

SINGLE-PARTICLE ELECTROPRODUCTION AT HIGH ENERGY

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Abstract: Single-particle electroproduction is discussed in the framework of the covariant parton model. It is found that the discussion of the process is model dependent, but that two contributions are likely to occur. One corresponds to a term found by Weis in a different discussion, and the other resembles a parton-model term suggested by Roy. It is possible that both terms have similar phenomenological consequences, with the amplitude falling off in Bjorken limit at a rate equal to the fall-off of form factors at large momentum transfer. The relation of these terms to fixed poles and to light-cone limits is clarified.

1. INTRODUCTION

Inclusive deep inelastic electroproduction has been the subject of intensive theoretical study. Although the discussion may be phrased in a variety of ways, a large number of authors [1–4]* agree that the dominant contribution arises from diagrams like that of fig. 1. The non-perturbative parton model that we have developed [1] provides a general and covariant language for the discussion, which is also as close as possible to conventional field theory. The electromagnetic current is constructed from one or more parton fields ψ , as a sum of terms proportional to

$$\bar{\psi} \gamma^\mu \psi . \quad (1)$$

The choice of a spin $\frac{1}{2}$ field is motivated by the experimental smallness of the ratio of the longitudinal and transverse cross sections. The upper bubble in fig. 1 then represents the complete parton propagator and the lower bubble is the parton-hadron scattering amplitude. The latter amplitude is non-amputated, that is it effectively includes a propagator in each parton leg. An important dynamical postulate of the theory is that this amplitude goes to zero sufficiently rapidly as the squared momenta in the parton legs become large. Without this softness requirement, Bjorken scaling

* The papers of ref. [4] show how the light-cone approach to deep inelastic scaling corresponds to fig. 1.

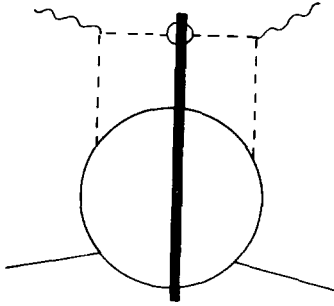


Fig. 1. The dominant type of diagram for deep inelastic electroproduction. The broken lines represent partons; the bar denotes a sum over intermediate states.

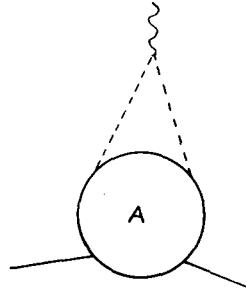


Fig. 2. The form factor in the parton model.

would be broken by logarithmic factors. The rate of decrease must be such that the form factor of fig. 2 is not logarithmically divergent, but otherwise none of the consequences deduced for inclusive deep inelastic electroproduction depend on the precise rate of decrease. This is satisfactory, since the determination of the off-shell behaviour of the parton field would require a specific model, such as would in principle be provided by postulating a Lagrangian density for its hadronic interactions, and we can scarcely hope to know what that should be.

However, the asymptotic behaviour of the amplitudes for exclusive electroproduction

$$e + h \rightarrow e + A + B \quad , \quad (2)$$

where h , A and B are hadrons, does prove to depend on the precise rate of decrease of the parton-hadron amplitudes for large values of the square of the parton momenta. Thus experimental data for these processes (2) will give a rather more detailed insight into the dynamics of the theory than do the inclusive data. In principle, our present theoretical understanding would allow almost any results to emerge, the only real constraint being that the cross section for each process (2) is less than the inclusive cross section. For this reason our present work contains a large element of speculation.

We are concerned with an amplitude

$$\gamma(q) + h(p) \rightarrow A(p_A) + B(p_B) \quad , \quad (3)$$

for large values of the energy $\nu = p \cdot q$ and of the virtual photon mass variable q^2 , such that

$$\omega = \frac{-2\nu}{q^2} \quad , \quad (4)$$

remains finite. Two recent papers [5,6] on this subject have reached different conclusions. We find that there are likely to be two contributions. One is similar to that obtained by Weis [5], who did not use a parton approach. The other corresponds to a mechanism considered by Roy [6], though, despite his use of a parton model (phrased in a manner different from our own), we obtain a somewhat different result.

2. PARTON-MODEL CALCULATIONS

The simplest type of contribution to (3) arises in the case where the parton propagator has a pole corresponding to one of the final-state hadrons, as shown in fig. 3. In these circumstances the exclusive process (3) contributes a non-zero fraction to the inclusive cross section. However, there is a reason [7] to think that the parton field ψ carries quark quantum numbers. In the absence of quarks as particles the propagator will have no pole, and fig. 3 will not occur. (The calculation [1] of the inclusive cross section does not assume the existence of a pole in the parton propagator.) In this case the leading contribution must come from the connected diagram, fig. 4.

In this figure, T is the connected, non-amputated two-parton three-hadron amplitude. It depends on seven independent variables, of which q^2 , $s = (p_A + p_B)^2$ and $t = (p - p_B)^2$ are fixed and the remaining four vary, corresponding to the four-dimensional integration over the parton loop momentum k . In this discussion, we ignore all spins.

We may analyse the asymptotic behaviour of the diagram with techniques previously used in our parton-model calculations [1,8]. There are two alternative methods, one using Sudakov variables [1] and the other involving Fourier transforms [8]. We give an account of the first method in the appendix, since it gives more insight into what is going on, though in the present problem it is easier to make the second method rigorous.

It is found in the appendix that the squared momenta in both the parton lines cannot be kept finite as $q^2 \rightarrow \infty$, and the leading contribution arises from that part

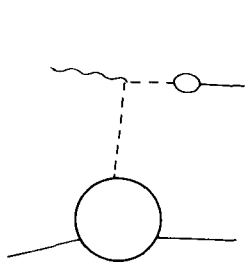


Fig. 3. A parton propagator pole contribution to the exclusive process.

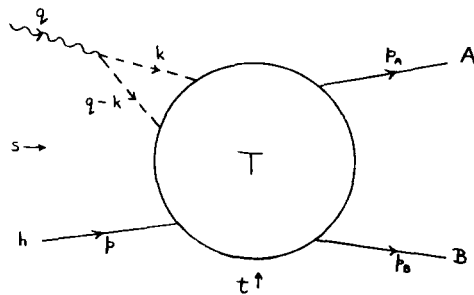


Fig. 4. Connected contribution to the exclusive process.

of the region of integration where one squared momentum is infinite and the other finite. The asymptotic behaviour of T under such conditions is not fixed by any familiar principles; in particular it need not be associated with Regge exchange. However, in order to have a definite model, we suppose that Regge exchange is appropriate, in just the same way as when the squared momenta in the parton lines are finite, and that account of the large squared momentum is taken by using an assumed asymptotic form for the reggeon coupling function concerned. Thus when k^2 is large, the coupling function $\beta(k^2, (p_A - k)^2)$ at the upper vertex of fig. 5a is taken to have the form $(k^2)^{-\gamma_1}$, and when $(q - k)^2$ is large the two-reggeon/parton coupling [9]* corresponding to the centre vertex of fig. 5b is taken to be $((q - k)^2)^{-\gamma_2}$.

It turns out that there are four contributions. In fig. 5a, k^2 is infinite and $(p + q - k)^2$ is finite; in fig. 5b, $(q - k)^2$ is infinite and so is $(p + q - k)^2$. The other two terms are obtained by interchanging the kinematic roles of the partons in fig. 5. Notice that if the parton field carries quark quantum numbers, then so does the reggeon in fig. 5a and the upper of the two reggeons in fig. 5b. Fig. 5a is similar to that considered by Roy [6], while fig. 5b resembles the contribution discussed by Weis [5].

As outlined in the appendix, fig. 5a results in a term

$$s^{\alpha_1(0)-\gamma_1} (\log s)^{\gamma_1-2} f(\omega, t) \quad (5)$$

where $\alpha_1(0)$ is the reggeon intercept. It is interesting to notice that this is valid even for finite q^2 : there is a fixed pole in the complex angular momentum plane, with residue independent of q^2 . From fig. 5b we obtain

$$s^{\alpha(t)+\alpha_1(0)-\gamma_2} (\log s)^{\gamma_2-\alpha(t)-2} \tilde{f}(\omega, t) \quad (6)$$

where $\alpha(t)$ is the trajectory of the lower reggeon. This term does not survive in this form for finite q^2 .

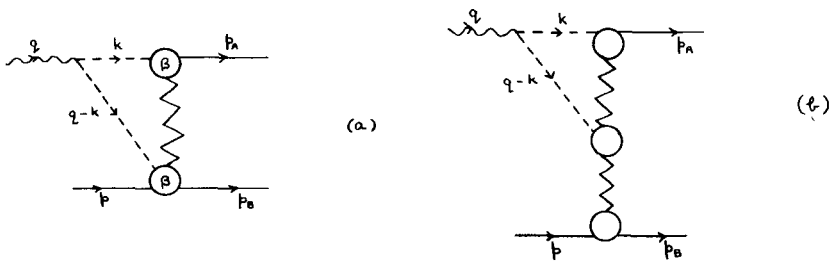


Fig. 5. Two diagrams giving the contributions to exclusive electroproduction in the Bjorken limit. Wavy lines denote reggeons. In (a) k^2 is large; in (b), $(q - k)^2$ is large.

* The variable η , associated with the Toller angle, is infinite in the present application.

The results (5) and (6) may be expressed in terms of elastic form factors. With the current (1), the elastic form factor corresponds to fig. 2. At large momentum transfer, the dominant contribution arises [1] from large values of the squared momenta of the partons. Again there is no reason to suppose that under such conditions the parton/hadron amplitude A must be dominated by reggeon exchange, but if we make such an assumption for T in fig. 4 it is natural also to make it for A . That is, we use fig. 6, with two coupling functions β identical with the one in fig. 5a. Then at large q^2 the asymptotic behaviour of the elastic form factor $F_A(q^2)$ of hadron A is found to be *

$$(q^2)^{\alpha_1(0)-\gamma_1} (\log q^2)^{\gamma_1-1} \quad (7)$$

Thus we may write (5) in the form

$$F_A(q^2) f'(\omega, t)/\log q^2. \quad (8)$$

This is similar to the result of Roy [6], except that he has $[F_A F_B]^{1/2}$ in place of F_A .

In order to deal similarly with (6), it is necessary to introduce a further assumption, as proposed by Weis [5]. This is that the numbers γ_1 and γ_2 depend on the reggeon trajectories attached to the vertices with which they are associated, in the way suggested by field-theory models [11]. This dependence says that

$$\gamma = \sum \alpha_i + \bar{\gamma} \quad (9)$$

where the sum is taken over all the trajectories at the vertex and $\bar{\gamma}$ is the value of γ when all these trajectories are at zero. Then (6) may be written

$$F_{OA}^{(1)}(q^2) \tilde{f}'(\omega, t) \quad (10)$$

where $F_{OA}^{(1)}$ is one of the terms present in the asymptotic behaviour of the excitation

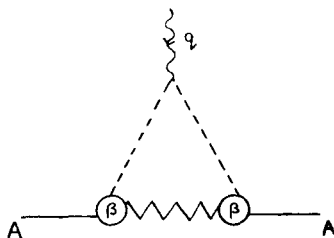


Fig. 6. A model for a vertex part contribution.

* This calculation was first performed by Cardy [10].

form factor connecting the spin zero particle on the lower trajectory of fig. 5b to the particle A. In a model for the form factor of the type of fig. 2 this term corresponds to the behaviour associated with the large momentum in the asymptotic limit being in the parton leg nearer the spin-zero particle. There is a second term in the asymptotic behaviour of the form factor corresponding to the large momentum being in the parton leg nearer A. To within logarithmic factors this second term gives the same asymptotic behaviour as that associated with $F_A(q^2)$. It is therefore possible to find models in which the two terms (8) and (10) combine to give a total asymptotic behaviour of the form

$$F_{OA}(q^2) f(\omega, t) \quad , \quad (11)$$

where F_{OA} is the complete excitation form factor, as suggested by Weis [5]. An example of such a model is provided by the Feynman integral models briefly discussed in the next section.

Finally, we note that we have not so far explicitly considered the dependence of the amplitude on the photon polarization μ at the photon-parton vertex. The calculations of the appendix readily show that gauge invariance is satisfied to the leading order in $\log \nu$.

3. DISCUSSION

Our arguments have been model dependent in a way that is not true of the discussions of inclusive electroproduction. In particular there is no compelling reason to accept the extensive use of Regge models in regions where both energy and masses are large so that the effective variable $\cos\theta_t \sim s/q^2$ is not asymptotic. Nevertheless we think it not unreasonable to hope that the results expressed in the form of eq. (8) and (10), connecting exclusive electroproduction with form factor behaviour, might prove to be reliable (though we would not attach any significance to the logarithmic factors). This hope is strengthened by considerations based on a Feynman diagram model given below.

If there is a universal rate of fall-off of form factors then the Weis and Roy type terms would coincide in their phenomenological consequences. There is, however, an interesting difference in status between the two terms which is probably most readily seen in diagrammatic terms [12].

If one considers by the standard methods [13] the asymptotic behaviour of a Feynman diagram like fig. 7 in the limit (4) one sees that there are end point contributions corresponding to the two types of d-lines indicated by the broken lines of figure. One type of d-line (the upper in the figure) joins the current vertex to the vertex at which particle A is emitted; the second type of d-line (the lower in the figure) joins the current vertex to another point. The rule of interpretation of α -space methods into momentum space language is that the sets of lines contracted in the

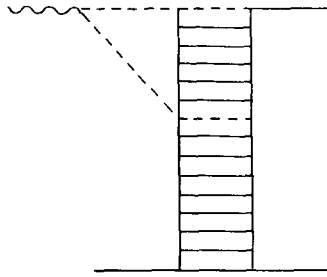


Fig. 7. A Feynman diagram.

one method correspond to those lines which carry the large momenta in the other. Thus the upper d-line corresponds to the Roy term, the lower to the Weis term* . The difference in status which results from this observation is that the Roy term corresponds to the contribution which would dominate in a light cone limit whilst the Weis term does not. The possible presence of both terms confirms that the process [3] is not in general light cone dominated.

To understand this observation one must recall that a light cone dominated process [4] results from a $Q_+ \rightarrow \infty$ limit, where Q is a momentum fed into and out of the process. If this momentum Q entered with the photon and left with particle A this would require not only that q^2 and ν tended to infinity but also m_A^2 . It is the fixed value of m_A^2 that prevents light cone dominance. However it is easy to see [13] that in the light cone limit it is only d-lines of the upper or Roy type which would count [3].

One can immediately confirm in these diagram models relations of the form of eq. (8) and (10). Finally it is easy to see from the diagrams that the Roy terms are related to fixed pole contributions in the Regge limit for electroproduction [14]. We had noted this already in sect. 2, and it is another illustration of the intimate connection between q^2 independent fixed Regge poles and light cone limits [8].

This fixed pole has a different character from the polynomial-residue fixed poles of Compton scattering previously discussed [8]. There is no obvious general reason why its position given by (5), $J = \alpha(0) - \gamma_1$, should correspond to an integer, though it will certainly be negative. In general similar poles would be expected in Compton amplitudes, for their absence would require dynamically intricate conditions to be satisfied. On the other hand, in very simple perturbation theory models [15] the pole we are considering occurs at a negative integer and its residue is proportional to the matrix element of the commutator of the electromagnetic current with the source of the A particle field. The vanishing of this commutator for neutral particles would make it possible that in this case only polynomial-residue fixed poles were present in Compton amplitudes.

* The absence of scalings over the rungs of the ladders shows the adventitious character of the Regge analysis in sect. 2.

APPENDIX

Write the final state momenta p_A and p_B in the form

$$p_i = x_i p + y_i q + \kappa_i, \quad i = A, B, \quad (\text{A.1})$$

where the κ_i are orthogonal to both p and q , and so are spacelike, $\kappa_i^2 \leq 0$. The mass-shell conditions for these momenta, together with energy-momentum conservation and the requirement that

$$t = (p - p_B)^2,$$

remains finite as $\nu \rightarrow \infty$, imply that

$$\begin{aligned} x_A &= \omega^{-1} + \bar{x}_A/2\nu, & y_A &= 1 + \bar{y}_A/2\nu, \\ x_B &= 1 - \omega^{-1} - \bar{x}_A/2\nu, & y_B &= -\bar{y}_A/2\nu, \end{aligned} \quad (\text{A.2})$$

where \bar{x}_A and \bar{y}_A are non-infinite. Similarly write the integration variable k in fig. 4 as

$$k = xp + yq + \kappa. \quad (\text{A.3})$$

Then

$$\begin{aligned} k^2 &= 2\nu y(x - y\omega^{-1}) + x^2 M^2 + \kappa^2, \\ (k - q)^2 &= 2\nu(y - 1) \left(x - \frac{y-1}{\omega} \right) + x^2 M^2 + \kappa^2. \end{aligned} \quad (\text{A.4})$$

By hypothesis, the amplitude T goes to zero fairly rapidly as the variables (A.4) become large, so that one at first sight expects the dominant contribution as $\nu \rightarrow \infty$ to come from the part of the integration region where both these variables are finite. This requires that

$$x = y\omega^{-1} + \bar{x}/2\nu, \quad y = 1 + \bar{y}/2\nu, \quad (\text{A.5})$$

where \bar{x} and \bar{y} are non-infinite; alternatively, there is another solution where the kinematic roles of k and $(q - k)$ are interchanged. With (A.5), we find that the variable $(p + q - k)^2$ is non-infinite, though of course q^2 and $s = (p_A + p_B)^2$ are large. According to usual ideas, in this kinematic domain the amplitude T is dominated by the contribution from the exchange of a single reggeon. Inserting this in fig. 4, we arrive at fig. (5a). However, when we make the change of variables (A.5) and take the

limit $\nu \rightarrow \infty$ under the integral, we find that the variable \bar{x} survives only in the expression for k^2 :

$$k^2 \sim \bar{x} + M^2\omega^{-2} + \kappa^2 \quad . \tag{A.6}$$

According to the usually-assumed analyticity properties of T , the singularities in k^2 are all below the real axis; hence we may close the contour of integration in the upper-half \bar{x} plane and so obtain zero from this integration. That is, the region of integration where both variables (A.4) are finite does not after all give the leading asymptotic behaviour. However, as we explained in the text, the behaviour of the amplitude T outside this region is very model dependent.

In these circumstances, the best way to analyse the asymptotic behaviour of fig. 4 is the Fourier-transform method that we have used in a previous work [8]. However, here we continue with the development of the Sudakov-variable approach, since although it is more difficult to make it rigorous, it offers more insight. The Fourier-transform method is found to lead to the same result. For definiteness, we consider the simple model where, even for large values of the square of the parton momentum, the amplitude T is dominated by simple reggeon exchange in exactly the same way that the on-shell amplitude is. We take account of the large mass of the virtual parton by simply replacing the appropriate reggeon coupling function by a postulated asymptotic form of that function.

Either \bar{x} or \bar{y} in (A.5) has to be large. Consider the case of large \bar{x} first; Then $(p + q - k)^2$ is still non-infinite, and fig. 5a is still appropriate. Because k^2 is now large, we replace the reggeon coupling function $\beta(k^2, (p_A - k)^2)$ at the upper vertex by its asymptotic form, which we suppose to be $(k^2)^{-\gamma_1}$ with γ_1 constant. Another factor in the integrand is $s^{\alpha_1(\omega_A - k)^2}$. We suppose that the Regge trajectory α_1 is linear, so that this is

$$s^{\alpha_1(0)} e^{\alpha_1' \log s [\bar{x}\bar{y}/2\nu + \bar{x}^2/4\nu^2 + \kappa^2]} \quad . \tag{A.7}$$

From this we determine just how large the variable \bar{x} is to be. Make what is essentially a Wick rotation, that is rotate the contour of the \bar{x} integration so that it runs parallel to the imaginary axis. Then \bar{x} cannot be larger than $O(2\nu/\log\nu)$, for otherwise the exponential would oscillate rapidly. So we make the change of variable

$$\bar{x} = \frac{2\nu}{\log\nu} \xi \quad , \tag{A.8}$$

and take the limit under the integral. Then (A.7) becomes $s^{\alpha_1(0)} e^{\kappa^2 \alpha_1' \log s}$ so that (remember that $\kappa^2 \leq 0$) the κ -integration gives a factor that is $O(s^{\alpha_1(0)}/\log s)$. The factor $(k^2)^{-\gamma_1}$ contributes $O((\log s)^{\gamma_1})$, and there is a further factor $(\log s)^{-1}$ from the Jacobian of the transformation of integration d^4k to $d\xi dy d^2\kappa$. Thus we have the result (5).

If instead \bar{y} is large, $(p + q - k)^2$ is now large, and so we must consider the exchange of another reggeon, fig. 5b. Now k^2 is finite, but $(q - k)^2$ is large, so we replace the two-reggeon/parton coupling [9] corresponding to the centre vertex by its asymptotic form, which we suppose to be $((q - k)^2)^{-\gamma_2}$, with γ_2 constant. The factors associated with the reggeons are

$$(q^2)^{\alpha_1((p_A - k)^2)} ((p + q - k)^2)^{\alpha(t)} \quad (\text{A.9})$$

If we suppose that α_1 is linear, an analysis similar to that above leads to the result (6).

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