duces the SCD prescription to the usual form.

In the breaking of the nonlinear realization of chiral symmetry we have obtained the mass relation $m_\chi = 2\epsilon m_\pi$, where ϵ is the normalization one chooses for the chiral covariant derivative. As long as the chiral-symmetry-breaking term transforms as a $(\frac{1}{2}, \frac{1}{2})$ representation of SU(2)×SU(2), this mass relation is independent of the particular nonlinear chiral variation of the π field. The mass relation will depend only on the scale dimension of the chiral-breaking term.

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Electromagnetic Currents in Dual Hadrodynamics*

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We discuss a possible scheme of introducing electromagnetic currents in the dual-resonance model in such a way as to maintain duality in many-photon amplitudes. It turns out that the currents build up a pair of trajectories (Pomeranchukon-like and photon-like) with intercept one. A more specific ansatz for the current vertex gives generalized ρ-dominance form factors and a scaling behavior of the inelastic structure function νW_2.

I. INTRODUCTION

The problem of electromagnetic interaction (and other external perturbations) in the dual-resonance model remains an unsolved theoretical question in spite of numerous papers written on the subject.[1] This is because several independent difficulties stand in our way, as may be summarized as follows.

(a) *Lack of factorization property for arbitrary external momenta of a dual amplitude.* The best way to interpret the dynamical content of the Veneziano model would be in terms of the Lagrangian formulation, which leads to a natural factorization property.[2] This method, however, has always had trouble with the trajectory intercepts. The set of four-dimensional harmonic oscillators ("rubber string" or "rubber band") enables one to reproduce self-dual amplitudes only when the intercept is 1. Otherwise one is led to introduce extra scalar oscillators in an *ad hoc* and clumsy way. But even this ceases to work when the external masses are allowed to vary individually, as the coupling of the scalar modes has a nonfactorizable mass dependence.

(b) *Problem of gauge invariance.* The Lagrangian formulation again is the most convenient way of discussing gauge invariance and current conservation. Because the four-dimensional harmonic oscillators may be interpreted as the coordinates of a rubber string, it is easy to write down a gauge principle for them. But the scalar modes do not admit such a natural treatment. In fact, their presence seems to be in general incompatible with gauge invariance, the reason being that the current vertex refuses to vanish even for a pure scalar mode excitation (0-0 transition) on the photon mass shell.

(c) *Form factors*. The electromagnetic current of a string is not uniquely defined, owing to the freedom of arbitrarily distributing the charge over it. But any reasonable distribution leads to a Gaussian form factor typical of harmonic oscillators. Such a form factor may be reasonable for a complex system when the wavelength is not small compared to the size of the system; but the actual nucleon form factors are certainly not of this type. Analytically it is undesirable and would be catastrophic in the timelike region. As an alternative, the vector-meson-dominance type is generally favored, but the problem of securing current conservation is not trivial.

The purpose of the present paper is to propose a scheme which seems to resolve these difficulties to a large extent. We will consider a hadron system described by a specific Lagrangian with an electromagnetic interaction only. Thus we are considering processes of the type

$$\text{hadron} + n\gamma \to \text{hadron} + n'\gamma$$

going through the assumed hadronic states (input trajectories). The γ's may be virtual. The duality will induce in the crossed channels,

$$\text{hadron pair} + n\gamma \to n'\gamma\,,$$

new resonances (output trajectories) which are different from the input. The input trajectories are the standard ones in the Veneziano model, having an arbitrary leading intercept and a fixed slope (~1 GeV^{-2}). But we find that the output trajectories must have a leading intercept unity, though the slope might be in general arbitrary. They are naturally divided into two classes of exchange-degenerate trajectories with opposite C (charge conjugation) properties. In particular, the parent trajectory will consist of what we may call a photon-like (odd C) trajectory and a Pomeranchukon-like (even C) trajectory.

We still have a considerable degree of arbitrariness in the choice of the current vertex. This enables one to adopt, as an example, a vertex of the vector-dominance type, which in addition satisfies the scaling law for the structure function νW_2 in the deep-inelastic region. Moreover, if the ρ (or ω) meson (intercept $=\frac{1}{2}$) couples to the vertex, the slope of the output trajectories comes out to be $\frac{1}{2}$.

There remain some characteristic difficulties and problems, such as the spurious poles and an abnormal behavior of the function W_1.

II. CONSERVED CURRENTS IN THE DUAL LAGRANGIAN

We begin by a general discussion of conserved currents in the Lagrangian formulation of duality. The dynamical variables are the Minkowski coordinates $Z^\mu(\zeta^\alpha)$ of the medium labeled by a pair of internal coordinates $(\zeta^0, \zeta^1) \equiv (\tau, \xi)$, where τ plays the role of an internal time. The free Lagrangian of the system is[2]

$$\bar{L} = \frac{1}{4\pi}\int_0^\pi \frac{\partial Z^\mu}{\partial \zeta^\alpha}\frac{\partial Z_\mu}{\partial \zeta_\alpha}\, d\xi\,. \qquad (1)$$

The metric here is $(+---)$ and $(+-)$ for the external and the internal space, respectively. As usual, however, we work effectively in Euclidean spaces for actual calculational purposes.

Let us assume that there is a conserved current $J^\alpha(\zeta, Z(\zeta))$ in the internal 2-space:

$$\partial J^\alpha/\partial \zeta^\alpha = 0\,. \qquad (2)$$

J^α may or may not actually depend on Z^μ. We can then construct a conserved current $j^\mu(x)$ in the external 4-space[3]:

$$j^\mu(x) = \iint \left[\frac{\partial}{\partial \zeta^\alpha}(J^\alpha Z^\mu)\right]\delta^4(Z-x)\,d^2\zeta$$
$$= \iint J^\alpha \partial_\alpha Z^\mu \delta^4(Z-x)\,d^2\zeta. \qquad (3)$$

Its divergence,

$$\partial_\mu j^\mu(x) = \iint \partial_\alpha(J^\alpha Z^\mu)\partial_\mu \delta^4(Z-x)\,d^2\zeta$$
$$= -\iint \partial_\alpha\left[J^\alpha \delta^4(Z-x)\right]d^2\zeta\,, \qquad (4)$$

will be zero provided that the boundary integral is zero, in particular, if

$$J^1(\tau, \xi=0) = J^1(\tau, \xi=\pi) = 0$$
and $\qquad (5)$
$$Z^0(\tau=\pm\infty, \xi) = \pm\infty.$$

There always exists a class of trivial solutions for such J^α given by

$$J_0 = \frac{\partial\chi}{\partial\zeta^1}, \quad J_1 = \frac{\partial\chi}{\partial\zeta^0}. \qquad (6)$$

$\chi(\zeta)$ is an arbitrary field which has fixed values at $\xi = 0$ and π. As a special case we have

$$J_0 = f(\xi), \quad J_1 = 0. \qquad (7)$$

We can now write down the electromagnetic interaction Lagrangian density as

$$L_{\text{em}}(x) = j_\mu(x) A^\mu(x). \qquad (8)$$

As can be seen by taking the action integral $\int L_{\text{int}}\, d^4x$, Eq. (8) corresponds to a Lagrangian density in the internal space

$$L_{\text{em}}(\zeta) = J^\alpha \frac{\partial Z^\mu}{\partial \zeta^\alpha} A_\mu(Z(\zeta)). \qquad (9)$$

To test gauge invariance we make the replacement $A_\mu(Z)=\partial_\mu\Lambda(Z)$, and obtain

$$J^\alpha \frac{\partial Z_\mu}{\partial \zeta^\alpha}\partial_\mu \Lambda(Z)=\frac{\partial(J^\alpha\Lambda)}{\partial\zeta^\alpha}. \tag{10}$$

For this to have no effect for arbitrary $\Lambda(Z)$ it is necessary that J^α have zero normal to the boundary, which is in fact satisfied by the choice we have made in Eqs. (5)–(7).

With the form of J^α given by Eqs. (6) and (7) we find, respectively,[4]

$$L_{\rm em}(\zeta)=\frac{\partial(Z^\mu,\chi)}{\partial(\tau,\xi)}A_\mu(Z), \tag{11a}$$

$$L_{\rm em}(\zeta)=f(\xi)\,\frac{dZ^\mu}{d\tau}A_\mu(Z). \tag{11b}$$

Equation (11b) corresponds to a charge distribution over the string with a weight function $f(\xi)$. The special case

$$f(\xi)=e_1\delta(0)+e_2\delta(\pi) \tag{12}$$

is noteworthy because this is the form of interaction that one assumes for strong vertices, and also it may be interpreted in terms of quarks located at the ends of the string. Moreover, such an interaction is independent of the internal reference frame (hence conformally invariant) since the action integral can be written as

$$\overline{L}_{\rm em}=e_1\int dZ^\mu(0)A_\mu(Z(0))+e_2\int dZ^\mu(\pi)A_\mu(Z(\pi)). \tag{13}$$

There are a few generalizations one can make beyond the above formulation, and we will utilize them later on. First, in writing down the basic formula (3) it is not necessary that the dynamical variable Z^μ be the same as that appearing in Eq. (1). We could replace it with[5]

$$\overline{Z}^\mu(\zeta)=\iint D(\zeta,\zeta')Z^\mu(\zeta')d^2\zeta'. \tag{14}$$

Here $D(\zeta,\zeta')$ is a smearing function invariant under the time displacement $\tau\to\tau+a$, $\tau'\to\tau'+a$. In all subsequent equations the same replacement may be made.

This would correspond to a nonlocal distribution of charge in the internal space, which might arise, for example, as a result of the radiative (higher-order) correction to the electromagnetic vertex due to strong interactions. The effect of such smearing is different from that which is associated with the distributed charge over the string.

The third kind of smearing is the more familiar one, which may be called external smearing. This is obtained by the substitution

$$j_\mu(x)\to\int D(x-x')j_\mu(x')d^4x', \tag{15}$$

where $D(x)$ represents a form factor of the usual variety.

Finally we will discuss the case where J_α itself depends on Z^μ. In general, an arbitrary function J_α will not be conserved. However, we can still define a conserved current by writing [in a Fourier decomposition of $A_\mu(Z)$]

$$\begin{aligned}L_{\rm em}(\zeta)&=\int\left[J_\alpha(\zeta,Z,q^2)\frac{\partial Z^\mu}{\partial\zeta_\alpha}-i\left(\frac{\partial}{\partial\zeta_\alpha}J_\alpha\right)\frac{q^\mu}{q^2}\right]e^{iq\cdot Z}A_\mu(q)d^4q\\&=\int J_\alpha\frac{\partial Z^\mu}{\partial\zeta_\alpha}\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)e^{iq\cdot Z}A^\nu\,d^4q+(\text{a divergence}),\end{aligned} \tag{16}$$

provided that near $q^2=0$ we have the expansion

$$J_\alpha(\zeta,Z,q^2)=J_\alpha(\zeta)+q^2J'_\alpha(\zeta,Z)+\cdots,\quad \partial_\alpha J^\alpha(\zeta)=0. \tag{17}$$

Under the substitution $A_\mu(q)\to q_\mu\Lambda(q)$, Eq. (16) properly reduces to a complete divergence. The condition (17) is necessary to eliminate the singularity as $q^2\to 0$. The q dependence in J_α may be understood to mean

$$J_\alpha(\zeta,Z,q^2)\to J_\alpha\left(\zeta,Z,\frac{-\partial^2}{\partial Z^\mu\partial Z_\mu}\right)$$

operating on $A_\mu(Z)$.

The discussion so far is based on the c-number theory. In going over to the q-number theory, care must be taken with respect to the ordering of operators to maintain gauge invariance. This may also be done by first going over to the Hamiltonian and invoking the gauge principle. The relation between Lagrangian and Hamiltonian, and the corresponding forms of S-matrix expansion are all according to the standard theory, and will not be elaborated on here.

III. CONSTRUCTION OF DUAL AMPLITUDES

We begin by considering an $(N+2)$-point dual amplitude A_n for scalar particles. As is well known, it can be written in the form[2]

$$A_n = \int_0^1 x_1^{-s_1+m_0^2-1} x_2^{-s_2+m_0^2-1} (1-x_1)^{-2q_1\cdot q_2 - C_2} (1-x_2)^{-2q_2\cdot q_3 - C_2} \cdots (1-x_{N-1})^{-2q_{N-1}\cdot q_N - C_2}$$
$$\times (1-x_1x_2)^{-2q_1\cdot q_3 - C_3} (1-x_2x_3)^{-2q_2\cdot q_4 - C_3} \cdots (1-x_{N-2}x_{N-1})^{-2q_{N-2}\cdot q_N - C_3} \cdots (1-x_1x_2\cdots x_{N-1})^{-2q_1\cdot q_N - C_N} dx_1 dx_2 \cdots dx_{N-1} \tag{18}$$

corresponding to the multiperipheral configuration (Fig.1). Here

$$\begin{aligned} &s_n = (p+q_1+\cdots+q_n)^2, \\ &C_n = \alpha_{n-2}(0) - 2\alpha_{n-1}(0) + \alpha_n(0), \quad n \geq 2 \\ &\alpha_0(0) = 1, \quad \alpha_1(0) = -m^2, \\ &m^2 = q_1^2 = q_2^2 = \cdots q_N^2. \end{aligned} \tag{19}$$

$\alpha_n(0)$ $(n \geq 2)$ is the intercept of the n-particle leading trajectory coupled to n momenta taken out of $(q_1, q_2, \ldots, q_N)$. The direct channels (with s_{12}, s_{123}, ...) are assumed to have a different intercept $-m_0^2$ because we are going to regard $q_1, \ldots, q_N$ as external currents.

If all C_n's in Eqs. (19) are zero, A_n can be constructed from the Lagrangian (1) and an interaction vertex

$$\Gamma(q) = :e^{iq\cdot Z(\xi=0)}: \tag{20}$$

with the constraint for the free states[6]

$$\begin{aligned} &(\overline{H}_0 - m_0^2)\psi(p) = (p^2 - M^2)\psi(p) = 0, \\ &\overline{H}_0 = \frac{1}{4\pi}\int_0^\pi :\left(\frac{\partial Z_\mu}{\partial \tau}\frac{\partial Z^\mu}{\partial \tau} + \frac{\partial Z_\mu}{\partial \xi}\frac{\partial Z^\mu}{\partial \xi}\right): d\xi \\ &\quad = p_0^2 + \tfrac{1}{2}\sum_{n=1}^\infty (4p_n^2 + \tfrac{1}{4}n^2 x_n^2) \\ &\quad = p^2 + \sum_{n=1}^\infty a_{n\mu}^\dagger a_n^\mu \equiv p^2 - M^2 + m_0^2, \end{aligned} \tag{21}$$

with

$$Z_\mu(\xi) = \sum_{n=0}^\infty x_{n\mu} \cos n\xi,$$

$$\left.\begin{aligned} &x_{n\mu} = (2/n)^{1/2}(a_{n\mu} + a_{n\mu}^\dagger), \\ &p_{n\mu} = i(\tfrac{1}{8}n)^{1/2}(a_{n\mu} - a_{n\mu}^\dagger), \\ &[a_{n\mu}, a_{m\nu}^\dagger] = -g_{\mu\nu}\delta_{nm} \end{aligned}\right\} (n \neq 0). \tag{22}$$

The condition $C_n = 0$ determines the intercepts $\alpha_n(0)$ once m^2 is fixed. In particular, if $m^2 = 0$, then

$$\alpha_n(0) = -n + 1. \tag{23}$$

Let us now vary the individual momenta $q_1, \ldots, q_N$ arbitrarily. Equation (19) must be generalized in this case, but it can be seen that each vertex contributes an extra factor

$$\left[\frac{(1-x_{n-1})(1-x_n)}{1-x_{n-1}x_n}\right]^{q_n^2} \quad (x_0 = x_n = 0) \tag{24}$$

in Eq. (18). Since this factor reduces to 1 for $q^2 = 0$, it should be possible to write down a conserved vector current, instead of a scalar vertex, according to the prescription of Sec. I provided that we can factorize the amplitude. It is indeed possible to do so by introducing new scalar excitations. But it looks rather artificial, and besides still seems to run into trouble with gauge invariance. We will therefore discuss it in Appendix A, and consider here the following alternative.

The Regge behavior of A_N with respect to any of the direct-channel energies $s_n \to -\infty$ may be obtained directly from the integrand of Eq. (18) by setting the corresponding parameter[7]

$$\begin{aligned} &x_n = e^{-\beta_n} \sim 1 - \beta_n, \\ &\beta_n \sim -1/s_n. \end{aligned} \tag{25}$$

Equation (24) then reduces to

$$\sim \left(\frac{1}{\beta_n} + \frac{1}{\beta_{n+1}}\right)^{q_n^2} \sim |s_n + s_{n+1}|^{q_n^2} \quad (s_n \text{ or } s_{n+1} \to \infty). \tag{26}$$

For the sake of obtaining the Regge behavior it is then sufficient to modify the vertex (20) by a multiplication factor. We will thus postulate the modification

$$\begin{aligned} &\Gamma(q) \to \tilde{\Gamma}(q), \\ &\langle m|\tilde{\Gamma}(q)|n\rangle = \langle m|\Gamma|n\rangle\langle m|V|n\rangle, \\ &\langle m|V|n\rangle \sim (n+m)^{-q^2} \quad (n \text{ or } m \text{ or both} \to \infty), \end{aligned} \tag{27}$$

where n and m stand for the level number.

There is the question whether the modified vertex (27) will not only produce the correct Regge asymptotic power behavior, but also correct poles in the crossed channels. The answer is yes. To see this we observe that these poles arise in Eq.

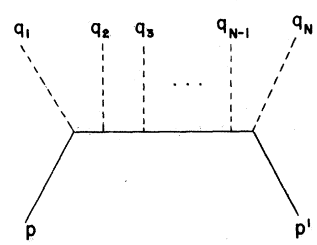

FIG. 1. The multiperipheral diagram. The broken lines are external currents.

(19) from the divergence of the integral near the upper end of the appropriate set of variables $\{x_m\}$. But this is where the expansion (25) becomes valid, and the singular factors in the integrand have the correct form to yield poles in the Regge variable $\alpha(t)$. As for the leading power behavior of A_N, however, the argument would be valid only if $\alpha(t)>-1$. For $\alpha(t)<1$ the integral fails to diverge near the upper end, hence it cannot be estimated in this way. In fact we should expect the presence of terms $\sim s_n^{-l}$ corresponding to fixed poles at $\alpha=-l$, $l=1, 2, \ldots$, in addition to the moving poles. Some more details of this problem will be given in Appendix B.

The intercepts of the output (crossed-channel) trajectories satisfy Eq. (23). Their slopes are all equal to that of the input hadron trajectory (=1). But actually this need not be the case. We could modify the vertex $\tilde{\Gamma}(q)$ by the substitution

$$q \to \tilde{q} = \sqrt{k}\, q \tag{28}$$

except for the factor which couples to the center-of-mass motion. This would correspond to a special case of the operation (14). The output trajectories then would acquire a slope k.

The price we have to pay for the above-mentioned multiplicative modification scheme is that the factorization property in the crossed channels seems to get lost because we can no longer go back to Eq. (18) except when $q_N^2=0$. We do not know whether complete factorization is possible in principle within the present framework.

IV. ELECTROMAGNETIC CURRENTS

We will now apply the results of Sec. III to electromagnetic vertices. The method developed in Sec. II is applicable in order to secure gauge invariance. Consulting Eqs. (16), (20), and (27), we find the current vertex to be

$$\langle m|\Gamma_\mu(q)|n\rangle = \left\langle m \left| :\frac{1}{2}\left\{\frac{\partial Z^\nu(0)}{\partial\tau}, \Gamma(q)\right\}: \right| n\right\rangle \langle m|V(q)|n\rangle \Lambda_{\mu\nu}\,,$$

$$\Lambda_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu/q^2. \tag{29}$$

$e\Gamma_\mu(q)A_\mu(q)$ is to be interpreted as the electromagnetic Lagrangian in the interaction representation.

We are concerned with the changes in the Regge behavior of the amplitude A_N resulting from the vector character of the vertices. Since each current carries spin ±1 or 0, the function $\alpha_n(t)$ no longer will represent the true leading Regge trajectory coupled to n currents; the latter is given by

$$\tilde{\alpha}_n(t) = \alpha_n(t) + n. \tag{30}$$

In view of Eq. (23), we then have

$$\tilde{\alpha}_n(0) = 1 \quad \text{for all } n. \tag{31}$$

In other words there is a set of exchange-degenerate trajectories passing through 1. Even n and odd n trajectories are even and odd under charge conjugation, respectively. It is not clear whether all even (or odd) $\tilde{\alpha}_n$'s can be regarded as a single trajectory, but we are tempted to call $\tilde{\alpha}_{\text{even}}$ and $\tilde{\alpha}_{\text{odd}}$, respectively, a Pomeranchukon-like and a photon-like trajectory.

Beyond the condition (27), the scalar vertex factor $\tilde{\Gamma}$ is not uniquely determined at this stage yet. We will next try to make it more specific. Accordingly we consider the following ansatz:

$$\langle m|V(\tilde{q})|n\rangle = \frac{\Gamma(\frac{1}{2}(n+m)+\beta)\Gamma(\gamma-kq^2)}{\Gamma(\frac{1}{2}(n+m)+\beta-kq^2)\Gamma(\gamma)}, \tag{32}$$

the substitution (28) being understood in all other factors too. Equation (32) has the properties: (a) For large $n+m$ it produces the required factor in Eq. (27), (b) it has vector-meson poles in q^2 at

$$kq^2 = \gamma,\ \gamma+1, \ldots \tag{33}$$

as well as zeros which depend on $n+m$, (c) for fixed $n+m$ it has a form factor going like

$$\sim (kq^2)^{\gamma-\beta-(n+m)/2} \quad (-q^2 \to \infty), \tag{34}$$

and (d) the deep-inelastic electron-hadron cross section exhibits a scaling behavior.

Only the last statement would require a demonstration. We accordingly compute the absorptive part of the forward virtual Compton amplitude

$$F_{\mu\nu} = \frac{1}{2m_0}\sum_{N_\lambda=N}\langle 0|\tilde{\Gamma}_\mu(-\tilde{q})|\lambda\rangle\langle\lambda|\tilde{\Gamma}_\nu(\tilde{q})|0\rangle$$

$$= -\Lambda_{\mu\nu}W_1 + (1/m_0^2)(\Lambda\cdot p)_\mu(\Lambda\cdot p)_\nu W_2, \tag{35}$$

where $|0\rangle$ is the ground state of the hadron with momentum p_μ. The summation in (35) is over all states with level number $N=(p+q)^2-m_0^2$.[8]

Using Eq. (29) we find

$$2m_0F_{\mu\nu} = k\sum_{N_\lambda=N}\langle 0|\Gamma(-\tilde{q})(K^\rho - i\sum_n\sqrt{2n}\,a^\rho{}_n)|\lambda\rangle\langle\lambda|(K^\sigma + i\sum_n\sqrt{2n}\,a^\sigma{}_n{}^\dagger)\Gamma(\tilde{q})|0\rangle|\langle\lambda|V|0\rangle|^2\Lambda_{\mu\rho}\Lambda_{\nu\sigma},$$

$$K_\mu = 2p_\mu + q_\mu. \tag{36}$$

$\Gamma(q)$ only excites oscillators in the direction q. It is therefore clear that the tensors $p_\mu p_\nu$ and $g_{\mu\nu}$ come, respectively, only from $K_\mu K_\nu$ and the contractions $\langle a_{\mu n}a^\dagger_{\nu n}\rangle$.

We find in this way

$$m_0^{-1}W_2(N,q^2)=2\sum_{N_\lambda=N}\langle 0|\tilde{\Gamma}(-\bar{q})|\lambda\rangle\,\langle\lambda|\tilde{\Gamma}(\bar{q})|0\rangle,$$
$$m_0W_1(N,q^2)=k\sum_{n=1}^{N}n\sum_{N_\lambda=N-n}\langle 0|\Gamma(-\bar{q})|\lambda\rangle\,\langle\lambda|\Gamma(\bar{q})|0\rangle\,\langle 0|V(-\bar{q})|\lambda\rangle\,\langle\lambda|V(\bar{q})|0\rangle. \tag{37}$$

From Eqs. (18), (27), and (32) follows then

$$m_0^{-1}W_2=2\sum_{n_\lambda=N}\langle 0|\Gamma(-\bar{q})|\lambda\rangle\,\langle\lambda|\Gamma(\bar{q})|0\rangle\,\langle 0|V(-\bar{q})|N\rangle\,\langle N|V(\bar{q})|0\rangle=\frac{2\Gamma(N-2kq^2)}{\Gamma(N+1)\Gamma(-2kq^2)}\left(\frac{\Gamma(\frac{1}{2}N+\beta)\Gamma(\gamma-kq^2)}{\Gamma(\frac{1}{2}N+\beta-kq^2)\Gamma(\gamma)}\right)^2. \tag{38}$$

In the Regge region $N\to\infty$ and fixed q^2, Eq. (38) behaves as $\sim N^{-1}\sim s^{-1}$. To be more exact,

$$m_0^{-1}W_2\sim 2N^{-1}2^{-2kq^2}\frac{[\Gamma(\gamma-kq^2)/\Gamma(\gamma)]^2}{\Gamma(-2kq^2)}. \tag{39}$$

In the "scaling" region we put as usual

$$\frac{N}{-q^2}=\frac{s-m_0^{\,2}}{-q^2}=\omega-1,$$
$$\omega=\frac{-2p\cdot q}{q^2}=\frac{-2m_0\nu}{q^2}. \tag{40}$$

In this case we have

$$m_0^{-1}W_2\sim C_\gamma N^{-1}\left(\frac{2kN}{\omega-1}\right)^{2\gamma-1/2}\left(1+\frac{2k}{\omega-1}\right)^{1/2-2\beta}$$
$$\equiv N^{2\gamma-3/2}f(\omega),\quad C_\gamma=4\pi^{1/2}[\Gamma(\gamma)]^{-2}, \tag{41}$$

which shows that $N^{3/2-2\gamma}W_2=(s-m_0^{\,2})^{3/2-2\gamma}W_2$ is a function of ω only. If we demand that W_2 behave as $\sim s^{\alpha(0)}f(\omega)$, we have to choose

$$\gamma=\tfrac{1}{4}. \tag{42}$$

$f(\omega)$ will then approach $C_{1/4}=0.54$ as $\omega\to\infty$. With this value of γ, Eq. (33) gives the relation

$$km_V^{\,2}=\tfrac{1}{4}, \tag{43}$$

between k and the mass of the first vector meson. Taking the ρ (or ω) meson, k is then $\approx\frac{1}{2}$. Further,

$$f(\omega)=0.54\left(1-\frac{1}{\omega}\right)^{2\beta-1/2}=0.54\left(1-\frac{1}{\omega}\right)^{2l}. \tag{44}$$

Here l is the asymptotic power $[\sim(-q)^{-l}]$ of the elastic form factor according to Eq. (34). Equation (44) implies that the usual function νW_2 becomes

$$\nu W_2\sim 0.27\left(1-\frac{1}{\omega}\right)^{2l-1}. \tag{45}$$

This power law agrees with that given by Drell and Yan.[9] Applied formally to the e-p scattering, Eq. (45) is in rough agreement with the experimental data.[10]

Let us now turn to the function W_1. From Eq. (37) we can derive the following relation:

$$W_1(N,q^2)=\frac{k}{2m_0}\,\frac{N(N-2kq^2)}{-2kq^2(-2kq^2+1)}\,W_2(N,q^2)$$
$$=\frac{1}{4m_0}\,\frac{N(\omega-1)(\omega-1+2k)}{2kN+\omega-1}\,W_2(N,q^2), \tag{46}$$

as is shown in Appendix C. In the Regge limit $N\to\infty$, q^2 fixed, W_1 properly goes like $\sim N^2W_2\sim N$. As a special case take the photoabsorption cross section. From Eqs. (37) and (46), we find in the limit $q^2\to 0$

$$W_1(N,0)=kN\quad[W_2(N,0)=2\delta_{N,0}] \tag{47}$$

from which the absorption cross section is found to be

$$\sigma=2\pi e^2m_0W_1/N=8\pi^2\alpha k\quad(\alpha=\tfrac{1}{137}), \tag{48}$$

in units of the standard trajectory slope $\approx 1\,\mathrm{GeV}^{-2}$. This constant behavior is rather peculiar, but with $k=\frac{1}{2}$ we get $\sigma\approx 110\ \mu$b. The agreement with high-energy γ-p data[11] is interesting.

In the scaling region, on the other hand, we seem to run into a problem. We get, from Eq. (46),

$$W_1\sim\frac{1}{8m_0^{\,2}k}(\omega-1)(\omega-1+2k)W_2. \tag{49}$$

Put in other words, the ratio R of longitudinal to transverse cross sections goes as

$$1+R\sim\frac{2kN\omega^2}{(\omega-1)^2(\omega-1+2k)}\to\infty\quad(N\to\infty). \tag{50}$$

Equations (49) and (50) are in contradiction with the e-p data at least for small values of ω (or large $-q^2$).

It is to be emphasized that the last difficulty is independent of the specific ansatz we have made for V. In fact it is not hard to see the underlying reason. The basic vertex $\Gamma(q)$ couples only to the modes parallel to q_μ. The transverse excitations come only through the vector factor $dZ_\mu/d\tau$. The nature of $\Gamma(q)$ is such that it enhances longitudinal excitations for large $-q^2>0$. It is therefore not surprising that R should go to infinity in the scaling region. If so, we must regard this result as a general property of the dual model. Introduction of quark spins, for example, would not help the situation.

Deferring further discussion on this question, we point out that our model can be generalized to a charge distribution like Eq. (11b) or (12). The total current will be a sum of terms coming from charges localized at different ξ. One sees easily that the high-energy behavior is dominated only by the diagonal terms, i.e., products of vertices at the same ξ. Consequently the W functions become a sum

$$W \to \sum_i e_i{}^2 W(\xi_i). \qquad (51)$$

In the case of Eq. (12) (quarks localized at both ends) we have $W(\xi=0)=W(\xi=\pi)$.[12]

V. DISCUSSION

We have described a possible scheme of dualizing electromagnetic amplitudes. It is well to recognize that some of the results are of more general validity than the others. First we note that duality (and hence Regge behavior) for electromagnetic processes requires the presence of C-even and C-odd trajectories passing through one. That they must coexist and be degenerate is understandable if we recall the infrared phenomena; they would inevitably make the two trajectories indistinguishable.

It is interesting to speculate that the Pomeranchukon-like trajectory we have might, in fact, be the Pomeranchukon that appears in purely hadronic processes. Factorization (applied to the leading trajectory) implies that both even and odd trajectories can contribute to the latter. Their coupling strengths to the hadron will be of order 1 instead of e, but they cannot be uniquely determined from the electromagnetic amplitudes because only the products of electromagnetic and strong coupling constants enter there.

We will next turn to the various unsatisfactory features that we have noted already.

(1) We have not achieved complete factorization of amplitudes in all channels or configurations. This arises from the presence of two multiplicative factors V and $\dot{Z}_\mu$ in the vertex. Thus we can formulate electromagnetic interaction only in the direct channels. Whether complete factorization and gauge invariance are compatible or not is an open question.

(2) The emergence of $\alpha(0)=1$ leading trajectories naturally invokes the specter of unwanted poles at $kt=0$ (spin 1 and 0) and $kt=-1$ (spin 0). However, we first note that the promotion of the trajectories $\tilde{\alpha}_n$ to α_n occurs only through that part of the vector factor $\dot{Z}_\mu$ in the vertex $\tilde{\Gamma}_\mu$, which belongs to the internal ($n\neq 0$) modes. The part proportional to the momentum p_μ does not modify the internal matrix elements of Γ. This means that in the virtual Compton amplitude the dispersive part associated with W_2 does not have those spurious poles. In the dispersive part of W_1, on the other hand, we expect in general spurious poles having spin 0 and even signature, of the form

$$\frac{a(q^2,q'^2)}{kt+1}+\frac{b(q^2,q'^2)}{kt}. \qquad (52)$$

On the photon mass shell $q^2=q'^2=0$, we find $a=-k$, $b=0$. Thus Eq. (52) would not vanish identically. However, it might be suggested that spin-0 elementary fields may be introduced in the t channel to produce counterterms to eliminate these poles. As we go to many-photon processes, we will encounter more and more of this kind of problem. Certainly it is not clear that we can dispose of them all in a similar way.

(3) As we have remarked already, the W_1 function in the scaling region does not behave in the desirable fashion (i.e., small R for all ω), and this seems to be a rather general property of the dual-resonance model. On the other hand, it is by no means a well established fact that W_1 does scale in the e-p scattering.[13] Thus a more precise determination of W_1 should be very important as a crucial test of the present model.

(4) Unitarity and ghosts are a general problem with the Veneziano model, but we address ourselves here only to the question of positivity of the W functions. These functions are constrained to satisfy the well-known relation

$$W_1 \geq 0, \quad \frac{W_1}{W_2}=\frac{1-\nu^2/q^2}{1+R}, \quad R\geq 0. \qquad (53)$$

As can be seen from our result, especially Eqs. (38) and (46), both W_1 and W_2 are positive for $-q^2>0$, and satisfy Eq. (53). On the other hand, they oscillate in sign in the timelike region $q^2>0$. The first time W_1 changes sign is when $2kq^2=2$, beyond which then our result would be unphysical.

(5) Our willingness to take $k\neq 1$ creates the problem that the Compton amplitude grows exponentially for fixed u and large $t\sim -s>0$. Similar effects will also appear in n-point amplitudes. However, this could be remedied by suitably modifying the current vertices.

It must be remarked that the various troubles associated with dualizing electromagnetic amplitudes may be avoided simply by dropping duality while still maintaining our basic dynamic picture. The origin of crossed-channel poles lies in the singular nature of the vertices, i.e., the fact they are local in the internal space. Smearing out the variable Z^μ as in Eq. (14) can cause a damping of vertices at high energy, and hence the elimination of both the Regge behavior and the associated crossed-channel poles. A fixed-power behavior lower than 1 is still possible, but such a solution does not seem to be supported by the data on high-energy electromagnetic processes. On the other hand, it can provide a simple scaling function which very well reproduces the data.[14] (See Appendix C.) Thus the smearing effect may exist as a correction to the Born approximation we are dealing with.

The quantitative results we obtained in Sec. IV are based on a rather arbitrary ansatz regarding the vertex function. But it is conceivable that this or a similar form might actually follow from a more elementary physical picture of the dynamics of electromagnetic interaction. Such a picture would be necessary for understanding, for example, the origin of the scaling behavior. A slightly different form of the vertex would not have led to scaling. The suggestive analogy that exists between the parton model and the dual-resonance model still remains to be clarified.

APPENDIX A. AN ALTERNATIVE SCHEME FOR FACTORIZATION OFF THE MASS SHELL

This is a mathematical trick to produce the factor (24) and thereby factorize the amplitude (19) for arbitrary ${q_n}^2$. We introduce a pair of scalar harmonic oscillators such that a state labelled by the occupation numbers n and m contributes a term $n+m$ to the mass operator M^2 in Eq. (21), and a multiplicative factor

$$\langle n,m|V|n',m'\rangle = C_{nm'}(q^2) \quad \text{for all } n' \text{ and } m \tag{A1}$$

to the vertex [cf. Eq. (27)], where

$$\left[\frac{(1-x)(1-y)}{1-xy}\right]^{q^2} = \sum C_{nm}(q^2)x^n y^m. \tag{A2}$$

One can check easily that the ansatz (A1), when inserted at each vertex of an amplitude, gives the desired factor provided that the initial and final hadron states belong to $(n,m)=(0,0)$. Equation (A1) is not Hermitian, but it may be said to be self-adjoint with respect to the operator P which interchanges the two oscillators:

$$P|n,m\rangle = |m,n\rangle\,. \tag{A3}$$

A trouble with this scheme is that the peculiar factor (A1) does not reduce to a C number for $q^2=0$, in conflict with the gauge-invariance prescription given by Eqs. (16) and (17).

APPENDIX B. HIGH-ENERGY LIMITS OF AN AMPLITUDE

The typical 4-point Veneziano amplitude

$$\begin{aligned} A(s,t) &= \int_0^1 x^{-\alpha(s)-1}(1-x)^{-\alpha(t)-1}dx \\ &= \int_0^\infty e^{\beta\alpha(s)}(i-e^{-\beta})^{-\alpha(t)-1}d\beta \end{aligned} \tag{B1}$$

may be evaluated by the method of steepest descent in the limit $\alpha(s)\to-\infty$.[7] The maximum of the integrand occurs at

$$\beta_0 \sim \frac{\alpha(t)+1}{\alpha(s)} \quad \text{if } \alpha(t)+1<0; \tag{B2a}$$

hence the Regge behavior

$$A \sim {\beta_0}^{-\alpha(t)} \sim |\alpha(s)|^{\alpha(t)}. \tag{B2b}$$

Now consider the Breit-Wigner expansion of Eq. (B1),

$$A = \sum_{N=0}^\infty \frac{C_N(t)}{\alpha(s)-N} = \int_0^\infty F(\beta)e^{\beta\alpha(s)}d\beta, \tag{B3}$$

where

$$F(\beta) = (1-e^{-\beta})^{-\alpha(t)-1} = \sum_{N=0}^\infty e^{-N\beta}C_N(t). \tag{B4}$$

$F(\beta)$ is thus the Laplace transform, or the thermodynamic function to use a more physical term, associated with the absorptive part of A. For small β, we have

$$F(\beta) \sim \beta^{-\alpha(t)-1}, \tag{B5}$$

which produces a leading pole of A at $\alpha(t)=0$. If $\alpha(t)+1>0, F(\beta)$ diverges at $\beta=0$, so that we can conclude from Eqs. (B4) and (B5) that

$$C_N(t) \sim \left[\frac{N^{\alpha(t)}}{\Gamma(\alpha(t)+1)}\right] \sim \beta_0 F(\beta_0) \tag{B6}$$

without knowing the exact form of $F(\beta)$. This is consistent with Eq. (B2).

Suppose we modify $C_N(t)$ so that

$$C_N(t) \to \tilde{C}_N(t) \sim C_N(t)N^l. \tag{B7}$$

Obviously we get

$$\tilde{F}(\beta) \sim \left(-\frac{\partial}{\partial\beta}\right)^l F(\beta) \sim \beta^{-\alpha(t)+1-l}. \tag{B8}$$

The amplitude $\tilde{A}$ will then behave like $|\alpha(s)|^{\alpha(t)+l}$. and its leading pole will shift to $\alpha(t)=-l$. In this way we can change the Regge behavior by a multiplicative modification of the Breit-Wigner residues as long as that power is >-1.

On the other hand, if $\alpha(t)<-1$ $F(\beta)$ converges at $\beta=0$, so we cannot make an estimate of $C_N(t)$ from $F(\beta)$. This also means that the real part of the amplitude goes as

$$A \sim F(\beta=0)\alpha(s)^{-1}, \quad \alpha(s)\to-\infty, \tag{B9}$$

indicating the presence of fixed poles if $F(0)\neq 0$. The same phenomenon will occur as we further cross negative integers.

The above analysis can be applied to multi-Regge limits of an n-point amplitude. The ansatz (27) for the vertices is seen to have satisfactory properties. We also remark that the method of the thermodynamic average (B4) gives an elegant way of reformulating finite-energy sum rules.

APPENDIX C. COMPTON AMPLITUDE FOR VIRTUAL PHOTONS

First we will derive a relation between W_1 and W_2 from Eq. (37). We note that

$$w_2(N,q^2)=\sum_{N_\lambda=N}\langle 0|\Gamma(-\bar{q})|\lambda\rangle\langle\lambda|\Gamma(\bar{q})|0\rangle$$
$$=\frac{(1/2m_0)W_2(N,q^2)}{[\langle N|V(\bar{q})|0\rangle]^2} \tag{C1}$$

is the coefficient of x^N in $(1-x)^{2kq^2}$. Then

$$w_1(N,q^2)=\sum_{n=1}^{N} n w_2(N-n,q^2)$$
$$=\frac{(m_0/k)W_1}{[\langle N|V(\bar{q})|0\rangle]^2} \tag{C2}$$

is the coefficient of x^N in

$$\sum_{n=0}^{\infty} nx^n(1-x)^{2kq^2}=x(1-x)^{2kq^2-2}. \tag{C3}$$

In other words

$$w_1(N,q^2)=w_2(N-1,q^2-1/k), \tag{C4}$$

from which follows Eq. (45). As $N\to\infty$, $-q^2\to\infty$, we have naturally $w_1/w_2=2m_0^2W_1/W_2k=O(1)$.

To see the generality of such a relation we will consider a vertex $\Gamma(q)$ of the form

$$\exp[iq\cdot\bar{Z}(0)]=\exp[iq\cdot x_0+iq\cdot\sum_{n=1}^{\infty}(2/n)^{1/2}(a_n+a_n^\dagger)f_n], \tag{C5}$$

with an arbitrary set of smearing parameters f_n. The corresponding $d\bar{Z}/d\tau$ is then

$$\frac{d\bar{Z}}{d\tau}=\dot{x}_0-i\sum_{n=1}^{\infty}\sqrt{2n}\,(a_n-a_n^\dagger)f_n. \tag{C6}$$

As in Appendix B, we construct the transforms $F_1(\beta)$, $F_2(\beta)$ of the corresponding w_1 and w_2. We easily find that

$$F_2(\beta)=\exp[-2q^2\sum_n e^{-n\beta}f_n^{\,2}/n]\equiv\exp[-q^2h(\beta)],$$
$$F_1(\beta)=(\sum 2ne^{-n\beta}f_n^{\,2})F_2(\beta)=h''(\beta)F_2(\beta). \tag{C7}$$

Suppose $h(\beta)$ has a leading logarithmic behavior for small β, $h(\beta)\sim c\ln(1/\beta)$. Then

$$F_2(\beta)\sim\beta^{cq^2},\quad F_1(\beta)\sim\beta^{cq^2-2}. \tag{C8}$$

From Eq. (B6) we see that

$$w_2\sim\frac{N^{-cq^2}}{\Gamma(-cq^2+1)},\quad w_1\sim\frac{N^{-cq^2+2}}{\Gamma(-cq^2+3)}$$

or

$$w_1/w_2\sim(N/q^2)^2\sim\omega^2, \tag{C9}$$

in agreement with the previous results.

If, on the other hand, $h(\beta)$ is less singular than $\ln\beta$, the normal Regge behavior will be lost. For example, a form $h(\beta)=c_0\beta+c_1\beta^{1+\epsilon}$, $1>\epsilon>0$, would make

$$w_2\sim q^2N^{-\epsilon-2}\exp[q^2h(\beta)]\sim q^2N^{-\epsilon-2}\exp(\lambda q^2/N+\cdots),$$
$$w_1\sim N^{-\epsilon}\exp(\lambda q^2/N+\cdots). \tag{C10}$$

They have a fixed-power behavior which is lower than would correspond to the Pomeranchukon dominance, although the leading residue function, $\exp(\lambda q^2/N)=\exp(-\lambda/\omega-1)$, has a nice scaling form. The rather *ad hoc* formula $W_2\sim CN^{-1}\exp(-\lambda/\omega-1)$, which one could obtain by *assuming* the relation $C_N\sim\beta F(\beta)$, $\beta\sim 1/N$ [see Eq. (B6)] to hold in general, can fit the W_2 data for protons very well in the range $1.5\lesssim\omega\lesssim 5$ with the choice $C\approx 0.7$, $\lambda\approx 1.5$.[14]

Finally, a word about the contact term in the Compton amplitude. With the vertex (C5) it is given by

$$g_{\mu\nu}\left(1+2\sum_{n=1}^{\infty}f_n^{\,2}\right)=g_{\mu\nu}[1-h'(\beta)]_{\beta=0} \tag{C11}$$

for real photons. This diverges as $f_n\to 1$. However, it is a subtraction term in the dispersion relation, and may be fixed by means of the low-energy theorem.

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[3]For each conserved quantity we can make a similar construction. Further discussion on this point will be made elsewhere.

[4]Equation (11a) is equivalent, up to a divergence, to the Pauli-type interaction

$$L'_{\text{em}}(\zeta)=\tfrac{1}{2}\chi(\zeta)\sigma_{\mu\nu}(Z)F_{\mu\nu}(Z),\quad \sigma_{\mu\nu}(Z)=\frac{\partial(Z_\mu,Z_\nu)}{\partial(\tau,\xi)},$$

where $\sigma_{\mu\nu}$ represents the surface element of the "rubber membrane" embedded in the external space. Dirac used this form with $\chi=$ const in his discussion of magnetic monopoles [P.A.M. Dirac, Phys. Rev. 74, 818 (1948)].

[5]It is a nonlocal substitution in the internal space. Therefore we may treat it only in perturbation theory.

[6]The normalization of $\overline{H}_0$ is chosen in such a way that the canonical momentum $p_{0\mu}$ coincides with the physical momentum p_μ and the vertex (20) gives a trajectory slope of unity.

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[8]Equation (35) is to be considered as representing the smoothed out average of cross sections around N.

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Inverse-Amplitude (K-Matrix) Method for Unitarizing Veneziano Amplitudes, with Application to $\pi\pi$ Scattering*

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The so-called K-matrix method for unitarizing Veneziano amplitudes is analyzed and shown to consist of proposing models for the differences between inverses of physical partial-wave amplitudes and the inverses of Veneziano partial-wave amplitudes. A precise formulation of the method is given, and a detailed analysis is made of the implications of analyticity, elastic and inelastic unitarity, crossing symmetry, Pomeranchukon exchange, and Regge asymptotic behavior. The difficulty of avoiding ghosts within such a framework is discussed, and the positions and residues of several ghosts present in a unitary model proposed earlier by Lovelace are given. It is argued that ghosts are likely to be present in a substantial number of the models for hadronic scattering amplitudes which exist in the literature. From our detailed discussion of the aforementioned topics, we are led to propose a set of equations for inverse partial-wave amplitudes whose solutions may afford an accurate description of $\pi\pi$ elastic scattering for center-of-mass energies up to 1 GeV or more. A numerical procedure for obtaining approximate solutions to the equations is proposed, but the actual computation of solutions is deferred to a later work.

I. INTRODUCTION

Although formulas of the type proposed by Veneziano[1] have many features of scattering amplitudes, they fail to satisfy the relations between real and imaginary parts of amplitudes implied by unitarity. As Lovelace[2] has pointed out, one way to obtain unitary amplitudes from the Veneziano formula is to interpret Veneziano amplitudes as elements of a K matrix.[3] However, a strict K-matrix interpretation of Veneziano amplitudes is not tenable, and the loose interpretation proposed by Lovelace was acknowledged[2] to be inconsistent with crossing symmetry.

Several authors[4] have attempted to improve agreement between crossing symmetry and unitar-