Reggeization of meson trajectories in quark-gluon theories*

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We identify the meson trajectories in the gauge theories of quarks and gluons, within the framework of perturbation-theory models, by a set of Feynman diagrams different from that for the Pomeron. The method has several attractive qualitative features: it produces a degenerate set of trajectories ρ, A_2, P', etc. with the "bare" P' trajectory separated from and having a lower intercept compared to the Pomeron. We discuss the implications of a logarithm-squared contribution to the process of Reggeization.

Perturbation-theory models, where Feynman diagrams are summed in the high-energy limit, have provided valuable insight into Regge theory.[1] Much attention recently has been focused on obtaining the Pomeron trajectory in the non-Abelian gauge theory of quarks and gluons, quantum chromodynamics (QCD). The appropriate diagrams are given in Fig. 1, whose sum in the leading-logarithm approximation is presumed to generate the Pomeron trajectory in the t channel. The result up to the eighth order is consistent with a moving Regge pole.[2,3,4]

However, this method may suffer from two complications. The first one is associated with the fact that the Pomeron is generally thought to be a reflection (through unitarity) of scattering in the inelastic channels (e.g., shadow effects) and *not* a bound state of $q\bar{q}$. Thus it is quite possible that the Pomeron singularity (be it a pole or a cut) obtained in $q+q\to q+q$ (or $q+\bar{q}\to q+\bar{q}$) will not be the same as in hadron+hadron → hadron+hadron[5]; the complicated two- and three-body interquark forces that make up a (color-singlet) hadron could very possibly make a difference. The second complication is that of infrared divergence in the limit of $\lambda\to 0$, where λ is the gluon mass.

In the following we outline a method for obtaining the non-Pomeron meson trajectories within the framework of perturbation-theory models. By non-Pomeron trajectories we mean those which correspond to a bound state of the $q\bar{q}$ system, e.g., the $I=1$ states of ρ, A_2, etc., or the $I=0$ states ω, f, etc, and so on. The method does not suffer from the two complications mentioned above. It has the attractive qualitative feature that the bound-state trajectories are separated from the Pomeron and it has, as a natural consequence of the gluons being I-spin singlets, the degeneracy between the $I=1$ and $I=0$ trajectories. However, as we shall see later, there is a new technical difficulty in the process of Reggeization that prevents us from obtaining an explicit form of the meson trajectories. We discuss several possibilities that may overcome this difficulty.

Consider the $I=1$ mesons, e.g., ρ, A_2, etc. which are the bound states of $q\bar{q}$. Their Regge trajectory can be obtained through the quark-antiquark process $\mathcal{P}+\bar{\mathcal{P}}\to\mathfrak{N}+\bar{\mathfrak{N}}$ by summing the diagrams in Fig. 2 which are different from those in Fig. 1. We note that diagrams in Fig. 1 do not contribute to $\mathcal{P}+\bar{\mathcal{P}}\to\mathfrak{N}+\bar{\mathfrak{N}}$, which has $I_t=1$ in the t channel because the gluons have $I=0$. The diagrams in Fig. 2 are of lower order in s, for each order of the quark-gluon coupling, than the Pomeron-producing diagrams in Fig. 1.

If we keep the same diagrams as in Fig. 2 but look at $\mathcal{P}+\bar{\mathcal{P}}\to\mathcal{P}+\bar{\mathcal{P}}$ we would get the (non-Pomeron) $I_t=0$ trajectories which will be degenerate with the $I_t=1$ trajectory because gluons couple the same way to $\mathcal{P}\bar{\mathcal{P}}$ and $\mathfrak{N}\bar{\mathfrak{N}}$. These $I_t=0$ trajectories can be identified with the "bare" ω, f, etc. trajectories whose intercept will automatically be smaller than that of the trajectory given by Fig. 1; the lowest-order term in the former behaves like s^{-1} while for the latter it behaves like s^{+1}.

The above procedure for the $I_t=1$ trajectory,

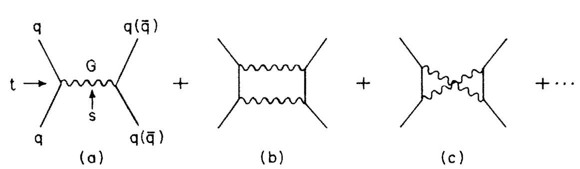

FIG. 1. Feynman diagrams for the Pomeron trajectory.

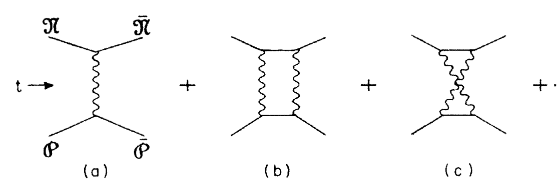

FIG. 2. Feynman diagrams for the leading $I_t=1$ meson trajectory.

and its degenerate $I_t=0$ trajectories, does not suffer from the possible complications present in the Pomeron calculation. Firstly, because the mesons are, indeed, bound states of the quark-antiquark system, the trajectory in $q+\bar{q}\to q+\bar{q}$ will be the *same* as in hadron + hadron → hadron + hadron; only the residue functions will change. Secondly, there is no infrared divergence in the amplitude corresponding to Fig. 2 obtained in the high-energy limit.

In other words, by writing the contributions from Fig. 1 and Fig. 2 separately, we have the interesting consequence of generating two different "bare" trajectories P and P' (for the $I_t=0$ case). The "bare" P' trajectory is degenerate with ρ, A_2. We note that the "physical" P' trajectory will be shifted by diagrams such as Fig. 3 (they can be separated into two parts by a t-channel intermediate state containing only gluon lines), which only contribute to $I_t=0$ processes, or by including the lower-order contributions of Fig. 1.

In what follows, we illustrate the questions involved by first considering the Abelian case and then come back to discuss possible ramifications in the non-Abelian case. Using the standard prescription for calcuating the diagrams we obtain the following results.[2,3,4]

Figure 2(a) is the pole term for the scattering $q(p_1)+\bar{q}(p_1')\to q(p_2)+\bar{q}(p_2')$:

$$g^2\frac{\bar{v}(p_1')\gamma_\mu u(p_1)\bar{u}(p_2)\gamma_\mu v(p_2')}{s}, \tag{1}$$

where g is the quark-gluon coupling and the gluon is assumed to have zero mass. Leaving out the spinor factors, the asymptotic behavior of this term is

$$g^2\frac{1}{s} \tag{2}$$

Diagram 2(b), after introducing Feynman parameters, is found to be

$$g^4\int\prod_{i=1}^{4}d\alpha_i\delta\Big(1-\sum_i\alpha_i\Big)\int d^4q'\{\bar{v}(p_1')\gamma_\nu[\gamma\cdot q'-\gamma\cdot(R+r_1)+m]\gamma_\mu u(p_1)\}$$
$$\times\{u(p_2)\gamma_\mu[\gamma\cdot q'-\gamma\cdot(R-r_1)+m]\gamma_\nu v(p_2')\}/[q'^2+\alpha_1\alpha_3 s+\alpha_2\alpha_4 t-(\alpha_2+\alpha_4)^2m^2]^4, \tag{3}$$

where $r_1=\frac{1}{2}(p_2-p_1)$, $r_2=\frac{1}{2}(p_1+p_2)$, $r_3=\frac{1}{2}(p_1'+p_2')$, and $R=-\alpha_1 r_2+\alpha_3 r_2+(\alpha_2-\alpha_4)r_1$. The contribution of Fig. 2(c) is obtained from Fig. 2(b) by $t\longleftrightarrow u$ and s fixed.

To obtain the asymptotic behavior of the above expression we note that there are two types of contributions, one coming from the product of two $\gamma\cdot q'$ and the other from the remaining products (the crossed terms give a vanishing contribution after integration). Calculating the product *not* involving $\gamma\cdot q'$ first, we find its asymptotic behavior, again leaving out the spinor factors, to be

$$g^4K(t)\frac{1}{s}\ln s, \tag{4}$$

where $K(t)$ is a function of t given by

$$K(t)=\int d\alpha_2 d\alpha_4\delta(1-\alpha_2-\alpha_4)$$
$$\times\frac{1}{\alpha_2\alpha_4|t|-(\alpha_2+\alpha_4)^2m^2}. \tag{5}$$

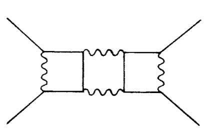

FIG. 3. Example of a diagram that breaks the degeneracy between the "bare" $I_t=0$ and $I_t=1$ trajectories.

The term involving a product of two $\gamma\cdot q'$ in (3) gives, in contrast to (4), a $\ln^2 s$ term,

$$g^4C\tfrac{1}{2}\ln^2 s, \tag{6}$$

where $C=(1/16\pi^2)[\bar{v}(p_1')\gamma_\mu u(p_1)\bar{u}(p_2)\gamma_\mu v(p_2')]$ apart from the same spinor-field factor as (1) is independent of t.[6] Such a term cannot be canceled by any other fourth-order diagrams.

The presence of the log-squared term in (6) above is of crucial significance. The same log-squared term appears in QED in $e^-e^+\to\mu^-\mu^+$ near the forward direction (or $e^-\mu^-\to e^-\mu^-$ near the backward direction) as pointed out some time ago by Gorschkov, Gribov, Lipatov, and Frolov (GGLF).[6,7] Their diagrams are identical to our Fig. 2 for $\mathcal{P}+\bar{\mathcal{P}}\to\mathcal{N}+\bar{\mathcal{N}}$ in the Abelian case.

Since $\ln^2 s$ dominates over $\ln s$ asymptotically, the series we have to consider will have the form

$$\frac{g^2}{s}+\frac{g^4C}{s}\ln^2 s+\cdots. \tag{7}$$

The presence of the log-squared term and its powers creates problems for Reggeization. The sum *cannot* add to a Regge pole which requires a power series in single log.[8]

In QED, for $e^-e^+\to\mu^-\mu^+$, it was found by GGLF that the sum of the type (7) gave rise to a *fixed* cut in the j plane of the t channel; the partial-wave amplitude was given by[6,7]

$$f(j) \sim \frac{1}{j+(j^2-\gamma^2)^{1/2}} \,, \tag{8}$$

where

$$\gamma = \left(\frac{2\alpha}{\pi}\right)^{1/2} .$$

In QCD the nature of the fixed j-plane singularity can be more complicated, since the ladder diagrams (and the associated bremsstrahlung diagrams) are not necessarily dominant in the high-energy limit as in QED.

It is perhaps surprising that in contrast to the trajectory in the Pomeron channel which seems to Reggeize at least to the eighth order, the meson channels have the log-squared complication in the fourth order. All the experimental evidence points to mesons being simple poles. That is, mesons, as bound states of $q\bar{q}$, should be moving j-plane singularities. On the other hand, a Pomeron singularity as indicated by experiments and by Reggeon field theories may well be more complicated. It is conceivable that the mechanism which produces confinement of quarks may also alter the log-squared contribution. One must then go to the non-Abelian case of QCD where confinement is presumably a natural outcome. In this context we note that because the log-squared contribution in (3) comes from the region $q_\perp'^2 \gg m^2$ (where $q_\perp'$ is perpendicular to p_1, p_1'), one can consider the case of infinite bare quark mass (m) as a reflection of confinement and, therefore, possibly of a moving Regge pole. However, it is difficult to implement $m \to \infty$ in an unambiguous way within the summation technique.

We also note that asymptotic freedom may soften the asymptotic behavior as indicated in a recent extension by Polkinghorne[9] of earlier[10] works investigating the ϕ^3 theory in six dimensions. However, based on these investigations, it is likely that some fixed singularity will remain and the moving pole can only be isolated from lower-order contributions.

In conclusion, our method of identifying the meson trajectories by a set of Feynman diagrams different from that for the Pomeron does not suffer from the problems of infrared divergence, etc., inherent in the Pomeron calculation and has several attractive qualitative features, namely, it produces a degenerate set of trajectories ρ, A_2, P', etc., with the "bare" P' trajectory separated from and having a lower intercept compared to the Pomeron. It would be interesting if any of the above-mentioned mechanisms implicit in QCD can remove the log-squared complication that will allow us to obtain an explicit form for the mesons as moving Regge poles.

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[8] We note that if the log-squared term were absent, then one would have

$$\frac{g^2}{s} + \frac{g^4}{s} K(t) \ln s + \cdots .$$

Such a series has the possibility of summing to a moving Regge pole (depending, of course, on how the higher-order terms behave) with the trajectory function $\alpha(t) = -1 + g^2 K(t)$.

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