SCALING, DUALITY, AND THE BEHAVIOR OF RESONANCES IN INELASTIC ELECTRON-PROTON SCATTERING*

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We propose that a substantial part of the observed behavior of inelastic electron-proton scattering is due to a nondiffractive component of virtual photon-proton scattering. The behavior of resonance electroproduction is shown to be related in a striking way to that of deep inelastic electron-proton scattering. We derive relations between the elastic and inelastic form factors and the threshold behavior of the inelastic structure functions in the scaling limit.

High energy inelastic electron-nucleon scattering provides a unique way to probe the instantaneous charge distribution of the nucleon and to search for possible substructure.[1] If one observes only the scattered electrons' energy and angle, then the results of such scatterings are summarized in the structure functions W_1 and W_2, which depend on the virtual photon's laboratory energy ν and invariant mass squared q^2. Considered as a collision between the exchanged virtual photon and the proton, one is studying the total cross section for the process "γ"$+p\to$ hadrons, where the hadrons have an invariant mass W which is related to ν and q^2 by $W^2=s=2M\nu+M^2-q^2$.

Experiments have revealed a very large cross section for inelastic ep scattering—a cross section which when integrated over ν at fixed q^2 is the same order of magnitude as the Mott cross section for scattering from a point proton.[1] This has led to descriptions of the scattering in terms of pointlike constituents of the proton (partons), and to the proposal of scaling,[2] i.e., as ν and $q^2\to\infty$, $W_1(\nu,q^2)$ and $\nu W_2(\nu,q^2)$ are to become functions of the single variable $\omega=2M\nu/q^2$. If we restrict ourselves to the region $W\geqslant 2.0$ GeV (above the prominent resonances) and $q^2\geqslant 0.5$ GeV^2, then the resulting subset of data is consistent with scaling, i.e., with a single smooth curve for νW_2 (and W_1) as a function of ω. This curve (for νW_2) starts at zero at $\omega=1$, the position of the elastic peak, rises to a maximum at $\omega\simeq 5$, and then appears to fall off at large ω.[13] Since νW_2 is proportional to the virtual photon-proton total cross section, such a falloff of νW_2 at large ω implies the presence of a nondiffractive (non-Pomeranchukon exchange) component of virtual photon-proton scattering. In hadronic reactions, at least, such a nondiffractive component at high energy is correlated with the presence and behavior of resonances at low energy. For example, the K^+p total cross section, which shows no prominent resonance bumps at low energy, is constant at high energy, while the K^-p total cross section, with many Y^* resonances at low energy, falls at high energy. This correlation between resonances at low energy and non-Pomeranchukon exchanges (falling total cross sections) at high energies is part of the more general concept of duality, and takes quantitative form in terms of finite-energy sum rules. We then direct our attention to the behavior of the resonances in electroproduction and the comparison of their behavior with that of νW_2 in the scaling limit, ν and $q^2\to\infty$. In particular we want to investigate whether the resonances disappear at large q^2 relative to a "background" which has the scaling behavior, or whether the resonances and any "background" have the same behavior, which might then be related to scaling and the apparent falloff in νW_2 at large ω.

When νW_2 is considered as a function of ω, the resonances occur at values of $\omega>1$, with the position of any given resonance moving towards $\omega=1$ as q^2 increases. On the other hand, the zeroth resonance or nucleon pole, corresponding to elastic scattering, always occurs at a fixed value of $\omega=1$. There have been recent attempts[4] within the framework of parton models to derive a connection between the q^2 dependence of the elastic form factors and the behavior of νW_2 in the scaling limit near $\omega=1$. But when νW_2 is considered as a function of ω the elastic peak is always at $\omega=1$ where νW_2 vanishes in the scaling limit. With the nucleon pole always at $\omega=1$, and the resonances at varying values of $\omega>1$, from a duality point of view it is difficult to compare either elastic scattering or resonance electroproduction with the scaling behavior of νW_2.

One can easily see the behavior of the resonances and of elastic scattering in comparison with νW_2 in the scaling limit by plotting the data for νW_2 versus the variable $\omega'=(2M\nu+M^2)/q^2=1+s/q^2=\omega+M^2/q^2$ (or more generally, ω'

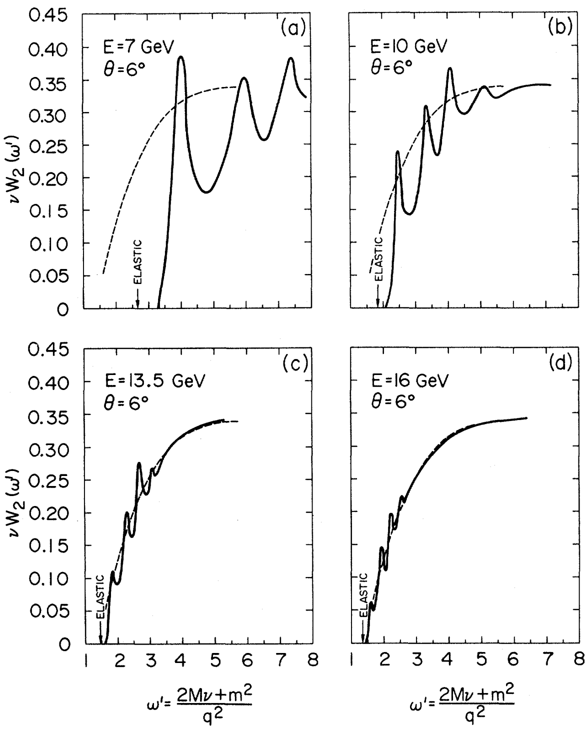

FIG. 1. The function νW_2 plotted versus $\omega' = (2M\nu + m^2)/q^2$, with $m^2 = M^2$. The solid lines are smooth curves drawn through the $\theta = 6°$ data at various incident electron energies. The dashed curve is the same in all cases and is a smooth curve through large ν and q^2 ($3 < q^2 < 7$ GeV^2, $W \geq 2$ GeV), $\theta = 10°$ data. All data are plotted assuming $R = \sigma_S/\sigma_T = 0$ (see Ref. 1). Note that the $E = 7$ GeV, $\theta = 6°$ data involve values of q^2 all of which are outside the scaling region.

$= \omega + m^2/q^2$ with $m^2 \simeq 1$ GeV^2). This variable originally arose in the analysis[5] of the large-angle inelastic ep data near $\omega = 1$. In the scaling limit where ν and $q^2 \to \infty$, the variables ω' and ω are clearly the same. For finite values of q^2 there is a difference; in particular, the elastic peak is no longer at $\omega' = 1$, but appears at $\omega' = 1 + m^2/q^2 > 1$, and moves to smaller values of ω' as q^2 increases, just as the other resonances do.[6]

The results of making such a plot versus $\omega' = 1 + s/q^2 = \omega + M^2/q^2$ are shown in Fig. 1. The dashed line, which is the same in all cases, is a smooth curve through the high energy $\theta = 10°$ data[7] in the region beyond the prominent resonances ($W > 2.0$ GeV) and with large q^2 ($3 < q^2 < 7$ GeV^2). This is a region where the scaling behavior has occurred experimentally, and we call this the "scaling limit curve," $\nu W_2(\omega')$. The solid lines are smooth curves through 6° data at incident electron energies of 7, 10, 13.5, and 16 GeV, and typical values of q^2 of 0.4, 1.0, 1.7, and 2.4 GeV^2, respectively. As q^2 increases the resonances move toward $\omega' = 1$, each clearly following in magnitude the smooth scaling-limit curve. As similar graphs of the 10° data in the resonance region also show, the prominent resonances do not disappear at large q^2 relative to a "background" under them, but instead fall at roughly the same rate as any "background" and closely, resonance by resonance, follow the scaling-limit curve. We emphasize that this behavior of the resonances, which is of central importance in our arguments, can be seen by careful examination of the data when they are plotted with respect to other variables; with respect to ω' it just becomes obvious at a glance.

Thus the resonances have a behavior which is closely related to that of νW_2 in the scaling limit. For large values of ω', the data for νW_2 with $q^2 > 0.5$ GeV^2 are consistently on a single curve which falls with increasing ω', just as when plotted versus ω. We therefore propose that the resonances are not a separate entity but are an intrinsic part of the scaling behavior of νW_2, and that a substantial part of the observed scaling behavior of inelastic electron-proton scattering is nondiffractive in nature. Appropriately averaged, the nucleon and the resonances at low energy build, in the duality sense, the relevant non-Pomeranchukon exchanges at high energy, which result in a falling νW_2 curve.[8]

What is unique to electroproduction is the experimentally observed scaling behavior which allows us to consider points at the same ω' arising from different values of q^2 and $s = W^2$, both within and outside the low-energy resonance region. If we choose ν_m and q^2 in the region where νW_2 scales, i.e., beyond the region of prominent resonances and where $\nu W_2(\nu, q^2) = \nu W_2(\omega') =$ a smooth function of ν (see Fig. 1), then a finite-energy sum rule for νW_2 at fixed q^2 tells us that

$$\frac{2M}{q^2}\int_0^{\nu_m} d\nu\, \nu W_2(\nu, q^2) = \int_1^{(2M\nu_m + m^2)/q^2} d\omega'\, \nu W_2(\omega'), \qquad (1)$$

since the integrands are the same for $\nu > \nu_m$ or $\omega' > (2M\nu_m + m^2)/q^2$ (by the assumption that ν_m and q^2 are in the region where νW_2 scales). Equation (1) states that for $\nu < \nu_m$, $\nu W_2(\omega')$ acts as a smooth function which averages $\nu W_2(\nu, q^2)$ in the sense of finite-energy sum rules. Thus, because we can vary the external photon mass in electroproduction and have scaling, we can directly measure a smooth curve which averages the resonances in the sense of finite-energy sum rules. High-energy electroproduction thus becomes a beautiful example of the duality between resonances and non-Pomeranchukon exchanges at high energy.

Looked at the other way, by appropriate averages over the resonances we would build up the curve for νW_2 in the scaling limit. But how can resonances, which have form factors which fall rapidly with q^2, be consistent with a scaling-limit curve which is supposed to characterize a very slow q^2 variation? Let us fix $s=M_R^2$, the mass squared of a given resonance (possibly this could be the zeroth resonance, the nucleon) and vary q^2. Then if $G(q^2)$ is the excitation form factor of the resonance,

$$\nu W_2 = 2M\nu[G(q^2)]^2\delta(s-M_R^2) = (M_R^2-M^2+q^2)[G(q^2)]^2\delta(s-M_R^2) \quad (2)$$

is its contribution to νW_2 in the narrow-resonance approximation. For large q^2, the form factor falls off as some power, say

$$G(q^2)\to(1/q^2)^{n/2}. \quad (3)$$

As q^2 increases, the resonance is pushed down toward $\omega'=1$, were $\nu W_2(\omega')$ can be parametrized by some power behavior,

$$\nu W_2 \xrightarrow[\omega'\to 1]{} c(\omega'-1)^p = c[(s-M^2+m^2)/q^2]^p. \quad (4)$$

If Eqs. (2), (3), and (4) are to be consistent and the resonances build up the scaling-limit curve locally, then we must have

$$n=p+1, \quad (5)$$

i.e., all resonances (including the nucleon) which are to follow the scaling limit curve as $q^2\to\infty$ must have the same power of falloff in q^2 for large q^2 and this power is related to the power with which $\nu W_2(\omega')$ rises at threshold.

Equation (5) is just the relation first derived by Drell and Yan[4] in the parton model for the elastic form factor $F_1(q^2)$. Here, by demanding that the resonances must locally build up the scaling-limit curve, we obtain it for the elastic and inelastic form factors. That all the resonance-excitation form factors have approximately the same behavior as the elastic form factor at large q^2 has been noted previously in nucleon-resonance electroproduction.[9]

Conversely, this experimental fact means that each resonance individually follows the scaling-limit curve in magnitude as $q^2\to\infty$ (i.e., as it approaches $\omega'=1$). Indeed, Fig. 1 suggests that for $m^2\simeq M^2$, the finite-energy sum-rule average of Eq. (1) can be made over a quite local region; i.e., the area under the scaling-limit curve $\nu W_2(\omega')$ equals the total area under a given resonance bump integrated over an energy region (in W) of a few hundred MeV below and above the resonance.

It is instructive to take the variable ω' on a more serious basis and to carry the idea of local averaging to an extreme: We make the very strong assumption that, in the sense of Eq. (1), the area under the elastic peak in νW_2 for large q^2 is also the same as the area under the scaling-limit curve, $\nu W_2(\omega')$, from $\omega'=1$ to an ω' corresponding to a hadron mass $W=W_t$ near physical pion threshold, i.e.,

$$\int_1^{1+(W_t^2-M^2+m^2)/q^2} d\omega'\,\nu W_2(\omega') = \left(\frac{2M}{q^2}\right)\int d\nu\,\nu W_2^{\mathrm{el}}(\nu,q^2) = [G(q^2)]^2 = [F_1(q^2)]^2 + (q^2\mu_A^2/4M^2)[F_2(q^2)]^2 = \frac{[G_E(q^2)]^2+(q^2/4M^2)[G_M(q^2)]^2}{1+q^2/4M^2}. \quad (6)$$

Taking the derivative with respect to q^2, we obtain

$$\nu W_2\left(\omega'=1+\frac{W_t^2-M^2+m^2}{q^2}\right) = \left(\frac{1}{\omega'-1}\right)\left(-q^2\frac{d}{dq^2}[G(q^2)]^2\right), \quad (7)$$

which allows us to calculate $\nu W_2(\omega')$ near threshold in terms of the elastic form factors once we have chosen $W_t^2-M^2+m^2$. If $G(q^2)\to(1/q^2)^{n/2}$ as $q^2\to\infty$, then from (7),

$$\nu W_2 \xrightarrow[\omega'\to 1]{} (\omega'-1)^p,$$

where $n=p+1$, so we again recover the relation (5) between the elastic form factor and threshold behavior of νW_2.

We might expect such radical assumptions to work when the elastic peak is pushed into the threshold region of $\nu W_2(\omega')$, i.e., when $q^2\gg 1$ GeV^2 and $\omega'-1=(W_t^2-M^2+m^2)/q^2\ll 1$. A value of $W_t^2-M^2+m^2\simeq 1.5$ GeV^2 results in a $\nu W_2(\omega')$ curve calculated from Eq. (7) which approximately averages the $\theta = 10°$, $E = 17.7$-GeV ($q^2 \simeq 7\ \mathrm{GeV}^2$) data with $W < 1.8$ GeV ($\omega' - 1 \lesssim 0.5$). Presently available data with $W > 2$ GeV do not extend into the region $\omega' - 1 \ll 1$, but preliminary indications from the large-angle data[5] indicate[10] a smooth scaling-limit curve which also approximately averages the $\theta = 10°$, $E = 17.7$-GeV data.

We note that similar assumptions applied to W_1 yield that $R = \sigma_S/\sigma_T$, the ratio of longitudinal to transverse total cross sections, goes to zero near threshold; and when applied to inelastic electron-neutron scattering, predicts that for large q^2,

$$\frac{\nu W_{2n}}{\nu W_{2p}} = \frac{(d/dp^2)[G_n(q^2)]^2}{(d/dp^2)[G_p(q^2)]^2} \xrightarrow[q^2 \to \infty]{} \left(\frac{\mu_n}{\mu_p}\right)^2, \tag{8}$$

or approximately one-half near threshold.[11] A difference between the neutron and proton is generally to be expected if a substantial part of the inelastic electron-nucleon scattering is nondiffractive, as we propose. If we take the nondiffractive parts of the $q^2 = 0$, γp and γn total cross sections[12] as a guide, then we expect in general that νW_{2n} will be smaller than νW_{2p}.

Finally, we note that in this paper we have not predicted scaling, but have found correlations of other observations with the experimentally observed scaling behavior. The connection between the behavior of the resonances and scaling which we propose hints again at a common origin for both in terms of a pointlike substructure of the nucleon. Translating this "hint" at a common origin into a real quantitative theory remains, as before, an unsolved problem.

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[1]For a review of the kinematics and of the experimental and theoretical aspects of inelastic electron-proton scattering, see R. E. Taylor and F. J. Gilman, in *Proceedings of the International Symposium on Electron and Photon Interactions at High Energies, Liverpool, 1969,* edited by D. W. Braben (Daresbury Nuclear Physics Laboratory, Daresbury, Lancashire, England, 1970). We use M = mass of the nucleon, and spacelike values of q^2 are positive.

[2]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[3]M. Breidenbach *et al.*, Phys. Rev. Lett. 23, 935 (1969).

[4]S. D. Drell and T. M. Yan, Phys. Rev. Lett. 24, 181 (1970); see also G. West, Phys. Rev. Lett. 24, 1206 (1970).

[5]G. Miller, E. D. Bloom, G. Buschhorn, D. H. Coward, H. DeStaebler, J. Drees, C. L. Jordan, L. W. Mo, and R. E. Taylor, to be published; J. I. Friedman, G. C. Hartmann, and H. W. Kendall, to be published. Preliminary results from these data have been presented by R. E. Taylor in Ref. 1, and in *Proceedings of the International Conference on Expectations for Particle Reactions at the New Accelerators, University of Wisconsin, Madison, Wisconsin, 20 March-1 April 1970* (Univ. of Wisc., Physics Dept., Madison, Wisc., 1970). We thank the members of the Stanford Linear Accelerator Center–Massachusetts Institute of Technology collaboration for discussions.

[6]A rationale for the use of the variables $q_0 \pm q_z$ to obtain double dispersion relations on the light cone has been suggested by R. P. Feynman. The variable $q_0 - q_z$ is very similar to $M\omega'$, but with m^2 varying from M^2 to 0 as ω' goes from 1 to ∞. We thank Professor Feynman for stimulating discussions.

[7]E. D. Bloom *et al.*, Phys. Rev. Lett. 23, 930 (1969); M. Breidenbach, thesis, Massachusetts Institute of Technology, 1970 (unpublished).

[8]In our proposal that a substantial part of the scaling behavior is due to a nondiffractive component of virtual photon-proton scattering we differ considerably from a number of other authors (see, for example, the discussion in Ref. 1). In particular, in our interpretation of the data for resonance electroproduction and the behavior of νW_2 for large ω, and correspondingly in the predictions following from our proposal of a substantial nondiffractive component [for example, the difference between electroproduction on neutrons and protons discussed after Eq. (8)], we differ from the previous application of duality ideas to inelastic electron scattering by H. Harari, Phys. Rev. Lett. 22, 1078 (1969).

[9]See, for example, W. K. H. Panofsky, in *Proceedings of the Fourteenth International Conference on High Energy Physics, Vienna, Austria, 1968,* edited by J. Prentki and J. Steinberger (CERN Scientific Information Service, Geneva, Switzerland, 1968), p. 23.

[10]We thank G. Miller for discussions of the threshold region in the large-angle data.

[11]E. D. Bloom and F. J. Gilman, to be published. Detailed neutron-proton and σ_S/σ_T predictions can be extracted and eventually can be compared with future large-angle, high-q^2, electron-deuteron experiments. Relations similar to Eq. (7) relate the threshold behavior of inelastic neutrino-nucleon scattering to $g_A(q^2)$.

[12]D. O. Caldwell *et al.*, to be published.