\Gamma(1-\alpha_t)}{\Gamma(2-\alpha_s-\alpha_t)} = \sum_{n=0}^{\infty}\left(\frac{R_n}{n+1-\alpha_s}+\frac{R_n}{n+1-\alpha_t}\right), \tag{1}$$

(2) P. ANNINOS, L. GRAY, P. HAGERTY, T. KALOGEROPOULOS, S. ZENONE, R. BIZZARRI, C. CIAPETTI, M. GASPERO, I. LAAKSO, S. LICHTMAN and G. C. MONETI: *Phys. Rev. Lett.*, **20**, 402 (1968).

(*) See also ref. (3). Strictly speaking, the model there violates the final-state interaction theorem. Nevertheless, it provides a description of the annihilation in terms of resonance poles.

(3) A. GLEESON, W. MEGGS and M. PARKINSON: *Phys. Rev. Lett.*, **25**, 74 (1970).

(4) C. LOVELACE: *Phys. Lett.*, **28** B, 265 (1968).

(5) C. ALTARELLI and H. RUBINSTEIN: *Phys. Rev.*, **183**, 1469 (1969).

(6) G. P. GOPAL, R. MIGNERON and A. ROTHERY: *Phys. Rev. D*, **3**, 2262 (1971); and Imperial College preprint ICTP/69/24 (Sept. 1970).

(7) R. ODORICO: *Phys. Lett.*, **33** B, 489 (1970).

(8) S. POKORSKI and G. H. THOMAS: University of Helsinki preprint TFT 17-70 (October 1970).

where

$$R_0 = 1 \,,$$

$$R_n = \frac{(-1)^n}{n!}\,(2-\alpha_s-\alpha_t)\ldots(n+1-\alpha_s-\alpha_t)\,,$$

and $\sqrt{s}$ and $\sqrt{t}$ are the two $\pi^+\pi^-$ invariant masses. This expansion converges whenever $\alpha_s + \alpha_t > 1$.

It is known that both the Lovelace model and the Altarelli-Rubinstein model are well approximated by the first few terms of this expansion [8,9]. In this case the physical significance of these dual models is that of a sum of terms plus (possibly) a nonresonant background. Crossing symmetry and duality then give constraints on the residues of the resonance terms as well as on any background terms.

For dual models which are sums of Veneziano amplitudes, the total amplitude, by means of eq. (1), can be expressed as (*)

$$A(s,t) = \sum_R \frac{(2L+1)\,C_R(s)\,P_L(\cos\theta)}{s-m_R^2} + (s\leftrightarrow t) + B(s,t)\,, \tag{2}$$

where the sum extends over all parent and daughter resonances R, and $B(s,t)$ represents the nonresonant background. The resonance masses m_R are determined by the straight-line trajectory

$$\alpha(s) = 0.483 + 0.885s\,. \tag{3}$$

For example, at $\alpha = 1$ there can be an S-wave (ε) and a P-wave (ρ) resonance; at $\alpha = 2$ there can be an S-wave (ε′), a P-wave (ρ′) and a D-wave (f) resonance, and so on.

For the purpose of comparison, the coupling strengths $C_R(m_R^2)$ for previous dual models are summarized in Table I. It is to be noted that at the ρ position ($\alpha = 1$), the Lovelace amplitude is pure S-wave, whereas in both the Altarelli-Rubinstein model and the similar model of Gopal *et al.* there is appreciable P-wave present. In the f-mass region although the authors agree that the D-wave is very weak, they disagree about the relative magnitudes of the S and P waves; ref. [4,5] predict that the S-wave is dominant while ref. [6] predicts that the P-wave is strongly dominant. The different relative couplings in these models may be traced to the total widths used for each resonance. All authors use the Lovelace prescription for introducing an imaginary part, namely

$$\alpha(s) = 0.483 + 0.885s + i0.28\sqrt{s-4\mu^2}\,.$$

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[9] J. Boguta: *Nucl. Phys.*, **13** B, 577 (1969); Bonn University preprint (December 1969).

(*) Isospin factors are not included.

Table I. – *The relative magnitudes $C_R(m_R^2)$ as defined by eq.* (2) [a].

	C_ε	C_ρ	$C_{\varepsilon'}$	$C_{\rho'}$	C_f
Lovelace	1	0	1.0	−0.2	0
Altarelli and Rubinstein	0.2	1	−2.1	1.2	−0.22
Gopal *et al.* [b]	1.8	1	−1.7	2.6	0.42
Our fit	1	−0.052	1.0	−0.56	0.072
ππ amplitude (Lovelace)	1.0	−0.2	2.0	−0.5	0.04

(*a*) The relative contribution of each resonance, neglecting interference effects, is $(2l+1)\,|C_R(m_R^2)|^2/M_R\Gamma_R$ when a finite width is given to the resonance.

(*b*) These numbers correspond to their first fit obtained with $\mathrm{Im}\,\alpha = 0.28\sqrt{s-4\mu^2}$ for the ρ trajectory. The couplings for the second fit differ only slightly.

This is reasonable for the Lovelace amplitude, which contains only daughters, but it is, however, against the principle of the final-state interaction model for the amplitudes containing appreciable ρ. The above formula gives the ρ a total width of 280 MeV, which, if reduced to a smaller value, would imply also a smaller ρ coupling.

From the above discussion we conclude that what should be tried is a dual model in which realistic total widths for parents and daughters are given. We start, as in ref. [5], with the general dual and crossing-symmetric amplitude

$$A(s,t) = \sum_{n,m} C_{nm}\,\frac{\Gamma(n-\alpha_s)\,\Gamma(n-\alpha_t)}{\Gamma(n+m-\alpha_s-\alpha_t)}\,, \tag{4}$$

keeping the terms C_{10}, C_{11}, C_{20}, C_{21} and C_{22} (*), which alone produce poles at $\alpha = 1, 2$, and also C_{30}, which contributes to the background. This choice should allow us sufficient freedom to obtain a reasonable fit to the data.

Next, (4) is split into a sum of resonance poles and a nonresonant background using expansion (1). Although there is no unique separation, the following is a natural choice:

$$\begin{aligned}
A(s,t) = {} & [C_{11} + C_{10}(1-\alpha_s-\alpha_t)]/(1-\alpha_s) + \\
& + [-C_{10}(1-\alpha_s-\alpha_t)(2-\alpha_s-\alpha_t) - C_{11}(2-\alpha_s-\alpha_t) - \\
& - C_{20}(1-\alpha_s)(1-\alpha_t)(2-\alpha_s-\alpha_t) - C_{21}(1-\alpha_s)(1-\alpha_t) + C_{22}]/(2-\alpha_s) + \\
& + (s\leftrightarrow t) + C_{21} - C_{22} + (2-\alpha_s-\alpha_t)\,C_{20} + (3-\alpha_s-\alpha_t)(\alpha_s+\alpha_t-\alpha_s\alpha_t)\,C_{30}\,.
\end{aligned} \tag{5}$$

(*) A C_{22}-term is included here since B_5 models for annihilation would in general contain these terms, unless specific assumptions about satellite terms are made. We thank Dr. M. Chaichian and Prof. H. R. Rubinstein for several communications concerning their B_5-model [10].

[10] H. Rubinstein, E. Squires and M. Chaichian: *Phys. Lett.*, **30** B, 189 (1969).

The nonresonant background defined in this way is independent of the number of resonant terms kept.

To make the connection between our model and the final-state interaction model as clear as possible, we expand $A(s, t)$ in partial waves as in (2), and require that each resonance term have the same phase as the corresponding $\pi\pi$ elastic phase. This can be done by replacing the denominator $s-m_R^2$ by $s-m_R^2+im_R\Gamma_R$, where m_R and Γ_R are determined in principle from the $\pi\pi$ scattering amplitude. In practice, the masses and total widths are known from production experiments. To compare the model with the data, the masses m_R will be taken from (3). These predictions are consistent with what is known experimentally. The ρ and f total widths are taken from experiment and parametrized as

$$M\Gamma=\frac{0.12}{0.885}\sqrt{s-4\mu^2}\,.$$

The daughter resonance total widths (ε, ε' and ρ') are less well known and so will be determined by the fit. These widths are parametrized as

$$M\Gamma=\frac{A}{0.885}\sqrt{s-4\mu^2}\,,$$

where, to a first approximation, A is the same for all the daughter resonances.

The model contains six unknown parameters excluding the overall normalization. These parameters were fitted to the data, as obtained by extracting by hand the numbers from the published cosine distributions. The data, divided into 120 bins, were fitted by means of the maximum likelihood method using the minimization program MINUIT [11]. To measure the « goodness of fit », used (with $\nu=113$ degrees of freedom). Since both χ^2 and ν are large the the χ^2-test was quantity $\sqrt{2\chi^2}-\sqrt{2\nu-1}\equiv SD$ is approximately normally distributed about zero with unit variance, and is hence a convenient measure of the goodness of fit.

We should like to make a comment about what values of SD we expect On the one hand, a value $SD>2$ would, with data containing only statistical errors, rule out any model. On the other hand, if there are systematic errors as well as statistical errors, this may be too stringent a requirement; SD is a measure both of the systematic error and the true error between the model and the experiment. Therefore we shall content ourselves with comparing our results to those of previous models to see if we obtain a significantly better description. In particular, we compare our results to those of final-state interaction models. For later reference, ANNINOS *et al.* obtain $SD=8.2$, and GLEESON, MEGGS and PARKINSON obtain $SD=8.9$.

(11) F. JAMES and M. ROOS: CERN Program Library.

3. – Results and discussion.

The best fit we were able to obtain is given in Fig. 1-3. It corresponds to the following values for the free parameters, taking $C_{11}\equiv 1$:

$$(6)\qquad \begin{cases} A = 0.43\pm 0.02\ (^*)\,,\\ C_{10}=-0.17\pm 0.01\,,\\ C_{20}= 0.9\ \pm 0.1\,,\\ C_{21}= 0.5\ \pm 0.3\,,\\ C_{22}=-1.0\ \pm 0.1\,,\\ C_{30}= 0.6\ \pm 0.2\,,\end{cases}$$

where the errors correspond to a change of 0.5 in the logarithm of the likelihood function. The corresponding coupling strengths are given in Table I. The measure of « goodness of fit » SD has the value 6.0 ($\chi^2=220$), *i.e.* 6.0 standard deviations from the mean. A slightly better fit was obtained by fixing $A=0.4$ and varying m_ε and the C_{nm}. This fit gave similar values for the C_{nm}, and $m_\varepsilon=0.84$; the χ^2 was 213 ($SD=5.6$).

The first point we wish to discuss is the quality of the fit obtained. Bin by bin, the fit is rather good except in the region $0.9\leqslant m_{+-}\leqslant 1.140$, which corresponds to the hole region of the Dalitz plot. The inability of our model to reproduce the depth of the hole, as well as the peak for $\cos\theta$ near 1, may be a general feature of Veneziano models (**). As pointed out by ODORICO, the difficulty is that these models produce unwanted zeros. In the present instance there seems to be some effect of a zero near $\cos\theta=1$ ($\alpha_s+\alpha_t=4$). This zero may be the reason for our not being able to obtain as deep a hole as given by the data. However, we note that without some zero structure in the amplitude, it is not possible to obtain as good a fit as Fig. 1.

The next point concerns the connection between our model and the simple final-state interaction model used by ANNINOS *et al.* (2). Our model differs

(*) This gives the ε a total width of 450 MeV, which is in rough agreement with the predictions of the Veneziano model for $\pi\pi$ elastic scattering using elastic unitarity.

(**) This difficulty exists also in the Altarelli-Rubinstein model which, due to the average treatment of resonance widths, is able to obtain the maximum effect of having zeros in the amplitude. The fit to the entire Dalitz plot with this model by GOPAL, MIGNERON and ROTHERY is comparable to the one here. We calculate that their χ^2 is 230 for 115 degrees of freedom, corresponding to $SD=6.2$. A comparison of the data as presented in Fig. 1 and their model also reveals some discrepancy in the region in question.

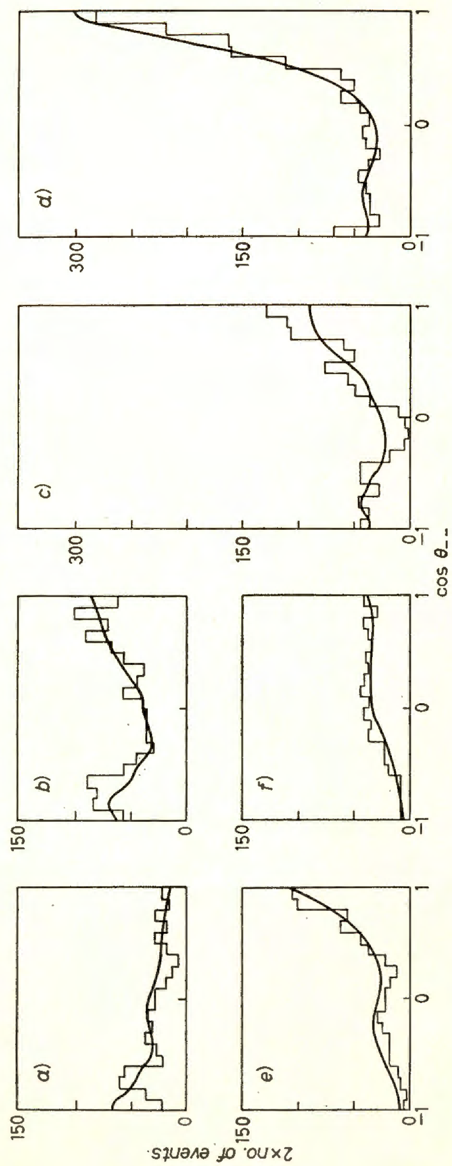

Fig. 1. – The experimental cosine distributions $\cos\theta$ of the π^- relative to the $(\pi^+\pi^-)$ dipion line of flight, in the dipion centre of mass for $\bar{p}n \to \pi^+\pi^-\pi^-$. The curves correspond to the best fit (6): *a*) $0.28 < M_{+-} < 0.64$, *b*) $0.64 < M_{+-} < 0.90$, *c*) $0.90 < M_{+-} < 1.14$, *d*) $1.14 < M_{+-} < 1.37$, *e*) $1.37 < M_{+-} < 1.50$, *f*) $1.50 < M_{+-} < 1.68$.

from this model in two ways: *a*) there is a nonresonant background, and *b*) the energy-dependence of the coupling strengths is not the simple threshold behaviour $C_R(m_R^2)(pq)^l/(p_0 q_0)^l$. To investigate these differences we have considered several additional fits with $A = 0.4$ and the C_{nm} and m_ε free. In this

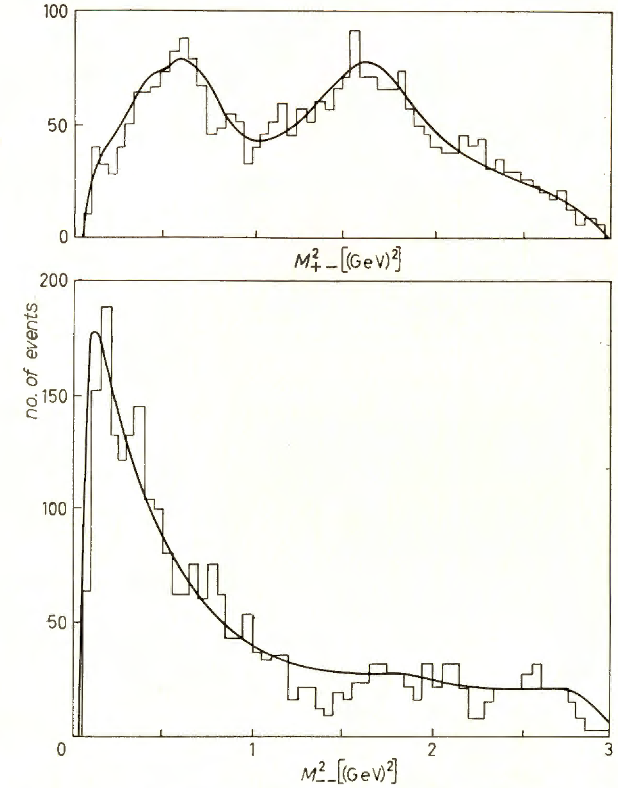

Fig. 2. – The experimental mass-squared distributions $M^2(\pi^+\pi^-)$ and $M^2(\pi^-\pi^-)$ compared with the best fit (6).

case, as noted above, one can obtain a slightly better χ^2. Using only the simple threshold behaviour and no background term we obtain a best fit of $SD = 9.6$, which is near the value of ref. (2). The main change from fit (6) is that C_ε and C_f are twice as strong while C_ρ is half as strong. These values are at least in qualitative agreement with those of our dual model. In this connection, we note that C_{30} in (6) is consistent with zero and contributes only to the background. Hence the C_{nm} parameters in the dual model are in a one-to-one relationship with the coupling strengths $C_R(m_R^2)$: the free parameters in our dual model are the parameters usually chosen as free in final-state interaction models.

To judge the importance of the energy dependence of the couplings, a best fit was made using the above simple threshold behaviour, and a nonresonant background present in (5). Qualitatively the result ($SD = 7$) is more similar to the dual description (6) than to the above description having no background term present. We conclude that the significant feature of our dual model is

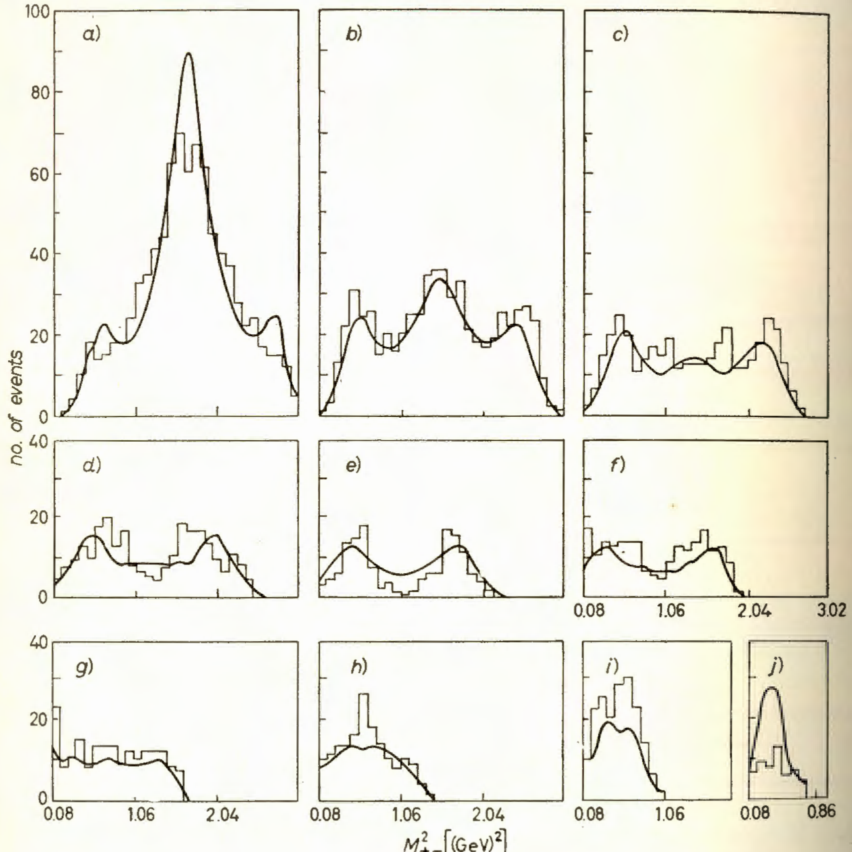

Fig. 3. – Experimental $\pi^+\pi^-$ mass-squared distribution (from ref. (6)) in intervals of $M^2(\pi^-\pi^-)$ as indicated in the Figure. The curves correspond to the best fit (6): *a*) $0.078 \div 0.372$, *b*) $0.372 \div 0.660$, *c*) $0.660 \div 0.960$, *d*) $0.960 \div 1.254$, *e*) $1.254 \div 1.548$, *f*) $1.548 \div 1.842$, *g*) $1.842 \div 2.136$, *h*) $2.136 \div 2.430$, *i*) $2.430 \div 2.724$, *j*) $2.724 \div 3.000$.

the presence of the nonresonant background (*), which is determined by the coupling strengths $C_R(m_R^2)$.

The third point we wish to make concerns the connection between our model and the $\pi\pi$ elastic-scattering amplitude of LOVELACE (C_{10} alone). On the one hand, it is seen from (6) that our amplitude is much closer to Lovelace's

(*) The zeros of the amplitude are due to the cancellation between the pole terms and the background.

annihilation amplitude (C_{11} alone) than his $\pi\pi$ amplitude. Note especially (Table I) that the ε and ε' couplings are approximately equal, and that the ρ and f are approximately decoupled. On the other hand, if the C_{10} term dominated, although the ρ and f would still have small couplings, $C_{\varepsilon'}$ would be approximately $2C_\varepsilon$ (8). This result appears still to be true if one forces the Adler condition (4) on the terms other than C_{10} (which is assumed to satisfy it). Using this condition we find much poorer fits than (6) ($SD \geqslant 11$). Thus we do not find any but a very qualitative connection between our model and a $\pi\pi$ elastic-scattering model which satisfies the Adler condition. Nevertheless, we should like to stress that the important role of daughter resonances in the annihilation channel is predicted already by the $\pi\pi$ amplitude. We have therefore an interesting example of the behaviour of the scattering amplitude in the unphysical region.

Lastly we wish to mention some difficulties related to five-point functions. Models with the Adler condition built in (12), such as $(1-\alpha_s-\alpha_t)B_5$, when evaluated at a pole in the $\bar{p}n$ channel, reduce to a sum of B_4 amplitudes as above. These models do not seem to give a reasonable description of the data. A comparison of our C_{nm} values with those predicted by RUBINSTEIN, SQUIRES and CHAICHIAN (10) using a B_5 model with satellites also reveals considerable disagreement. Finally we note that $\varepsilon(1234)\cdot B_5$ models seem to work well only for peripheral collisions (13). Therefore it seems that the five-point function model for $\bar{p}n$ annihilation still has unsolved problems.

4. – Conclusions.

We conclude that a more reasonable description of the data can be obtained using a dual model than with a pure final-state interaction model, although there are close similarities between the two. The main difference is found to be the addition of a nonresonant background, which is due to the duality and crossing symmetry of the starting amplitude. The elements of the model which most contribute to the fit are 1) the presence of daughter states, notably ε and ε', and 2) the presence in the amplitude of zeros, one of which gives the observed hole in the Dalitz plot. Since these elements occur in dual models in a natural way, we feel that such models are conceptually superior to final-state interaction models.

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We are very grateful to Drs. B. PETERSSON, H. SATZ and N. A. TÖRNQVIST for several useful conversations.

(12) S. POKORSKI, M. SZEPTYCKA and A. ZIEMINSKI: *Nucl. Phys.*, **27** B, 568 (1971).
(13) CHAN HONG-MO, R. O. RAITIO, G. H. THOMAS and N. A. TÖRNQVIST: *Nucl. Phys.*, **19** B, 173 (1970).

● RIASSUNTO (*)

Si usa uno sviluppo polare delle ampiezze di quattro punti di Veneziano convergente nel canale di annichilazione per *a*) analizzare il contenuto di risonanze della descrizione duale della reazione $\bar{p}n \to \pi^+\pi^-\pi^-$ in quiete proposto precedentemente, e *b*) costruire un modello duale di questa reazione che presenti una chiara connessione col modello di interazione dello stato finale. Le predizioni importanti della dualità e della simmetria incrociata sono l'esistenza delle risonanze figlie nel sistema $\pi\pi$ (in particolare ε e ε') e la presenza di un fondo non risonante nell'ampiezza di annichilazione. Con questo modello si trova una ragionevole approssimazione ai dati.

(*) *Traduzione a cura della Redazione.*

Резюме не получено.

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Sliced Extensions and Gell-Mann–Nishijima Formula.

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Summary. — A vector space, generated by the set of Hilbert spaces of one-particle states of the different hadrons, is proposed as representation space of groups which allow for classifications of hadrons in multiplets. With the aid of methods of Lie algebra extension theory, it is proved that the Gell-Mann–Nishijima formula is a particular case of an expression characterizing a class of classification schemes. This expression appears as the eigenvalue version of a formula in terms of operators on the Hilbert space of one-particle hadronic states.

1. – Introduction.

The Gell-Mann–Nishijima formula

$$Q_e = T_3 + Y/2 , \tag{1.1}$$

which gives the electric charges of hadrons in units of e, is a familiar expression in elementary particle physics. As we know, it was obtained from the phenomenological strangeness scheme [1], *i.e.* essentially from the hypothesis of T-spin (isospin) conservation in strong interactions. However, although this hypothesis is sufficient, it is certainly not necessary in order to get (1.1). In fact, we may change the T-spin assignments without affecting the Gell-

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(1) M. GELL-MANN: *Phys. Rev.*, **92**, 833 (1953); T. NAKANO and K. NISHIJIMA: *Progr. Theor. Phys.* (*Kyoto*), **10**, 581 (1953); K. NISHIJIMA: *Progr. Theor. Phys.* (*Kyoto*), **12** 107 (1954); **13**, 285 (1955).