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e^+e^- annihilation into hadrons

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This review is concerned with the various theoretical approaches to the problem of e^+e^- annihilation into hadrons in the light of the new experimental data obtained in 1973–4. Discussions are given of the behavior of the total cross section for e^+e^- annihilation into hadrons, the form of the inclusive spectra, the dependence of the particle form factors in the timelike region on the square of the momentum transfer, and the effect of the process of e^+e^- annihilation into hadrons on scattering by positrons and on the process $e^+e^- \to \mu^+\mu^-$. Other topics considered include the relation between inclusive annihilation and electroproduction, as well as a number of theoretical models: the parton model, the statistical model, etc.

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CONTENTS

1. INTRODUCTION

During the past few years, we have steadily gained confidence in the possibility of describing hadron structure in terms of quarks. There have been two main reasons for this confidence: first, the generally successful description of the data of baryon spectroscopy, and second, the success with which electroproduction and neutrino reactions have been explained on the basis of the quark model of current algebra on the light cone.

As is well known (see, e.g.,[1]), experimental studies of deep inelastic electroproduction and neutrino-nucleon scattering[1)] have revealed that these processes differ markedly from the elastic scattering of electrons and neutrinos by nucleons. Whereas the nucleon form factors which determine the behavior of the elastic scattering cross sections are rapidly decreasing functions of q^2, the square of the momentum transferred to the nucleon, the invariant functions which determine the cross sections for deep inelastic processes, when summed over a large number of final hadronic states, have been found to have a weak dependence on q^2. This fact, in analogy with the well-known problem in nonrelativistic quantum mechanics concerning the scattering of electrons by a weakly bound system of particles (e.g., by an ion or an atomic nucleus), has led to the idea that the nucleon in such processes can be described as a combination of point-like constituents—partons. The parton model (see, e.g.,[2]), in particular in the specific form of the parton-quark model, in which the partons appear as quarks, has been highly successful in describing deep inelastic electroproduction and neutrino-nucleon scattering: an explanation has been given for the experimentally observed scale-invariant behavior of the cross sections, the small ratio of the cross sections for the scattering of longitudinally and transversely polarized virtual photons by nucleons, and the relation between the cross sections for electroproduction and neutrino scattering. These same results have been obtained in a somewhat more general form by making use of the fact that the interaction in a deep inelastic lepton-nucleon scattering processes occurs in the vicinity of the light cone (see, e.g.,[3]) and assuming a definite form for the current commutators on the light cone[4].

The success of the parton model has led to very specific and reliable predictions regarding the behavior of the total cross section for e^+e^- annihilation into hadrons in colliding beams, predictions which are clearly not in agreement with the latest data obtained at CEA and SPEAR[2)]. Similarly, the scale-invariant behavior of the electroproduction process (and of the inclusive hadron spectra in this process) has led to the hypothesis of scale invariance for inclusive hadron production in e^+e^- annihilation[3)]. This scale invariance is also clearly violated. In this situation, it is clear that we now face a period of careful tests of both the theoretical techniques which we have been using and the facts which we believe the experiments are teaching us. We must ascertain whether the difficulties are due to the basic hypothesis about the quark structure of hadrons or to an inordinate confidence in the theoretical assumption of scale invariance, or, finally, whether these difficulties are simply caused by a "background" of additional "exotic" processes in e^+e^- annihilation into hadrons.

2. EXPERIMENTAL DATA

Measurements of the total cross section for the annihilation $e^+e^- \to$ hadrons at Frascati[5] have yielded a large value for the cross section for producing mutli-hadron final states at e^+e^- center-of-mass energies $E_{c.m.}$ in the range 1.5 GeV $< E_{c.m.} <$ 3 GeV (but with a large systematic dispersion in the experimental data). The cross section for hadron production was found to be larger by a factor 1–2 than the cross section for the process $e^+e^- \to \mu^+\mu^-$—the theoretical scale for cross sections which is usually employed for the class of pheno-

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mena under consideration: $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-) \sim 1\text{–}2$. (The theoretical values of the effective cross sections for the process $e^+e^- \to \mu^+\mu^-$ have been confirmed experimentally at SPEAR with good accuracy up to energies $E_{c.m.} \approx 5$ GeV).

At higher energies, measurements of the total cross section for the annihilation $e^+e^- \to$ hadrons carried out at CEA[6] and confirmed at SPEAR[7] give a roughly constant total cross section $\sigma_{tot} \approx 20$ nb for $E_{c.m.}$ between 3 GeV and $E_{c.m.} \approx 4.8$ GeV. At the highest energy, this corresponds to a total cross section which is about 5 times as large as the cross section for muon pair production: $R \approx 5$.

A number of highly preliminary results regarding the spectrum of produced hadrons have also been obtained at SPEAR[7]. These results may be formulated briefly as follows:

a) The inclusive cross section $Ed\sigma/d^3p$ for producing charged hadrons with energy E can be well described (with an accuracy $\sim 20\%$) by an exponential $e^{-E/T}$ with $T \approx 170$ MeV at the two e^+e^- beam energies $E_{c.m.} \approx 3$ GeV and $E_{c.m.} \approx 4.8$ GeV. A break appears in the curve above $E \approx 1.2$ GeV, leading to a growth in the cross section by about a factor of 3 at $E \approx 1.7$ GeV.

b) The angular distribution of charged particles (for $|\cos\theta| < 0.5$) is isotropic for both small momenta ($p/p_{max} < 0.45$) and large momenta, at each of the energies $E_{c.m.} = 3$ GeV and $E_{c.m.} \approx 4.8$ GeV. For angular distributions represented in the form $1 + \cos^2\theta$, values of α of the type $\alpha \approx 0.0 \pm 0.3$ were obtained (this number should be interpreted as a general indication of the accuracy of the preliminary results).

c) For hadron momenta $p < 0.6$ GeV in the inclusive spectrum, the contributions of charged pions, charged kaons and protons were separated (by time of flight measurements). After the energy dependence of $Ed\sigma/d^3p$ was constructed for each of these spectra, it was found that they all lie on a single exponential curve constancy of the e^+e^- beam energies $E_{c.m.} = 3$ GeV and $E_{c.m.} = 4.8$ GeV. When these spectra were integrated, it was found that the protons, kaons and pions were, in order of magnitude, in the ratio $p : K : \pi = 1 : 10 : 100$.

d) The constancy of σ_{tot} and $Ed\sigma/d^3p$ as a function of the total energy, together with the assumed complete isotropy of the angular distribution and the rapid fall-off of $Ed\sigma/d^3p$ with E, imply that the multiplicity of charged particles, $\bar{n}_{ch}$, must be constant. More direct measurements indicate a slow growth of $\bar{n}_{ch}$ between $E_{c.m.} = 3$ GeV and $E_{c.m.} = 4.8$ GeV ($\bar{n}_{ch} \approx 4$ and increases by $\Delta\bar{n}_{ch} \approx 0.5$).

e) The constancy of σ_{tot} and $Ed\sigma/d^3p$ as a function of the total energy, together with the isotropy of the angular distribution and the relation $Ed\sigma/d^3p \sim e^{-E/T}$, would imply that the total average energy of the charged hadrons in the final state is independent of $E_{c.m.}$. In other words, the fraction of the initial energy carried by the charged hadrons is a decreasing function of $E_{c.m.}$. An experiment which is more direct than the foregoing argument indicates that this fraction falls off from $\approx 2/3$ at $E_{c.m.} \approx 3$ GeV to $\approx 1/2$ at $E_{c.m.} = 4.8$ GeV. The precise number associated with this "energy crisis" is weakly dependent on the production model, since the solid angle subtended by the measuring apparatus is $\approx 2\pi$.

f) The foregoing results, which are so typical of purely hadronic processes, might suggest an interpretation of the data in terms of a two-photon mechanism for $e^+e^- \to e^+e^-$ + hadrons in which the electron and positron emit virtual photons, whose collision gives rise to the hadrons. However, there exist certain experimental facts which indicate that less than $\sim 10\%$ of the events can be due to the two photon mechanism. Luminosity monitors in the SPEAR experiment placed near $\theta = 0°$ and sensitive to e^+ and e^- indicate no coincidences with multi-hadron reactions, apart from several coplanar e^+e^- and $\mu^+\mu^-$ pairs with small effective masses, in accordance with theoretical expectations; improved measurements of the two-photon contribution will be feasible in 1975 at SPEAR and at the new e^+e^- rings DORIS at DESY[4)].

g) For the exclusive channels, measurements at high energies have been made only at Frascati[8]. At $E_{c.m.} \approx 2.1$ GeV, the total cross sections for e^+e^- annihilation into the final states $\pi^+\pi^-$, K^+K^- and $\bar{p}p$ are comparable and the form factors are of the same order of magnitude:

$$|F_\pi|^2 \approx |F_K|^2 \approx 0.02, \quad |F_{p\,\mathrm{eff}}|^2 \approx 0.014.$$

3. GENERAL DISCUSSION OF THE EXPERIMENTAL DATA

a) "**Exotic**" **explanations**. Although it is highly probable that multi-hadron production proceeds via the exchange of a single time-like virtual photon, it is also important to test other possibilities. The two-photon hypothesis encounters difficulties when compared with the experimental data. However, even if this were not so, the observed cross section is at least an order of magnitude greater than what would be expected in the case according to the vector-dominance model. The hypothesis that there is a direct electron-hadron interaction (a "zero-photon process") via a new semi-weak interaction must provide answers to the following questions: 1) Will there be a large violation of the scale-invariant behavior in electroproduction at values of order $Q^2 \sim 10\text{–}25$ GeV^2? 2) Why does the neutrino not have such an interaction? 3) If muons have such an interaction, does it affect the decays $\eta \to \mu^+\mu^-$, $\eta \to e^+e^-$ and $K_L \to \mu^+\mu^-$ or the level shift in mu-mesonic atoms?

Greenberg and Yodh[9a] and Nanopoulos and Vlassopulos[9b] ascribed completely hadronic properties to the electron and claimed that the observed effect is "diffraction scattering"; they assumed a sharp forward peak in the angular distribution of hadrons. Since any object exchanged in the t-channel or the u-channel of the amplitude for the process $e^+e^- \to$ hadrons carries lepton number, this process is very different from an ordinary hadronic process. (It is apparently much more local in impact parameter space.) Any similarity between such a zero-photon process and ordinary hadronic physics is purely accidental.

Within the framework of the single-photon exchange mechanism, there also exist "exotic" explanations, such as production of heavy lepton pairs which decay predominantly into multi-hadron states or production of arbitrary pairs of charged non-hadronic objects (particularly bosons with $J = 1$) which, however, decay predominantly into hadrons. A good test of this hypothesis would be strict constancy of $\bar{n}$ as a function of $E_{c.m.}$ and (for large $E_{c.m.}$) exact scaling behavior of the inclusive distribution.

We shall not consider the "exotic" alternatives here.

Thus, we shall assume that the experimentally measured quantity (apart from correction terms) is the square of the matrix element of the hadronic electromagnetic current operator between the vacuum and the appropriate final hadron states.

b) **Consequences of $\sigma_{tot} \sim$ const.** If σ_{tot} remains constant at $\sim$20 nb with increasing $E_{c.m.}$, the contribution to the photon propagator from the hadronic vacuum polarization becomes large and quantum electrodynamics using perturbation theory proves to be inapplicable. This fact is discussed in detail in Sec. 4g and Sec 8. However, a large value of R in any particular case can lead to measurable corrections to $\mu^+\mu^-$ production and to the cross section for e^+e^- scattering (Bhabha scattering).

For example, if σ_{tot} remained constant up to $E_{c.m.} \approx 50$ GeV, then the correction to the Bhabha cross section at 90° for $E_{c.m.} \approx 8$ GeV would be $\sim$3%.

c) **Distant scaling?** Many theorists have put forward the idea that there exists a scaling behavior for σ_{tot}(R = const) but that the approach to scaling is slow ("senile scaling"), at least about 20 times as slow as the approach to scaling in electroproduction. However, it is also possible that a scaling behavior exists for $2\,\text{GeV}^2 \lesssim q^2 \lesssim 10\,\text{GeV}^2$ (for example, the constant R is $\sim$2 and the form of the inclusive spectrum satisfies scaling). The principal region of space-like Q^2 for which a scaling behavior has been established is of just this order of magnitude. Thus, the fact that we have slipped into the "scaling" region $2 < q^2 < 10\,\text{GeV}^2$ in e^+e^- annihilation (in the experimental sense) almost without noticing it is evidence of the power of the colliding-beam technique in achieving high energies. In the corresponding time-like region, there exist only the Frascati data for σ_{tot} with a large dispersion in the experimental points, and there is as yet no measurement of the inclusive spectrum.

d) **The "energy crisis."** As we have already mentioned, the neutral particles carry away a larger fraction of the energy than what might be expected naively on the basis of equal π^0, π^+ and π^- production, with a ratio (neutral energy)/(charged energy) ≈ 0.5. At $E_{c.m.} \approx 3$ GeV, the data are in rough agreement with this estimate. However this ratio is approximately equal to unity (with errors $\sim$20%) at $E_{c.m.} \approx 4.8$ GeV. Before drawing far-reaching conclusions from this fact, it is important to note that the purely annihilation process $\bar{p}p \to$ mesons (with no baryons) is also characterized by a surplus[10]

$$\left(\frac{\text{natural energy}}{\text{charged energy}}\right)_{\bar{p}p} \sim 0.7$$

for $10 \le s \le 15\,\text{GeV}^2$.

This large surplus can be explained by assuming that inclusive η production is comparable with inclusive kaon (or pion) production, since over 80% of the energy in η decay goes into the neutral channels. But the energy dependence of the experimental effect remains unexplained, particularly because the ratio K/π (at a fixed hadron energy) is weakly dependent on $E_{c.m.}$ and the kaons must be produced in pairs. Thus, an explanation based on the energy dependence of η production as a consequency of a threshold effect becomes difficult.

However, after allowance is made for the effect of η production, the magnitude of the remaining "energy crisis" is quite small and may even disappear completely as a result of the more careful analysis of the data which is now in progress.

e) **The ratios of particle yields.** Wherever the $\pi : K : p$ production ratios have been measured at the same hadron energy, they have been found to be of order unity. This can be regarded as a natural consequence of the statistical hydrodynamic picture of production processes[11,12]. Having been generalized to cases of inclusive processes involving large final energies (in conjunction with the ideas of "duality" or "correspondence"[13,14]), this statement would imply, however, that the ratio of the exclusive $\bar{p}p$ and $\pi\pi$ cross sections must also be of order unity everywhere, i.e., that the pion form factor $F_\pi(q^2)$ and the proton magnetic form factor $G_{M_p}(q^2)$ must have the same dependence on q^2. This is not in accordance with the theoretical ideas (or experimental tendencies) which assume that $F_\pi \gg G_{M_p}$ at large q^2[5]. For example, for $F_\pi \sim (1 - q^2/m^2\rho)^{-1}$ and $G_{M_p} \sim (1 - q^2/0.7)^{-2}$, the ratio $\bar{p}p/\pi\pi$ would decrease by about a factor of 5 as q^2 varies from $10\,\text{GeV}^2$ to $q^2 \approx 25\,\text{GeV}^2$ and by about a factor of 25 from the Frascati energy ($q^2 \approx 4.4\,\text{GeV}^2$, where $\bar{p}p/\pi\pi \sim 1$) to the highest energies at SPEAR.

f) **Scaling in inclusive processes.** As we shall describe later, a number of theoretical ideas lead to the conclusion that the E-dependence of $(E/\sigma_{tot})(d\sigma/d^3p)$ must have a scale-invariant form (together with $\sigma_{tot} \sim 1/q^2$), i.e.,

$$q^2 E_{\max}\left(\frac{d\sigma}{dE}\right) = f\left(\frac{E}{E_{\max}}\right). \tag{1}$$

When $q^2 E\, d\sigma_{\max}/dE$ is represented as a function of $\omega = E/E_{\max}$, approximate scaling behavior is actually observed for $\omega \gtrsim 0.45$. Moreover, the form of the structure function for $q^2 \sim 10\,\text{GeV}^2$ (where R $\sim$ 2 and the ratio of the energy of the neutral particles to that of the charged particles is $\sim$0.5) is in rough agreement with the theoretical expectations, $\sim 2(1-\omega)$[15]. This result may be an indication that the effect has a two-component character, where the growth of R is a consequence of a new process in which only low-energy hadrons are produced. However, the scaling hypothesis in the general case also assumes an anisotropic angular distribution (of the type $1 + \cos^2\theta$) of the energetic hadrons[6]. No sign of such an angular dependence has been observed, even at the highest energies, which makes it difficult to justify this two-component hypothesis in the absence of a large contribution of longitudinal photons (which does not occur in the electroproduction process). However, some care must be exercised here, since the data are preliminary and the measurements are extremely delicate, requiring a good knowledge of the efficiency of the detectors as a function of the angle.

4. THE TOTAL CROSS SECTION FOR e^+e^- ANNIHILATION INTO HADRONS. THEORETICAL EXPECTATIONS

It is well known that the single-photon approximation gives a total cross section for e^+e^- annihilation into a pair of non-interacting particles of spin 1/2 or 0 at large beam energies E (i.e., E $\gg$ the masses of all the particles) which is proportional to $1/s$, where $s = q^2 = 4E^2$. In particular, in the case of annihilation of an e^+e^- pair into a pair of fermions of spin 1/2 (e.g., $e^+e^- \to \mu^+\mu^-$) we have

$$\sigma_{\text{ann}} = \frac{4\pi\alpha^2}{3}\frac{1}{s}, \tag{2}$$

while for e^+e^- annihilation into a pair of spin-0 bosons we have

$$\sigma_{\text{ann}} = \frac{\pi\alpha^2}{3}\frac{1}{s}. \tag{2'}$$

There exist a number of arguments and demonstrations[16-21] based on various approaches to show that this type of asymptotic dependence of the total cross section for the annihilation $e^+e^- \to$ hadrons, $\sigma_{tot} \sim 1/s$, holds even when allowance is made for the strong interactions, provided that the "bare" particles which appear in the Lagrangian of the electromagnetic interaction of the hadrons have spins 0 and 1/2. We shall consider these arguments in turn.

a) **Dimensional arguments and the utilization of Wilson's technique for expanding a product of operators at small distances.** Considering the fact that the theory involves no dimensioned constants for large masses q^2 of the virtual photon, it is an obvious consequence of dimensional arguments that $\sigma_{tot} \sim 1/s$. This same result can be derived[18] by using the method of Wilson[22]. The total cross section σ_{tot} can be represented in the form

$$\sigma_{tot} = -\frac{8\pi^2\alpha^2}{3q^4} \int \langle 0 | [j_\mu(x), j_\mu(0)] | 0 \rangle e^{iqx} d^4x, \tag{3}$$

where $j_\mu(x)$ is the electromagnetic current of the hadrons. By virtue of the causality condition, the values $x \lesssim 1/q_0 = 1/\sqrt{q^2}$ are important at large q^2 in the integral (3) in the c.m.s. According to Wilson, the behavior of σ_{tot} as $q^2 \to \infty$ is determined by the dimensions of the product of current operators. Since the dimensions of the current operators cannot be changed by the interaction, owing to the conservation of charge, we have $\langle 0 | [j_\mu(x), j_\mu(0)] | 0 \rangle_{x \to 0} \sim x^{-6}$ and $\sigma_{tot} \sim 1/s$. In the case of deep inelastic electroproduction, we note that similar arguments based on the analysis of the important space-time region in this process lead to a behavior[23] $\sigma_{\gamma N}(q^2, \nu) \lesssim 1/|q^2|$ of the total cross section for the absorption of a virtual photon of mass $\sqrt{-q^2}$ by a nucleon, in agreement with the experimental data on ep scattering. We mention here an interesting relationship obtained by Crewther[24, 25] on the basis of the above-mentioned assumption about the behavior of the field operators at small distances. This is a relationship between the low-energy parameter—the $\pi^0 \to 2\gamma$ decay constant S—and the high-energy parameters, and takes the form

$$3S = KR'.$$

Here R' is a quantity analogous to $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ and differing from R only by the fact that R' contains the axial-vector current instead of the vector hadronic current; K is a constant which determines the magnitude of the difference between the cross sections for the scattering of electrons polarized along the beam and opposite to the beam, for deep inelastic scattering of electrons by polarized protons in the beam[16]. Although the quantity R' is not directly measurable experimentally, we can expect it to be close to R, since the equality $R = R'$ is a necessary condition for the convergence of Weinberg's first sum rule for the spectral functions[26].

b) **An argument based on the analysis of the Schwinger term in the current commutator**[17]. Using only Lorentz invariance, the spectral condition and current conservation, one can derive the sum rule parton lifetime

$$\int_0^\infty \sigma_{tot}(q^2)\, q^2\, dq^2 = -i16\pi^3\alpha^2 \int x_i \langle 0 | [j_0(x), j_i(0)] | 0 \rangle_{x_0=0}\, d^3x, \tag{4}$$

which relates an integral of σ_{tot} with respect to q^2 to a single time commutator of the time and space components of the currents (the so-called Schwinger term). This commutator is quadratically divergent in theories in which $j_\mu(x)$ is a current of charged spin-1/2 fermions and/or spin-0 bosons. This implies that the integral on the left-hand side of (4) is also quadratically divergent, i.e., that $\sigma_{tot}(s) \sim 1/s^?$.

c) **The parton model**[19-21]. In the parton model, the annihilation $e^+e^- \to$ hadrons takes place in such a way that the e^+e^- pair first annihilates into a non-inter-into various hadronic states. The possibility of such a description is based on the previously mentioned fact that the important interval of time during which the parton must be regarded as a free particle (in order to make use of the results of calculations of σ_{tot} for free fields) is of order $\tau \sim q_0^{-1}$ in relation to the parton lifetime $T \sim 1/M_{eff}$. It is assumed that the effective parton masses do not increase with energy (or that they increase, but slowly), so that $\tau \ll T$. A confirmation of the smallness of the interaction which "dresses" the parton with momentum $\sim q_0$ in a time $\tau \sim q_0^{-1}$ is the scale-invariant character of the data on electroproduction, where the "dressing" of the parton is clearly small in the time $\tau' \sim \omega m^{-1}$ (ω is a dimensionless variable whose values are usually of order 2–20 in the electroproduction process).

It should be noted, however, that at the present time we have no clear understanding of the dynamics of the parton model and that the different kinematics for electroproduction and annihilation may lead to different results for these two processes. Putting these doubts aside, we obtain in the parton model the prediction

$$R = \frac{\sigma_{tot}}{\sigma(e^+e^- \to \mu^+\mu^-)} = \sum_{i,\, s=1/2} Q_i^2 + \frac{1}{4} \sum_{k,\, s=0} Q_k^2, \tag{5}$$

where Q_i and Q_k are the charges of the spin-1/2 and spin-0 partons, respectively. It follows from this result that $R = 2/3$ for the ordinary quarks, $R = 2$ for "colored" quarks[27], and $R = 10/3$ for a model involving three quartets of fractionally charged quarks[28]. None of these values are consistent with the experimental data.

For the model of Han and Nambu[29, 30] involving three triplets of quarks with integral charges $(0, -1, -1)$, $(1, 0, 0)$ and $(1, 0, 0)$, we have $R = 4$, in approximate agreement with the existing experimental data. We can expect[31] the "colored" degrees of freedom to be "frozen" at low energies ($q^2 \ll 15\ \text{GeV}^2$). Since all hadronic states are "colored" singlets, only the singlet part of the electromagnetic current is effective at these energies, and this gives the value $R = 2$, in agreement with the experimental data. Agreement of this model with the data requires in addition that half of the events of annihilation into hadrons at $\sqrt{s} \approx 4$–5 involve the production of "colored" states which decay into ordinary hadrons with the emission of either photons or lepton pairs. Therefore, from the point of view of this model, the situation which we termed the "energy crisis" in Sec. 3, if it is attributed to the surplus of energy carried away by the photons or neutrinos, can only be welcomed. Such a "color thaw" should lead to a similar increase in the structure functions for deep inelastic electroproduction (to about twice their scale-invariant values). Preliminary experimental data on the reaction $\mu^+p \to \mu$ + hadrons at $E_\mu = 150$ GeV and $Q^2 = 30\ \text{GeV}^2$ obtained at NAL have revealed instead a 30% decrease.

One can, of course, obtain agreement between the experimental and theoretical values of R by considering more complex models involving a large number of quarks. (For example, with three quartets of quarks with integral charges, we have R = 6.) But such models are justified only if they can also successfully explain a number of other phenomena.

To explain the behavior of the total cross section for the process $e^+e^- \to$ hadrons in the framework of the parton model, it has been proposed in several papers[32-34] that partons have a form factor with a resonant character of the type $\{q^2 - [\Lambda + i(\Gamma/2)]^2\}^{-1}$. This assumption clashes with the basic idea of the parton approach—the point-like character of partons and the local structure of the electromagnetic current. Nevertheless, only experiment can decide whether, in our journey through nuclei, nucleons and partons, we must pass still another level in hadron structure before reaching constituents (Weisskopf's partinos) similar to leptons in their point-like properties. But it is not so easy to reconcile the data on deep inelastic electroproduction and annihilation, even with the hypothesis of parton structure: in addition to the finite size of the parton, we must introduce its anomalous magnetic moment. There is also no explanation of the large violation of scale invariance in inclusive processes.

Next, we shall consider variants of the theory which lead to asymptotic behaviors of σ_{tot} other than $\sigma_{tot} \sim 1/s$.

d) **The theory involving charged strongly interacting vector bosons.** As is well known, the cross section for e^+e^- annihilation into a pair of non-interacting vector bosons having zero anomalous magnetic moment κ behaves like a constant as $E \to \infty$ (if $\kappa \neq 0$, we have $\sigma \sim E^2$ as $E \to \infty$). It is therefore natural to attribute the experimentally observed constancy of σ_{tot} to a contribution to the electromagnetic hadron current from charged vector bosons. A non-decreasing cross section $\sigma(e^+e^- \to V^+V)$ as $E \to \infty$ is a consequence of the growth of the electromagnetic interactions of the vector bosons as a function of energy (the nonrenormalizability of the electrodynamics of vector bosons). Thus, it is very important to determine whether exact allowance for the strong interactions will lead to a suppression of the electromagnetic interactions of vector bosons and a decrease in σ_{tot}. If it turned out that $\sigma_{tot}(E)_{E\to\infty} \to$ const when exact allowance is made for the strong interactions, this would mean that the strong interactions do not suppress the growth of the electromagnetic interactions of vector bosons, with its ensuing profound consequences for the theory. A study of this problem based on an analysis of the Schwinger term in the current commutator gave the result[35] (with $\kappa = 0$)

$$\sigma_{tot} = \frac{\text{const}}{s^\gamma}, \quad 0 \leqslant \gamma \leqslant 1, \tag{6}$$

where $\gamma = 0$ corresponds to a bare mass of the V boson which remains finite (apart from logarithmic terms) when allowance is made for the strong interactions.

Thus, the theory involving charged strongly interacting vector bosons can in principle describe the experimentally observed behavior of $\sigma_{tot}(s)$.

In[36, 37] a study of the process $e^+e^- \to$ hadrons was made under the assumption that the vector bosons $V^\pm$ are partons, and definite results were obtained, including $\sigma_{tot} \to$ const as $E \to \infty$ and an angular distribution $\sim 1 + \cos^2\theta$ of the high-energy hadrons. In discussing these results, it should be borne in mind that the basic hypothesis of the parton model which is used in this approach—the assumption that the parton is not "dressed" during the important time interval $\tau \sim q_0^{-1}$—is a very dangerous one because of the more singular behavior of the amplitudes at small distances.

e) **The vector-dominance model (VDM) based on the algebra of fields**[38]. In this theory, it is assumed that the electromagnetic hadron current is proportional to the field of the neutral vector mesons:

$$j_\mu(x) = -\frac{m_V^2}{g_V} V_\mu(x). \tag{7}$$

An exact result has been obtained[39-41] for $\sigma_{tot}(s)$ in the VDM:

$$\sigma_{tot}(s)|_{s\to\infty} = \frac{\pi\alpha^2}{s} \frac{m_V^2}{s g_V^2} f(s), \tag{8}$$

where f(s) is a decreasing function of s. The relation (8) is clearly inconsistent with the experimental data, so that the VDM in this form[38] must evidently be rejected. Attempts have been made[42-47] to rescue the VDM by introducing a mass spectrum $\rho(m_V^2)$ of vector mesons in the theory. A satisfactory description of the experimental data on $\sigma_{tot}(s)$ can be obtained by choosing the dependence of $\rho(m_V^2)$ on m_V^2. Although the theory then loses the main attractions of the ordinary VDM, it is possible that new ideas connected with dual models[48, 49] may provide a justification for introducing a large number of particles with J = 1.

f) **Description of σ_{tot} using a model involving the production of a large number of different boson resonances**[50]. A good description of the experimental data on σ_{tot} can be obtained with this approach. However, one uses here a large number of undetermined constants, and it is necessary to adopt various assumptions about the form factors of the boson resonances.

To summarize the various theoretical descriptions of the experimentally observed behavior of the total cross section for the annihilation $e^+e^- \to$ hadrons, it should be said that there is currently no satisfactory explanation of the growth of R(E) as a function of energy, particularly if this growth is found to persist at higher energies (excluding, perhaps, the theory involving charged vector bosons, which, however, requires further theoretical and experimental study).

g) **Up to what maximum energies can the growth of R(E) continue?** The answer to this question can be obtained by using the Källén-Lehmann representation for the photon Green's function $D(q^2)$, which leads to the rigorous inequality[51]

$$\frac{1}{\pi} \int_{m_{\min}^2}^{\infty} \frac{\operatorname{Im} D(q^2)}{|D(q^2)|^2 q^4} dq^2 \leqslant 1. \tag{9}$$

The derivation of the inequality (9) made use of only a single property of the Källén-Lehmann representation for $D(q^2)$, namely the fact that $D(q^2)$ is an R-function in the complex q^2-plane. The condition (9) therefore remains valid in the more general case in which $D(q^2)$ is represented in the form of a once-subtracted dispersion relation instead of the usual unsubtracted one.

Assuming that the annihilation $e^+e^- \to$ hadrons proceeds via single-photon exchange and neglecting the lepton contribution to $\operatorname{Im} D(q^2)$, we obtain from (9) the result

$$\frac{\alpha}{3\pi} \int \frac{R(s)}{s} ds < 1. \tag{10}$$

Only the electromagnetic interaction of the leptons, and not that of the hadrons, is assumed to be weak in the inequality (10) (the single-photon approximation). The resulting inequality is therefore valid even if the electromagnetic interaction of the hadrons grows with energy (in this case, the final states in e^+e^- annihilation may contain photons emitted by the hadrons with a large probability).

Taking a linear growth for R(s) with the same slope as at the available energies, $R(s) \sim s/5m_{\text{nuc}}^2$, (10) leads to a maximum value $s = s_{\max}$ given by

$$\sqrt{s_{\max}} = 80\ \text{GeV} \tag{11}$$

Actually, we can expect the growth of R(s) to come to an end at much lower energies. As we discussed above, a rapid growth of R(s) as a function of s would mean that the strong interactions do not suppress the growth of the electromagnetic interactions of the hadrons with energy. By extending this result to virtual processes, we would conclude that there is a large violation of isotopic invariance,in the interactions of hadrons. As this is not so experimentally, we naturally expect the effective parameter $(\alpha/\pi)R(s)$ to be bounded by a value of the order of the violation of isotopic invariance, $(\alpha/\pi)R(s_{\max}) \lesssim (4\text{–}5 \times 10^{-2}$, and

$$\sqrt{s_{\max}} < 10\ \text{GeV}. \tag{12}$$

According to this estimate, the linear growth of R(s) must come to an end by the time the energies expected at SPEAR and DORIS are reached.

5. INCLUSIVE ANNIHILATION AND ELECTROPRODUCTION

In the single-photon approximation, the differential cross section for inclusive e^+e^- annihilation accompanied by the production of a single observed hadron h with energy E′ and scattering angle θ in the c.m.s. has the form

$$\frac{d\sigma}{dE'\,d\Omega} = \frac{2\alpha^2}{q^4}\frac{m\nu}{\sqrt{q^2}}\sqrt{1-\frac{m^2q^2}{\nu^2}}\left[2\bar{w}_1(q^2,\nu) + \frac{\nu^2}{q^2m^2}\left(1-\frac{q^2m^2}{\nu^2}\right)\sin^2\theta\,\bar{w}_2(q^2,\nu)\right] \tag{13}$$

where m is the mass of the hadron, $\nu = E_{\text{c.m.}}E'$, and $\bar{w}_1(q^2, \nu)$ and $\bar{w}_2(q^2, \nu)$ are functions of the invariants q^2 and ν analogous to the functions w_1 and w_2 in the case with $\omega = 2\nu/q^2 = 2E'/E_{\text{c.m.}}$, the result

$$\frac{d\sigma}{d\omega} = \frac{4\pi\alpha^2}{q^2}\omega m\sqrt{1-\frac{4m^2}{q^2\omega^2}}\left[\bar{w}_1(q^2,\omega) + \frac{\omega\nu}{6m^2}\left(1-\frac{4m^2}{q^2\omega^2}\right)\bar{w}_2(q^2,\omega)\right]. \tag{14}$$

If scale invariance holds, then the functions $\bar{w}_1(q^2, \nu)$ and $\bar{w}_2(q^2, \nu)$ have the form[20, 52]

$$\begin{aligned}\bar{w}_1(q^2,\nu) &= \frac{1}{m}\bar{F}_1(\omega),\\ \bar{w}_2(q^2,\nu) &= \frac{m}{\nu}\bar{F}_2(\omega).\end{aligned} \tag{15}$$

The experimental data discussed in Chap. 2 shows that scale invariance does not hold in the process $e^+e^- \to h^\pm + \text{all}$ for $10 < q^2 < 25\ \text{GeV}^2$ and $\omega < 0.45$. On the other hand, scale invariance has been established with good accuracy in electroproduction on nucleons for $|q^2| < 15\ \text{GeV}^2$ and $\omega = -2q^2/\nu > 1.5$. We shall therefore begin our survey of inclusive processes with a discussion of the relation between the invariant functions for the processes of inclusive annihilation and electroproduction.

The relation between the cross section for inclusive e^+e^- annihilation involving the production of a single proton and the cross section for electroproduction on the proton follows from the analysis of the amplitude $T_{\mu\nu}(s, u, t, q_1^2, q_2^2)$ for scattering of a virtual photon with initial momentum q_1 and final momentum q_2 by a proton with initial momentum p_1 and final momentum p_2, where s, t and u are the usual Mandelstam variables: $s = q_1^2 + 2\nu + m^2$, $u = q_1^2 - 2\nu + m^2$ and $t = (q_1 - q_2)^2$. The cross section for electroproduction can be expressed in terms of the functions

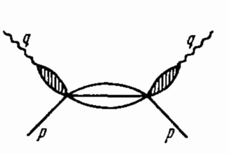

FIG. 1

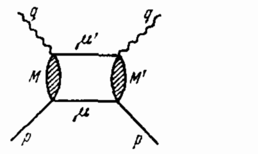

FIG. 2

$$w_{\mu\nu}(q^2,\nu) = \frac{1}{2i}\left[T_{\mu\nu}(s+i\varepsilon, u, 0, q^2, q^2) - T_{\mu\nu}(s-i\varepsilon, u, 0, q^2, q^2)\right] \tag{16}$$

with $q^2 < 0$ and $s \geq m^2$, while the cross section for inclusive annihilation involving the production of a proton with momentum p can be expressed in terms of the function[53]

$$\begin{aligned}\bar{w}_{\mu\nu}(q^2,\nu) = \frac{1}{2i}[&T_{\mu\nu}(s, u+i\varepsilon, 0, q^2+i\varepsilon_1, q^2-i\varepsilon_1)\\ &- T_{\mu\nu}(s, u-i\varepsilon, 0, q^2+i\varepsilon_1, q^2-i\varepsilon_1)]\end{aligned} \tag{17}$$

with $u > m^2$.

It follows from (16) and (17) that the function $\bar{w}_{\mu\nu}(q^2, \nu)$ cannot in general be obtained from $w_{\mu\nu}(q^2, \nu)$ by an analytical continuation, owing to the different signs of the imaginary parts added to the squares of the masses of the two photons. (The presence of different discontinuities in (16) and (17)—with respect to s and u—offers no difficulties for the analytic continuation because of the crossing relation.) The fact that $\bar{w}_{\mu\nu}(q^2, \nu)$ and $\bar{w}_{\mu\nu}(q^2, \nu)$ cannot be obtained by means of an analytic continuation can be seen diagrammatically by considering, for example, the diagrams of Figs. 1 and 2 (and their crossed diagrams).

The function $w(q^2, \nu)$ corresponding to the diagram of Fig. 1, for the case of electroproduction, has the form

$$w(q^2,\nu) = \Phi^2(q^2)f(s), \tag{18}$$

while the function $\bar{w}(q^2, \nu)$ corresponding to the crossed diagram of Fig. 1 is given by

$$\bar{w}(q^2,\nu) = |\Phi(q^2)|^2 f(u). \tag{19}$$

It is well known that the problem of determining the modulus of a function $\Phi(q^2)$ on its cut from its values off the cut is mathematically incorrect, since the solution is unstable with respect to small variations off the cut. The problem becomes even more complicated if w and $\bar{w}$ are taken to be sums of expressions of the type (18) and (19) with various intermediate states, i.e.,

$$w(q^2,\nu) = \sum_i \Phi_i^2(q^2) f_i(s), \tag{18′}$$

$$\bar{w}(q^2,\nu) = \sum_i |\Phi_i(q^2)|^2 f_i(u). \tag{19′}$$

It is obvious that (19′) cannot be expressed in terms of (18′) by means of relations such as dispersion relations. As has been shown in[54, 55], a similar situation occurs in the diagram of Fig. 2 whenever the masses M and M′ of the particles in the vertical lines and the values of q^2 are such that the particles M, M′, μ' and μ can be on the mass shell at the same time, i.e., the discontinuity in s (or in u) and the discontinuity in q^2 are simultaneously non-zero. A physical example of this situation is the inclusive pion spectrum in the process $e^+e^- \to \omega$ + hadrons followed by the decay $\omega \to 3\pi$. Calculations show[54, 55] that the situation is not improved in the case of the diagram of Fig. 2 if the analysis is restricted to the scaling region ($|q^2| \to \infty$, $\nu \to \infty$ and $|q^2|/\nu = \text{const}$): even in this region, the functions $\bar{w}(\omega)$ and $w(\omega)$ are not analytic continuations of each other.

It follows from the analysis of these two examples that it is impossible in principle to obtain a direct relationship between $\bar{w}(q^2, \nu)$ and $w(q^2, \nu)$ in the form of an analytic continuation or an expression for one function in terms of the other by means of relations such as dispersion relations. However, this does not exclude more complicated relationships such as sum rules which interrelate integrals of both functions[8)].

Direct relations between $\bar{w}$ and w can, of course, occur in definite models. Two such model-dependent relations should be mentioned. In a pseudoscalar meson theory with a cut-off in the transverse momenta (one of the variants of the parton model) in the scaling limit, Drell, Levy and Yan[20] obtained the following relation between the functions $\bar{F}_1(\omega)$ and $\bar{F}_2(\omega)$ defined in (15), with h = p (the proton), and the functions $F_1(\omega)$ and $F_2(\omega)$ for electroproduction on the proton (the variable ω for electroproduction is defined as $\omega = 2\nu/Q^2$, with $Q^2 = -q^2$):

$$\bar{F}_1(\omega) = -F_1(\omega),\qquad \bar{F}_2(\omega) = F_2(\omega). \tag{20}$$

Thus, in the model of Drell, Levy and Yan, the values of $\bar{F}_1(\omega)$ and $\bar{F}_2(\omega)$ for inclusive annihilation with proton production for $0 \le \omega \le 1$ are obtained by analytic continuation from the values of $F_1(\omega)$ and $F_2(\omega)$ for electroproduction on the proton for $1 < \omega < \infty$. (The signs in the relations (20) are reversed in the case of inclusive annihilation with production of a spin-0 boson.)

By summing the terms of order $(g^2 \ln q^2)^n$ in the scaling limit in a vector and pseudoscalar theory involving a neutral meson with a small coupling constant, Gribov and Lipatov[56] found an interesting relation between $\bar{w}$ and w:

$$\bar{w}_1(\omega, q_1^2) = \frac{1}{\omega} w_1\left(\frac{1}{\omega}, q^2\right), \quad \bar{w}_2(\omega, q^2) = -\frac{1}{\omega} w_2\left(\frac{1}{\omega}, q^2\right) \tag{21}$$

(in the Gribov-Lipatov model, there is no scale invariance, and the limit (15) does not exist, $-2m^2\bar{w}_1 = \omega\nu\bar{w}_2$). It should also be mentioned that the relation (21) holds in the Gribov-Lipatov model only in the case when the target particle is also the only virtual particle which interacts with the photon (i.e., this relation does not hold, for example, in the quark model or in a pseudoscalar symmetric theory).

It has been argued[57] that there exists a relation between the functions $\bar{w}$ and w near the point $\omega = 1$ in the scaling limit. The reasoning was based on an analysis of ladder-type diagrams with the exact propagators in the vertical lines and the exact vertex functions at the vertices (the diagram of Fig. 2 is the simplest diagram in this class). In this approximation, it was shown that, if $F_1(\omega)$ for $\omega \to 1$ has the form

$$F_1(\omega) \to A(\omega - 1)^p, \tag{22}$$

then

$$\bar{F}_1(\omega) \to A(1-\omega)^p. \tag{23}$$

(there is a minus sign for $\bar{F}_2$ in (23)).

For deep inelastic electroproduction, it is well known that the dependence (22) for $\omega \to 1$ follows from the parton model[58] or from an argument that the resonance region joins smoothly with the scaling region[14C], when $p = 2n - 1$, where n is the rate of fall-off of the elastic form factor $G(q^2) \sim (1/a^2)^n$.

Turning now to the discussion of the experimental data mentioned at the beginning of this section in the light of the foregoing remarks, it should be noted first of all that for kinematic reasons scale invariance can be tested at available energies only for pions. A necessary condition for scaling behavior is that the energy of the emitted hadron in the center-of-mass system must be large in comparison with its rest mass[9)]. This condition, expressed in terms of the scaling variable ω, means that ω must be much greater than $m/\sqrt{q^2}$ for scale invariance to hold. Consequently, we have the following possible explanations of the observed violation (of scaling) in inclusive annihilation.

a) Scaling holds for electroproduction and e^+e^- annihilation, but the approach to scaling for reactions involving an observed pion is slower than for reactions involving an observed nucleon.

b) Scaling holds for both eh $\to$ e + all and $e^+e^- \to$ h, but the approach to scaling is much slower in the case of inclusive annihilation (particularly at small ω). As we have already mentioned, this obviously holds for h = K, p, . . . only because of the kinematics.

c) Scaling holds for ep $\to$ e + all, but not for $e^+e^- \to$ h.

d) Scaling holds neither for inclusive annihilation nor for electroproduction (this means that it disappears in electroproduction in going to large values of q^2 and ν.

6. ARGUMENTS FOR AND AGAINST SCALING IN INCLUSIVE SPECTRA

Many arguments have been put forward in favor of a scale-invariant form of the inclusive spectra. The first such argument is based on the parton model. However, the hypothesis of parton fragmentation (Eq. (1)), which leads to scaling, is no more than a hypothesis. It is supported by the existence of scaling in the inclusive hadronic spectra in the electroproduction process, but data exist only for relatively small Q^2.

Another argument for scaling in inclusive e^+e^- annihilation into hadrons is based on the similarity between kinematic and diagrammatic structures of the effective cross sections for the reactions $e^+e^- \to$ h + hadrons and e^- + h $\to e^-$ + hadrons. This point is discussed in Sec. 5.

It is quite clear, however, that the structure functions νw_2 and w_1 for electroproduction (expressed in terms of $\langle p|[j_\mu(x), j_\mu(0)]|p\rangle$) have much more in common dynamically with the cross section $\sigma_{tot}(e^+e^- \to \text{hadrons})$, which is proportional to $\langle 0|[j_\mu(x), j_\mu(0)]|0\rangle$. The similarity of these quantities consists in the fact that both of them are determined by the behavior of the current commutators near the light cone. (There is also a distinction between these two quantities: in σ_{tot} small distances along the light cone $z \sim 1/q_0$ are important, while in νw_2 and w_1 the distances along the light cone are of the order of the inverse hadronic mass $z \sim \omega/m$ with $\omega > 1$.)

A third argument[59] for scaling in inclusive e^+e^- annihilation is based on the fact that the inclusive structure functions $\bar{w}_1$ and $\bar{w}_2$ can be expressed in terms of the quantity

$$\bar{w}_{\mu\nu}(\nu, q^2) \quad (24)$$
$$= \int d^4x\, d^4y\, d^4z e^{ip(x-y)} e^{iqz} \langle 0 | R\{j_\mu(z), \eta_\alpha(x)\} R\{\bar{\eta}_\beta(y), j_\nu(0)\} | 0 \rangle (\hat{p}+m)_{\alpha\beta},$$

where $\eta(x)$ is the source of the hadronic field[10] and R denotes the retarded commutator. An analysis of the exponential factors in (24) shows that the important region in the integrand is the region near the light cone, $z^2 \sim 1/q^2 \sim 0$. However, these exponential factors in (24) are multiplied by very complicated functions, and we cannot exclude the possibility of compensating oscillations. Moreover, scale invariance of the inclusive hadronic spectrum in e^+e^- annihilation does not follow from light-cone dominance alone, without the use of additional important assumptions. This statement also holds for electroproduction[23], which is determined by an expression for the type (24).

The foregoing arguments in favor of scale invariance are counterbalanced by arguments against it. One such argument is based on an analysis of the weak-coupling approximation in the framework of quantum field theory, in which $q^2 \ll 1$ but $g^2 \ln(q^2/m^2) \sim 1$[56, 60]. In this approximation, the functions $w_{\mu\nu}(\nu, q^2)$ for both electroproduction and annihilation do not have a scale-invariant form. This argument is not completely convincing, since the growth of $w(\omega, q^2)$ as a function of $|q^2|$ at fixed ω which occurs in this approach in a theory involving a vector gluon is a consequence of the growth of the interaction at small distances and a reflection of the well-known fact that the theory is internally inconsistent in this approximation (the limiting transition $|q^2| \to \infty$ is not possible). In the case of a theory with massless Yang-Mills gauge fields with asymptotic freedom, we can expect this difficulty to be absent and scale invariance to hold (apart from logarithmic terms). However, we are not yet able to produce a mass for Yang-Mills particles in theories with a non-Abelian symmetry group other than by the Higgs mechanism, which destroys the asymptotic freedom and must evidently lead to a breakdown of scale invariance.

Another argument against the existence of the scale-invariant limit (15) for the functions $\bar{w}_1(\nu, q^2$ and $\bar{w}_2(\nu, q^2)$ as $q^2 \to \infty$ with ω = const is based on an analysis of the consequences of the hypothesis of similarity in the strong interections in the presence of anomalous dimensions [18,61] The assumptions underlying this approach and their physical consequences will be considered later in Sec. 7. Here we shall mention only one of the results—sum rules for the functions $\bar{w}_1$ and $\bar{w}_2$:

$$\int_0^1 \omega^{j+1} \bar{w}_1(q^2, \omega)\, d\omega = f_1(j)(q^2)^{\rho(j)},$$
$$\int_0^1 \omega^{j+3} \bar{w}_2(q^2, \omega)\, d\omega = f_2(j)(q^2)^{\rho(j)-1}, \quad (25)$$

where $f_1(j)$, $f_2(j)$ and $\rho(j)$ are certain unknown functions of j, and in general $\rho(j) \neq 0$. It is obvious that (25) can be reconciled with (15) only if $\rho(j) \equiv 0$. Each of these arguments against scale invariance applies equally to both electroproduction and e^+e^- annihilation, while the experimentally observed behavior of the structure functions is different in the two cases.

We can conclude that the reasoning referring to scaling in inclusive e^+e^- annihilation is much less reliable than the reasoning in favor of scaling for $\sigma_{tot}(e^+e^- \to \text{hadrons})$ or for the structure functions νw_2 and w_1 in electroproduction.

A number of authors[62, 63] have attempted to explain the violation of scale invariance for σ_{tot} by first considering the inclusive spectrum and its scaling properties. As we pointed out earlier, we cannot expect scaling in inclusive processes for values $\omega \lesssim m/\sqrt{q^2}$. Consequently, if σ_{tot} is obtained by integrating the inclusive spectrum, a component of σ_{tot} which is not scale-invariant appears because of the low-energy contribution. Moreover, the arguments regarding the scale-invariant behavior of the inclusive spectrum are less conclusive than those used to demonstrate scaling in σ_{tot}. Scaling in the inclusive spectrum is a sufficient condition for the validity of scaling in σ_{tot}, but is by no means necessary. A comparable situation for electroproduction would be an attempt to obtain a scaling behavior for the structure functions νw_2 and w_1 by postulating a scaling behavior for the inclusive hadronic spectrum and determining νw_2 and w_1 by integrating it with respect to the momentum of the observed hadron. This method of analysis is much less reliable than the usual direct analysis using the properties of the current commutators on the light cone.

7. MODELS

In this section we shall discuss the predictions of various models for multiple and inclusive processes of e^+e^- annihilation into hadrons. We shall consider the following physical characteristics of multiple processes: a) the average multiplicity; b) the multiplicity distribution; c) energy distributions in inclusive processes; d) angular correlations.

We shall discuss the following models: 1) the parton model; 2) the model based on the hypothesis of similarity in the strong interactions; 3) the model involving strongly interacting charged vector bosons; 4) the model using light-cone dominance (as well as certain other assumptions); 5) the statistical and hydrodynamical models.

a) The parton model. The parton model (see, e.g.[2a, 19, 64, 65]) has no precise formulation, but includes the ideas that 1) hadrons consist of a certain number of point-like constituents, and 2) for a certain class of processes (again not precisely defined, but including deep inelastic electroproduction and neutrino processes), an incident high-energy hadron may be regarded as a beam of massless (or massive, with a fixed small mass) point-like non-interacting constituents—partons—when calculating the cross sections summed over all final hadronic states. The electromagnetic and weak interaction of these partons is given by an elementary coupling, in analogy with the interaction of leptons. These hypotheses make it possible to calculate the cross sections for processes such as

$e^\mp p \to e^\mp$ + hadrons, (26a)

$\mu^\pm p \to \mu^\pm$ + hadrons, (26b)

$\nu p \to \mu^-$ + hadrons, etc., (26c)

$\gamma p \to \gamma$ + hadrons (for a final γ with large $p_\perp$), (26d)

$pp \to \mu^+\mu^-$ + hadrons, etc., (26e)

$\mu p \to \mu\gamma$ + hadrons, etc. (for a final μ and γ with large $p_\perp$), (26f)

provided that the momentum distribution in the parton beam which replaces each incident hadron is known. Considering the physically intuitive relation to the more general scaling ideas discussed in Sec. 6, it is also very reasonable to include the process

$e^+e^- \to$ hadrons

in the foregoing list, with the prediction of the parton model given by Eq. (1). However, since the data are in obvious conflict with this prediction, we must reconsider this problem and ascertain whether there exists a distinction between the process $e^+e^- \to$ hadrons and the other processes listed above.

There is one obvious difference: in all other cases, partons are already present in the initial state, whereas there are none for annihilation. It is therefore possible that a parton in a nucleon interacts continuously with its environment so as to preserve the properties of the free field, while there is not enough time to establish the required environment if a parton pair is produced by a single virtual photon. A parton-anriparton pair in the annihilation process is emitted from the point of production with the velocity of light. The environment is created with a moderate velocity, and it is then more difficult to establish the equilibrium between the parton and the environment required for the assumed behavior of the massless free field.

Several different theoretical schemes have been proposed[66, 67] in which it is assumed that the environment of the nucleon contains a certain coordinate-dependent classical field such as a Higgs scalar field, which near the nucleon compensates for the large initial quark mass of an isolated parton. To retain the partons—in particular, the quarks—it is also possible to use massless Yang-Mills fields. In at least some of these works, the authors were restricted to some extent by the state of the colliding-beam data and the foregoing reasoning. However, we shall not continue with the discussion of this possibility; instead, we shall consider e^+e^- annihilation on the same basis as for other deep inelastic processes. None of the cases except e^+e^- annihilation exhibit a clear inconsistency with the parton picture (except possibly the very preliminary NAL data on deep inelastic scattering mentioned earlier in Sec. 4c). There has also been nothing to corroborate the parton picture, apart from the cases (26a)–(26c), which can be obtained under more general assumptions than those of the parton model. In the cases (26d)–(26f), the experimental values are larger than those predicted by the parton model and may be due to some kind of "background."

A more speculative extension of the ideas of the parton model, which is logically independent of the foregoing arguments, concerns the properties of the final hadronic states in deep inelastic processes. Roughly speaking, a parton is a quantum of the free Hamiltonian H_0. After a collision or after production in e^+e^- annihilation, it must become a quantum of the full Hamiltonian H, i.e., hadrons.

A hypothesis which is adopted here and which is confirmed to some extent by calculations in field-theoretical models with a cut-off[64] (but which is violated in other, more exact field-theoretic calculations[56, 68]) is that, if a produced parton or a parton after a collision has momentum p_μa(with large p_0 and $p^2 \sim 0$), then the characteristic momentum of a hadron emitted in the direction of the initial parton is $\sim xp_\mu$, while the momentum distribution is given by the formula

$$x\frac{dN}{dx}=\frac{x}{\sigma}\frac{d\sigma_{h,q}}{dx}=g_{h,q}(x). \tag{27}$$

This hypothesis of "parton fragmentation" is confirmed to a small extent by the data on electroproduction, and it also leads to the scaling behavior of $\bar{W}_1$ and $\bar{W}_2$ discussed in Sec. 6. For spin-1/2 partons, it also gives the relation $2m^2\bar{W}_1 = -\omega\nu\bar{W}_2$ at sufficiently high energies, leading to the angular distribution $1+\cos^2\theta$. As we have already discussed, these predictions are clearly in conflict with the experimental data. The virtue of the hypothesis of parton fragmentation expressed by Eq. (27) is its universality: the spectra of final hadrons in all deep inelastic processes depend only on a relatively small number of functions $g_{h,q}(x)$, the number of these functions being proportional to the number of different partons. Thus, measurements of the inclusive hadron spectrum in deep inelastic electroproduction and/or in neutrino processes largely determine the hadron spectrum in e^+e^- annihilation. If the isotopic spin of the partons does not exceed 1/2, there also exist various isospin constraints, such as

$$\frac{dN_{\pi^0q}}{dx}=\frac{1}{2}\left(\frac{dN_{\pi^+q}}{dx}+\frac{dN_{\pi^-q}}{dx}\right), \tag{28}$$

i.e., the π^0 spectrum in e^+e^- annihilation must coincide with the spectrum of charged pions. Many such relations among inclusive spectra can be found in the literature.

Another consequence of the hypothesis of parton fragmentation is the prediction of a two-jet structure in e^+e^- annihilation[20, 21]. This means that the energetic hadrons must be emitted roughly parallel to the axis determined by the direction of emission of the parton-antiparton pair. We expect the transverse momentum of a hadron (with respect to the axis of the jet) to be limited, of the order of 300–400 MeV, as in hadron-hadron collisions. The entire configuration of produced hadrons may then resemble the configuration which occurs, for example, in $\pi\pi$ collisions at the same energy in the center-of-mass system. In particular, the rapidity distribution of emitted hadrons of sufficiently high energy (always measured along the axis of the jet, event by event) may have the form of a plateau like that found in hadron-hadron collisions. If the partons have quark quantum numbers, we cannot expect two groups of leading particles separated by a rapidity dip, since conservation of charge would imply that each group has fractional charge. For this reason, we can expect something like a plateau region[19, 69]. In terms of Eq. (27), this means that $g(0) \neq 0$ and $g(0) < \infty$ for the plateau. A consequence of this hypothesis is the prediction that the average hadron multiplicity is $\bar{n} \sim \ln q^2$ at sufficiently high energies. However, the momenta of the secondary hadrons must be much greater than $\langle p_\perp \rangle \sim 0.4$ GeV for the occurrence of a structure in the form of two jets. Thus, the available energies are rather low for the study of these jets. (This statement is also confirmed by more detailed investigations.) The next generation of experiments with $E_{c.m.} \sim 8$ GeV will evidently be able to provide a good test of the jet structure. However, even if the jets miraculously emerge in the future out of the present confusion, revealing a central plateau, very large values of $E_{c.m.}$ comparable with the energies of the intersecting rings at CERN will be required. Thus, the logarithmic growth of the multiplicity should not, strictly speaking, set in until very high energies are reached. However, if we believe that the hadronic inclusive distributions are similar to those found in strong-interaction processes, we should also expect the multiplicity to be similar to that found in the strong interactions (approximately logarithmic).

The jet structure also has important consequences regarding the nature of two-particle (or higher-order) correlations. These correlation functions have been studied in detail by various authors, in particular by Gatto and Preparata[70], who employed the Mueller-Regge formalism, which is in agreement with the picture of parton fragmentation (including the plateau structure) which we have been discussing. As before, the strong correlation expected from the jet structure appears only at energies that are somewhat higher than those available at present.

A more speculative application of the idea of the parton model concerns the production of hadrons with large $p_\perp$ in hadron-hadron collisions. The previous hypothesis must now be supplemented by certain assumptions about the strong parton-parton interaction. In spite of the great uncertainty involved in these assumptions, there remains an interesting relation between the hadron-hadron process at large $p_\perp$ and e^+e^- annihilation. The π:K:p ratios for large $p_\perp$ in hadron collisions and for ω close to 1 in e^+e^- annihilation must be intimately related if these hadrons are proton fragments of some kind. In fact, the K/π and $\bar{p}/\pi$ ratios are very large in such pp collisions. For e^+e^- annihilation, these ratios increase with the momentum in the same way as the ratios observed in pp collisions. However, this agreement with the predictions should not be regarded as serious evidence in favor of the parton model.

b) The model based on the hypothesis of similarity in the strong interactions[18, 61]. The starting point of this approach is the assumption that the strong interactions are invariant with respect to scale transformations $x \to \lambda x$ at small distances. The various field operators then transform like $\varphi \to \lambda^{-\Delta}\varphi$, where Δ gives the dimensions of φ (generally anomalous, i.e., different from the usual canonical dimensions of the field φ). The various Green's functions transform in accordance with the number and type of field operators which appear in them. A second very important assumption which is used in this approach is that at small distances, i.e., at large momenta of the external particles off the mass shell, all of the same order of magnitude k^2, the unitarity conditions for the Green's functions G and the vertex parts Γ, expressed in terms of G and Γ, are saturated by those terms which are of order unity (i.e., independent of k^2). This implies that anomalous dimensions must occur.

These assumptions, when applied to the analysis of the imaginary part of the photon polarization operator due to the hadrons, leads to the following physical picture of e^+e^- annihilation into hadrons. A heavy virtual quantum first decays into a small number of virtual hadronic products (fragments). Each of these fragments then decays in turn into several fragments of smaller mass, and this continues until the masses of the fragments are of the order of the masses of the real hadrons. Since the number of fragments appearing in each decay is of order unity, we have

$$m \sim c^{-L}\sqrt{s}, \tag{29}$$

where L is the number of decays, m is a quantity of the order of the hadron mass, and $c > 1$. Hence $L \sim \ln(\sqrt{s}/m)/\ln c$. The energy of a fragment which appears after the n-th decay, in the rest system of the parent fragment, is obviously of order $\bar{E}_{n+1} \sim m_n$, and the energy in the c.m.s. is $E_{n+1} \sim b_{n+1}\bar{E}_{n+1}$, where b_{n+1} is the Lorentz factor. Consequently, the average energy of the real hadrons is of order

$$\bar{E} \sim (b/c)^L\sqrt{s} \sim \sqrt{s}(s/m^2)^{-\delta},\ \delta = -2\ln(b/c)/\ln c,\quad 0 < \delta < 1/2.$$

By virtue of the relation $\overline{En} = \sqrt{s}$, the power-law behavior for $\bar{E}$ implies a power-law behavior for the average multiplicity, $\bar{n} \sim (s/m^2)^\delta$. This behavior is a natural one for the model under consideration, in which it is assumed that all the Green's functions behave as powers asymptotically. In this approach, it is also easy to determine the form of the cross section $\sigma_n(s)$ for producing n particles as a function of n and s. Since we expect $\sigma_n(s)$ to have a power-law dependence on s, the form of $\sigma_n(s)$ must be

$$\sigma_n(s) = \frac{1}{s^a}\chi\left(\frac{n}{s^b}\right). \tag{30}$$

The powers a and b are determined by the conditions

$$\sigma_{\text{tot}}(s) = \sum_n \sigma_n(s) \approx \int \sigma_n(s)\,dn \sim \frac{1}{s}, \tag{31}$$

$$\bar{n} = \frac{\int dn\,n\sigma_n(s)}{\sigma_{\text{tot}}(s)} \sim \left(\frac{s}{m^2}\right)^\delta. \tag{32}$$

(In this model, $\sigma_{\text{tot}} \sim 1/s$; see Sec. 4a.) From (29)–(32), we obtain $b = \delta_1$ and $a = 1 + \delta$, i.e.,

$$\sigma_n(s) = \frac{1}{s^{1+\delta}}\chi\left(\frac{n}{s^\delta}\right). \tag{33}$$

It is rather difficult to say anything definite about the angular and energy distributions in this model. The tree-like structure of the diagrams for fragmentation in this model must lead to fractional charges (or triality) in the final states if the initial fragments had fractional charges (or triality). The problem of neutralizing these quantum numbers has not been studied.

c) The model involving charged strongly interacting vector bosons. As regards its physical consequences, this model seems similar to the model discussed in the preceding section, since, as in the previous case, we can expect a power-law asymptotic behavior for the model. Since the growth of $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ in this model is due to the interaction of the photon with charged vector bosons, and the angular distribution of the produced (free) vector bosons is proportional to $1 + \cos^2\theta$, we should expect a dependence $1 + \alpha\cos^2\theta$ with $\alpha \lesssim 1$ for the angular distribution of fast hadrons. In addition, the hadron distribution in the model with charged vector bosons should evidently have a two-jet character. A quantitative analysis of this theory (apart from the behavior of the total cross section for the annihilation $e^+e^- \to$ hadrons) has so far not been carried out, so that all the foregoing remarks have a qualitative character.

d) The light-cone dominance model (LCDM)[4a, 59, 71]. In contrast with the case of electroproduction, where a number of results have been obtained on the basis of this model and the LCDM was found to be equivalent to the parton model, physical results can be obtained in the LCDM for e^+e^- annihilation only if some important additional assumptions are made in addition to that of light-cone dominance, which holds in e^+e^- annihilation. These assumptions concern the type of singularity of Eq. (24) as $z^2 \to 0$ and the dimensions of the operators containing the singularity as a factor. After introducing these assumptions, the result[71] is unfortunately rather ambiguous; depending on the assumed dimensions of the operators, the multiplicity behaves like a power $n \sim (s/m^2)^\delta$ with $0 < \delta < 1/2$, a logarithm $n \sim \ln(s/m^2)$, or a constant.

e) The statistical and hydrodynamical models. Of the various models that have been proposed, those of the statistical type give the best description of the experimentally observed near-exponential fall-off of the inclusive spectrum $(E/\sigma_{tot})(d\sigma/d^3p)$ with energy and the universal character of this spectrum for π, K and p. In fact, if it is assumed that all the particles are emitted from the interaction region with the same temperature T_k, the distribution of particles of the i-th type (π, K, p, etc.) is described in the statistical theory by the formula[72]

$$\frac{E}{\sigma_{\text{tot}}}\frac{d\sigma}{d^3p}=g_iA_i\,(e^{E/T_k}\pm 1)^{-1}\approx g_iA_ie^{-E/T_k},\quad E\gg T_k, \tag{34}$$

where g_i are the spin and isospin weights for the particles of type i, and the signs "+" and "−" refer to Fermi and Bose particles respectively. To good accuracy, the constants A_i are equal for π, K and p. The equality $A_\pi = A_p$ follows from the condition of baryon conservation (neglecting antihyperon production), which requires that the sum of the chemical potentials of the nucleons and antinucleons is equal to zero: $\mu_N + \mu_{\bar{N}} = 0$, i.e., $\mu_N = -\mu_{\bar{p}}$. On the other hand, the p, n and $\bar{p}$, $\bar{n}$ spectra must be the same, owing to the charge and isotopic symmetry of the problem. This implies that $\mu_p = \mu_{\bar{p}} = 0$, i.e., $A_\pi = A_p$. Similarly, the equality $A_\pi = A_K$ is established by using strangeness conservation.

Although Eq. (34) gives a good description of the experimentally observed energy distribution in the region of relatively small momenta, it is not likely that this description would also be applicable in the region of large momenta, where, in analogy with the distribution in $p_\perp$ in hadronic collisions, we should expect more of a power-law than an exponential fall-off. This last statement is also supported by the fact that, in the case of an exponential fall-off of $d\sigma/d^3p$, the inclusive spectrum would not match smoothly at its end with the power-law decrease of the exclusive spectrum, as we might expect[14] and as is the case in electroproduction. A power-law fall-off of the energy spectrum at large momenta, if observed, would not imply that the statistical model is inapplicable, but only that its domain of applicability is limited. This situation is quite natural, since, in measuring particles with large momenta, we are selecting those which came from the initial event and did not manage to undergo a sufficient number of collisions. (An analogy with the problem of retardation of neutrons in matter may be useful here. If we have a point source of fast neutrons but observe only slow neutrons, their radial distribution will have a Gaussian character because of the retardation by the atoms of the medium. If, however, we are interested in the fast neutrons with energy of the order of the initial energy, the number of them will be greater than that given by the theory of diffusion, owing to the distances which they traverse with no interaction.) If this reasoning is correct, the domain of applicability of the statistical model should become larger with increasing beam energy. The energy dependence of the multiplicity in this model should be different in character in the regions of high and ultra-high energies (see[72]). In the region of high but not ultra-high energies, if the process of thermodynamic expansion does not last very long and if the total multiplicity is not very large, we should expect to find the Pomeranchuk regime[73], where the statistical equilibrium described by the formulas for an ideal gas sets in for volumes V of the system proportional to the number of particles, $V_c = nV_\pi$ with $V_\pi \approx m_\pi^{-3}$. Since the total energy is then $\sqrt{s} = V_cT_k^4$ with $T_k \sim m_\pi$, the quantity n is proportional to $\sqrt{s}$, and the average energy per particle is independent of the total energy. At ultra-high energies, we find the hydrodynamic Landau regime[74], in which the hydrodynamic pressure p plays a major role in the expansion process. The boundary between the two regimes seems to be at $n \sim 10$[72]. The process of hydrodynamic expansion takes place adiabatically. If, following Landau, we take the ultrarelativistic equation of state of matter in this process to be $p = \epsilon/3$ (where ϵ is the energy density), the entropy will be proportional to

$$S\sim VT^3. \tag{35}$$

The total number of produced particles is $n \sim S$[74]. Since entropy is conserved in the expansion process, we find, by applying (35) to the initial instant and using the conservation of energy $E_{c.m.} = V_0T_0^4$,

$$n\sim E_{\text{c.m.}}^{3/4}\,V_0^{1/4}=s^{3/8}V_0^{1/4}, \tag{36}$$

where V_0 is the volume of the system at the initial instant. It is usually assumed[11a, c, 72, 74, 75] that $V_0 \sim \text{const} \sim 1\ F^3$. The average multiplicity is then

$$n\sim s^{3/8}=E_{\text{c.m.}}^{3/4}. \tag{37}$$

However, on the basis of dimensional arguments or an estimate of the important distances in this process (see Sec. 4a), it seems just as reasonable to assume that $V_0 \sim (q^2)^{-3/2}$. In that case,

$$n\sim\text{const}. \tag{38}$$

We note that the determination of the effective initial volume for deep inelastic electroproduction based on the estimate of the important distances in the electroproduction process also leads to values of the volume and the multiplicity which differ from those that are usually assumed[76]. For electroproduction at large $|q^2|$, the transverse distances are $r^2 \sim 1/|q^2|$ and the longitudinal distances in the laboratory system are $z \sim \nu/|q^2|m$, so that the initial volume in the laboratory system is $V_{0L} \sim \nu/(q^2)^2$. In the c.m. system with $\nu \gg m^2$,

$$V_0=\frac{\nu}{(q^2)^2}\frac{E_{\text{c.m.}}}{\nu}=\frac{E_{\text{c.m.}}}{(q^2)^2}. \tag{39}$$

Thus, with our estimate of the volume for deep inelastic electroproduction, we should expect a multiplicity

$$n=a\frac{E_{\text{c.m.}}}{\sqrt{|q^2|}}=a\frac{\sqrt{q^2+2\nu+m^2}}{\sqrt{|q^2|}}\approx a\sqrt{\omega-1} \tag{40}$$

(where a is a numerical coefficient), in contrast with the usual assumption[75] that $n \sim E_{c.m.}/\nu^{1/4}$[11].

In[11a, c] calculations of the angular and energy distributions of the outgoing hadrons were made in the hydrodynamical model. A more general equation of state than $p = \epsilon/3$ was also considered in these works. It should be noted that all the calculations in the statistical and hydrodynamical models are phenomenological in character, and, in particular, the problem of how the total cross section for e^+e^- annihilation into hadrons behaves in this model remains, in general, beyond the scope of the analysis. The statistical and hydrodynamical approach is now being used by experimental groups to analyze their data, and its successes and limitations will become much more clearly defined when this work has been completed.

8. e^+e^- ANNIHILATION INTO HADRONS AND QUANTUM ELECTRODYNAMICS

As we have already mentioned, the monotonic growth of the ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ should lead to a major change in the photon propagator at energies for which $R \gtrsim 137$ and hence to the inadequacy of quantum electrodynamics based on perturbation theory. However, there may be observable effects in electron-positron scattering (Bhabha scattering) and the process $e^+e^- \to \mu^+\mu^-$ even at low energies[69, 77]. If only the hadron contribution to the vacuum polarization is taken into account, the general expression for the photon propagator has the form (for space-like $Q^2 = -q^2$)

$$D(Q^2) = -\frac{1}{Q^2}\left[1 - \frac{\alpha}{3\pi} Q^2 \int\limits_{4m_\pi^2}^{\infty} \frac{R(s)\,ds}{s(s+Q^2)}\right]^{-1}. \tag{41}$$

If

a) $$R(s) \approx \left(\frac{s}{M^2}\right)^n, \quad 0 < n < 1,$$

then

$$D(Q^2) = -\frac{1}{Q^2}\left[1 - \frac{\alpha}{3\sin\pi n} R(Q^2)\right]^{-1}. \tag{42}$$

If

b) $$R(s) \approx \begin{cases} \dfrac{s}{M^2}, & s < \Lambda^2, \\ \dfrac{\Lambda^2}{M^2}, & s \gg \Lambda^2 \end{cases}$$

(according to the existing data, $M^2 \approx 5$ GeV^2), then for $\Lambda^2 \gg Q^2$, with an accuracy up to terms of order $(\ln \Lambda^2/Q^2)^{-1}$

$$D(Q^2) \approx -\frac{1}{Q^2}\left[1 - \frac{\alpha}{3\pi} R(Q^2) \ln\frac{\Lambda^2}{Q^2}\right]^{-1}. \tag{43}$$

For time-like q^2, a power-law growth of R, as in case a), leads to an additional phase factor $e^{-i\pi n}$. The quantity Re $D(q^2)$, which is of special interest from the experimental point of view (since it interferes with the lowest order) is given by

$$\mathrm{Re}\, D(q^2) = \frac{1}{q^2}\left[1 + \frac{\alpha}{3}\mathrm{ctg}\,\pi n R(q^2)\right]^{-1} \qquad (0 < n < 1) \tag{44}$$

In case b) (again neglecting terms of order $\ln(\Lambda^2/q^2)^{-1}$), Re $D(q^2)$ remains the same as in (43). In discussing the relation of the problem of e^+e^- annihilation to the general problems of quantum electrodynamics, we mention an interesting possible way out of the difficulties of quantum electrodynamics, which was pointed out by Landau and Pomeranchuk[78] (see also[77]). If, at very high energies $s \gg \Lambda^2$, the total effective cross section for single-photon e^+e^- annihilation into all particles is $\sigma \sim 1/s$ and if $R = R_0 = \sigma(e^+e^- \to$ hadrons or any other particles except e and $\mu)/\sigma(e^+e^- \to \mu^+\mu^-)$ = const, then the photon Green's function

$$D(Q^2) = -Q^2\left[1 - \frac{\alpha}{3\pi}\left(R_0 \ln\frac{Q^2}{\Lambda^2} + \ln\frac{Q^2}{m_e^2} + \ln\frac{Q^2}{m_\mu^2}\right)\right]^{-1} \tag{45}$$

has an unphysical pole at $\ln(Q^2/\Lambda^2) \approx (3\pi/\alpha)(R_0 + 2)^{-1}$. To avoid this difficulty, Landau and Pomeranchuk assumed that the pole appears at energies for which the gravitational interaction becomes important, i.e., $Q^2 \sim 1/\kappa$, where $\kappa = 6 \times 10^{-39}$ m^{-2} is the gravitational constant. This position of the pole corresponds to $R_0 = 12$, a value which can be attained in the next generation of experiments at SPEAR and DORIS. If the assumption of Landau and Pomeranchuk is correct, the growth of R should come to an end no later than this value $R = R_0 = 12$.

The experimental consequences of the change in the photon propagator due to the hadronic polarization of the vacuum are as follows (see also[79]).

1) The angular distribution for Bhabha scattering is modified in accordance with (42) and (43). A comparison of the small-angle data with the large-angle data can already give observable effects in the following generation of experiments with $s \sim 60$ GeV^2. In this case, the effective cross section for Bhabha scattering at a scattering angle of 90° is determined by the exchange of a space-like photon with $Q^2 \sim 30$ GeV^2 and $R \approx 5$. The corrections for case a) are $\sim (1\%)(\sin \pi n)^{-1} \gtrsim 1\%$ for $D(Q^2)$ and $\sim 2\%$ for the cross section. For case b) with $\Lambda \sim 80$ GeV, the correction to the cross section is $\sim 4\%$.

2) The ratio $\sigma(e^+e^- \to \mu^+\mu^-)/\sigma(e^+e^- \to e^+e^-)$ at 90° is most sensitive to the change in the photon propagator in the time-like region (in the annihilation diagram). In case a), the correction is very sensitive to the power n and vanishes for $n = 1/2$. In case b), however, we can expect $R \sim 12$ for $s \sim 60$ GeV^2 and, according to (43), roughly a 10% correction to the cross section.

Thus, precise measurements of purely electrodynamic processes are capable of providing definite information about the ratio R at energies which are still unattainable at the present time. It is therefore vital to improve the accuracy of these measurements even beyond the very impressive values reached at Frascati and at SPEAR.

9. THE p, π AND K FORM FACTORS IN THE TIME LIKE REGION

The simplest of the exclusive processes in e^+e^- annihilation into hadrons is the process $e^+e^- \to \bar{h}h$, measurements of the cross section for which provides information about the form factor $F_h(q^2)$ of the hadron h in the time-like region $q^2 > 0$. A knowledge of $F(q^2)$ for both $q^2 < 0$ and $q^2 > 0$ makes it possible to determine the analytic properties of $F(q^2)$. As is well known (see, e.g.,[80]), the analytic properties of the form factor, which follow directly from the causality condition (and which have been rigorously proved for $F_\pi(q^2)$), are as follows: $F(q^2)$ is an analytic function of q^2 in the entire complex q^2-plane, with a cut along the real axis from M^2 to infinity, where M is the mass of the lowest hadronic state with the quantum numbers of the photon and $\bar{h}h$. Therefore a test of these properties might serve as a test of the condition of microcausality[12]. On the other hand, the use of analyticity enables us to make definite predictions about the behavior of $F(q^2)$.

Experimentally, we now know the proton (and neutron) form factors quite well in the region of space-like q^2 up to $q^2 \approx -25$ GeV^2. These proton form factors can be approximated by the dipole formula $F(q^2) = 1/[1-(q^2/m_0^2)]^2$ with $m_0^2 = 0.7$ GeV^2. For $q^2 > 0$, measurements of the process $e^+e^- \to \bar{p}p$ have been made[8a] only at the value $Q^2 = 4.4$ GeV^2, with the result

$$\frac{|G_{M_p}|^2 + (2m^2/q^2)\,|G_{E_p}|^2}{\mu^2} \approx 0.014 \tag{46}$$

($G_M(q^2)$ and $G_E(q^2)$ are the magnetic and electric form factors of the nucleon, with $G_M(0) = \mu$ and $G_{E_p}(0) = 1$). If, at $q^2 = 4.4$ GeV^2, we put $G_{M_p} = G_{E_p}$[13], we have $|G_{M_p}/\mu| = 0.10 \pm 0.01$ and $|G_{E_p}| = 0.27 \pm 0.04$.

These values are significantly smaller (by a factor 5–10) than those that would be obtained if we used the same dipole formula in the region $q^2 > 0$ as for $q^2 < 0$.

Measurements of the form factors for π and K mesons in the time-like region have been made up to $q^2 \approx 9$ GeV^2 [8b]. These measurements have shown that the form factors for π and K mesons for $q^2 > 1.5$ GeV^2 are quite similar and that at the point $q^2 = 4.4$ GeV^2 we have $|F_\pi|^2 \approx |F_K|^2 \sim 0.02$, i.e., they are of the same order of magnitude as the effective form factor of the proton, Eq. (46). For $q^2 > 1.5$ GeV^2, the π and K form factors fall off like $1/q^2$, i.e., much more slowly that the proton form factor for $q^2 < 0$. The data on the pion form factor in the space-like region obtained indirectly by measuring pion electroproduction involve relatively small $-q^2(|q^2| < 1.2$ GeV^2); therefore we shall not consider them below. (In principle, it is possible experimentally to measure directly the π and K form factors for $q^2 < 0$ and large $|q^2|$ [81].) Considering the rapid progress in the physics of colliding beams, will it be possible to say anything now or in the near future about the analytic properties of the form factors or about their asymptotic behavior as $|q^2| \to \infty$? Let us begin with a discussion of boson form factors, in particular the pion form factor, for which all values $q^2 > 4m_\pi^2$ on the cut are in the physically observable region. For the process $e^+e^- \to h^+h^-$, it is not the form factor but the square of its modulus which is measured. The value of the square of the modulus of a function on the cut is insufficient for its determination throughout the complex plane; this also requires a knowledge of the positions of all its zeros. However, on the basis of only the above-mentioned analytic properties of $F_\pi(q^2)$ (and the assumption that $|\ln F_\pi(q^2)|$ grows more slowly than $|q^2|^\alpha$ with $\alpha < 1/2$ as $|q^2| \to \infty$ in the complex plane), it is possible to derive the rigorous inequality[82]

$$\int_{4m_\pi^2}^{\infty} \frac{\ln |F_\pi(q^2)|^2}{q^2 \sqrt{q^2 - 4m_\pi^2}}\, dq^2 \geqslant 0. \qquad (47)$$

The currently available experimental data are inadequate for a test of the inequality (47); a major uncertainty comes from large values $4 < q^2 < 9$ GeV^2, where the experimental accuracy is poor, as well as from the very small values $0.08 < q^2 < 0.3$ GeV^2 and the very large values $q^2 > 9$ GeV^2, where there are no experimental data. Conversely, if we assume that the inequality (47) is valid, we can use it to derive a bound on the rate of fall-off of $|F_\pi(q^2)|$ at large $|q^2|$. For example, by using the experimental data on $|F_\pi(q^2)|$ for $0.3 < q^2 < 4$ GeV^2, it can be shown that a fall-off of the form factor according to the law $F_\pi \sim 1/(q^2)^2$ starting with $q^2 = 4$ GeV^2 is in conflict with (47) (provided that $|F_\pi(q^2)|^2$ is not very large for $4m_\pi^2 < q^2 < 0.3$ GeV^2, so that, on the average, $|F_\pi|^2 < 4$ there). There also exist a number of other rigorous inequalities which relate integrals of $|F_\pi(q^2)|^2$ for $q^2 > 0$ to the values of $F_\pi(q^2)$ for $q^2 < 0$[83].

In the case of the proton form factor, for which there is abundant experimental information for $q^2 < 0$ and quite meager data for $q^2 > 0$, it is expedient to adopt another formulation of the problem: what can be said about the behavior of the form factor for $q^2 > 0$ on the basis of the data for $q^2 < 0$? Now $F(q^2)$ is an analytic function of q^2, so that $F(q^2)$ is in principle determined throughout the entire complex plane by specifying $F(q^2)$ on any segment of the real axis for $q^2 < 0$, where $F(q^2)$ is real. In practice, however, the problem of determining a function on the cut from its values off the cut is unstable, since small oscillating contributions off the cut may give a large contribution on thecut. It is therefore actually impossible to write dispersion-type relations which express $F(q^2)$ for $q^2 > 0$ in terms of integrals of $F(q^2)$ for $q^2 < 0$. (A fantastic accuracy of the experimental data would be required for such integrals to be meaningful.) Instead, we can write relations of the sum-rule type, which relate integrals of $F(q^2)$ for $q^2 > 0$ and $q^2 < 0$. Bearing in mind the fact that only $|F(q^2)|$ is measured for $q^2 > 0$, it is convenient to consider for this purpose the function

$$\varphi(z) = \frac{f(z)\ln F(z)}{z\sqrt{z - z_0}}, \quad z = \frac{q^2}{4m^2}, \quad z_0 = \frac{m_\pi^2}{m^2}, \qquad (48)$$

where $f(z)$ is a function which is analytic in the complex z-plane, with a cut for $z < 0$. By considering the integral of $\varphi(z)$ around a contour consisting of both edges of the cuts of $F(z)$ and $f(z)$ and a large circle and assuming that the form factor has no zeros in the complex plane, it is easy to derive the sum rule[84]

$$\int_{z_0}^{\infty} f(z) \frac{\ln |F(z)|^2}{z\sqrt{z - z_0}}\, dz = 2\int_0^{\infty} \operatorname{Im} f(\rho) \frac{F(-\rho)}{\rho\sqrt{z_0 + \rho}}\, d\rho. \qquad (49)$$

In the case of the nucleon form factor, $f(z)$ can be chosen so that the contribution from the unphysical region $4m_\pi^2 < q^2 < 4m^2$ to the left-hand side of (49) is small. Although the experimental information on the proton form factor for $q^2 > 0$ which is currently available to us is extremely limited, we can draw certain physical conclusions from the sum rule (49) even now. Evidently we can say[84] that the dipole behavior of $F(q^2)$ for $q^2 < 0$ can be reconciled with the data for $q^2 > 0$ only if the form factor has not less than two zeros inthe complex plane, i.e., if the $\bar{p}p$ system has at least 4 broad resonances with the quantum numbers of the photon (the ρ, ρ' and two more unknown resonances). Another possible way of reconciling the data for $q^2 < 0$ and $q^2 > 0$ is to assume an exponential behavior of the proton form factors as $q^2 \to \infty$. Thus, for example, the dependence

$$F(q^2) = \frac{1}{1 - (q^2/a)} e^{-b\left(\sqrt{4m_\pi^2 - q^2} - 4m_\pi^2\right)/2m} \qquad (50)$$
$$(a = 0.33\ \mathrm{GeV}^2, \quad b = 1.28)$$

provides a good description of the experimental data for $q^2 < -3$ GeV^2 and $q^2 = 4.4$ GeV^2.

We can hope for a significant improvement in our understanding of this problem as new experimental data on the form factors for $q^2 > 0$ become available.

10. CONCLUSIONS

We have already learned much by studying the annihilation process $e^+e^- \to$hadrons, and undoubtedly will learn more in the near future. We have established that the "orthodoxy" (i.e., the fall-off of σ_{tot} like $1/q^2$ and scaling in inclusive processes), which is put forward on the basis of various theoretical approaches and is considered very reliable, apparently does not exist at currently available energies. It is possible that the most direct interpretation of the data is that, since σ_{tot} does not have a scale-invariant behavior, the behavior of products of current operators for very short times $\Delta t \sim (q^2)^{-1/2}$ is not that of the free fields (of spins 1/2 and 0). In terms of the parton model, this means that the partons in e^+e^- annihilation interact or decay during a time which is small or comparable with $(q^2)^{-1/2}$ and hence that an environment containing many partons appears quickly. But regardless of whether this behavior will persist at high energies or whether the orthodox expectations will be realized, we shall learn a good deal about the strong and electromagnetic interactions of hadrons at small distances from e^+e^- annihilation. And if new surprises appear in e^+e^- annihilation at higher energies, we will be compelled to reconsider many of our ideas about the laws of nature at small distances. It is customary to conclude reviews such as this by offering suggestions to the experimenters on what to measure and to the theorists on what to ponder upon. In our case, this is unnecessary: it is obvious to everybody that the experimenters should make the best measurements they can at both existing and higher energies, and that the theorists should not cease to be surprised.

Note added in proof (March 24, 1975). Since the writing of this review (June 1974), notable discoveries in physics have been made, which may significantly alter our ideas about the physics of elementary particles in general and about the problem of e^+e^- annihilation into hadrons in particular. New particles have been discovered—narrow resonances with masses 3.1 and 3.7 GeV. The particle with mass 3.1 GeV was discovered almost simultaneously by two experimental groups: at Brookhaven (U.S.A.) in a study of the mass spectrum of electron pairs in the reaction $p + Be \to e^+e^- + \text{all}$[85] and at SPEAR in a further study of the cross section for the annihilation $e^+e^- \to$ hadrons with an improved resolution in the beam energy[86]. The second particle with mass 3.7 GeV was discovered somewhat later at SPEAR in the reaction $e^+e^- \to$ hadrons[87]. The new particles have also been observed in experiments on e^+e^- annihilation at Frascati and at DORIS and in the reactions $\gamma + Be \to \mu^+\mu^- + \text{all}$ and $n + Be \to \mu^+\mu^- + \text{all}$ at Batavia. Systematic studies of the new particles (ψ mesons in the terminology of the SPEAR group) carried out at SPEAR have yielded the following results.

$\psi(3.1)$: Mass M = 3095 ± 5 MeV, width Γ = 77 ± 20 keV, widths for e^+e^+ and $\mu^+\mu^-$ decays $\Gamma_{ee} \approx \Gamma_{\mu\mu} = 5.2 \pm 1.3$ keV, width for decays into hadrons $\Gamma_h \approx 67 \pm 20$ keV, spin, parity and charge parity $J^{PC} = 1^{--}$, isotopic spin apparently T = 0, and conservation of parity in the decays $\psi \to e^+e^-$ and $\psi \to \mu^+\mu^-$.

$\psi(3.7)$: Mass M = 3684 ± 5 MeV, $\Gamma_{\mu\mu} \approx \Gamma_{ee} = 2.2 \pm 0.5$ keV, and 200 keV < Γ < 800 keV; it is assumed that the quantum numbers of the $\psi(3.7)$ are the same as for the $\psi(3.1)$, i.e., $J^{PC} = 1^{--}$ and T = 0.

The decay $\psi(3.7) \to \psi(3.1)\pi^+\pi^-$ has been observed, with a width ~30% of the full width of the $\psi(3.7)$. A remarkable feature of the new particles is the extremely small width for their decay into hadrons. In addition, the most plausible conclusion which follows from the set of experimental data and, in particular, from the decay $\psi(3.7) \to \psi(3.1)\pi^+\pi^-$, whose effective coupling constant is of order unity, is that ψ mesons are hadrons of a new type, whose decays into ordinary hadrons are strongly suppressed for some reason. It is quite possible that ψ mesons, in the quark language, consist of a quark and antiquark, $\bar{c}c$, which possess a new quantum number—charm. (The hypothesis that there exists a new "charm" quantum number (or supercharge) was put forward theoretically in 1970 to resolve the problems of weak-interaction physics[88] (see the review[89]). The strong interactions then have SU(4) symmetry.) In this case, we should expect production of charmed particles at sufficiently high energies. Charmed particles would be expected to decay into ordinary hadrons and leptons with a lifetime $\sim 10^{-13}$ sec, and their decay products would, as a rule, contain strange particles. In the process $e^+e^- \to$ hadrons, their production may set in at $E_{c.m.} \sim$ 4–5 GeV. The new measurements of $\sigma_{tot}(e^+e^- \to \text{hadrons})$, which in general confirm the previous data, have demonstrated the presence of a broad resonance with a width ~200–300 MeV at $E_{c.m.}$ = 4.15 GeV. With the hypothesis that charmed particles are produced, it is natural to expect that the $\psi(4.15)$ should decay predominantly into charmed particles and that the growth of R(s) with s for $\sqrt{s} > 4$ GeV is due to the production of charmed particles. We should then also expect a growth of the ratio of the number of kaons to the number of pions. Whether or not this is so will be shown by future experiments.

[1)]It is conventional to use the terminology "deep inelastic" for lepton-nucleon scattering processes in which the momentum transferred from the leptons to the hadrons and the mass of the produced hadronic state are large in comparison with the mass of the nucleon.

[2)]CEA and SPEAR are electron-positron colliding-beam accelerators (U.S.A.). CEA is the Cambridge Electron Accelerator, and SPEAR is the Stanford Positron-Electron Acceleration Ring.

[3)]We shall henceforth speak of a scale-invariant behavior of the total cross section to mean a proportionality $\sigma_{tot} \sim 1/E^2_{c.m.}$ and use the term "scale-invariant behavior of inclusive processes" to mean the behavior of the cross sections described by Eqs. (14) and (15), where the invariant functions depend only on the ratio of the energy of a particle to the total energy. Instead of the term "scale invariance," we shall sometimes employ its synonym, "scaling."

[4)]DESY stands for Deutsches Elektronen Synchrotron, the German electron synchrotron at Hamburg (West Germany); DORIS is the system of electron and positron storage rings under construction there.

[5)]It should be noted, however, that experimentally we know the proton form factor mainly for $q^2 < 0$ and the pion form factor mainly for $q^2 > 0$. Thus, if the form factors have different asymptotic behaviors in the time-like and space-like regions, it may be that $F_\pi \sim G_{M_p}$ asymptotically. This problem will be discussed in greater detail in Sec. 9.

[6)]The violation of scaling due to the fact that $E_{c.m.}$ is still small reduces the anisotropy in the angular distribution. A crude estimate gives a distribution of the form $1 + (\frac{1}{2})\cos^2\theta$ at $E_{c.m.} \approx 3$ GeV.

[7)]It should be noted that, for a theory in which the electromagnetic current is a quark current, the right-hand side of (4) can be expressed in terms of the matrix element $\lim_{\epsilon\to 0} \langle 0|\psi^+(x+\epsilon)\gamma\epsilon\psi(x)|0\rangle$, which can be represented as an integral of the spectral function of the Källén-Lehmann representation for the quark propagator. Only the physical states with the quark quantum numbers contribute to the spectral function. Consequently, the cut-off parameter must be chosen to be larger than the masses of the physical quarks. In this case, the argument given in the text may not be relevant to the presently available energies.

[8)]We shall consider examples of such relations for exclusive reactions in Sec. 9 below.

[9)]For pions, E_π must also be large in comparison with the characteristic momentum ~0.4 GeV, corresponding to the finite size of the hadron.

[10)]Eq. (24) is written for the case in which the observed hadron is a fermion.

[11)]We should expect that the statistical theory may hold only for central collisions, for which the orbital angular momentum l is small. Since $l \sim kr$, where k is the momentum in the center-of-mass system given by $k \approx \nu/\sqrt{s}$ and $r \sim 1/\sqrt{|q^2|}$, we have $l \sim (\nu/\sqrt{s})/\sqrt{|q^2|} \approx \omega/\sqrt{\omega - 1}$, and the condition $l \sim 1$ means that ω must be small: $\omega \lesssim$ 2-3. Thus (provided that a is not anomalously large), (40) holds only for relatively small multiplicities, and this may mean that there is no domain of applicability of the hydrodynamical theory for electroproduction processes.

[12)]An investigation of whether $F(q^2)$ is an analytic function of q^2 at large $|q^2|$ corresponds to a test of causality at small distances. In this respect, this method of testing causality differs from another method based on tests of dispersion relations for forward scattering at high energies, where the foregoing statement cannot be made without ambiguity.

[13)]The equality $G_M = G_E$ at $q^2 = 4m^2$ follows from the expressions for G_E and G_M in terms of the Pauli form factors, $G_E = F_1 + (q^2/4m^2)F_2$ and $G_M = F_1 + F_2$.

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