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INCLUSIVE PHOTO- AND ELECTROPRODUCTION OF PIONS IN THE TRIPLE REGGE REGION

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Abstract: We develop a Regge pole expansion including spin for inclusive photo- and electroproduction of pions in the photon fragmentation region and see to what extent the available data at intermediate energies can be explained in terms of Regge poles alone. It is shown that a pure Regge pole model is not adequate, although some qualitative features are reproduced.

1. Introduction

A great deal of effort has been devoted recently to high energy electroproduction experiments in which a single hadron is detected in coincidence with the outgoing electron (fig. 1) [1]

$$\mathrm{e}+\mathrm{p}\rightarrow \mathrm{e}'+\text{hadron}+\mathrm{X}, \tag{1.1}$$

where X is any hadron state, which has been summed over in the experiment. Such experiments are important if we want to understand the challenging results of the total inclusive experiments [2]

$$\mathrm{ep}\rightarrow \mathrm{e}'\mathrm{X}. \tag{1.2}$$

In the one-photon exchange approximation we can describe (1.1) by the corresponding photon reaction

$$\gamma+\mathrm{p}\rightarrow \text{hadron}+\mathrm{X}, \tag{1.3}$$

where γ stands for the virtual photon of varying mass $q^2 (q^2 < 0$, spacelike) and linear polarization ε. Then single hadron production on protons with real photons is just a special case of (1.3), with $q^2 = 0$. In this sense we shall put electroproduction in the foreground with the understanding that the relevant quantities for photoproduction will result as a special case for which $q^2 = 0$. After separation of the lepton vertex which is given by QED the cross section still depends on four invariants q^2, s, t and M^2, where $s = (p+q)^2$ is the total c.m. energy squared, $(-t)^{\frac{1}{2}}$ is the invariant momentum transfer of the virtual photon to the hadron and M is the missing mass of the unobserved system X. In this paper we shall consider only the inclusive photo- and electroproduction of pions. It is well known that inclusive cross sections behave differently in different kinematical regions; these being usually classified as the beam and target fragmentation region ($|x| \approx 1$, where $x = k/k_{\max}$ and k is the longitudinal c.m. momentum of the observed particle) and the central region ($x \approx 0$)*. Although data are available in all three regions we shall consider beam fragmentation only. This is particularly interesting in connection with the results from the single-arm experiments (1.2). If the general ideas about the photon fragmentation region are correct we might expect a different dependence of the cross sections with the photon mass q^2 in this region than has been found for the cross section for (1.2)**.

In sect. 2 the kinematics of reaction (1.1) are reviewed and the relevant structure functions defined and related to the cross sections of interest. In sect. 3 we discuss the Regge pole structure of the inclusive cross section in the photon fragmentation region, in particular its triple Regge limit. Results of numerical estimates are given in sect. 4, together with a comparison with the data. The contribution to the total inclusive structure function will also be discussed.

2. Kinematics

Let q, k and p be respectively the momenta of the incoming virtual photon, the outgoing pion and the target proton (fig. 1) and s, t, and M^2 be the following invariants $s = (p+q)^2$, $t = (q-k^2)$, $M^2 = (q+p-k)^2$. The momenta of the incoming and outgoing electrons are l and l' respectively. The lepton mass will consistently be ignored and m shall denote the target mass. The cross section for production of single pions on protons is obtained from:

$$\frac{\mathrm{d}^6\sigma}{\dfrac{\mathrm{d}^3 l'}{2l'_0}\dfrac{\mathrm{d}^3 k}{2k_0}} = \frac{2\alpha^2}{-q^2(2\pi)^4 m l_0}|T|^2, \tag{2.1}$$

where $|T|^2$ is given by

$$|T|^2 = t^{\mu\nu} W_{\mu\nu}; \tag{2.2}$$

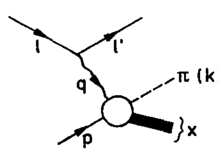

Fig. 1. The one photon exchange diagram for e+p→e′+π+anything.

* For a review of the kinematics of inclusive processes, see for example ref. [3].

** Here we are referring to the intermediate q^2 region, where the total inclusive cross section drops with q^2 slower than one expects from typical electromagnetic form factors [4].

$t_{\mu\nu}$ is the tensor coming from the lepton vertex in fig. 1:

$$t_{\mu\nu} = \tfrac{1}{2} \sum_{\text{spins}} \bar{u}(l')\gamma_\mu u(l)(\bar{u}(l')\gamma_\nu u(l))^* , \tag{2.3}$$

and $W_{\mu\nu}$ is the hadron tensor which describes the "reaction" $\gamma_\mu \mathrm{p} \to \pi \mathrm{X}$ which is defined in our case by

$$W_{\mu\nu} = \tfrac{1}{4} \sum_{\text{spin of p}} \sum_n \int \prod \mathrm{d}^4 p_j \delta(p_j^2 - m_j^2)(2\pi)^4 \, \delta^{(4)}(p+q-k-p_n)$$
$$\times \langle \mathrm{p} | J_\mu(0) | \pi(k) \mathrm{X}_n \rangle \langle \mathrm{X}_n \pi(k) | J_\nu(0) | \mathrm{p} \rangle . \tag{2.4}$$

The definition of $W_{\mu\nu}$ in (2.4) contains an extra factor $\frac{1}{2}$, so that it is the forward discontinuity of the corresponding six point function (see fig. 2).

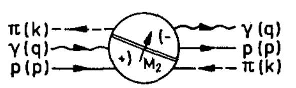

Fig. 2. The Mueller discontinuity diagram for $\gamma_v + p \to \pi +$anything.

As is usual, we factorize the cross section (2.1) into the equivalent photon spectrum [5] Γ where

$$\Gamma = \frac{\alpha}{16\pi^2 m^2 l_0^2(-q^2)} \frac{s-m^2}{1-\varepsilon}, \quad \varepsilon^{-1} = 1 + \frac{2\boldsymbol{q}^2}{-q^2} \mathrm{tg}^2(\tfrac{1}{2}\theta_e), \tag{2.5}$$

and the one-pion distribution $2k_0 \mathrm{d}^3\sigma/\mathrm{d}^3 k$ resulting from the absorption of a virtual photon with transverse polarization ε:

$$\frac{\mathrm{d}^6\sigma}{\mathrm{d}^3 k/2k_0 \mathrm{d}q^2 \mathrm{d}s \mathrm{d}\phi} = \Gamma \frac{\mathrm{d}^3\sigma}{\mathrm{d}^3 k/2k_0}, \tag{2.6}$$

where

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}^3 k/2k_0} = \frac{\alpha(1-\varepsilon)}{(2\pi)^2(s-m^2)(-q^2)} |T|^2 . \tag{2.7}$$

The experimental data is parametrized by the cross section of particular virtual photon polarization, in terms of which (2.7) is written in the form

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}^3 k/2k_0} = \frac{\mathrm{d}^3\sigma}{k\,\mathrm{d}k_0\,\mathrm{d}\cos\theta\,\mathrm{d}(\frac{1}{2}\phi)}$$
$$= \frac{1}{\pi}\left\{\frac{\mathrm{d}^2\sigma_{\mathrm{U}}}{k\,\mathrm{d}k_0\,\mathrm{d}\cos\theta} + \varepsilon\frac{\mathrm{d}^2\sigma_L}{k\,\mathrm{d}k_0\,\mathrm{d}\cos\theta} + \varepsilon\frac{\mathrm{d}^2\sigma_T}{k\,\mathrm{d}k_0\,\mathrm{d}\cos\theta}\cos 2\phi \right. \tag{2.8}$$
$$\left. + \sqrt{2\varepsilon(\varepsilon+1)}\,\frac{\mathrm{d}^2\sigma_{\mathrm{I}}}{k\mathrm{d}\,k_0\,\mathrm{d}\cos\theta}\cos\phi\right\}.$$

The term $2\mathrm{d}^2\sigma_{\mathrm{U}}/k\mathrm{d}k_0\mathrm{d}\cos\theta$ is the cross section for unpolarized transverse virtual photons, $\varepsilon\cos 2\phi\, 2\mathrm{d}^2\sigma_{\mathrm{T}}/k\mathrm{d}k_0\mathrm{d}\cos\theta$ is due to the transverse linear polarization of the photon, $\varepsilon\, 2\mathrm{d}^2\sigma_L/k\,\mathrm{d}k_0\,\mathrm{d}\cos\theta$ accounts for the longitudinal polarized photons and $\sqrt{2\varepsilon(\varepsilon+1)}\, 2\mathrm{d}^2\sigma_I/k\,\mathrm{d}k_0\,\mathrm{d}\cos\theta$ describes the interference between the longitudinal and transverse matrix element. For $q^2=0$, $2\mathrm{d}^2\sigma_U/k\,\mathrm{d}k_0\,\mathrm{d}\cos\theta$ is identical to the cross section for the absorption of unpolarized photons whereas $2\mathrm{d}^2\sigma_T/k\,\mathrm{d}k_0\,\mathrm{d}\cos\theta$ describes the difference of cross section with the linear polarization of the photon respectively parallel and perpendicular to the production plane. The various differential cross section sometimes will be also given in the Lorentz invariant forms:

$$\frac{2\,\mathrm{d}^2\sigma}{k\,\mathrm{d}k\,\mathrm{d}\cos\theta} = 8\sqrt{s}\,q\,\frac{\mathrm{d}^2\sigma}{\mathrm{d}M^2\,\mathrm{d}t} = 4\,\frac{k_0}{k_{\max}}\,\frac{\mathrm{d}^2\sigma}{\mathrm{d}x\,\mathrm{d}k_\perp^2}, \tag{2.9}$$

where k, q and k_0 are the momenta and energies in the c.m.s., $k_{\max}$ is the maximal momentum of the pion $k_{\max}=(1/2\sqrt{s})\,\lambda(s, m^2, m_\pi^2)$ and x is the Feynman variable $x=k_\parallel/k_{\max}$, $k_\parallel$ is the component of k in the direction of q.

The three cross sections defined in (2.8) can be related to special helicity structure functions

$$H^{ab} = \varepsilon_\mu^{(a)}\varepsilon_\nu^{(b)*}\,W^{\mu\nu}, \tag{2.10}$$

where ε_μ^a are the polarization vector of the virtual photon in a given polarization state $a(\pm, 0)$. Then

$$\frac{\mathrm{d}^2\sigma_{(U,L,T,I)}}{\mathrm{d}x\,\mathrm{d}k_\perp^2} = \frac{\pi k_{\max}}{2k_0}\,\frac{2\alpha}{(2\pi)^2}\,\frac{1}{s-m^2}\,(H_{++},\, H_{00},\, -H_{+-},\, -\sqrt{2}\,\mathrm{Re}\,H_{+0}). \tag{2.11}$$

To incorporate gauge invariance and to avoid unwanted kinematical singularities when making Regge expansions it is convenient not only to work with the helicity structure functions but also to introduce invariant structure functions as one does for the total inclusive cross section $\gamma\mathrm{p}\to\mathrm{X}$. A covariant decomposition of the spin averaged hadronic tensor $W_{\mu\nu}$ is

$$\begin{aligned} W_{\mu\nu} &= \tfrac{1}{4}\sum_{\text{spins}}\sum_n \langle \mathrm{p}|J_\mu(0)|\pi,\, \mathrm{X}_n\rangle\langle\pi,\, \mathrm{X}_n|J_\nu(0)|\mathrm{p}\rangle(2\pi)^4\,\delta(p+q-k-p_n) \\ &= \sum_{i=1}^{5} \Gamma_{\mu\nu}^i\, V_i(s, q^2, t, M^2), \end{aligned} \tag{2.12}$$

where the covariants are defined by

$$\begin{aligned} \Gamma_{\mu\nu}^1 &= q_\mu q_\nu - q^2 g_{\mu\nu}, \\ \Gamma_{\mu\nu}^2 &= (p_\mu q_\nu + p_\nu q_\mu)\,pq - p_\mu p_\nu q^2 - g_{\mu\nu}(pq)^2, \\ \Gamma_{\mu\nu}^3 &= (k_\mu q_\nu + k_\nu q_\mu)\,kq - k_\mu k_\nu q^2 - g_{\mu\nu}(kq)^2, \\ \Gamma_{\mu\nu}^4 &= (p_\mu k_\nu + k_\mu p_\nu)\,qk\,pq - k_\mu k_\nu (pq)^2 - p_\mu p_\nu (kq)^2, \\ \Gamma_{\mu\nu}^5 &= i\left[\left(k_\mu - \frac{kq}{q^2}q_\mu\right)\left(p_\nu - \frac{pq}{q^2}q_\nu\right) - \left(p_\mu - \frac{pq}{q^2}q_\mu\right)\left(k_\nu - \frac{kq}{q^2}q_\nu\right)\right], \end{aligned} \tag{2.13}$$

so that $q^{\mu} W_{\mu\nu} = 0$ and since $(W_{\mu\nu})^* = W_{\nu\mu}$ all five structure functions are real for $q^2 < 0$. The four cross sections $d\sigma_x (x = \mathrm{U, L, T}$ and I) depend only on for structure functions V_1, V_2, V_3 and V_4.

The relation between the H^{ab} and the V_i is given in table 1.

Table 1

H^{ab} \ V_i	1	2	3	4
$++$	q^2	ν^2	$(kq)^2 - \frac{1}{2}k_{\perp}^2 q^2$	$-\frac{1}{2}k_{\perp}^2 \nu^2$
$+-$			$\frac{1}{2}q^2 k_{\perp}^2$	$\frac{1}{2}k_{\perp}^2 \nu^2$
$+0$			$-\sqrt{-\frac{1}{2}q^2}\, k_{\perp} a$	$\sqrt{-\frac{1}{2}q^2}\, k_{\perp} \nu b$
00	$-q^2$	$-q^2 m^2$	$-q^2(k_{\perp}^2 + m_{\pi}^2)$	$q^2 b^2$

In this table the variables $k_{\perp}$ and k are the transverse and longitudinal momentum of the outgoing pion in the c.m.s. and $\nu = pq$. The relation to the variables s, t and M^2 is

$$\begin{aligned} s &= m^2 + q^2 + 2\nu, \\ t &= q^2 + m_{\pi}^2 - 2q_0 k_0 + 2|q|k, \\ M^2 &= s - 2\sqrt{s}\, k_0 + m_{\pi}^2, \end{aligned} \tag{2.14}$$

where q_0 is given in the c.m. frame:

$$\begin{aligned} q_0 &= \frac{1}{2\sqrt{s}}(s - m^2 + q^2), \quad p_0 = \frac{1}{2\sqrt{s}}(s + m^2 - q^2), \\ |q|^2 &= q^2 - q_0^2, \quad k_0^2 = m_{\pi}^2 + k^2 + k_{\perp}^2, \\ a &= k_0|q| - kq_0, \quad b = k_0|q| + kp_0. \end{aligned} \tag{2.15}$$

For k we also use the Feynman variable $x = k/k_{\max}$.

3. The Regge limit

For a fixed missing mass M^2 the process

$$\gamma \mathrm{p} \to \pi \mathrm{X} \tag{3.1}$$

may be considered as a quasi two-body reaction. One therefore expects the corresponding cross section to be described in terms of the usual Regge pole expansions involved in two body reactions. The detailed way in which Regge expansions arise in the case of inclusive distributions has been much discussed recently [6]. However a number of points have still to be clarified. We shall discuss these formal problems in the case of electroproduction elsewhere [8], where we also discuss the inclusion of Regge cuts. Here we concentrate on the phenomenological aspects of a pure Regge pole model.

Thus in the kinematic region of large s and q^2, M^2 and t being fixed and small, i.e.

$$s,\ -u \gg m^2,\ t,\ q^2,\ M^2,$$

we shall assume that the process (3.1) is dominated by Regge exchanges in the $\gamma\bar{\pi}$ channel (see fig. 3). Possible candidates for charged pion production are: π, ρ, A_2, A_1 and B exchange whereas for neutral pion production we have ω, ϕ, ρ and B exchange. In this kinematic region if we put spin complications aside for the moment the inclusive cross section has the form [6, 7]

$$s\frac{d^2\sigma}{dt\,dM^2} = \frac{1}{s-m^2}\sum_{i,j} \beta_{\pi i\gamma}(t,q^2)\,\beta_{\pi j\gamma}(t,q^2)\,\xi_{\alpha_i}(t)\,\xi^*_{\alpha_j}(t)\left(\frac{s}{M^2}\right)^{\alpha_i(t)+\alpha_j(t)} \times A_{jp,\,ip}(M^2,t), \tag{3.2}$$

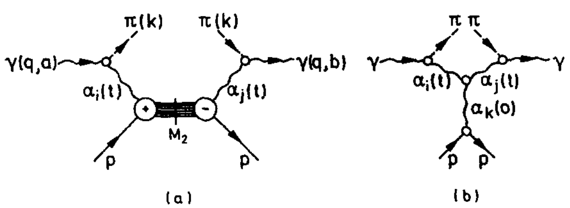

Fig. 3. (a) A Regge pole contribution to the $\gamma_v + p \to \pi +$ anything in the photon fragmentation region. (b) Triple Regge contribution.

In (3.2) $\beta_{\pi i\gamma}(t,q^2)$ is the coupling of the reggeon i to the $\gamma\pi$ system, which depends on the mass $\sqrt{q^2}$ of the virtual photon and the mass $\sqrt{t}$ of the reggeon i, $\xi_i(t)$ its signature factor

$$\begin{aligned}\xi_{\alpha_i}(t) &= \alpha_i'\,\Gamma(-\alpha_i(t))\,\tfrac{1}{2}(1\pm e^{-i\pi\alpha_i(t)}) \\ &= \frac{\pi\alpha_i'}{\sin\pi\alpha_i(t)\,\Gamma(\alpha_i(t)+1)}\,\tfrac{1}{2}(1\pm e^{-i\pi\alpha_i(t)}),\end{aligned} \tag{3.3}$$

and $s^{\alpha_i(t)}$ its Regge propagator. $A_{jp,\,ip}(M^2,t)$ is the forward imaginary part of the amplitude for the reaction (reggeon i)+p→(reggeon j)+p, which depends on the mass $\sqrt{t}$ of the reggeon and $M^2 = (p_j + p)^2$.

We notice that in this form the inclusive cross section if given in terms of the variables q^2, t, M^2 and s, the q^2-dependence appears only through the coupling of the reggeons to the $\gamma\pi$ system. Therefore in the photon fragmentation region the q^2 dependence of the inclusive cross section should be similar to the q^2 dependence of particular two-body reactions as for instance $\gamma p \to \pi N$ or $\gamma p \to \pi \Delta$ etc.

In (3.2) we considered also interference terms (i, j) between different Regge exchanges. It follows immediately from (3.2) that the interference terms of exchanges with exchange degenerate trajectories cancel if they have opposite signature, independent of the strength of the residues $\beta_{\pi i \gamma}(t, q^2)$. This way the interference terms of ρ and A_2 and π and B respectively drop out in (3.2). To simplify the discussion we shall in the following consider only non-interfering Regge exchanges. We shall at the end briefly mention situations, in which the interference terms might play a role.

Together with s/M^2 also M^2 is large compared to m^2, q^2 and t we have the so-called "triple regge region" [6, 7]. This is kinematically possible if s is sufficiently large. In this case the diagram in fig. 3a reduces further to the triple regge graph (fig. 3b). It amounts to approximating the reggeon-particle amplitude by a Regge expansion

$$A_{ip,\,ip}(M^2, t) = \beta_{Pp\bar{p}}\, g_{Pii}(t)(M^2)^{\alpha_P(0)} + \beta_{Rp\bar{p}}\; g_{Rii}(t)(M^2)^{\alpha_R(0)}. \tag{3.4}$$

The first term in (3.4) is the Pomeranchukon exchange. It produces the scale invariant contribution to the inclusive cross section, whereas the second term describes the scale breaking contribution.

One might expect (3.4) also to be a good approximation for smaller M^2 in the sense that (3.4) describes the reggeon-particle amplitude for low M^2 on the average (similar to the Dolen-Horn-Schmidt [9] type of semilocal duality hypothesis for two-particle reactions). However this need not to be the case since Regge particle amplitudes are known to satisfy fixed pole sum rules [10]. One should therefore distinguish (3.2) from the ansatz (3.4). This point will be elaborated on in ref. [8].

For one particular Regge pole exchange (i) in the $\gamma\pi$ channel the combination of (3.4) and (3.2) yields the following triple Regge expression for the inclusive one-particle distribution:

$$s\frac{d^2\sigma}{dt\,dM^2} = \frac{1}{s-m^2}\,(\beta_{\pi i\gamma}(t,q^2))^2\,|\xi_i(t)|^2\left(\frac{s}{M^2}\right)^{2\alpha_i(t)} \times \{\beta_{Pp\bar{p}}\, g_{Pii}(t)(M^2)^{\alpha_P(0)} + \beta_{Rp\bar{p}}(t)(M^2)^{\alpha_R(0)}\}. \tag{3.5}$$

Usually experimental data are presented in such a way that the inclusive cross section is plotted as a function of x for fixed $k_\perp$ and s and not as a function of M^2 for fixed t and s which would be much more transparent from the theoretical point of view. The relation between M^2 and t and x and $k_\perp$ respectively, is asymptotically ($s \to \infty$):

$$M^2 = s(1-x),$$
$$t = -[x(1-x)(-q^2)+(1-x)m_\pi^2+k_\perp^2]/x. \tag{3.6}$$

For photoproduction ($q^2 = 0$) and for electroproduction with q^2 small compared to $k_\perp^2/x(1-x)$ the main x-dependence comes from the factor $s^{2\alpha_i(t)}(M^2)^{\alpha_K(0)-2\alpha_i(t)}$ with $K = \text{P}$ or $K = \text{R}$ respectively. We have for the two types

$$K \equiv \text{P}, \quad s\frac{\mathrm{d}^2\sigma}{\mathrm{d}t\,\mathrm{d}M^2} \sim (1-x)^{1-2\alpha_i(t)}, \tag{3.7a}$$

$$K \equiv \text{R}, \quad s\frac{\mathrm{d}^2\sigma}{\mathrm{d}t\,\mathrm{d}M^2} \sim s^{-\frac{1}{2}}(1-x)^{\frac{1}{2}-2\alpha_i(t)}, \tag{3.7b}$$

where we assumed that $\alpha_\text{P}(0) = 1$ and $\alpha_\text{R}(0) = 0.5$. The magnitude of the R-term in (3.5) could be studied by comparing data with eq. (3.5) as a function of s for fixed x and $k_\perp^2$. This has been done by Moffeit et al. [11] for the over $k_\perp^2$ integrated distribution for $\gamma\text{p}\to\pi^-\text{X}$ with $E_\gamma = 2.8$, 4.7 and 9.3 GeV. Their data are consistent with scaling i.e. only a small contribution of the Regge-term compared to the Pomeranchuk-term. They also made a fit of their data to eq. (3.7a) and determined the trajectory as a function of t. The trajectory found is compatible with a Regge trajectory of slope 1 GeV^{-2} and intercept of zero. Of course, the candidate for this trajectory is the pion trajectory. Therefore inclusive photoproduction data for charged pions seem to be consistent with pion exchange and Pomeranchuk exchange in the respective t-channels. Of course, it is not ruled out that the tensor and vector trajectories still make important contributions. For example, near $x = 1$ the term (3.7b) diverges like $(1-x)^{-\frac{1}{2}-2\alpha'_\text{R}t}$ for $\alpha_\text{R} = \alpha_\rho = \alpha_{\text{A}_2} = 0.5+\alpha'_\text{R}t$. It is one of the purposes of this paper to find out whether we can expect such contributions in the photo- and electroproduction of charged pions.

The next step is to find out which contributions the exchanges π, $\rho(\omega)$ and A_2 in the $\gamma\pi$ channel make to the four cross sections $\mathrm{d}\sigma_\text{U}$, $\mathrm{d}\sigma_\text{L}$, $\mathrm{d}\sigma_\text{T}$ and $\mathrm{d}\sigma_\text{I}$. The Regge pole expansions are constructed so that, when one extrapolates to the particle poles in the $\gamma\bar{\pi}$ channels, the appropriate elementary spin structure emerges. To this end one makes use of the invariant structure functions V_i defined above and for example the gauge invariance of the pion contribution is assured, if we identify its contribution with V_4. The latter reduces to the Drell formula [12] for elementary pion exchange and preserves the kinematic constraints on H_{+0} and H_{00} in the limit $q^2 \to 0$. [In fact this prescription is similar to the usual gauge completion of the pion exchange graph in exclusive photoproduction.] The results for the above three exchange contributions will be listed below.

3.1. Pion exchange

We have

$$
\begin{aligned}
\tfrac{1}{2}(H_{++}+H_{+-}) &= 0,\\
\tfrac{1}{2}(H_{++}-H_{+-}) &= F_\pi^2(q^2)\,|\xi_{\alpha_\pi}(t)|^2\left(\frac{s}{M^2}\right)^{2\alpha_\pi(t)} \mathrm{Im}\ T_{\pi N}(t,M^2)2k_\perp^2,\\
H_{+0} &= F_\pi^2(q^2)\,|\xi_{\alpha_\pi}(t)|^2\left(\frac{s}{M^2}\right)^{2\alpha_\pi(t)} \mathrm{Im}\ T_{\pi N}(t,M^2)\left(-2\sqrt{2}\sqrt{-q^2}\,k_\perp\frac{b}{\nu}\right),\\
H_{00} &= F_\pi^2(q^2)\,|\xi_{\alpha_\pi}(t)|^2\left(\frac{s}{M^2}\right)^{2\alpha_\pi(t)} \mathrm{Im}\ T_{\pi N}(t,M^2)\left(-4q^2\frac{b^2}{\nu^2}\right).
\end{aligned}
\tag{3.8}
$$

In (3.8) the $T_{\pi\mathrm{N}}$ is the off-shell pion-nucleon forward scattering amplitude for pions with mass t and averaged over the helicities of the nucleon. We notice that the pion exchange contributes to the three cross sections $\sigma_{\|}$, σ_{I} and σ_{L}. It does not give a contribution to the cross section for photons polarized perpendicular to the scattering plane. This result is in line with the expectation that exchanges with odd normalities do not contribute to $\sigma_\perp$. The result (3.8) if inserted into (2.11) agrees with Drell's formula [12] for inclusive pion photoproduction if the limit to elementary pion exchange is taken. Furthermore we notice that $H_{++}\sim H_{+-}\sim\sigma_{\|}$ vanishes for $k_\perp\to 0$ like $k_\perp^2$. From spin conservation this cross section does not need to vanish in this limit, whereas H_{+0} must vanish like $k_\perp$ and H_{+-} like $k_\perp^2$ for kinematic reasons. The behaviour of $\sigma_{\|}$ was pointed out by Drell and Sullivan when discussing two body electro- and photoproduction amplitudes involving elementary or Regge exchange [13].

3.2. Rho (omega) exchange

We parametrize the forward ρ-nucleon scattering amplitude averaged over the nucleon helicities by

$$
\mathrm{Im}\ T_{\mu\nu}^{\rho N} = V_1\,p_\mu p_\nu - V_2\,g_{\mu\nu}\,, \tag{3.9}
$$

so that the helicity amplitudes in the ρN c.m.s. for virtual ρ's with mass $t<0$ are

$$
\begin{aligned}
\mathrm{Im}\ T_{++}^{\rho N} &= V_2\,,\\
\mathrm{Im}\ T_{00}^{\rho N} &= -V_2-[\lambda(M^2,m^2,t)/4t]\,V_1\,.
\end{aligned}
\tag{3.10}
$$

It is instructive first to calculate the helicity cross section for elementary ρ-exchange and then proceed to the Regge expressions. The result is

$$H_{++} = \frac{F^2_{\rho\pi\gamma}(q^2)}{(t-m_\rho^2)^2}\{V_2[-\tfrac{1}{2}k_\perp^2 q^2+(kq)^2-q^2m_\pi^2]+V_1[\tfrac{1}{2}k_\perp^2((pq)^2-q^2m^2)]\},$$

$$H_{+-} = \frac{F^2_{\rho\pi\gamma}(q^2)}{(t-m_\rho^2)^2}\{q^2V_2+V_1[(pq)^2-q^2m^2]\}\tfrac{1}{2}k_\perp^2,$$

$$H_{+0} = \frac{F^2_{\rho\pi\gamma}(q^2)}{(t-m_\rho^2)^2}\sqrt{\tfrac{1}{2}(-q^2)}\,k_\perp\{-aV_2\}, \qquad (3.11)$$

$$H_{00} = \frac{F^2_{\rho\pi\gamma}(q^2)}{(t-m_\rho^2)^2}(-q^2)k_\perp^2V_2.$$

In eq. (3.11) we observe the following facts. All four cross sections vanish for $k=q$ (then $k_\perp=0$) which is in agreement with the statement of Drell and Sullivan for natural parity exchange. Second in the high energy approximation $s\to\infty$ we have $H_{++}=H_{+-}$ and $H_{+0}=H_{00}=0$. This result is quite general and is valid for all trajectories with positive normality. In (3.11) $F_{\rho\pi\gamma}(q^2)$ stands for the transition form factor $\gamma(q^2)\to\rho\pi$.

It is simple now to write down the Regge expressions for the cross sections H_{ab}:

$$\tfrac{1}{2}(H_{++}+H_{+-}) = F^2_{\rho\pi\gamma}(q^2)\,|\xi_{\alpha\rho}(t)|^2\,M^4\,V_1(M^2,t)\left(\frac{s}{M^2}\right)^{2\alpha_\rho(t)}\tfrac{1}{8}k_\perp^2,$$

$$\tfrac{1}{2}(H_{++}-H_{+-}) = H_{+0} = H_{00} = 0. \qquad (3.12)$$

From (3.12) we see that the Pomeranchuk term in V_1 is proportional to M^{-2}, so that $H_{++}=H_{+-}$ have the appropriate behaviour M^2. In order that H_{++} is dominant for $s\to\infty$ that means $H_{++}\sim s^{2\alpha_\rho(t)}$ we must have $V_1\neq 0$. This is possible only, if $-\mathrm{Im}\ T_{00}^{\rho\mathrm{N}}\neq \mathrm{Im}\ T_{++}^{\rho\mathrm{N}}$. Therefore in order that the vector trajectories contribute to the inclusive cross section the off-shell ρN scattering amplitude is not allowed to obey helicity independence in the ρN c.m.s. [14]. Furthermore it should be noticed that the cross section for $\rho(\omega)$ exchange is proportional to $k_\perp^2$.

From (3.10) we see that $\mathrm{Im}\ T_{00}^{\rho\mathrm{N}}$ has a singularity for $t\to 0$ if V_1 does not vanish like t. It is not clear whether we should require that $V_1\sim t$ for $t\to 0$ also, since V_1 describes in (3.12) the discontinuity for reggeon-proton scattering in the forward direction.

The formulas (3.11) can also be used for γ-exchange (inclusive Primakoff effect). We only have to replace $F_{\rho\pi\gamma}(q^2)$ by $F_{\gamma\pi\gamma}(q^2)$ and the ρ-propagator $(t-m_\rho^2)^{-1}$ by t^{-1}, the photon propagator.

3.3. B-exchange

We parametrize the forward B-nucleon scattering amplitude averaged over the nucleon helicities by

$$\mathrm{Im}\ T_{\mu\nu}^{\mathrm{BN}} = U_1 p_\mu p_\nu - U_2 g_{\mu\nu}, \qquad (3.13)$$

and the $\gamma B\pi$-vertex by

$$(2\pi)^3 \langle \pi(k)\, B(P) | J_\mu(0) | 0 \rangle = g_{\pi B\gamma}(q^2)(Pq\, g_{\mu\nu} - P_\mu q_\nu), \tag{3.14}$$

with $P = q - k$. In principle (3.14) should contain a second coupling which accounts for the d-wave part of the transition $B \to \pi\gamma$. We neglect this part because very little is known about these coupling constants [15].

Then we obtain for the cross sections in the limit $s \to \infty$ and for B-Regge exchange

$$\begin{aligned}
&\tfrac{1}{2}(H_{++} + H_{+-}) = 0, \\
&\tfrac{1}{2}(H_{++} - H_{+-}) = g^2_{\pi B\gamma}(q^2)\, |\xi_{\alpha_B}(t)|^2 M^4 U_1(M^2, t) \left(\frac{s}{M^2}\right)^{2\alpha_B(t)} \tfrac{1}{8} k_\perp^2, \\
&H_{+0} = g^2_{\pi B\gamma}(q^2)\, |\xi_{\alpha_B}(t)|^2 M^4 U_1(M^2, t) \left(\frac{s}{M^2}\right)^{2\alpha_B(t)} \frac{\sqrt{-q^2}\, k_\perp}{4\sqrt{2}\, qk} \left(a + (q^2 - qk)\frac{b}{\nu}\right), \\
&H_{00} = g^2_{\pi B\gamma}(q^2)\, |\xi_{\alpha_B}(t)|^2 M^4 U_1(M^2, t) \left(\frac{s}{M^2}\right)^{2\alpha_B(t)} \frac{-q^2}{4qk} \\
&\qquad \times \left(qk - (k_\perp^2 + m_\pi^2) - (q^2 - qk)\frac{b^2}{\nu^2}\right);
\end{aligned} \tag{3.15}$$

H_{+0} and H_{00} can be simplified for $s \to \infty$ if we use the variable x i.e.

$$\begin{aligned}
&H_{+0} = g^2_{\pi B\gamma}(q^2)\, |\xi_{\alpha_B}(t)|^2 M^4 U_1(M^2, t) \left(\frac{s}{M^2}\right)^{2\alpha_B(t)} \tfrac{1}{4}\sqrt{-\tfrac{1}{2}q^2}\, k_\perp (1 - x), \\
&H_{00} = g^2_{\pi B\gamma}(q^2)\, |\xi_{\alpha_B}(t)\, M^4 U_1(M^2, t) \left(\frac{s}{M^2}\right)^{2\alpha_B(t)} (-q^2)\tfrac{1}{4}(1 - x)^2 .
\end{aligned} \tag{3.16}$$

The result $H_{++} = -H_{+-}$ agrees with the general statement that trajectories with negative normality do not contribute to $d\sigma_\perp$. Furthermore we observe again that H_{++} is proportional to $k_\perp^2$.

3.4. A_2 exchange

The forward A_2-nucleon off-shell scattering amplitude will be parametrized by

$$\begin{aligned}
\mathrm{Im}\, T^{A_2 N}_{\mu\nu,\rho\sigma} &= W_1\, p_\mu p_\nu p_\rho p_\sigma \\
&+ W_2 (g_{\mu\rho} p_\nu p_\sigma + g_{\mu\sigma} p_\nu p_\sigma + g_{\nu\rho} p_\mu p_\sigma + g_{\nu\sigma} p_\mu p_\rho) \\
&+ W_3 (g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho}).
\end{aligned} \tag{3.17}$$

The $\gamma A_2 \pi$ vertex has the form

$$(2\pi)^3 \langle \pi(k)\, A_2(q-k)\,|J_\mu(0)|\,0\rangle = F_{\pi A_2 \gamma}(q^2)\,\varepsilon_{\mu\alpha\beta\gamma}\, q^\alpha k^\beta k^\delta\,, \tag{3.18}$$

where γ and δ are the indices of the A_2 polarization vector.

Asymptotically ($s \to \infty$) only the part proportional to W_1 contributes. In this limit we have for the A_2-Regge exchange

$$\tfrac{1}{2}(H_{++} + H_{+-}) = F^2_{\pi A_2 \gamma}(q^2)\,|\xi_{\alpha_{A_2}}(t)|^2\, M^8\, W_1(M^2, t)\left(\frac{s}{M^2}\right)^{2\alpha_{A_2}(t)}$$

$$\times \tfrac{1}{8} k_\perp^2 \left(\frac{pk}{pq} - \frac{qk - m_\pi^2}{m_{A_2}^2}\left(1 - \frac{pk}{pq}\right)\right)^2, \tag{3.19}$$

$$\tfrac{1}{2}(H_{++} - H_{+-}) = H_{+0} = H_{00} = 0.$$

The last factor in (3.19) is for $s \to \infty$ equal to

$$\left(x - \frac{qk - m_\pi^2}{m_{A_2}^2}(1-x)\right)^2,$$

which is roughly proportional to x^2 for moderate q^2 and $k_\perp^2$. Of course, also this contribution is proportional to $k_\perp^2$ and obeys the rules for natural parity exchange.

The vanishing of H_{++} with $k_\perp^2$, follows necessarily in a pure Regge pole model, since it predicts for symmetry reasons that $H_{++} = \pm H_{+-}$ and H_{+-} vanishes with $k_\perp^2$ for kinematic reasons. If however $H_{++} \sim \mathrm{d}\sigma_U$ does not vanish at $k_\perp^2 = 0$, as seems to be the case experimentally, then necessarily $\mathrm{d}\sigma_U = \mathrm{d}\sigma_\perp = \mathrm{d}\sigma_\parallel$ at $k_\perp^2 = 0$. The vanishing with $k_\perp^2$ of H_{++} and H_{+-} is completely similar to the vanishing with t in a conventional Regge pole model for exclusive photoproduction. There one took, as a possible solution conspiring Regge exchanges, the idea being that the conspirators contribute in the same way, but have opposite normality. Of course Regge cuts have the same effect, since they, in general, do not correspond to the exchange of definite normality.

Up to now we have not considered interference terms resulting from different exchanges in the two $\gamma\bar{\pi}$ channels. We already mentioned that exchange degenerate trajectories do not interfere with each other. Further if one does not measure the spin of the target proton only interference terms involving the same normality are possible*. Therefore only $\rho - \omega$ interferences are possible in the case of $\gamma p \to \pi^0 X$. This can make a small difference in the π^0 distribution near $x = 1$.

* This is similar to the situation in two-body photo- and electroproduction of pions.

4. Inclusive cross sections for the Regge pole model

In this section we shall give the results for the various inclusive cross sections which have been calculated on the basis of the Regge pole model discussed in sect. 3. For this purpose we inserted the H_{ab} as given by (3.8) to (3.15) into (2.11) and computed the contributions of the different exchanges.

Let us start with the one-pion exchange term (3.8) which is the dominant Regge pole contribution for photo- and electroproduction of charged pions.

For the off-shell pion nucleon forward scattering amplitude Im $T_{\pi N}(M^2, t)$ we either take the Regge expansion (3.4), where we replaced the Regge residues by their on-shell values [17] or put in the actual measured pion nucleon total cross section at the appropriate M^2 value. With the first case we obtain the picture as plotted in fig. 4 where the invariant cross section $2k_0 d^3\sigma/d^3k$ for photoproduction ($q^2 = 0$) is shown as a function of $k_\perp^2$ from 0 up to $5m_\pi^2$ and as a function of x between $x = 0.75$ and $x = 1$. As a function of $k_\perp^2$ all distributions vanish for $k_\perp^2 = 0$, have a maximum near $k_\perp^2 = m_\pi^2$ and fall off with $k_\perp^2$ quite strongly. This strong fall-off comes from the pion propagator squared which in our case is contained in the signature factor $|\xi_{\alpha_\pi}(t)|^2$. This is proportional to $(\pi\alpha_\pi'/\sin \pi\alpha_\pi(t))^2$. Another important factor in (3.8) is

$$\left(\frac{s}{M^2}\right)^{2\alpha_\pi(t)} M^2 = s(1-x)^{1+2m_\pi^2/x} \exp\left[2k_\perp^2 \frac{\ln(1-x)}{x}\right], \tag{4.1}$$

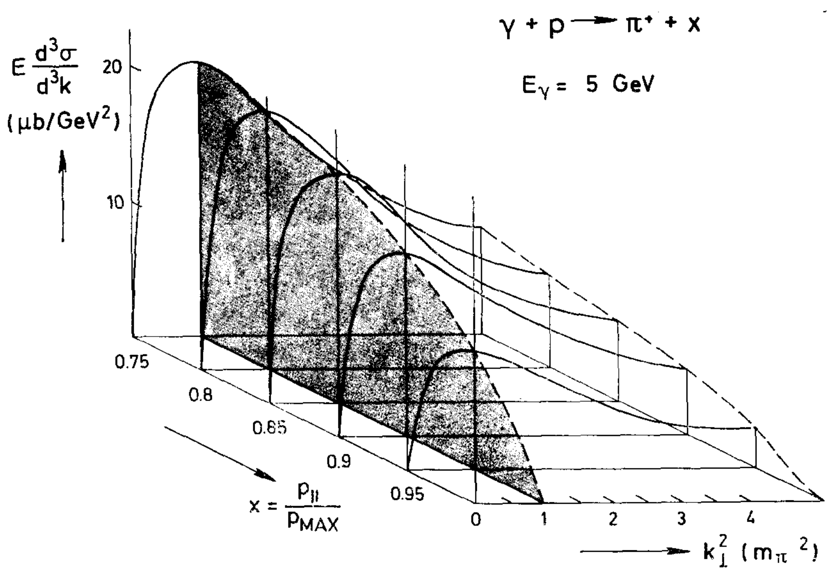

Fig. 4. $k_\perp^2$ and x distribution for $\gamma p \to \pi^+ X$, $E_\gamma = 5$ GeV and $q^2 = 0$, based on reggeized one-pion exchange and asymptotic form for $\pi^- p$ total cross section.

if we take for the pion trajectory $\alpha_\pi(t) = t - m_\pi^2$ so that $k_\perp^2$ and m_π^2 in (4.1) are measured in GeV^2. The second factor in (4.1) has the characteristic effect that the $k_\perp^2$ distribution antishrinks with decreasing x. This is quite clearly seen in fig. 4. For fixed $k_\perp^2$ the distribution in x is approximately described by (4.1) also. This distribution vanishes for $x = 1$ and increases for decreasing $x < 1$ as is seen in fig. 4 explicitly for $k_\perp^2 = m_\pi^2$. In fig. 4 we calculated the inclusive distribution for the process $\gamma p \to \pi^+ X$. In the triple Regge approximation the distribution for $\gamma p \to \pi^- X$ differs only little from the distribution in fig. 4. Since the total cross section in the intermediate energy region for $\pi^+ p$ scattering is smaller than that for $\pi^- p$, the distribution for $\gamma p \to \pi^- X$ will be somewhat smaller (roughly between 10–20%). In fig. 5 we plotted the distribution for $\gamma p \to \pi^- X$ using the second possibility for Im $T_{\pi N}(M^2, t)$. Since, even for photon energies of $E_\gamma = 5$ GeV, the missing mass M^2 is in the region of the $\Delta(1236)$ resonance for $x \sim 1$, the strong peak in $\pi^+ p$ total cross section caused by this resonance must enhance the inclusive distribution for the appropriate x value. This low mass enhancement is much larger in $\gamma p \to \pi^- X$ because the Δ-peak is largest in $\pi^+ p$ total cross section. One also notices in fig. 5 that the resonance structure is reduced with increasing $k_\perp^2$ because of the antishrinking of the $k_\perp^2$ distribution with increasing missing mass M.

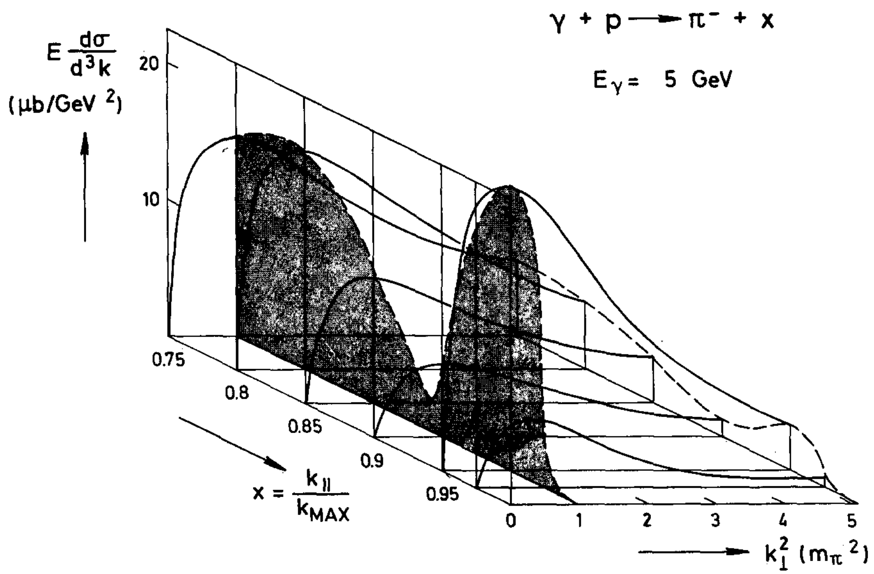

Fig. 5. $k_\perp^2$ and x distribution for $\gamma p \to \pi^- X$, $E_\gamma = 5$ GeV and $q^2 = 0$, based on reggeized one-pion exchange and measured $\pi^+ p$ total cross sections.

Before looking at other exchanges for inclusive photoproduction we shall discuss the pion exchange contribution for inclusive electroproduction. Of course, for very small q^2 the distributions for electroproduction will look very similar to the distribution for photoproduction, the longitudinal cross section (H_{00} or $\mathrm{d}\sigma_L/\mathrm{d}x\mathrm{d}k_\perp^2$) will be small and also other terms depending on q^2 are negligible. The mass scale for this similarity is the pion mass. This is clearly seen if one compares in (3.8) the formulas for H_{++} and H_{00}. In H_{00} the $k_\perp^2 \sim m_\pi^2$ in H_{++} is replaced by q^2 and in the pion propagator $\alpha_\pi(t) = t - m_\pi^2$ we have now the extra term $(-q^2)\,x(1-x)$ i.e.

$$m_\pi^2 - t = \frac{1}{x}\left((-q^2)\,x(1-x) + m_\pi^2 + k_\perp^2\right), \tag{4.2}$$

which must be small compared to m_π^2 if it should not play any role. On the contrary if q^2 is increased to values large compared to m_π^2 the inclusive electroproduction is determined predominantly by the longitudinal cross section (of course only if ε is of order one, see (2.8)), and furthermore the extra term $(-q^2)\,x(1-x)$ influences its behaviour quite strongly. We see this clearly in fig. 6, where we show the inclusive distribution for $E_\gamma = 5$ GeV, $q^2 = 0.2$ GeV^2 and $\varepsilon = 0.8$ as a function of x and $k_\perp^2$.

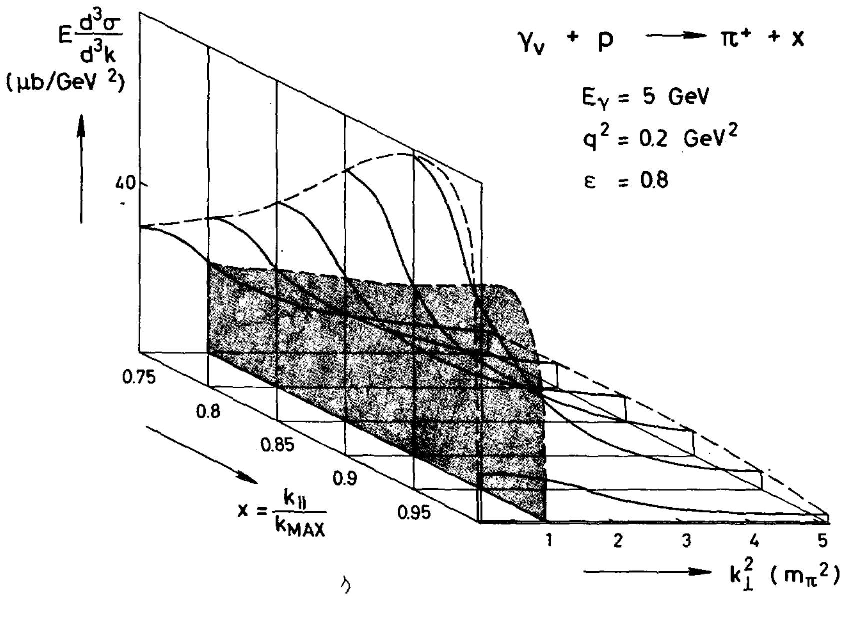

Fig. 6. $k_\perp^2$ and x distribution for $\gamma p \to \pi^+ X$, $E_\gamma = 5$ GeV, $q^2 = 0.2$ GeV^2, $\varepsilon = 0.8$, based on reggeized one-pion exchange and asymptotic form for $\pi^- p$ total cross section.

Now the cross section does not vanish any more for $k_\perp^2 = 0$ (see (3.8)). The fall-off with increasing $k_\perp^2$ is still determined by the pion propagator and by the factor $(s/M^2)^{2\alpha_\pi(t)}$ but now the pion propagator and $\alpha_\pi(t)$ change with x, because of the term

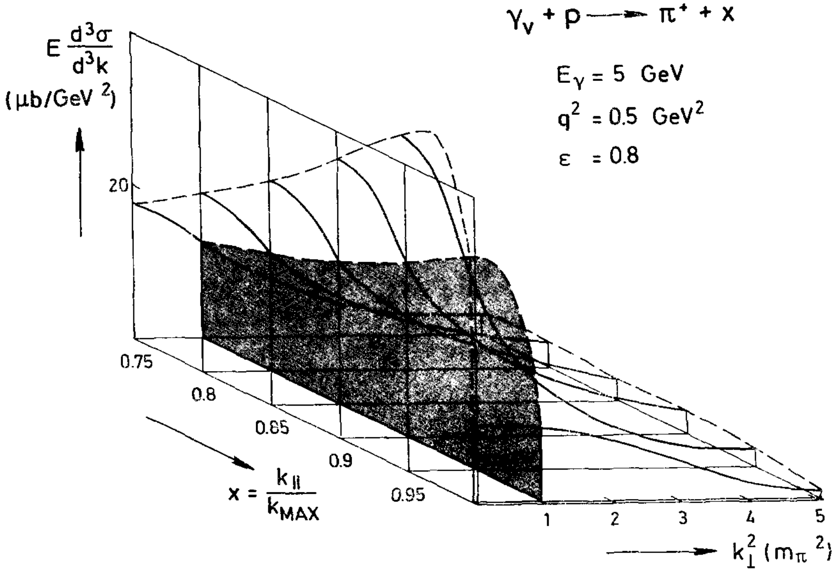

Fig. 7. As fig. 6 except $q^2 = 0.5$ GeV^2.

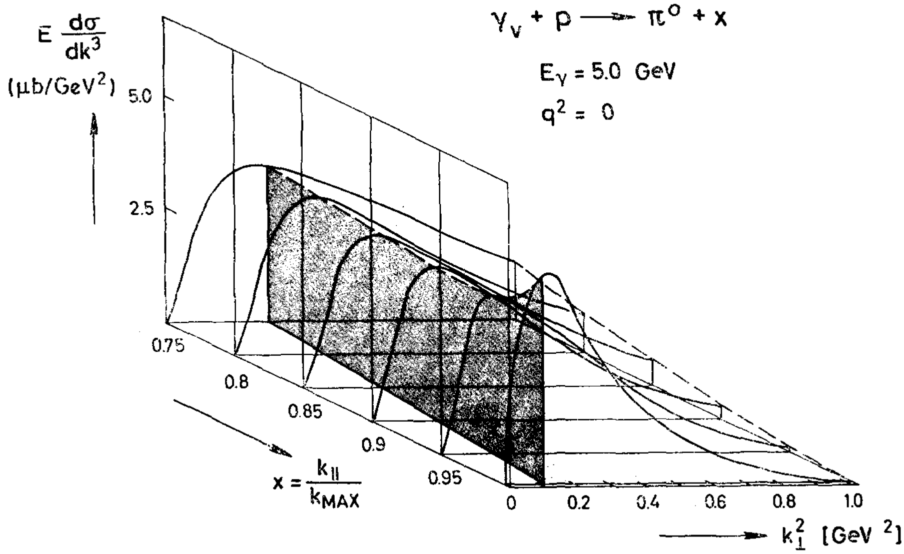

Fig. 8. $k_\perp^2$ and x distribution for $\gamma p \rightarrow \pi^0 X$, $E_\gamma = 5$ GeV, $q^2 = 0$ based on Regge ω exchange and an asymptotic form for ωN cross section.

$(-q^2)x(1-x)$ in (4.2). Therefore one has a strong fall-off with $k_\perp^2$ near $x = 0.95$ and for decreasing x the cross section changes with $k_\perp^2$ much less dramatically. Also compared to photoproduction the x-distribution is different. It still vanishes for $x = 1$, but near $x \approx 1$ it increased strongly and then falls down slowly with decreasing x. Fig. 7 exhibits the same picture for $q^2 = 0.5$ GeV^2. To get an impression of the contribution of the other exchanges we show in fig. 8 the x- and $k_\perp^2$ distribution for ω-exchange with a normalization which is given by the known $g_{\pi\omega\gamma}$ coupling constant and the function $M^4 V_1(M^2,t) = 0.5 \text{ Im } T_{\pi N}(M^2, m_\pi^2)$ in (3.12) so that it corresponds to the prediction for inclusive π^0 photoproduction. We observe the forward dip in $k_\perp^2$ and maxima of the $k_\perp^2$ distribution at $k_\perp^2$ of the order of 0.1 GeV^2 depending on x. Of course ω, ρ and A_2 exchange contribute only to $H_{++} + H_{+-}$. The distribution for ρ and A_2 exchange look similar to the one in fig. 8, only with different normalization. Since the maximum in $k_\perp^2$ occurs much further out than for pion exchange and the absolute normalization is much less than in pion exchange the ρ^- and A_2 contribution are negligible for very small $k_\perp^2 \sim m_\pi^2$ but can contribute further out near $k_\perp^2 \sim 10 m_\pi^2$ since the $k_\perp^2$ slope of these distributions is less than for the pion exchange (see fig. 11).

Now we shall compare the Regge pole model with recent experimental data [11,18]. Here we shall use the usual vector dominance argument and insert the ρ(or ω) pole for the $\gamma\pi$ reggeon form factor. In fig. 9 we show a comparison with photo- and electroproduction data (x distribution) with $k_\perp^2 \approx 0$ and photon energies near 3 GeV from the DESY groups Burfeindt et al. ($q^2 = 0$), Dammann et al. ($-q^2 \approx 0.2$ GeV^2) and Alder et al. ($-q^2 = 1.15$ GeV^2). The $q^2 = 0$ data (fig. 9a) are compared with the theory with $k_\perp^2 = m_\pi^2$, because we expect that the forward dip is filled in by Regge cut contributions. The theoretical curves for $q^2 \neq 0$ are for $k_\perp^2 = 0$, since in this case the longitudinal cross section dominates. We show two curves, a solid curve, for which the imaginary part of the pion-nucleon forward amplitude is presented by the asymptotic expression (3.4), the other, a dotted curve, where we replace the $\pi - N$ amplitude by actual data from π^-p total cross sections. The theoretical curves are for $\gamma p \to \pi^+ X$ and in fig. 9b the discrete one-neutron contribution is taken out in the dashed curve as has been done in the data analysis. In fig. 9c the data of Alder et al. contain the one-neutron contribution and it has been included also in the theoretical prediction (dashed curve). Concerning the comparison between experimental data and theoretical curves we see that the different shape of the x-distribution if we go from photoproduction $q^2 = 0$ to electroproduction with $q^2 \gtrsim 0.2$ GeV^2 is well accounted for. The one-pion exchange with asymptotic forms for the $\pi - N$ amplitude describes the data in average quite well including the single nucleon contribution. Structures for x values near one are well reproduced by the corresponding structures in π^-p total cross section. For larger q^2($\gtrsim$ 1 GeV^2) the triple Regge mechanism, we have considered, contributes appreciably only in the small region $x > 0.85$, thus leaving the large q^2 data for $x < 0.85$ unaccounted for. The $\rho - A_2$ contribution could be a candidate for the latter region, which is fairly constant in x, see fig. 9c. However, the normalization would have to be at least an order of magnitude larger than our estimate.

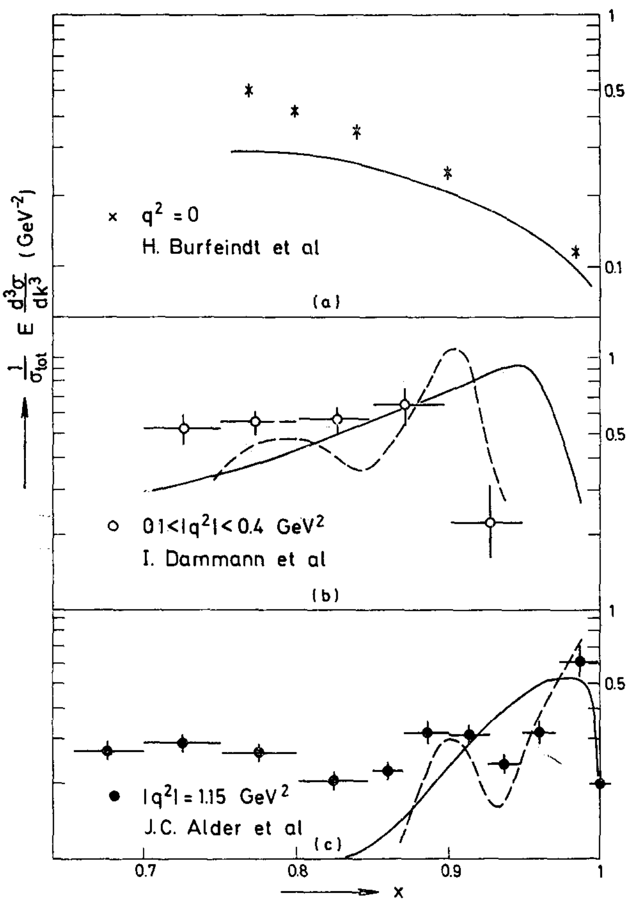

Fig. 9. Comparison of pure Regge pole model for $q^2 = 0.0$, 0.2 abd 1.0 GeV^2 with x distribution of the inclusive data for $\gamma p \to \pi^+ X$ from ref. [18]: (a) Burfeindt et al., (b) Dammann et al., and (c) Alder et al. [The theoretical curves are described in the text.]

We now turn to the $k_\perp^2$ dependence of the inclusive cross sections for fixed x, in particular the question of the forward dip. In fig. 10 we compare data from Burfeindt et al., taken for a lab. scattering angle of $\theta = 2°$ (which is equivalent to $k_\perp = 0.11x$ GeV), with our prediction. We see that the theoretical curves for $\gamma p \to \pi^- X$ and $\gamma p \to \pi^+ X$ as a function of x agree qualitatively with the x-behaviour of the experimental distribution, in particular the cross over in the π^+ and π^- distribution near $x = 0.85$ is reproduced. However the absolute normalization of the theoretical prediction is wrong by a factor around 3. The origin is the $k_\perp^2$ factor in H_{++} in eq. (3.8). We conclude from this, that the data in π^+ and π^- inclusive photoproduction do not show the forward dip. The difference in normalization would be partially remedied if one replaced $k_\perp^2$ in (3.8) by m_π^2. This suggests that the inclusion of Regge cuts might

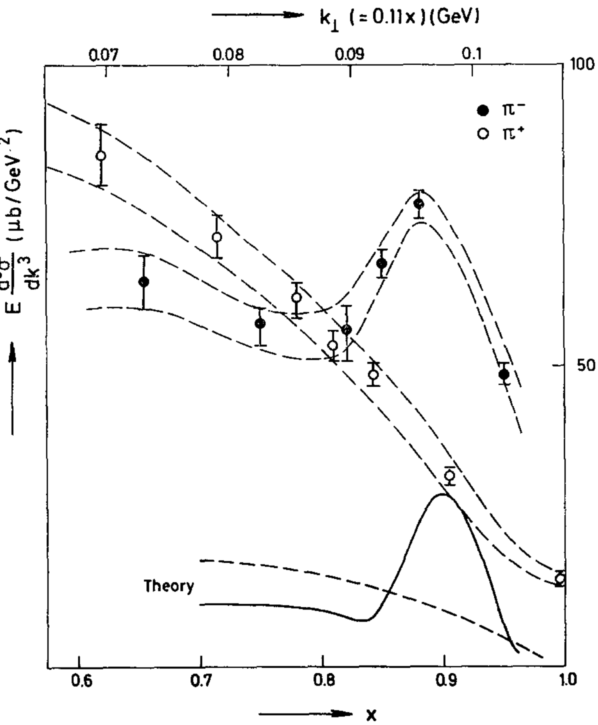

Fig. 10. Comparison of pure one-pion exchange model for $q^2 = 0.0$ and $k_\perp = 0.11\ x$ for $\gamma p \to \pi^- X$ (full curve) and $\gamma p \to \pi^+ X$ (dashed curve) with the data from Burfeindt et al. [18]. The off-shell πN cross section is replaced by measured values of $\pi^\pm p$ total cross section.

account for this change. The nonvanishing of the cross section at $k_\perp^2 = 0$ is also seen in the data of Moffeit et al. [11]. In fig. 11 we show their data at $E_\gamma = 9.3$ GeV for $\gamma p \to \pi^- X$ as a function of $k_\perp^2$ up to $k_\perp^2 \approx 1$ GeV^2 and compare them with the Regge pole model. The theoretical curves are computed for $x = 0.7$ whereas the data represent an average over the interval $0.7 < x < 1.0$. The data clearly do not show the forward dip present in the theoretical curve. But otherwise the $k_\perp^2$ dependence up to $k_\perp^2 = 0.2$ GeV^2 is well reproduced by the pion exchange contribution. For larger $k_\perp^2$ the data shows a change of slope in the $k_\perp^2$ distribution. This we can explain by adding the ρ and A_2 terms which have a smaller slope (~ 3 GeV^{-2}) similar to the ω-contribution which determines the π^0 inclusive photoproduction. Our prediction for the π^0 inclusive photoproduction is confronted with recent data of Berger et al. [19] in fig. 12. Also these data have no forward dip, in disagreement with the factor $k_\perp^2$ in the formula (3.12) (and (3.15) if we would include also B-exchange). The theoretical curve has therefore a maximum for $k_\perp^2 = 0.3$ GeV^2 which is certainly not present in the data. On the other hand the different fall-off of the $k_\perp^2$ distribution compared to the π^+ or π^- distributions (see for example in fig. 11) is well accounted for by the fall-off determined

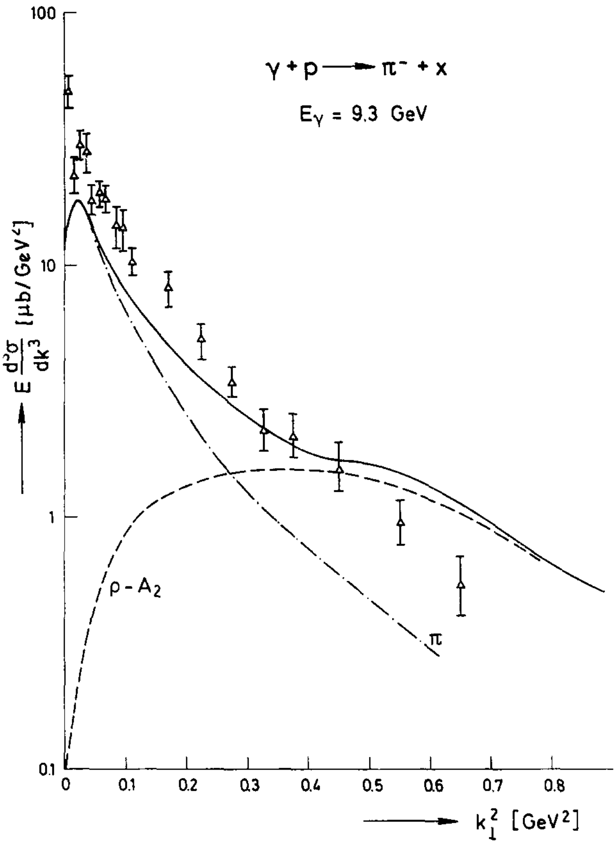

Fig. 11. Data for $\gamma p \to \pi^- X$ from Moffeit et al. [11], $E_\gamma = 9.3$ GeV and $0.7 < x < 1.0$ compared to π, ρ and A_2 exchange model with $x = 0.7$ as a function of $k_\perp^2$.

by the ω-trajectory and the absolute normalization is reproduced. This suggests that ω-exchange is responsible for the π^0 distribution in the photon fragmentation region, however apparently with a significant absorption correction. In this context we might mention that the process $\gamma p \to \pi^0 p$ (or $\gamma p \to \pi^0 n$) are rather well described by ω-exchange but here the cross section vanishes for $t \to 0$ (except for the Primakoff effect) [20], so that the necessity of other contributions as for example Regge cuts is not so apparent in this case if we look at the cross section alone.

If one approximates the $k_\perp^2$ dependence of the π^+ distributions by $k_0 d^3\sigma/d^3k \sim \sim e^{-ak_\perp^2}$, then the change in the slope parameter a as one goes from photo- to electroproduction can be understood in terms of the formula (3.8). In fig. 13 the theoretical curve corresponds to $x = 0.7$ whereas the experimental data of Dammann et al. is averaged between $0.7 < x < 0.9$ with the one-neutron contribution removed which has the effect of weighting the average more towards $x = 0.7$.

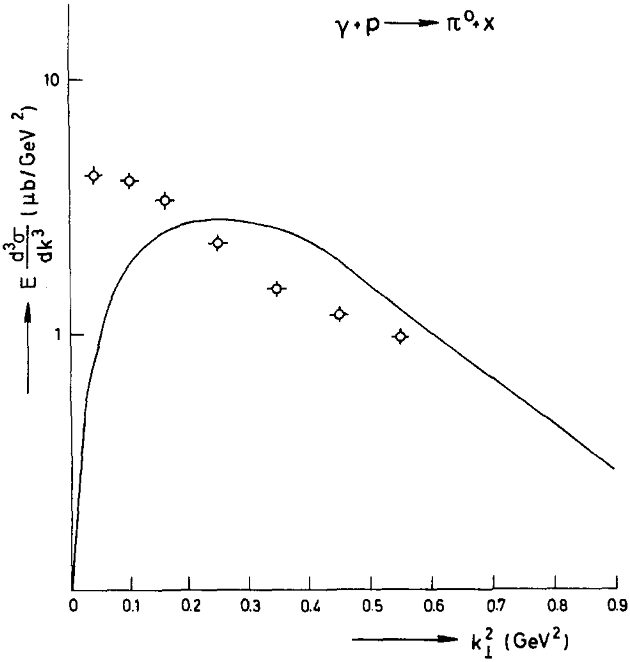

Fig. 12. Data for $\gamma p \to \pi^0 X$ from ref. [19], $E_\gamma = 6$ GeV and $x = 0.85$ compared to ω-exchange model.

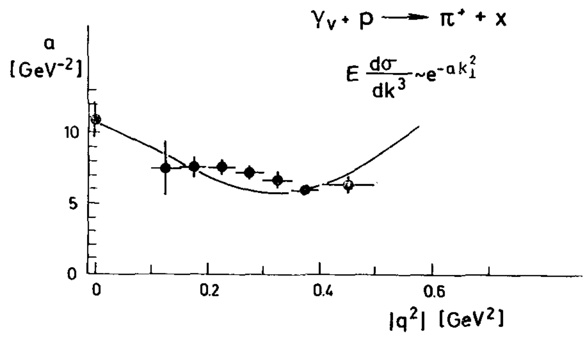

Fig. 13. Slope a of the form exp $(-ak_\perp^2)$ fitted to the theoretical distribution of the invariant cross section as a function of q^2 for $x = 0.7$, compared with the experimental slopes taken from Dammann et al. [18] ($\gamma p \to \pi^0 X$).

Finally we mention that in the work of Moffeit et al. [11] the cross sections $d\sigma_\perp$ and $d\sigma_\parallel$ have been measured separately by using a polarized photon beam. Their result in the interval $0.3 < x < 1.0$, for the photon energy of 9.3 GeV and integrated over $k_\perp^2$ gives $\sigma_\perp/\sigma_\parallel = 0.68$. If this result would be characteristic for $x > 0.7$ and also for the $k_\perp^2$ region below 0.2 GeV^2 we must conclude that the Regge pion term alone is not compatible with the data in that region since it would predict this ratio to be zero see eq. (3.8) $H_{++} = H_{+-}$) and $d\sigma_\parallel$ vanishes with $k_\perp^2 \to 0$. The fact, that this is not seen in the data, indicates disagreement with the pure Regge pole model and we think that significant absorption corrections must be included. The fact that additional contributions to one-pion exchange should play a role in determining the charged pion distribution was emphasized recently in connection with the gauge problem by Mütter et al. [16].

5. Conclusions

We have analysed the one-pion distributions in the photon fragmentation region of inclusive photo- and electroproduction within the framework of a Regge expansion. With only Regge pole contributions in the $\gamma\bar{\pi}$ channels, we have seen that spin and symmetry requirements play an essential role in determining the structure of the polarized cross sections. Furthermore the Regge pole model reproduces the over all scale of the normalization and some qualitative features of the data. At no stage were free parameters introduced, but instead we made estimates, by for example relating off shell quantities to their physical limits. The q^2 dependence of the $\gamma\pi$-reggeon form factors was approximated by the ρ (or ω) pole contribution and it was seen for $0.8 < x < 1.0$ this accounts for the change in the normalization of the $\pi^\pm$ distributions, as one goes from $q^2 = 0$ to $q^2 = 1$ GeV^2 (see fig. 9). This suggests the triple Regge pole contribution has a stronger q^2 dependence than the total inclusive structure function and gives an increasingly negligible contribution to the latter for larger q^2. This is not surprising, since the Regge pole model predicts that, for larger q^2, the $\pi^\pm$ distributions are predominantly determined by the longitudinal structure function H_{00}, whereas the total inclusive cross section is known to be mostly transverse.

If one were to introduce free parameters within the context of a purely Regge pole expansion, a number of features of the data would still remain unaccounted for. In particular the behaviour for small $k_\perp^2$ would not be explained. This leads us to conclude that Regge cut contributions play a non-negligible role. The nature and effect of absorption corrections in inclusive photo- and electroproduction will be discussed elsewhere [8].

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