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Hadronic Phases and Isospin Amplitudes in $D(B) \to \pi\pi$ and $D(B) \to K\bar{K}$ Decays

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Abstract

Hadronic phases in $\pi\pi$ and $K\bar{K}$ channels are calculated à la Regge. At the D mass one finds $\delta_{\pi\pi} \simeq \frac{\pi}{3}$ and $\delta_{K\bar{K}} \simeq -\frac{\pi}{6}$ in good agreement with the CLEO data while at the B mass these angles are predicted to be, respectively, 11° and -7°. With the hadronic phase $e^{i\delta_{K\bar{K}}}$ taken into account, a quark diagram decomposition of the isospin invariant amplitudes in $D \to K\bar{K}$ decays fits the data provided the exchange diagram contribution is about 1/3 of the tree level one.

1 Introduction

To extract information on weak interaction parameters from non leptonic two body decays of the D and B mesons, it is crucial to understand the final hadronic effects which are at work in these decays. For $(K\pi)$, $(\pi\pi)$ and $(K\bar{K})$ decay modes, the important hadronic parameter is an angle δ which is the difference between s-wave phase shifts in the appropriate isospin invariant amplitudes.

In a previous paper [1] we used a Regge model to determine $\delta_{K\pi}$ as a function of energy. Good agreement with data at the D mass [2] was obtained and $\delta_{K\pi}$ is predicted to be around 20° at the B mass.

In this letter we extend this Regge analysis to $(\pi\pi)$ and $(K\bar{K})$ channels and determine $\delta_{\pi\pi}(s)$ and $\delta_{K\bar{K}}(s)$. At the D mass, we find $\delta_{\pi\pi}(m_D^2) \simeq \frac{\pi}{3}$ and $\delta_{K\bar{K}}(m_D^2) \simeq -\frac{\pi}{6}$ once again in agreement with the data[3][2]. At the B mass these angles are predicted to be of the order of 11° and -7° respectively, implying that hadronic effects in B decays remain important. Details of the derivation of these results are given in Section 2.

With hadronic phases thus determined it becomes interesting to compare a quark diagram decomposition of isospin invariant amplitudes with the data. In Section 3, we argue that for $D \to K\bar{K}$ decays a fit to the data [2] implies that the contribution of exchange quark diagrams be of the order of 1/3 of the tree-level one.

2 $\delta_{\pi\pi}(s)$ and $\delta_{K\bar{K}}(s)$ in a Regge Model

In $\pi\pi \to \pi\pi$ scattering, isospin eigenamplitudes $(I = 0, 1, 2)$ in the s, t, u-channels are related by the crossing matrices

$$\begin{pmatrix} A_0^s \\ A_1^s \\ A_2^s \end{pmatrix} = \begin{pmatrix} 1/3 & 1 & 5/3 \\ 1/3 & 1/2 & -5/6 \\ 1/3 & -1/2 & 1/6 \end{pmatrix} \begin{pmatrix} A_0^t \\ A_1^t \\ A_2^t \end{pmatrix} = \begin{pmatrix} 1/3 & 1 & 5/3 \\ -1/3 & -1/2 & 5/6 \\ 1/3 & -1/2 & 1/6 \end{pmatrix} \begin{pmatrix} A_0^u \\ A_1^u \\ A_2^u \end{pmatrix} \tag{1}$$

while for $K\bar{K} \to K\bar{K}$ scattering $(I = 0, 1)$ the $s - t$ crossing matrix reads

$$\begin{pmatrix} \tilde{A}_0^s \\ \tilde{A}_1^s \end{pmatrix} = \begin{pmatrix} 1/2 & 3/2 \\ 1/2 & -1/2 \end{pmatrix} \begin{pmatrix} \tilde{A}_0^t \\ \tilde{A}_1^t \end{pmatrix}. \tag{2}$$

The basic physical idea of a Regge model is that the high energy behaviour of s-channel amplitudes is determined by "exchanges" in the crossed channels. For $\pi\pi$ scattering, the dominant exchanges in the t-channel are the Pomeron (P) and the exchange degenerate $\rho - f_2$ trajectories while in $K\bar{K}$ scattering one must also add the exchange degenerate $\omega - a_2$ trajectories. The u-channel exchanges in $\pi\pi$ scattering are identical to the t-channel ones (P, ρ, f_2) while in $K\bar{K}$ scattering there are no exchanges in the (exotic) u channel $(KK \to KK)$.

In the energy range ($3\ \mathrm{GeV}^2 \lesssim s \lesssim 35\ \mathrm{GeV}^2$) which is of interest to us, the Pomeron (P) contribution to the isoscalar t-channel amplitude is phenomenologically well described by the formula

$$A_P = i\beta_P(0) e^{ib_p t} s \tag{3}$$

where the residue $\beta_P(0)$ and slope b_P depend on the scattering process considered. The ρ, f_2, ω, a_2 Regge trajectories are all degenerate i.e.

$$\alpha_\rho(t) = \alpha_{f_2}(t) = \alpha_\omega(t) = \alpha_{a_2}(t) = \frac{1}{2} + t \tag{4}$$

The ρ and ω trajectories have negative signatures while the f_2 and a_2 trajectories are of positive signature.

The effective ρ trajectory contribution to the isovector t-channel amplitude is written as

$$A_\rho = \frac{\bar{\beta}\rho}{\sqrt{\pi}}(1 + ie^{-i\pi t})s^{0.5+t} \tag{5}$$

while the a_2 contribution to $\widetilde{A}_1^t$ reads

$$A_{a_2} = \frac{\bar{\bar{\widetilde{\beta}}}a_2}{\sqrt{\pi}}(-1 + ie^{-i\pi t})s^{0.5+t}. \tag{6}$$

Similar expressions are used for the effective ω and f_2 trajectory contributions to the isoscalar t-channel amplitude.

The residues of these trajectories are related as follows :

a) in $\pi\pi$ scattering

$$\bar{\beta}_{f_2} = \frac{3}{2}\bar{\beta}_\rho \tag{7}$$

b) in $K\bar{K}$ scattering

$$\bar{\bar{\widetilde{\beta}}}_{f_2} = \bar{\bar{\widetilde{\beta}}}_\rho \tag{8}$$

$$\bar{\bar{\widetilde{\beta}}}_{a_2} = \bar{\bar{\widetilde{\beta}}}_\omega. \tag{9}$$

Furthermore, $SU(3)$ symmetry and ideal mixing give the additionnal relation

$$\bar{\bar{\widetilde{\beta}}}_\rho = \bar{\bar{\widetilde{\beta}}}_\omega. \tag{10}$$

Eqs (7)-(9) follow from “duality” : the scattering processes $(\pi^+\pi^+ \to \pi^+\pi^+)(A_2^s)$ and $(KK \to KK)$ are purely diffractive, hence the imaginary part of the Regge trajectory contributions to these processes must cancel [4].

Using Eqs (1) (3)-(5) and (7) our Regge model for $\pi\pi$ scattering near the forward direction (t small) reads :

$$A_0^s(s\ \text{large}, \text{small}\ t) = \frac{i}{3}\beta_P(0)e^{b_p t}s + \frac{1}{2}\frac{\bar{\beta}_\rho}{\sqrt{\pi}}s^{0.5+t} + \frac{3i}{2\sqrt{\pi}}\bar{\beta}_\rho e^{-i\pi t}s^{0.5+t} \tag{11}$$

$$A_2^s(s \text{ large, small } t) \;=\; \frac{i}{3}\beta_P(0)e^{b_P t}s - \frac{\bar{\beta}_\rho}{\sqrt{\pi}}s^{0.5+t}. \tag{12}$$

In the backward direction (u small) exactly the same formulae hold with t replaced by u.

Similarly, for $K\bar{K} \to K\bar{K}$ scattering one obtains from Eq.(2), using the relations Eqs (8)-(10), that

$$\widetilde{A}_0^s(s \text{ large, small } t) \;=\; \frac{i}{2}\widetilde{\beta}_P(0)e^{\widetilde{\beta}_P t}s + \frac{4i\widetilde{\bar{\beta}}_\rho}{\sqrt{\pi}}e^{-i\pi t}s^{0.5+t} \tag{13}$$

$$\widetilde{A}_1^s(s \text{ large, small } t) \;=\; \frac{i}{2}\widetilde{\beta}_P(0)e^{\tilde{\beta}_P t}s. \tag{14}$$

From Eqs (11)-(14) we compute the $l = 0$ partial wave amplitudes and find, up to irrelevant overall real factors

$$a_0(s) \;=\; \frac{i}{3}\frac{\beta_P(0)}{b_P}s + \frac{\bar{\beta}_\rho}{2\sqrt{\pi}}\frac{s^{1/2}}{\ln s} + \frac{3i}{2\sqrt{\pi}}\bar{\beta}_\rho\frac{(\ln s) + i\pi}{(\ln s)^2 + \pi^2}s^{1/2} \tag{15}$$

$$a_2(s) \;=\; \frac{i}{3}\frac{\beta_P(0)}{b_P}s - \frac{\bar{\beta}_\rho}{\sqrt{\pi}}\frac{s^{1/2}}{\ln s} \tag{16}$$

$$\widetilde{a}_0(s) \;=\; \frac{i}{2}\frac{\widetilde{\beta}_P(0)}{\widetilde{b}_P}s + \frac{4i\widetilde{\bar{\beta}}_\rho}{\sqrt{\pi}}\frac{(\ln s) + i\pi}{(\ln s)^2 + \pi^2}s^{1/2} \tag{17}$$

$$\widetilde{a}_1(s) \;=\; \frac{i}{2}\frac{\widetilde{\beta}_P(0)}{\widetilde{b}_P}s. \tag{18}$$

The u-channel contributions in Eqs. (15)-(16) are identical to the t-channel ones and we have dropped a (common) factor of 2 in these equations.

Clearly the phases $e^{i\delta_0}$ and $e^{i\delta_2}$ of $a_0(s)$ and $a_2(s)$ depend on the phenomenological parameter

$$x_{\pi\pi} = \frac{\sqrt{\pi}\beta_P(0)}{\bar{\beta}_\rho(0)}\frac{1}{b_P} \tag{19}$$

and similarly $e^{i\widetilde{\delta}_0}$ and $e^{i\widetilde{\delta}_1}$ depend on

$$x_{K\bar{K}} = \frac{\sqrt{\pi}\widetilde{\beta}_P(0)}{\widetilde{\bar{\beta}}_\rho(0)}\frac{1}{\widetilde{b}_P}. \tag{20}$$

From the fits given in references [5] and [6][7] we extract the values

$$x_{\pi\pi} \;=\; 0.69 \pm 0.10 \tag{21}$$

$$x_{K\bar{K}} \;=\; 1.72 \pm 0.30. \tag{22}$$

With these values we obtain respectively

$$\delta_{\pi\pi}(m_D^2) = \delta_2(m_D^2) - \delta_0(m_D^2) = 60^\circ \pm 4^\circ \tag{23}$$

$$\delta_{K\bar{K}}(m_D^2) = \tilde{\delta}_1(m_D^2) - \tilde{\delta}_0(m_D^2) = -29^\circ \pm 4^\circ \tag{24}$$

and predict

$$\delta_{\pi\pi}(m_B^2) = 11^\circ \pm 2^\circ \tag{25}$$

$$\delta_{K\bar{K}}(m_B^2) = -7^\circ \pm 1^\circ. \tag{26}$$

At the D mass, the experimental values given by the CLEO collaboration [3][2] are,

$$\delta_{\pi\pi}(m_D^2) = 82^\circ \pm 10^\circ \tag{27}$$

$$\delta_{K\bar{K}}(m_D^2) = \pm(24^\circ \pm 13^\circ). \tag{28}$$

Clearly our Regge model calculation of these phases is in good agreement with the data as announced previously.

In summary, for $(\pi\pi)$ and $(K\bar{K})$ decay channels as well as for $(K\pi)$ channels, hadronic angles are correctly predicted at the D mass by a Regge model and are found to be quite sizeable at the B mass : hadronic effects simply cannot be ignored in B decays.

3 Isospin Amplitudes and Quark Diagrams in $(D \to K\bar{K})$ Decays

Having determined the hadronic phase $\delta_{K\bar{K}}$, we now illustrate the strategy advocated in Ref.[8] to analyze the $D \to K\bar{K}$ data.

In lowest order the weak Hamiltonian responsible for $(D \to K\bar{K})$ decays contains an isodoublet $(H_W^{1/2})$ and an isoquadruplet $(H_W^{3/2})$ part. With the reduced matrix elements

$$w_1 = \ll D \mid H_W^{1/2} \mid (K\bar{K})I = 1 \gg \tag{29}$$

$$w_0 = \ll D \mid H_W^{1/2} \mid (K\bar{K}) I = 0 \gg \quad (30)$$

$$v_1 = \ll D \mid H_W^{3/2} \mid (K\bar{K}) I = 1 \gg \quad (31)$$

and the *hadronic* angle $\delta_{K\bar{K}} = \delta_1 - \delta_0$ one readily obtains, up to an overall phase factor

$$A(D^+ \to K^+ \bar{K}^0) = -\frac{v_1}{2} + w_1 \quad (32)$$

$$A(D^0 \to K^+ K^-) = \frac{v_1}{2} + \frac{w_1}{2} + \frac{w_0}{2} e^{-i\delta_{K\bar{K}}} \quad (33)$$

$$A(D^0 \to K^0 \bar{K}^0) = -\frac{v_1}{2} - \frac{w_1}{2} + \frac{w_0}{2} e^{-i\delta_{K\bar{K}}}. \quad (34)$$

We assume again [8] all reduced matrix elements to be real and expressed in terms of *quark* diagrams classified following their ($1/N$-inspired) topology.

In this "phenomenological" picture where we keep the explicit $(V-A)$ times $(V-A)$ $W^{\pm}$ propagations, the annihilation diagram (A) is helicity-suppressed and the b, s and d quarks are exchanged in the penguin diagrams (P_q). On the contrary, in the "formal" language [9] the effects of $W^{\pm}$ and b would be hidden in the short-distance Wilson coefficients of local operators built out of the u, d, s and c quarks only.

The contributions from the tree-level (T), annihilation (A), penguins (ΔP)[1], exchanges with either a $u\bar{u}$ or a $d\bar{d}$ pair created (E) and, finally, exchanges with a $s\bar{s}$ pair created (E_s) lead to the relations

$$w_0 = T + \Delta P + 2E - E_s \quad (35)$$

$$w_1 = T + \Delta P + \frac{1}{3}E_s - \frac{2}{3}A \quad (36)$$

$$v_1 = \frac{2}{3}E_s + \frac{2}{3}A. \quad (37)$$

If one assumes

$$E = E_s \quad (38)$$

[1] If we neglect the (multi-) Cabibbo-suppressed b quark contribution, there are two diagrams to consider with opposite signs: the "chin" of the penguin is either a d quark or a s quark. In the limit where $m_d = m_s, \Delta P \equiv P_s - P_d = 0$

which is what one expects in the $SU(3)$ limit, then Eqs (35)-(37) imply

$$w_0 = w_1 + v_1 \tag{39}$$

and Eqs (32)-(34) now read

$$\begin{aligned} A(D^+ \to K^+\bar{K}^0) &= w_0 - \frac{3v_1}{2} & (40)\\ A(D^\circ \to K^+K^-) &= \frac{w_0}{2}(1 + e^{-i\delta_{K\bar{K}}}) & (41)\\ A(D^\circ \to K^\circ\bar{K}^\circ) &= -\frac{w_0}{2}(1 - e^{-i\delta_{K\bar{K}}}). & (42) \end{aligned}$$

Eqs (41)-(42) are in good agreement with experimental data when Eq.(24) is used.

It is difficult to imagine a more spectacular illustration of final state hadronic effects [10] than Eqs (41)-(42). Furthermore, from the experimental value

$$\frac{\Gamma(D^0 \to K^+K^-)}{\Gamma(D^+ \to K^+\bar{K}^0)} \simeq 1.6 \tag{43}$$

one deduces $v_1 \simeq \frac{1}{6}w_0$ or, in terms of quark diagrams,

$$\frac{E + A}{T + \Delta P + E} \simeq \frac{1}{4}. \tag{44}$$

Since A and ΔP are expected to be quite small in our phenomenological picture, Eq.(44) entails

$$\frac{E}{T} \simeq \frac{1}{3}. \tag{45}$$

This result may be at odds with some theoretical prejudices but is required by the data : with $\delta_{K\bar{K}}$ given by Eq.(24), all the $D \to K\bar{K}$ data are indeed very nicely fitted by Eqs (40)-(42) provided Eq.(44) holds.

The detailed analysis of $D \to K\bar{K}$ decays presented in this section can be repeated for $(D \to \pi\pi)$ and $(D \to K\pi)$ channels. In these channels, sizeable color-suppressed quark diagrams (C) operate and nothing as striking as Eq.(42) or as unexpected as Eq.(45) emerges from such an analysis.

To summarize our earlier work on $(K\pi)$ channels as well as the results of the present paper on $(\pi\pi)$ and $(K\bar{K})$ decays let us insist on the following points:

- at the D mass, hadronic phases are rather well estimated in the context of a Regge model. Note that the hierarchy

$$\delta_{K\pi} \simeq \frac{\pi}{2}, \quad \delta_{\pi\pi} \simeq \frac{\pi}{3}, \quad \delta_{K\bar{K}} \simeq -\frac{\pi}{6} \tag{46}$$

follows from the difference in u-channel exchanges for the corresponding scattering processes combined with different Clebsch Gordan coefficients weighing the relative contributions of the Pomeron and the $I = 1$ Regge trajectories. In this paper, we have ignored inelastic channels such as $\{K\eta\} \to \{K\pi\}$ or $\{\pi\eta\} \to \{K\bar{K}\}$. In fact, our Regge analysis shows that they have little effect on phases at least at the D mass.

- at the B mass, hadronic phases are predicted to be non negligible in the three channels considered so far:

$$\delta_{K\pi} \approx 17^\circ, \quad \delta_{\pi\pi} \simeq 11^\circ, \quad \delta_{K\overline{K}} \approx -7^\circ. \tag{47}$$

We have assumed that inelastic channels have a small overall effect on these hadronic phases [11]. Whether this is true or not is an experimental question. But clearly final state hadronic phases remain large in B decays and the prospect for CP-asymmetries looks particularly promising in the $K\pi$ channel.

- the parametrization suggested in Ref [8] works very nicely as exemplified by our analysis of $(D \to K\bar{K})$ decays. When hadronic phases are important, quark-diagram absorptive parts and inelastic effects on the phases seem to be negligible. These latter conclusions may not hold for decay processes where hadronic phases are quite small.

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