for $|\arg x| < \pi$, $|x| < 1$, $k = 1, 2, 3, \ldots$. This gives, for example, Eq. (3.11) below when $\xi = 0$ and $l \to 1$,

$$1 = \frac{R(0)}{16\pi^3}\left[\frac{1}{6} + \left(\ln\frac{\mu^2}{m^2} + \frac{7}{3}\right)\left(\frac{\mu^2}{m^2}\right) + 6\left(\ln\frac{\mu^2}{m^2} + \frac{17}{12}\right)\left(\frac{\mu^2}{m^2}\right)^2 + \cdots\right].$$

[12]H. W. Wyld, Jr., Phys. Rev. D 3, 3090 (1971).

[13]We computed the slope to order $[(\mu^2)/(m^2)]^2$ because the series X in Eq. (3.15) and Y in Eq. (3.18) converge less rapidly than that in Eq. (3.11). Although the $O([(\mu^2)/(m^2)]^2)$ terms are in magnitude only about 15% of the $O(1) + O((\mu^2)/(m^2))$ terms, they contribute with opposite signs to X and Y, making the quotient X/Y change by about 30%.

[14]G. Tiktopoulos and S. B. Treiman, Phys. Rev. 137, B1597 (1965).

[15]Our approximate solution shown in Fig. 3 lies slightly above the exact solution for $\alpha(0) \lesssim \frac{1}{3}$. This is due to the fact that we have computed $\alpha(0)$ from Eq. (3.11) only up to first order in $(\mu^2)/(m^2)$. Inclusion of higher-order $(\mu^2)/(m^2)$ terms will make it lie below the exact solution in that region also.

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Feynman-Diagram Models of Fermion Regge-Pole Conspiracies*

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A Feynman-diagram model of spin-J fermion Regge poles is developed for meson-nucleon scattering and is used to study the conspiracies arising from two types of Lorentz-invariant couplings. For a completely symmetric coupling, the model automatically leads to a conspiracy relation ($M=\frac{1}{2}$) between two leading trajectories of opposite parity which are MacDowell partners. The second type of coupling, which is antisymmetric in two indices, leads to an $M=\frac{3}{2}$ conspiracy relation between four leading trajectories. At high energies, the $M=\frac{1}{2}$ conspiracy favors the scattering of spin-1 mesons with zero helicity in both initial and final states while the $M=\frac{3}{2}$ conspiracy favors mesons of helicity ±1. Since the $M=\frac{3}{2}$ trajectory chooses nonsense coupling at $J=\frac{1}{2}$, the lowest spin of a particle on the trajectory is $\frac{3}{2}$.

I. INTRODUCTION

In Regge-pole theory Feynman-diagram models have proven to be a useful tool in studying the analytic structure of the scattering amplitudes. They provide a convenient method of coupling a Regge pole to the external particles so that the basic notions of analyticity are satisfied. In these models, the introduction of daughter and conspirator Regge trajectories to restore analyticity to the scattering amplitude[1] arises in a natural way. The numerator of the high-spin off-mass-shell Feynman propagators carries lower-spin components which combine to cancel the singular parts of the spin-J projection operators. Models of this type were first studied by Van Hove and Durand.[2] They have been used to study fermion and boson daughter trajectories[3,4] and have been extended to incorporate boson conspiracies.[5,6]

This paper studies fermion conspiracies within the framework of the Feynman-diagram models. It will be shown that the amplitude for Regge-pole exchange automatically contains the necessary conspiring Regge trajectories to maintain analyticity. Since we are studying conspiring Regge trajectories, the masses of the external particles will be taken to be equal, so that the daughter trajectories which arise from unequal external masses will decouple from the scattering amplitude. Signature will be ignored since it can be trivially included at the end of the calculation.

Within the framework of the one-particle-exchange (OPE) model of Van Hove and Durand, we will see that the conspiracy relations for π-N scattering are automatically satisfied by MacDowell-symmetric[7] baryon trajectories. We will then look at the meson-baryon scattering in a more general four-point coupling model in which the coupling is via Lorentz-invariant tensors. For ρ-N scattering this leads to more complicated conspiracy relations among the leading Regge trajectories and makes definite predictions about the ρ's density matrix. These predictions depend only on the form of the Lorentz-invariant couplings and should be independent of the dynamical details of the model.

The angular momentum part of the coupling between the external particles and the Regge pole is obtained by use of Lorentz-invariant tensors. The dynamical part of the coupling is taken as a constant since we are studying the spin structure of a Regge-pole's contribution to the scattering amplitude. Two types of Lorentz-invariant couplings are considered: a completely symmetric tensor and a symmetric tensor which is however antisymmetric in its last two indices. We shall see that the π-N amplitudes only couple to the completely symmetric tensor while the ρ-N amplitudes couple to both. For the symmetric coupling, the two leading trajectories (MacDowell partners) conspire to satisfy angular momentum conservation at $u=0$ in the crossed channel.[8] For the antisymmetric coupling there are four leading trajectories which conspire among themselves to conserve angular momentum in the crossed channel.

In Sec. II the Van Hove-Durand model for the back scattering of a scalar boson and a fermion via the exchange of a spin-J elementary-particle fermion is developed. The necessary extra factors of spin are obtained from the relative angular momenta of the external particles. The conspiracy relations, first studied by Gribov,[8] are seen to be a statement of MacDowell symmetry.

In Sec. III the three-point couplings of the OPE model is generalized to a four-point coupling model analogous to that of Blankenbecler and Sugar.[6] This has the advantage that the spin-J poles in the scattering amplitude arise dynamically from the interaction itself and are not postulated *a priori* as in the Van Hove model. The spin-J poles arise from the requirement that the four-point interaction satisfy two-particle unitarity in the u channel. The external mesons are given a unit of spin so that both the symmetric and antisymmetric couplings can be studied. The symmetric coupling leads to an $M=\frac{1}{2}$ type conspiracy in which the mesons preferentially have zero helicity at high energy. The antisymmetric coupling leads to an $M=\frac{3}{2}$ type conspiracy, which favors meson helicities of ± 1.

II. ONE-PARTICLE-EXCHANGE MODEL

Before beginning the analysis of the fermion Regge conspiracies, we introduce the OPE model to establish notation. The spin-0 meson is taken to be a scalar π, the spin-$\frac{1}{2}$ fermion to be a N, and the spin-1 meson to be a ρ. We shall set all external masses equal to μ since we are not studying daughter trajectories. The exchanged particle of spin J has mass $m=m(J)$. The Mandelstam variables s, t, and u are used. Since we are looking at backward scattering in the s channel, s and u (as shown in Fig. 1) are the relevant variables.

At each vertex an effective interaction Hamiltonian density for scalar coupling may be taken to be

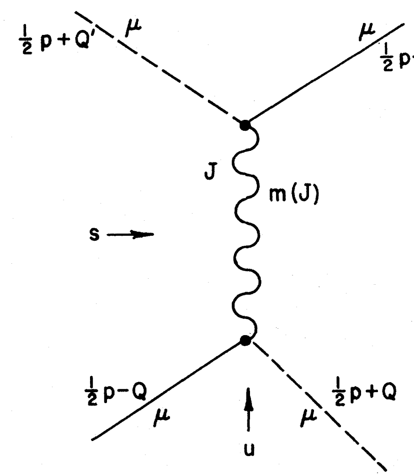

FIG. 1. u-channel spin-J pole.

$$\mathcal{H}_S^J = g(J)[p(l)]^{1/2}\bar{\psi}_\alpha(x)(\overleftrightarrow{\partial}_\mu)^l\phi(x)\psi^J_{\alpha\mu}(x) + \text{H.c.}, \tag{2.1}$$

where

$$\overleftrightarrow{\partial}_\mu = \tfrac{1}{2}(\overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu),$$

$$J = l + \tfrac{1}{2},$$

$$\mu = \mu_1\mu_2\cdots\mu_l,$$

$$p(l) = \frac{\Gamma(2l+2)}{2^l\Gamma^2(l+1)}.$$

$\psi_\alpha(x)$ is the nucleon field, $\phi(x)$ the pion field, and $\psi^J_{\alpha\mu}(x)$ the field of the spin-J fermion. The factor $[p(l)]^{1/2}$ has been extracted from $g(J)$ for convenience. This form of the Hamiltonian corresponds to minimal derivative coupling.

The u-channel amplitude for π-N elastic scattering through the spin-J pole is obtained by using the interaction Hamiltonian at each vertex and

$$\sum_\lambda \psi^{J,\lambda}_{\alpha\mu}\bar{\psi}^{J,\lambda}_{\beta\nu} = (-1)^l\Gamma^J_{\alpha\mu;\nu\beta}(m^2),$$

where $\Gamma^J_{\mu;\nu}(m^2)$ is the numerator of the spin-J propagator and is discussed in the Appendix. We contract the factors of $(Q_\mu)^l$ as shown in the Appendix to obtain the Feynman amplitude:

$$\begin{aligned} F^J &= g^2(J)p(l)\bar{u}_\alpha(Q'_\mu)^l\,\frac{(-1)^l\Gamma^J_{\alpha\mu;\nu\beta}(m^2)}{m^2(J)-p^2}\,(Q_\nu)^l u_\beta \\ &= g^2(J)\,\frac{|\vec{Q}'|^l|\vec{Q}|^l}{m^2(J)-p^2}\,\bar{u}(\not{p}+m) \\ &\quad\times[P_{l+1}'(z)+\gamma\cdot G\cdot\hat{Q}'\gamma\cdot G\cdot\hat{Q}P_l'(z)]u, \end{aligned} \tag{2.2}$$

where $p=\sqrt{u}$ and $G_{\mu\nu}=g_{\mu\nu}-p_\mu p_\nu/m^2$.

The Feynman amplitude for pseudoscalar coupling is obtained from an interaction Hamiltonian like Eq. (2.1) with the substitution $(\overleftrightarrow{\partial}_\mu)^l \to i\gamma_5(\overleftrightarrow{\partial}_\mu)^l$. The amplitude is trivially obtained from Eq. (2.2) by the replacement $m(J) \to -m(J)$.

The identity

$$\not{p}+m = (p+m)\left(\frac{p+\not{p}}{2p}\right) - (p-m)\left(\frac{p-\not{p}}{2p}\right)$$

separates the amplitude F^J into states of definite parity. Assuming the proper "smoothness" properties and asymptotic behavior of $g^2(J)$ and $m(J)$, the sum over J of the parity amplitudes is done by using the Sommerfeld-Watson transformation. Each resultant parity amplitude has a moving pole at $J=\alpha(u)$ where $m^2(\alpha(u)) - u = 0$.

Although it is not essential to the results of this section, we shall do the two-particle unitarity calculation in order to facilitate comparison with the more sophisticated four-point model. Two-particle unitarity is satisfied, as expected, by summing over the self-energy bubbles of the spin-J propagator. In the case of unequal external masses, the unitarity calculation is essential in order that the fixed daughter poles be removed from the amplitude.[3,4]

As a model for the unitarity sum we assume that the dominant contribution to the full propagator comes from the two-particle intermediate states (Fig. 2). Since the calculation is similar to that in Sec. III, it will not be given here. The unitarized propagator is given by

$$T^{J\pm}_{\mu;\nu} = f^{J\pm}(p)\Gamma^{J\pm}_{\mu;\nu}(m^2),$$

where $f^{J\pm}(p)$ is the self-energy function and contains the propagator's pole. The resulting integral equation is easily solved.

$$f^{J\pm}(p) = \left[m \mp p - g^2(J)\left(\pm\frac{p}{2\mu} - 1\right)\mathcal{J}(p,l)\right]^{-1}, \quad (2.3)$$

$$\mathcal{J}(p,l) = i\mu\int \frac{d^4k}{(2\pi)^4}\frac{|\vec{k}|^{2l}}{[(\frac{1}{2}p+k)^2-\mu^2][(\frac{1}{2}p-k)^2-\mu^2]}. \quad (2.4)$$

A cutoff parameter should be introduced here since $\mathcal{J}(p,l)$ diverges for large k^2. If we were using a more realistic coupling constant, it would be profitable to introduce a cutoff. We shall not explicitly write the cutoff parameter for the self-energy integrals.

The unitarized spin-J amplitude is given by

$$F^{J\pm} = g^2(J)|Q'|^l|Q|^l\bar{u}\left(\frac{\not{p}\pm p}{2p}\right) \times[P_{l+1}'(z_u)+\gamma\cdot G\cdot\hat{Q}'\gamma\cdot G\cdot\hat{Q}P_l'(z_u)]uf^{J\pm}(p) \quad (2.5)$$

To obtain the full amplitude for Regge exchange, the signature factor should be included here. (It is an inessential complication and will be ignored.) Performing the Sommerfeld-Watson transformation, we get

$$F^{\pm} = g^2(\alpha_\pm)(|Q'||Q|)^{\alpha_\pm - 1/2}\frac{d\alpha_\pm}{dp}\frac{1}{\cos\pi\alpha_\pm}\bar{u}\left(\frac{\not{p}\pm p}{2p}\right) \times[P_{\alpha+1/2}'(z_u)+\gamma\cdot G\cdot\hat{Q}'\gamma\cdot G\cdot\hat{Q}P_{\alpha-1/2}'(z_u)]u, \quad (2.6)$$

where $\alpha_\pm(p)$ is the largest root of the equation

$$m\mp p - \left(\pm\frac{p}{2\mu} - 1\right)\mathcal{J}(p,l) = J - \alpha_\pm(p) = 0. \quad (2.7)$$

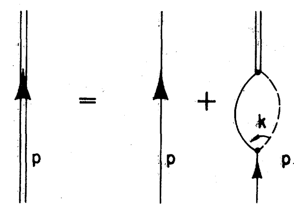

FIG. 2. Two-particle unitarity equation for the propagator.

We then take the standard limit of large physical s and small fixed u. The s-channel helicity amplitudes must exhibit the proper analyticity properties at $u=0$. This is most easily shown in terms of s-channel angular momentum conservation for back scattering. The s-channel helicity amplitudes in the limit $s\to\infty$ and $u=p^2\to 0$ are given in Table I.

Note that individually the nonflip parity amplitudes are nonvanishing for $\sqrt{u}=0$ and thus violate angular momentum conservation; however, the Regge trajectories of opposite parity conspire so that their sum does vanish. This conspiracy was first discussed by Gribov[8] and is simply a statement of MacDowell symmetry,

$$F^{J+}(p) = -F^{J-}(-p).$$

It is interesting, too, that the trajectory functions given by Eq. (2.6) are approximately linear for small p. However, it is easy to adjust the model so that $\alpha_-(p)$ does not rise high enough to produce physical poles for positive p. Thus the

TABLE I. Parity-conserving helicity amplitudes for π-N scattering. [Only the energy dependence from Eq. (2.6) is given; all other factors are the same.]

$\lambda_f \backslash \lambda_i$	$+\frac{1}{2}$	$-\frac{1}{2}$
$+\frac{1}{2}$	$\pm s^\alpha$	$-s^\alpha$
$-\frac{1}{2}$	s^α	$\pm s^\alpha$

problem of baryon resonances of opposite parity can be avoided. Although this seems to be a general feature of the model which comes from the $\not p+m$ term in the Feynman propagator, it depends crucially on the dynamical nature of the coupling constant and the integral $\mathcal{g}(p, l)$ [Eq. (2.4)].

The poles in the u-channel partial-wave amplitudes are not dynamical poles in the sense that they are not due to bound states or resonances. They arise because elementary spin-J particles were introduced into the interaction Hamiltonian and are analogous to Castillejo-Dalitz-Dyson poles.

III. FOUR-POINT COUPLING MODEL

In this section we shall look at the elastic scattering amplitude in the case where the external mesons carry a unit of spin and the interaction proceeds through a four-point coupling. The spin of the meson allows the external particles to couple via a symmetric and an antisymmetric Lorentz-invariant tensor. The four-point coupling model has the advantage that the elementary-particle spin-J poles of the OPE diagram are replaced by dynamical ones which arise from the interaction.

The interaction Hamiltonian which generates a symmetric four-point interaction between a meson and a nucleon may be taken to be

$$\mathcal{H}_I=(-1)^l\Lambda(J)\mu^{-1}p(l)\frac{l+1}{2l+3}\bar\psi(x)(\overleftrightarrow{\partial}_\mu)^{l-1}\gamma_{\mu_{l+1}} \times\epsilon'^{*}_{\mu_l}(x)S^{l+1}_{\mu;\nu}\epsilon_{\nu_l}(x)(\overleftrightarrow{\partial}_\nu)^{l-1}\gamma_{\nu_{l+1}}\psi(x)+\text{H.c.}, \tag{3.1}$$

where $\epsilon_{\mu_l}(x)$ is the meson's polarization vector, and $S^{l+1}_{\mu;\nu}$ is the totally symmetric unit matrix. This Hamiltonian can be obtained from that of the OPE model discussed in Sec. II by taking the formal limit $m(J)\to\infty$ so that $g^2(J)/m(J)\to\Lambda(J)/\mu$.

The Born term for the four-point amplitude in a state of spin J is given by

$$F^J=(-1)^l\Lambda(J)\mu^{-1}p(l)\frac{l+1}{2l+3}\bar u \times\epsilon'^{*}_{\mu_l}(Q'_\mu)^{l-1}\gamma_{\mu_{l+1}}S^{l+1}_{\mu;\nu}\gamma_{\nu_{l+1}}\epsilon_{\nu_l}(Q_\nu)^{l-1}u\,. \tag{3.2}$$

$S^{l+1}_{\mu;\nu}$ can be expanded in terms of the projection operators of spin J, $J-1,\ldots$.

We now require that the amplitude satisfy two-particle unitarity in the u channel. Equation (3.2) can be used as the irreducible kernel in the Bethe-Salpeter equation (Fig. 3). If we were to treat the dynamics realistically, the coupling constant $\Lambda(J)$ should also depend on p^2, Q^2, and/or Q'^2. By taking $\Lambda(J)$ to be constant, the integral equation can be solved exactly and clearly exhibits the spin structure of the scattering matrix. This structure would remain basically unaltered even if a more complicated kernel were used.

After assuming that the coupling constant can be pulled outside the self-energy integral, taking the trace, solving for the self-energy function, and taking the Regge limit, we obtain for the amplitude

$$F^{\pm}_{\lambda_f\lambda_i}=\tfrac{3}{2}N^{\pm}\frac{C^{\pm}_{\lambda_f\lambda_i}}{1-(1\pm p/2\mu)\mathcal{g}(p,l)}, \tag{3.3}$$

where

$$N^{\pm}=\frac{\Lambda(\alpha_\pm)}{16\sqrt{\pi}\,\mu^2}\frac{\Gamma(\alpha_\pm+\frac12)}{\Gamma(\alpha_\pm+1)}s^{\alpha_\pm},$$

$$\mathcal{g}(p,\alpha)=\tfrac12(\alpha+\tfrac12)\Lambda(\alpha) \times i\int\frac{d^4k}{(2\pi)^4}\frac{|\vec{k}|^{2\alpha-3}}{[(\frac12 p+k)^2-\mu^2][(\frac12 p-k)^2-\mu^2]}\,.$$

The $C^{\pm}_{\lambda_f\lambda_i}$ are given in Table II. Only those matrix elements which have the full s^α strength are given in Table II. If either external ρ has nonzero helicity the amplitude is down by a factor of s^{-1}. As before, the parity amplitudes which individually violate angular momentum conservation at $u=0$ conspire with their MacDowell partners. The totally symmetric ρ-N elastic scattering amplitude is identical (up to numerical factors) to the π-N amplitude discussed in Sec. II.

Now we turn our attention to ρ-N elastic scattering where the coupling is given by a symmetric Lorentz tensor with its last two indices antisymmetric. We replace the tensor $S^{l+1}_{\mu;\nu}$ in the interaction Hamiltonian of Eq. (4.1) by $\Theta^{l+2}_{\mu;\nu}$, an antisymmetric tensor of rank $l+2$.[5,9]

$$S^{l+1}_{\mu;\nu}\to\Theta^{l+2}_{\mu;\nu},$$

$$\Theta^{l+2}_{\mu;\nu}=\frac{1}{(l+1)!2}\sum_{\text{perm}}g_{\mu_1\nu_1}g_{\mu_2\nu_2}\cdots g_{\mu_l\nu_l} \times(g_{\mu_{l+1}\nu_{l+1}}g_{\mu_{l+2}\nu_{l+2}}-g_{\mu_{l+1}\nu_{l+2}}g_{\mu_{l+2}\nu_{l+1}})\,.$$

Let us define orthogonal projection operators of normal and abnormal parity. Since the ρ has negative intrinsic parity, states of normal parity formed from l factors of $\vec{Q}$ are given by $-(-1)^l$.

TABLE II. Leading parity-conserving helicity amplitudes for ρ-N scattering via a symmetric Lorentz coupling. (The amplitudes in which the ρ's polarization is ±1 are down by a factor s^{-1} and are not given.)

λ_f $(\epsilon,\bar u)$ \ λ_i (ϵ',u)	$0,+\frac12$	$0,-\frac12$
$0,+\frac12$	±1	-1
$0,-\frac12$	1	±1

$$\Theta^{l+2}_{\mu;\nu} = V^{l+1}_{\mu;\nu} + A^{l+1}_{\mu;\nu} + \text{(lower-order spins)}, \tag{3.4}$$

$$V^{l+1}_{\mu;\nu} = -\tfrac{1}{2}\Gamma^{l+1}_{\mu_1\cdots\mu_l\kappa;\nu_1\cdots\nu_l\kappa'}(p^2)\frac{p_\lambda p_{\lambda'}}{p^2} \times \epsilon_{\lambda\kappa\mu_{l+1}\mu_{l+2}}\epsilon_{\lambda'\kappa'\nu_{l+1}\nu_{l+2}}, \tag{3.5}$$

$$A^{l+1}_{\mu;\nu} = \tfrac{1}{2}\Gamma^{l+1}_{\mu_1\cdots\mu_l\kappa;\nu_1\cdots\nu_l\kappa'}(p^2)\frac{p_\lambda p_{\lambda'}}{p^2} \times (g_{\mu_{l+2}\kappa}g_{\mu_{l+1}\lambda} - g_{\mu_{l+2}\lambda}g_{\mu_{l+1}\kappa}) \times (g_{\nu_{l+2}\kappa'}g_{\nu_{l+1}\lambda'} - g_{\nu_{l+2}\lambda'}g_{\nu_{l+1}\kappa'}), \tag{3.6}$$

$$A^J \equiv \frac{l+1}{2l+3}\gamma_{\mu_{l+1}}\cdot A^{l+1}_{\mu;\nu}\cdot\gamma_{\nu_{l+1}},$$

$$V^J \equiv \frac{l+1}{2l+3}\gamma_{\mu_{l+1}}\cdot V^{l+1}_{\mu;\nu}\cdot\gamma_{\nu_{l+1}}.$$

Here $\epsilon_{\lambda\kappa\mu\mu'}$ is the totally antisymmetric unit tensor. In the c.m. frame, $V^{l+1}_{\mu;\nu}$ forms a state of spin J and parity $-(-1)^l$ from $\vec{Q}_{\mu_1}, \vec{Q}_{\mu_2}, \ldots, \vec{Q}_{\mu_l}\times\vec{\epsilon}_{\mu_{l+2}}$ and $A^{l+1}_{\mu;\nu}$ a state of parity $(-1)^l$ from $\vec{Q}_{\mu_1}, \ldots, \vec{Q}_{\mu_l}, (p\cdot\epsilon)$.

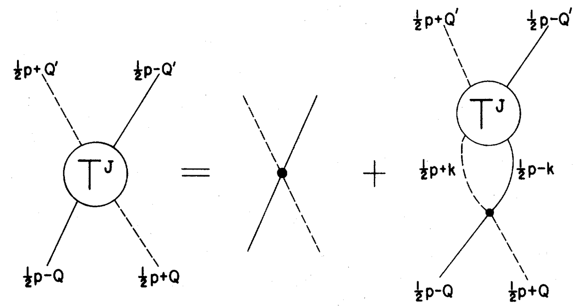

FIG. 3. Bethe-Salpeter equation.

The external factors of $\vec{Q}_\mu$ and $\epsilon_{\mu_{l+2}}$ are contracted to obtain the Born term for the antisymmetric coupling. This Born term is then used as the kernel in the Bethe-Salpeter equation to obtain the unitarized spin-J projection operator

$$T^J_{\alpha\mu;\nu\beta} = (-1)^l\Lambda(J)\mu^{-1}p(l)\frac{l+1}{2l+3}\gamma_{\mu_{l+1}} \times\left(\Theta^{l+2}_{\mu;\nu}\delta_{\alpha\beta} + i\int\frac{d^4k}{(2\pi)^4}\frac{\Theta^{l+2}_{\mu\sigma}\delta_{\alpha\rho}(k_\sigma)^l(\frac{1}{2}\not{p}-\not{k}+\mu)[g_{\sigma_{l+1}\lambda_{l+1}} - \mu^{-2}(\frac{1}{2}p-k)_\sigma(\frac{1}{2}p-k)_\lambda]\,T^J_{\rho\lambda;\nu\beta}(k_\lambda)^l}{[(\frac{1}{2}p+k)^2-\mu^2][(\frac{1}{2}p-k)^2-\mu^2]}\right)\gamma_{\nu_{l+1}}, \tag{3.7}$$

where α and β are spinor indices. T^J can be expressed in terms of the projection operators A^J and V^J:

$$T^J = (a+b\not{p})A^J + (v+w\not{p})V^J + \cdots. \tag{3.8}$$

To solve for the expansion coefficients a, b, v, and w, multiply T^J by $\not{p}A^J$, A^J, $\not{p}V^J$, and V^J, take the trace, and solve for the coefficients. The identity

$$a+b\not{p} = \left(\frac{\not{p}+p}{2p}\right)(a+bp) + \left(\frac{p-\not{p}}{2p}\right)(a-bp) \tag{3.9}$$

separates the states of definite parity from the nucleon spinor. We then obtain

$$T^J_{\mu;\nu} = T^{J+}_{\mu;\nu} + T^{J-}_{\mu;\nu} = (-1)^l p(l)\Lambda(J)\mu^{-1}\left[\left(\frac{(p+\not{p})/2p}{1-(p/2\mu+1)\mathfrak{g}_V}V^J_{\mu;\nu} + \frac{(p-\not{p})/2p}{1-(1-p/2\mu)\mathfrak{g}_A}A^J_{\mu;\nu}\right) + \left(\frac{(p+\not{p})/2p}{1-(1+p/2\mu)\mathfrak{g}_A}A^J_{\mu;\nu} + \frac{(p-\not{p})/2p}{1-(1-p/2\mu)\mathfrak{g}_V}V^J_{\mu;\nu}\right)\right], \tag{3.10}$$

$$\mathfrak{g}_A = \tfrac{1}{2}\Lambda(J)i\int\frac{d^4k}{(2\pi)^4}\frac{|\vec{k}|^{2l-2}[(1-p^2/4\mu^2)|\vec{k}|^2 - k_0^2(2l+1)/l]}{[(\frac{1}{2}p+k)^2-\mu^2][(\frac{1}{2}p-k)^2-\mu^2]}, \tag{3.11}$$

$$\mathfrak{g}_V = \frac{l+1}{2l}\Lambda(J)i\int\frac{d^4k}{(2\pi)^4}\frac{|\vec{k}|^{2l}}{[(\frac{1}{2}p+k)^2-\mu^2][(\frac{1}{2}p-k)^2-\mu^2]}. \tag{3.12}$$

The unitarized amplitude for ρ-N scattering is then given by

$$F^J = \bar{u}\epsilon'^{*}_{\mu_{l+2}}(Q'_\mu)^l\cdot T^J_{\mu;\nu}\cdot(Q_\nu)^l\epsilon_{\nu_{l+2}}u. \tag{3.13}$$

After contracting in the factors of $(Q_\nu)^l\epsilon_{\nu_{l+2}}$ it is convenient to divide F^J into four parts, $F^{V\pm}$ and $F^{A\pm}$. The parity states of F^J are given by

$$F^{J\pm}=F^{V\pm}+F^{A\mp}\,, \tag{3.14}$$

$$F^{A\pm}=\frac{\Lambda(J)}{2\mu}\,\frac{|Q'|^{l}|Q|^{l}}{1-(1\pm p/2\mu)\mathcal{g}_A}\,\frac{(p\cdot\epsilon'^{*})(p\cdot\epsilon)}{p^2}\,\bar{u}\left(\frac{p\pm\not{p}}{2p}\right)[P_{l+1}{}'(z_u)+\gamma\cdot\hat{Q}'\gamma\cdot\hat{Q}P_l{}'(z_u)]u\,, \tag{3.15}$$

$$F^{V\pm}=-\frac{\Lambda(J)}{2\mu l^2}\,\frac{\eta_{fi}|Q'|^{l}|Q|^{l}}{1-(1\pm p/2\mu)\mathcal{g}_V}\,\bar{u}\{\gamma\cdot G\cdot\epsilon'^{*}\gamma\cdot G\cdot\epsilon P_l{}'(z_u)+\vec{\epsilon}\,'^{*}\cdot\hat{Q}\vec{\epsilon}\cdot\hat{Q}[P_{l+1}{}'''(z_u)+\hat{\not{Q}}'\hat{\not{Q}}P_l{}'''(z_u)]$$
$$+\vec{\epsilon}\,'^{*}\cdot\vec{\epsilon}[P_{l+1}{}''(z_u)+\hat{\not{Q}}'\hat{\not{Q}}P_l{}''(z_u)]+(\vec{\epsilon}\,'^{*}\cdot\hat{Q}\hat{\not{Q}}'\gamma\cdot G\cdot\epsilon+\vec{\epsilon}\cdot\hat{Q}'\gamma\cdot G\cdot\epsilon'^{*}\hat{\not{Q}})P_l{}''(z_u)\}u\,, \tag{3.16}$$

where

$$\eta_{fi}=\pm 1=4\times(\text{product of nucleon helicities})\,. \tag{3.17}$$

By making a Wick rotation $k_0\to ik_4$ and calculating the angular part of the four-dimensional integrals, we can show that $\mathcal{g}_A(p,l)$ and $\mathcal{g}_V(p,l)$ are equal at $u=0$. This is necessary in order to have the poles in all the denominators occur at the same value of J and thus be able to form conspiracy relations.

Each parity amplitude $F^{J\pm}$ has been divided into two parts $(A^{\pm}+V^{\mp})$. The calculation and interpretation has been done in terms of the four trajectories so that the contributions of each term could be identified. Since the trajectory functions associated with $A^{\pm}$ and $V^{\pm}$ are different away from $u=0$, this interpretation of four different trajectories seems the most natural.

As before the Reggeized amplitudes are calculated in the limit of $s\to\infty$ for small fixed u:

$$F^{\pm}_{\lambda_f\lambda_i}=\frac{1}{l}\left(-\frac{N^{\pm}\mathcal{V}^{\pm}_{\lambda_f\lambda_i}}{1-(1\pm p/2\mu)\mathcal{g}_V}+\frac{N^{\mp}\mathcal{A}^{\mp}_{\lambda_f\lambda_i}}{1-(1\mp p/2\mu)\mathcal{g}_A}\right), \tag{3.18}$$

where $\mathcal{V}^{\pm}_{\lambda_f\lambda_i}$ and $\mathcal{A}^{\pm}_{\lambda_f\lambda_i}$ are given in Tables III and IV. The complete helicity amplitudes for $F^{\pm}$ are given in Table V. The amplitudes which contain extra factors of $(-u)^{1/2}$ and/or s^{-1} are not given exactly.

The expansion in powers of $\sqrt{u}$ around $\sqrt{u}=0$ shows that all the helicity amplitudes vanish with the correct power of $(-u)^{1/2}$. For example,

$$F_{3/2,3/2}\sim(\sqrt{-u})^3\frac{[\mathcal{g}_V{}'(0,l)-\mathcal{g}_A{}'(0,l)][1+\mathcal{g}(0,l)]}{[1-\mathcal{g}(0,l)]^3}+O((-u)^{5/2})\,. \tag{3.19}$$

The amplitudes of Table V are those for which the ρ has helicities of ± 1. If either ρ has zero helicity, the amplitudes are down by at least a factor of $(-u)$.

The Lorentz-invariant antisymmetric coupling leads to four leading trajectories: the normal- and abnormal-parity trajectories and their MacDowell-symmetric partners. If either the normal- or abnormal-parity amplitude appeared by itself, angular momentum conservation would be violated even though its MacDowell partners were also considered. (This is most easily seen by looking at the $F_{3/2,1/2}$ amplitude.) Then, in order to restore angular momentum conservation at $u=0$, we would be forced to require that $\Lambda(J)$ contain an evasive factor of u and all helicity amplitudes would then vanish at $u=0$.

Continuation of the helicity amplitudes down to

TABLE III. Parity-conserving helicity amplitudes for the axial-vector part of ρ-N scattering via an antisymmetric Lorentz tensor $(\beta\sim\sqrt{-u})$. $\mathcal{A}^{\pm}$ listed below are defined by Eq. (3.18).

λ_f $(\epsilon,\bar{u})$ \ λ_i (ϵ',u)	$1,-\frac{1}{2}$	$1,+\frac{1}{2}$	$0,-\frac{1}{2}$	$0,+\frac{1}{2}$	$-1,-\frac{1}{2}$	$-1,+\frac{1}{2}$
$1,-\frac{1}{2}$	$\pm i$	1	$\pm i\beta$	β	$\mp i$	-1
$1,+\frac{1}{2}$	-1	$\pm i$	$-\beta$	$\pm i\beta$	1	$\mp i$
$0,-\frac{1}{2}$	$\mp i\beta$	$-\beta$	$\mp i\beta^2$	$-\beta^2$	$\pm i\beta$	β
$0,+\frac{1}{2}$	β	$\mp i\beta$	β^2	$\mp i\beta^2$	$-\beta$	$\pm i\beta$
$-1,-\frac{1}{2}$	$\mp i$	-1	$\mp i\beta$	$-\beta$	$\pm i$	1
$-1,+\frac{1}{2}$	1	$\mp i$	β	$\mp i\beta$	-1	$\pm i$

TABLE IV. Parity-conserving helicity amplitudes for the vector part of ρ-N scattering via an antisymmetric Lorentz tensor. (The amplitudes in which the ρ's polarization is zero are down by a factor of s^{-1} and are not given.) $\mathcal{V}^{\pm}$ listed below are defined by Eq. (3.18).

λ_f $(\epsilon,\bar{u})$ \ λ_i (ϵ',u)	$1,-\frac{1}{2}$	$1,+\frac{1}{2}$	$-1,-\frac{1}{2}$	$-1,+\frac{1}{2}$
$1,-\frac{1}{2}$	$\pm il$	l	$\pm il$	l
$1,+\frac{1}{2}$	$-l$	$\pm i(l-1)$	$-l$	$\pm il$
$-1,-\frac{1}{2}$	$\pm il$	l	$\pm i(l-1)$	l
$-1,+\frac{1}{2}$	$-l$	$\pm il$	$-l$	$\pm il$

TABLE V. Full parity-conserving helicity amplitudes for ρ-N scattering via an antisymmetric Lorentz tensor near $(-u)=0$. [The amplitudes in which the ρ's polarization is zero contain extra factors of $(-u)$ and are not given.] The amplitudes are a combination of $\mathcal{A}^{\pm}$ and $\mathcal{V}^{\pm}$ as given by Eq. (3.18).

λ_f $(\epsilon,\bar{u})$ \ λ_i (ϵ',u)	$1,-\frac{1}{2}$	$1,+\frac{1}{2}$	$-1,-\frac{1}{2}$	$-1,+\frac{1}{2}$
$1,-\frac{1}{2}$	$\pm 2il$	0	0	$-2l$
$1,+\frac{1}{2}$	0	$\pm i(2l-1)$	$2l$	0
$-1,-\frac{1}{2}$	0	$-2l$	$\pm i(2l-1)$	0
$-1,+\frac{1}{2}$	$2l$	0	0	$\pm 2il$

$\alpha=\frac{1}{2}$ $(l=0)$ shows that the amplitude must choose nonsense in order to eliminate the pole in the $F_{3/2,3/2}$ amplitudes. Thus there is a zero in the residues of the $F_{1/2,1/2}$ amplitude and there are no $M=\frac{3}{2}$ particles[10] with spin less than $\frac{3}{2}$.

The symmetric Lorentz coupling leads to an $M=\frac{1}{2}$ trajectory which favors ρ's of zero helicity, while the antisymmetric coupling leads to an $M=\frac{3}{2}$ trajectory which favors ρ helicities of ± 1. The $M=\frac{3}{2}$ trajectory requires that the external particles have a combined spin of $\frac{3}{2}$. None of the presently identified baryon trajectories are of this type. In order to identify an $M=\frac{3}{2}$ trajectory one would have to look at reactions like

$$\pi N \to \rho\Delta\,,$$

$$\gamma N \to \rho N$$

and see if ρ helicities of ± 1 are favored in the backward hemisphere. At high energies, where the contribution of order s^{-1} would be relatively small, one could determine whether $M=\frac{3}{2}$ trajectories contribute to the scattering amplitude by looking at the density matrix of the final-state particles. Up to angular momentum factors of $(-u)$, the $\lambda_f\lambda_i=(\pm\frac{3}{2},\mp\frac{3}{2})$ amplitudes, and the $\lambda_f\lambda_i=(\pm\frac{1}{2},\mp\frac{1}{2})$ amplitudes of an $M=\frac{3}{2}$ trajectory would be roughly equal. For an $M=\frac{1}{2}$ trajectory the $(\pm\frac{3}{2},\mp\frac{3}{2})$ amplitudes would be suppressed.

High-energy polarization measurements would be crucial in determining whether the antisymmetric Lorentz coupling does occur.

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APPENDIX

This Appendix briefly discusses the projection operator for a fermion of arbitrary half-integral spin. The formulas for contracting it with external factors of angular momentum and spin are also given. The spin-J projection operators are discussed in detail by Fronsdal[11] and Steele.[4]

The spin-J wave function is defined as an irreducible representation of the Lorentz group.[10] The spin-J $(J=l+\frac{1}{2})$ wave function is constructed by taking the direct product of l spin-1 wave functions and a spin-$\frac{1}{2}$ spinor and projecting out the lower-spin components. The numerator of the on-mass-shell spin-J Feynman propagator is given by the spin-J projection operator. The projection operator is given by

$$\Gamma^{J}_{\alpha\mu;\nu\beta}(p^2)=(-1)^l\sum_{\lambda}\epsilon^{J,\lambda}_{\alpha\mu}\epsilon^{*J,\lambda}_{\beta\nu}$$

$$=\frac{l+1}{2l+3}\frac{\not{p}+m}{2m}\gamma_{\mu_{l+1}}\cdot\Gamma^{l+1}_{\mu;\nu}(p^2)\cdot\gamma_{\nu_{l+1}},\quad\text{(A1)}$$

where $\epsilon^{J,\lambda}_{\mu\,\alpha}$ is the spin-J wave function and $\Gamma^{l+1}_{\mu;\nu}(p^2)$ is the projection operator for integer spin $l+1$.

Contraction of the integer-spin projection operator with the external momentum vectors yields

$$(Q'_\mu)^l\cdot\Gamma^{l}_{\mu;\nu}(p^2)\cdot(Q_\nu)^l=(-1)^l\,\frac{2l+1}{p(l)}\,|Q'|^l|Q|^lP_l(z),\quad\text{(A2)}$$

where

$$z=\cos\theta=-\frac{Q'_\mu\cdot G_{\mu\nu}\cdot Q_\nu}{|Q'||Q|},\quad\text{(A3)}$$

$$G_{\mu\nu}=g_{\mu\nu}-p_\mu p_\nu/p^2,\quad\text{(A4)}$$

$$p(l)=\frac{\Gamma(2l+2)}{2^l\Gamma^2(l+1)},\quad\text{(A5)}$$

$$\mu=\mu_1\mu_2\cdots\mu_l\,.\quad\text{(A6)}$$

Since $\Gamma^{l}_{\mu;\nu}(p^2)$ is symmetric in all its indices, the scalar product can easily be calculated when one or more of the external factors are different from Q, by making the substitution

$$A_{\mu_l}(Q_\mu)^{l-1} \rightarrow \frac{1}{l}(A\cdot\partial)(Q_\mu)^l, \tag{A7}$$

where ∂ is the derivative with respect to Q_{μ_l}. Using this substitution we can calculate the matrix elements with the numerator of the spin-J Feynman propagator.

$$(Q'_\mu)^l\gamma_{\mu_{l+1}}\cdot\Gamma^{l+1}_{\mu;\nu}(p^2)\cdot\gamma_{\nu_l}(Q_\nu)^l = (l+1)^{-2}(\gamma\cdot\partial')(\gamma\cdot\partial)(Q'_\mu)^{l+1}\cdot\Gamma^{l+1}_{\mu;\nu}(p^2)\cdot(Q_\nu)^{l+1}$$
$$= (-1)^l\frac{2l+3}{(l+1)p(l)}(|Q'||Q|)^{l-1}[|Q'||Q|P_{l+1}'(z)+\not{Q}'\not{Q}P_l'(z)]. \tag{A8}$$

When the external mesons carry a unit of spin, there are two more factors different from Q which must be contracted with the propagator.

$$(Q'_\mu)^{l-1}A_{\mu_l}\gamma_{\mu_{l+1}}\cdot\Gamma^{l+1}_{\mu;\nu}(p^2)\cdot\gamma_{\nu_{l+1}}B_{\nu_l}(Q_\nu)^{l-1}\frac{(l+1)(-1)^l}{(2l+3)p(l)}$$
$$=\frac{|Q'|^{l-3}|Q|^{l-3}}{l^2(l+1)^2}(A\cdot Q'B\cdot Q\{-|Q'||Q|[P_{l+1}'''-(2l+3)P_l'']-\not{Q}'\not{Q}[P_l'''-(2l+1)P_{l-1}'']\}$$
$$-A\cdot QB\cdot Q'(|Q'||Q|P_{l+1}'''+\not{Q}'\not{Q}P_l''')+A\cdot G\cdot B(|Q'||Q|P_{l+1}''+\not{Q}'\not{Q}P_l'')|Q'||Q|$$
$$-|Q'|^2|Q|^2\gamma\cdot G\cdot A\gamma\cdot G\cdot BP_l'+(A\cdot QB\cdot Q+A\cdot Q'B\cdot Q')(|Q'||Q|P_l'''+\not{Q}'\not{Q}P_{l-1}''')$$
$$+(A\cdot Q\not{Q}'\not{B}+B\cdot Q'\not{A}\not{Q})P_l''|Q'||Q|-(A\cdot Q'\not{Q}'\not{B}|Q|^2+B\cdot Q\not{A}\not{Q}|Q'|^2)P_{l-1}''). \tag{A9}$$

These formulas are used for calculating the matrix elements in the text.

As was first emphasized by Durand,[1] the off-mass-shell Feynman propagator for integer spins carries lower-spin components. One takes the propagator off mass shell by making the substitution

$$G_{\mu\nu}=g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\rightarrow g_{\mu\nu}-\frac{p_\mu p_\nu}{m^2(J)}. \tag{A10}$$

When the propagator is written as $\Gamma^l_{\mu;\nu}(m^2)$, we mean that the p^2 in every factor of $G_{\mu\nu}$ is replaced by $m^2(J)$.

For half-integer Feynman propagators the off-mass-shell continuation is more difficult.[4] When one makes the normal continuation, the spin-J propagator also picks up an extra spin-J component in addition to the lower-spin components. Steele[4] has shown how to make an off-mass-shell continuation which does not have the extra spin-J component. This continuation has the effect of modifying the coupling of the secondary trajectories. In this calculation, we are only working to leading order in the spin and do not need to consider the complications in the secondary trajectories which arise from the off-mass-shell continuation.

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