J. Phys. G: Nucl. Phys., Vol. 3, No. 11, 1977. Printed in Great Britain. © 1977

Rescattering effects in the Deck model

I J R Aitchison† and M G Bowler‡
† Department of Theoretical Physics, 1 Keble Road, Oxford OX1 3NP, UK
‡ Nuclear Physics Laboratory, Keble Road, Oxford OX1 3RH, UK

Received 6 April 1977, in final form 11 May 1977

Abstract. We study resonant rescattering corrections to the Deck amplitude B for diffractive dissociation of hadrons. The results of numerical calculations for a variety of physically-reasonable forms of B show that provided B falls off faster than a simple pole the rescattered amplitude is well approximated by the form $Be^{i\delta}\cos\delta$, rather than by $Be^{i\delta}\sin\delta/k$. We conclude that the former expression is generally relevant to situations in which the Born amplitude B is spatially diffuse, while the latter applies if B is spatially localised.

1. Introduction

In the Deck model of diffraction dissociation (Deck 1964) an incident particle emits a virtual particle (usually a pion) and carries on either as itself (e.g. $p \to p + \pi$) or as low-mass quasi-stable particle (e.g. $\pi \to \rho + \pi$, $p \to \Delta + \pi$). The virtual particle is diffractively scattered on the target, becoming a real state. This process produces a low-mass kinematic enhancement in the effective mass spectrum of the fast particles, dominated by a relative s wave (Stodolsky 1967). The enhancement results from the rise of the phase-space factor from threshold combined with the fall of the amplitude for the Deck process. The faster the Deck amplitude falls off in mass, the more spatially diffuse is the production process to which it corresponds; this is discussed further below, following equation (9).

If the fast particles can interact among themselves, however, effects of rescattering must be taken into account, and the bare Deck amplitude must be regarded as a Born term. This is illustrated in figure 1. The rescattering effects are particularly important if the interaction is resonant, and this is the case we shall treat exclusively.

In previous work we have considered the effect of rescattering, and applied our results to the phenomenological analysis of data on diffractive production (Bowler

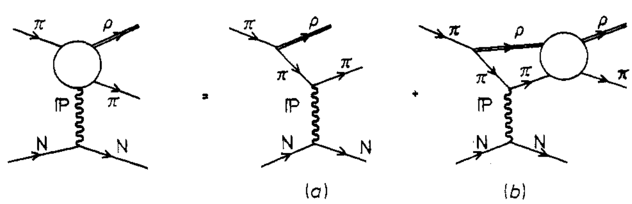

Figure 1. Rescattering following diffractive dissociation. The rescattered amplitude is the sum of the Deck term (a) and the rescattering process (b).

et al 1975). Our principal result was that for rescattering corrections to the Deck process the rescattered amplitude will be much closer to the form

$$F \sim B e^{i\delta} \cos \delta, \tag{1}$$

which actually *vanishes* at resonance, than to the conventional 'FSI enhancement' form

$$F \sim \frac{B\, e^{i\delta} \sin \delta}{k} \tag{2}$$

in which the full effects of the resonance are seen. In (1) and (2) B is the Deck amplitude and δ is the two-particle elastic phase-shift, assumed to be resonant; k is the two-particle phase-space factor.

We have been led to reconsider our earlier result (1) by a paper of Basdevant and Berger (1976). These authors consider rescattering effects through a single resonance coupled to both the $K^*\pi$ and ρK diffractively-produced components of the Q system. The bare Deck amplitudes are represented by a simple pole in the $K\pi\pi$ mass (Stodolsky 1967) and the resonance is inserted via a K matrix with a single pole in each channel. The advantage of these assumptions is that analytic expressions can be written down for the full amplitude; but if these assumptions are introduced in the one-channel, one-resonance case a result of the form (2) rather than (1) is obtained.

In this paper we therefore examine critically the assumptions we have made previously (Bowler *et al* 1975) and the assumptions made by Basdevant and Berger. We show that the assumptions of Basdevant and Berger are rather special, and that in most situations of rescattering following diffractive production (i.e. from a spatially-extended source) the rescattered amplitude is likely to be much closer to (1) than to (2).

2. The calculation of rescattering effects

Let us suppose that the elastic phase-shift δ returns asymptotically to zero: that is, the usual N and D functions satisfy the condition

$$D = 1 - \frac{1}{\pi} \int_R \frac{N'k'\, dm'^2}{m'^2 - m^2 - i\epsilon} \tag{3}$$

where R signifies the 'right-hand' unitarity cut along the real m'^2 axis from threshold. In this case, we say that the resonance is 'dynamically generated'. Then the rescattered amplitude is given unambiguously by the expression (Omnès 1958)

$$F = B + \frac{1}{\pi D} \int_R \frac{g'D'B'\, dm'^2}{m'^2 - m^2 - i\epsilon} \tag{4}$$

where $g = e^{i\delta} \sin \delta$.

We first note that (4) can also be written as

$$F = B\, e^{i\delta} \cos \delta + \frac{e^{i\delta} \sin \delta}{k} \left(\frac{\text{PV}}{N\pi} \int_R \frac{N'k'B'\, dm'^2}{m'^2 - m^2} \right) \tag{5}$$

where PV stands for principal value. Thus to obtain (1) the PV contribution must be very small, while for (2) it must dominate. We therefore analyse carefully the conditions under which the PV term is or is not large.

To facilitate comparison with the work of Basdevant and Berger it is useful to introduce an alternative expression for F, which may be obtained (Jackson 1961) by rewriting the integral in (4) as a contour integral in the m'^2 plane, with the contour running just below the real axis from $+\infty$, encircling the threshold, and returning to $+\infty$ just above the real axis. The contour can then be completed by a great circle at infinity, C_∞ say, and by a piece enclosing the singularities of B. Provided the integral over C_∞ vanishes, one obtains

$$F = \frac{1}{\pi D} \int_{\mathrm{L}} \frac{\mathrm{Im}\, B'D' \, \mathrm{d}m'^2}{m'^2 - m^2 - \mathrm{i}\epsilon} \tag{6}$$

where L is the 'left-hand' cut corresponding to the singularities of B.

If B is now assumed to have a simple pole at $m^2 = m_0^2$, the integral in (6) can be evaluated immediately and the result is

$$F = \frac{B(m^2)D(m_0^2)}{D(m^2)} = \frac{BD_0}{N} \frac{\mathrm{e}^{\mathrm{i}\delta} \sin \delta}{k}. \tag{7}$$

The same result can also be obtained from (4) provided that (3) is true, a condition which is equivalent to requiring that the integral over C_∞ vanishes. Thus (7) is undoubtedly *the* solution to the rescattering problem if B is a simple pole and the phase-shift is dynamically generated. For the case of a resonant phase-shift, N is approximately constant over the resonance region where $\sin \delta$ is large, and hence (7) is of the form (2) rather than (1). In fact, for a pure pole form of B, the principal-value term in (5) swallows up the $B\mathrm{e}^{\mathrm{i}\delta} \cos \delta$ term completely.

In the work of Basdevant and Berger, however, the resonance is represented by a Breit–Wigner formula, so that the phase-shift does not return to zero at infinity—that is, a CDD pole (Castillejo *et al* 1956) is present. In this case it is easy to verify that the integral over C_∞ is not zero, and thus the results of using (6) and (4) will differ. To illustrate the point, let us simplify the kinematics so that $m^2 = c + dk^2$ for some constants c and d. Equation (6) will necessarily give (7) again, while in this case (4) gives

$$F = B \frac{k_{\mathrm{r}}^2 - k^2 + \Gamma k_0}{k_{\mathrm{r}}^2 - k^2 - \mathrm{i}\Gamma k} \tag{8}$$

for a Breit–Wigner resonance with parameters k_{r} and Γ as indicated, where $\mathrm{i}k_0$ is the value of k at the pole position $m^2 = m_0^2$. Since for this case $D \propto (k_{\mathrm{r}}^2 - k^2 - \mathrm{i}\Gamma k)$, the forms (7) and (8) differ by

$$B \frac{(k_0^2 + k^2)}{k_{\mathrm{r}}^2 - k^2 - \mathrm{i}\Gamma k},$$

which is of the form constant/D, since $B \propto (k_0^2 + k^2)^{-1}$. Thus the forms (7) and (8) differ by a solution (Jackson 1961) of the homogeneous FSI equation.

We now observe that (8) can be written as

$$B\, \mathrm{e}^{\mathrm{i}\delta} \cos \delta + Bk_0 \frac{\mathrm{e}^{\mathrm{i}\delta} \sin \delta}{k}. \tag{9}$$

The Fourier transform of the distribution of relative momentum of the two fast particles may be interpreted as the spatial correlation between them. In practical cases of diffractive production, a sharp fall-off is seen in the mass distribution of the diffractively-produced fast pair (see §§3 and 4 below); this corresponds to a sharp fall in momentum space and thus to a diffuse spatial relation between the pair. In (9), the parameter k_0 controls the rate of fall-off of B in momentum space: equivalently, k_0^{-1} is a measure of the spatial correlation between the pair. Thus in diffractive production we expect the second term in (9) to be relatively small, and the final result using (4) to approximate to (1) rather than (2).

Yet using (6) in this case leads to the result (7), which is of the form (2), and so the essential question is: how is one to determine which is the physically correct solution in this case? We have tackled this question as follows. The ambiguity has arisen in the case of two special assumptions, namely that the phase-shift is given by a pure Breit–Wigner resonance, and that B is a pure pole. If one relaxes these assumptions a unique result will be obtained: then, as one moves closer to the special case, the physical solution will be indicated by continuity. We have investigated the effects of modifying B so that it falls off for large m^2 more quickly than m^{-2} (i.e. we add some damping), and we have also used dynamically-generated phase-shifts rather than Breit–Wigner ones. We can then allow the extra damping to vary systematically (so as to include, in particular, the pure pole case), and we can also mimic dynamically the Breit–Wigner phase-shift to a greater or lesser extent. In this way we think that we have arrived at a good understanding of the essential physics involved.

In summary, our results are as follows. Although (7) is indeed exact for a pure pole form of B, additional damping in B can very quickly bring about a drastic modification: for physically-reasonable damping of the Deck term, we find that it is the form (1) which is a good approximation to the exact result (4), and the form (2) is seriously wrong. We also find that the dynamically-generated phase-shift may be replaced by the Breit–Wigner one (i.e. N is treated as a constant in (5)) without significantly modifying the results.

These conclusions require some qualification. They hold if the dynamically-generated phase-shift is well approximated by a Breit–Wigner formula up to several full widths beyond the resonance. If, however, the phase-shift starts to return to zero soon after passing $\frac{1}{2}\pi$, then the rescattered amplitude is intermediate between the forms (1) and (2), unless an unreasonable amount of damping is introduced.

3. Numerical calculations

We have considered two possible forms of damping, introduced in each case through a modification of the simple pion propagator $(t - m_\pi^2)^{-1}$. The simple π-exchange Deck amplitude, with diffractive πN scattering in figure 1, is proportional to

$$\frac{s_{\pi N}}{m_\pi^2 - t}$$

where $s_{\pi N}$ is the square of the total centre-of-mass energy of the recoil nucleon and scattered pion, and t is the four-momentum transfer squared carried by the exchanged pion. At high energies and in the forward direction we have (see, for example, the review by Berger (1975))

$$s_{\pi N} \approx \frac{s(m_\pi^2 - t)}{m^2 - m_\pi^2}$$

where s is the square of the total centre-of-mass energy. Thus in the diffractive case the pion pole is cancelled in the final amplitude (Stodolsky 1967), and if the exchanged pion is described by the simple propagator only, the Deck amplitude B is given by a pole in m^2. Since the FSI problem is linear, the normalisation of B is irrelevant for our present purposes, and we take for the undamped B the simple form

$$B = \frac{1}{m^2 - m_\pi^2}.$$

If, however, an exponential damping factor (corresponding to form factor effects, for example) is introduced,

$$\frac{1}{t - m_\pi^2} \rightarrow \frac{\exp(at)}{t - m_\pi^2}, \tag{10}$$

then the Deck amplitude is no longer pure s wave, and projecting out the s-wave part

$$\frac{1}{m^2 - m_\pi^2} \rightarrow \frac{(\exp(at_{\min}) - \exp(at_{\max}))}{a(m^2 - m_\pi^2)(t_{\min} - t_{\max})}. \tag{11}$$

Alternatively, if the pion propagator is Reggeised via

$$\frac{1}{t - m_\pi^2} \rightarrow \frac{\exp(-\mathrm{i}\alpha t)}{t - m_\pi^2} \tag{12}$$

then

$$\frac{1}{m^2 - m_\pi^2} \rightarrow \frac{1}{m^2 - m_\pi^2} \frac{\sin[\frac{1}{2}\alpha(t_{\min} - t_{\max})] \exp[-\frac{1}{2}\mathrm{i}\alpha(t_{\min} + t_{\max})]}{\frac{1}{2}\alpha(t_{\min} - t_{\max})}. \tag{13}$$

In both cases the s-wave Deck amplitude is damped at large values of m^2 ($t_{\min}$ and $t_{\max}$ are the kinematic limits in t).

We have made calculations for a range of values of a, and for $\alpha = \frac{1}{2}\pi$. In addition, because Reggeisation at low mass is a dubious procedure, and because of the unitarity problem (Michael 1975, Morgan 1975), we have made calculations with $\alpha \rightarrow 0$ at low m^2 values, rising over a mass range $\sim 0{\cdot}5$ GeV/c^2 to $\frac{1}{2}\pi$ at a number of different masses, and we have done this for two different dynamically-generated phase-shifts, δ_A and δ_B. These phase-shifts, and the corresponding N functions, are shown in figure 2. (For δ_A we used a five-pole form for N, and for δ_B a seven-pole form.)

In figure 3 we show the rescattered Deck amplitudes for δ_A and δ_B, and for a range of values of a in (11). The form (1), the 'pure pole' form (7), and the results of replacing the dynamically-generated resonance in (4) by a Breit–Wigner formula are also shown. It will be observed that for the very moderate damping factors ($a > 1$ (GeV)$^{-2}$ for δ_A and $a > 0{\cdot}5$ (GeV)$^{-2}$ for δ_B) the form (1) is a good approximation to the full amplitude; the difference between (1) and the exact numerically-evaluated amplitude is the (numerically small) principal-value term in (5). For the phase-shift δ_B, replacing the dynamically-generated resonance by a Breit–Wigner makes little difference.

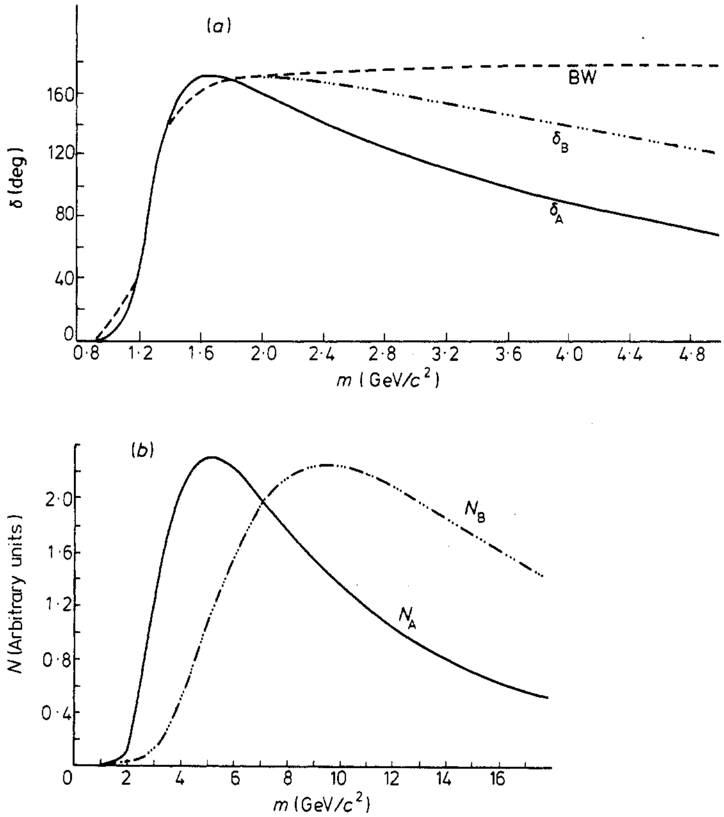

Figure 2. (*a*) The phase-shifts used in the calculations. δ_A and δ_B are the dynamically-generated phase-shifts, using respectively a five-pole and a seven-pole form for N (see figure 2(*b*)). The curve labelled BW is the Breit–Wigner phase, obtained from $k \cos\delta = (m_r - m)/\gamma^2$ with $m_r = 1{\cdot}28\ \mathrm{GeV}/c^2$, $\gamma^2 = 0{\cdot}29\ \mathrm{GeV}/c^2$. (*b*). The two N functions, N_A and N_B, which generate the phase-shifts δ_A and δ_B via equation (3).

In figure 4 we show the rescattered Deck amplitudes for the choice (13) for B, with an asymptotic value of α of $\frac{1}{2}\pi$, and various positions of the 'pointer' determining where the Reggeisation is switched on. It will be seen that with the pointer below $2\ \mathrm{GeV}/c^2$ for δ_A, and below $4\ \mathrm{GeV}/c^2$ for δ_B, the form (1) is again a good approximation to the exact amplitude.

4. Conclusions from the numerical results

The results shown in figure 4 are particularly suggestive. Since $k \cot\delta = \mathrm{Re}(D)/N$, the region where the phase-shift δ starts to fall back from the Breit–Wigner asymptotic value of π is correlated approximately to the region where N starts to increase (see figure 2). Referring now to equation (5), we would expect the principal-value term to contribute relatively little if the extra damping in B sets in at mass values less than or equal to those at which the rise in N commences. This is indeed the lesson to be drawn from figures 2 and 4: the amplitude approximates to the form (1) when

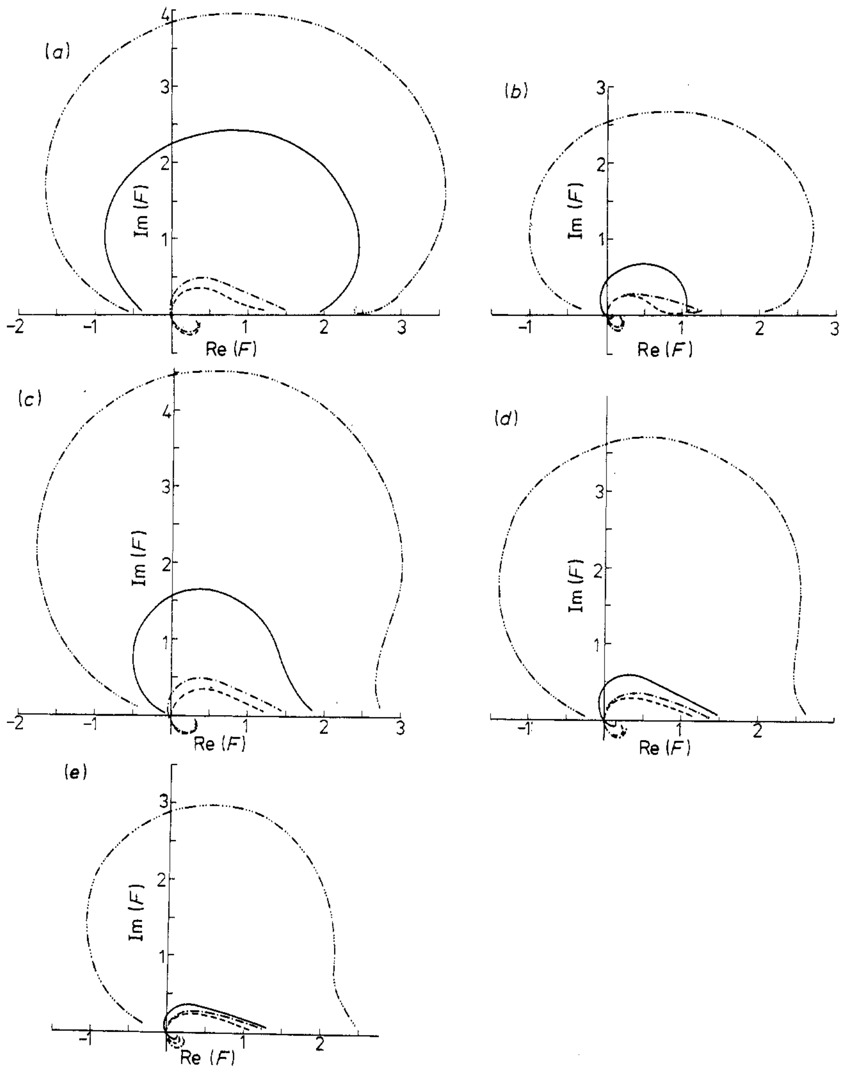

Figure 3. Results of calculations of the various amplitudes of interest, using the form (11) for the Deck amplitudes. (*a*) and (*b*) are for the phase-shift δ_A; in (*a*) the parameter a in (11) has the value 0·1 $(\text{GeV})^{-2}$ and in (*b*) it has the value 1·0 $(\text{GeV})^{-2}$. (*c*), (*d*) and (*e*) are for the phase-shift δ_B; in (*c*), $a = 0{\cdot}1$ $(\text{GeV})^{-2}$, in (*d*), $a = 0{\cdot}5$ $(\text{GeV})^{-2}$, in (*e*) $a = 1{\cdot}0$ $(\text{GeV})^{-2}$. In this figure, and in figure 4, the full curve is the exact solution (equation (4)), the broken curve is $Be^{i\delta}\cos\delta$ (equation (1)), the chain curve is the amplitude calculated using equation (4) with a Breit–Wigner resonance form for D and the chain curve with three dots is the 'pure pole' form equation (7). The curves are traversed in a counter-clockwise sense as m increases; the topmost point in all cases is at approximately the resonance mass (here taken to be about 1·3 GeV/c^2).

the damping is introduced at mass values of the order of, or less than, those at which δ starts to fall and N to rise. The longer the phase-shift stays up, the longer the introduction of the damping can be delayed.

The phase-shifts of well-established elastic resonances are well approximated by Breit–Wigner formulae, at least over some sensible energy interval. We are not

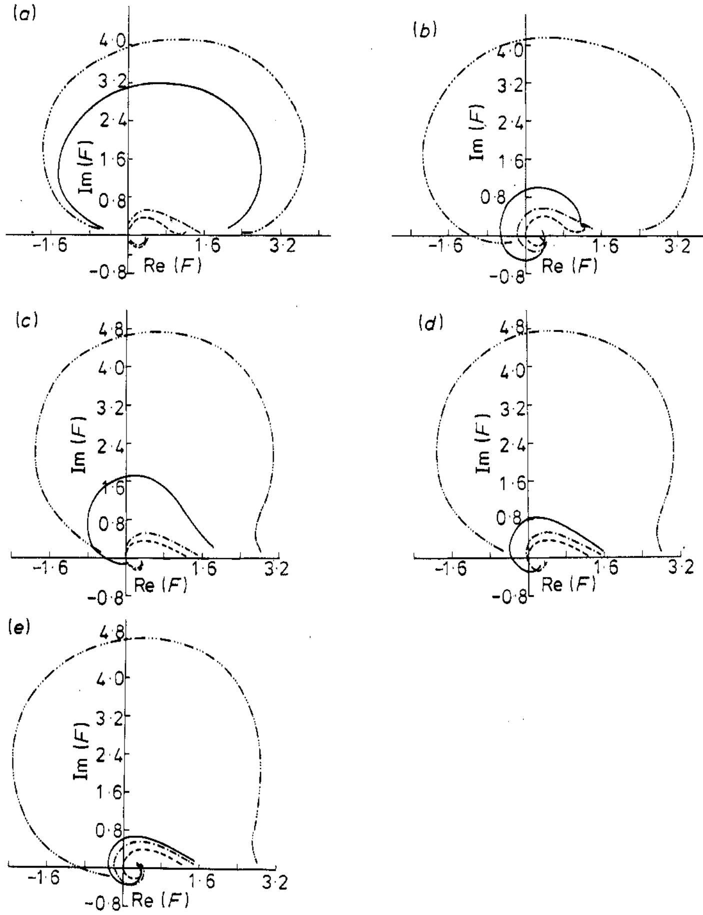

Figure 4. Results of calculations of the various amplitudes of interest, using the form (13) for the Deck amplitude. (*a*) and (*b*) are for the phase-shift δ_A. The Regge phase parameter α has a dependence on the 3π mass m given by $\alpha(m) = \frac{1}{2}\pi/[\exp 2(c - m) + 1]$. In (*a*) the value of the parameter c is $6\,\mathrm{GeV}/c^2$, in (*b*) it is $2\,\mathrm{GeV}/c^2$. (*c*), (*d*) and (*e*) are for the phase-shift δ_B; in (*c*), $c = 6\,\mathrm{GeV}/c^2$, in (*d*) $c = 4\,\mathrm{GeV}/c^2$ and in (*e*) $c = 2\,\mathrm{GeV}/c^2$.

required to speculate about whether these states are truly CDD poles (representing $q\bar{q}$ states, possibly), or whether they are dynamically-generated in the hadronic channels: indeed, it is not self-evident that these points of view are mutually exclusive. The full amplitude is well approximated by the form (1) when damping is introduced at physically-reasonable mass values and in physically-reasonable amounts; and in this case, the form (2) is grossly wrong. An independent empirical indication of the amount of damping to be expected is provided by the presence of waves other than the s wave in diffractively-produced 3π (Antipov *et al* 1973) and $K\pi\pi$ (Brandenburg *et al* 1976) systems, and indeed Reggeisation (with constant α) of the pion propagator accounts qualitatively for a large number of the features of the 3π data (Ascoli *et al* 1973).

We conclude that resonant rescattering corrections to diffractive production via the Deck mechanism will lead to an amplitude well approximated by the form (1), and that therefore our analysis (Bowler *et al* 1975) remains a valid possible interpretation of the data.

5. Inclusion of direct resonance production

Equation (4) is the solution of the inhomogeneous FSI equation. To such a solution we may add the general solution of the corresponding homogeneous equation

$$F' = P/D$$

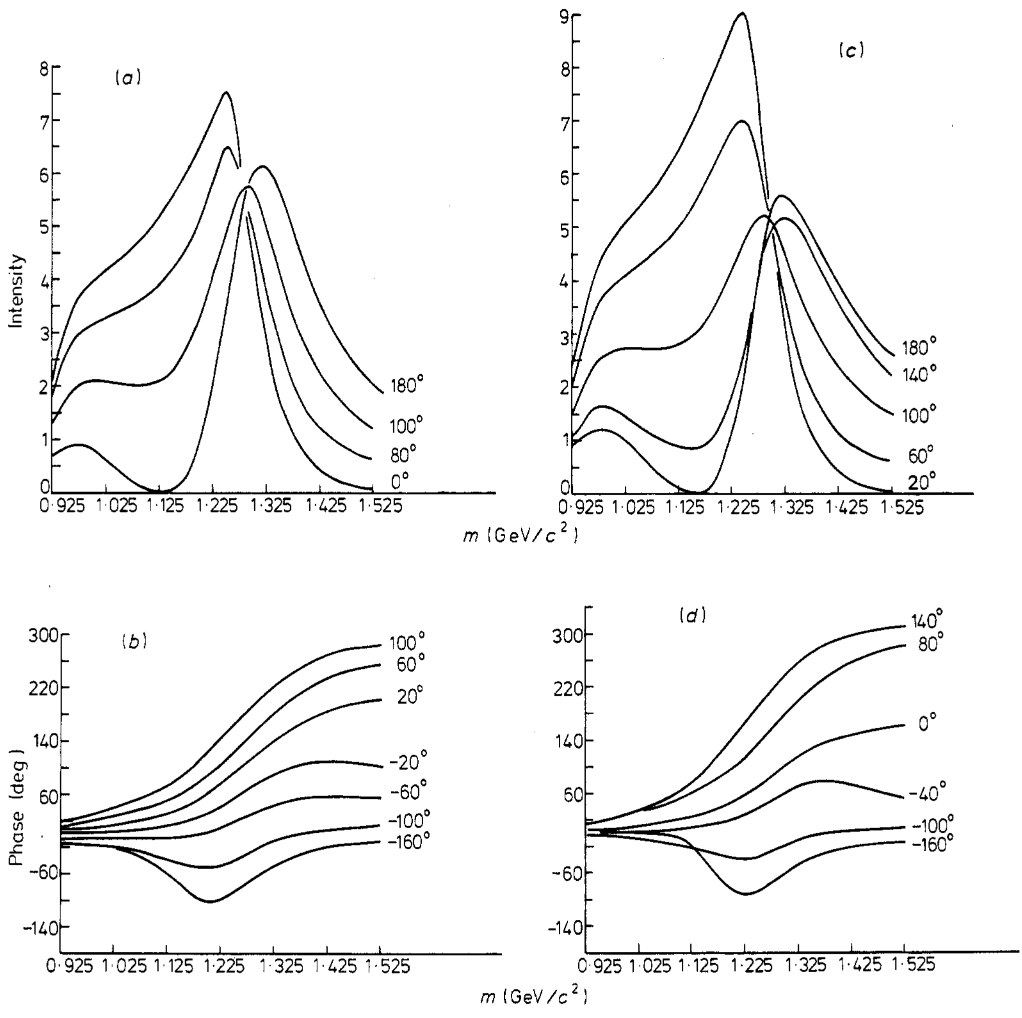

Figure 5. The intensity ($|F|^2 \times$ phase-space) and phase of the total amplitude given by equation (14). The Breit–Wigner phase-shift is used. (*a*) and (*b*) are for the choice (11) for the Deck amplitude, with $a = 0{\cdot}5\ \mathrm{GeV}^{-2}$; (*c*) and (*d*) are for the choice (13), with the parameter c in $\alpha(m)$ set equal to $4\ \mathrm{GeV}/c^2$. The different curves are labelled by the different values of the phase parameter ϕ in equation (14). The intensities corresponding to $+\phi$ and $-\phi$ are indistinguishable.

where P is a polynomial. In previous work (Bowler *et al* 1975) we have interpreted the presence of such terms as due to direct diffractive production of the resonance. In order to exhibit the rich structure of the total amplitude which is obtained when such terms are included, we show in figure 5 the intensity and phase corresponding to the total amplitude

$$F + p\exp(i\phi)/(m_r - m - i\gamma^2 k) \tag{14}$$

where F is given by (4) evaluated in the N = constant approximation, with the choices (11) ($a = 0{\cdot}5\ (\mathrm{GeV})^{-2}$) and (13) (Reggeisation at $4\ \mathrm{GeV}/c^2$) for B, where we took $m_r = 1{\cdot}28\ \mathrm{GeV}/c^2$, $\gamma^2 = 0{\cdot}29\ \mathrm{GeV}/c^2$, $p = 0{\cdot}12$ and various values of the relative phase ϕ. We should point out that diffractive resonance production is not properly understood and we do not know the relative phase ϕ. The diffractive part of the elastic scattering amplitude is imaginary, so that the production amplitude B is certainly imaginary. The phase of P has been measured for diffractive production of ρ mesons (Alvensleben *et al* 1971) and is $\approx 90°$, but the mechanism in that case is diffractive scattering of a virtual ρ into the real state: elastic scattering. Production of A1 or Q resonances involves s$\rightarrow$p-wave quark transitions and in a simple model these amplitudes would be real rather than imaginary, but would vanish in the forward direction. The study of diffractive meson production may thus yield important information on the structure of hadrons, but it is essential that good data ($\approx 10^5$ events) are obtained in each of a number of ranges ($\approx 0{\cdot}1\ \mathrm{GeV}/c^2$) of the momentum transferred to the diffractively-excited system.

Acknowledgments

We thank J L Basdevant and T Lasinski for useful correspondence.

References

Alvensleben H *et al* 1971 *Nucl. Phys.* B **25** 342
Antipov Yu M *et al* 1973 *Nucl. Phys.* B **63** 141, 153
Ascoli G *et al* 1973 *Phys. Rev.* D **8** 3894
Basedevant J L and Berger E L 1976 *Phys. Rev. Lett.* **37** 977
Berger E L 1975 *Proc. Daresbury Study Weekend on Three-Particle Phase-Shift Analysis and Meson Resonance Production* eds J B Dainton and A J G Hey (Daresbury Nuclear Physics Laboratory DL/R34) pp 35–64
Bowler M G, Game M V, Aitchison I J R and Dainton J B 1975 *Nucl. Phys.* B **97** 227
Brandenburg G W *et al* 1976 *Phys. Rev. Lett.* **36** 703, 706
Castillejo L, Dalitz R H and Dyson F J 1956 *Phys. Rev.* **101** 453
Deck R T 1964 *Phys. Rev. Lett.* **13,** 169
Jackson J D 1961 *Scottish Universities' Summer School 1960* ed G R Screaton (Edinburgh and London: Oliver and Boyd) pp 54–61
Michael C 1975 *Proc. Daresbury Study Weekend on Three-Particle Phase-Shift Analysis and Meson Resonance Production* eds J B Dainton and A J G Hey (Daresbury Nuclear Physics Laboratory DL/R34) p 134
Morgan D 1975 *Rutherford Laboratory preprint RL-75-100*, contribution to 1975 *EPS Int. Conf. on High Energy Physics, Palermo* ed A Zichichi (Bologna: Editrice Compositori) abstract number F1-01
Omnès R 1958 *Nuovo Cim.* **8** 316
Stodolsky L 1967 *Phys. Rev. Lett.* **18** 973