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Renormalizability of the critical limit of lattice gauge theory by BRS invariance

Daniel Zwanziger [1]

Physics Department, New York University, New York, NY 10003, USA

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The critical limit of lattice gauge theory obtained previously, and which contains a parameter with dimension of (mass)^4, reflecting the boundary of the fundamental modular region, is shown to be renormalizable. The proof relies on BRS symmetry. It is also proven that the exact propagator of the Fermi ghost possesses a $1/(q^2)^2$ singularity at $q = 0$. The relation of these results to confinement, the gluon condensate, and the fundamental modular region of gauge theory is discussed briefly.

1. Introduction

Wilson's lattice gauge theory provides a regularized form of euclidean gauge theory which is invariant under a local gauge group G. This group is compact, so lattice gauge theory has the celebrated property that there is no need to fix a gauge, for example in numerical simulations. However precisely because of this local gauge invariance, the Wilson ensemble defines a measure on the quotient space, U/G, of the space of configurations modulo gauge transformations, which is the physical configuration space. It would not be surprising if this space played an important role in the critical or continuum limit of lattice gauge theory.

Since the fundamental work of Gribov [1], it is known that this space is bounded by a horizon. It is frequently thought that this horizon cannot be accounted for in renormalizable perturbation theory. However in recent studies of continuum gauge theory [2] and the critical limit of lattice gauge theory [3], the constraint that the functional integral lie inside this horizon was implemented by a Boltzmann factor $\exp(-\gamma H)$. (The explicit formula is given in eq. (2.1) below.) Here "the horizon function", $H(A)$, is a function of the gauge connection A, and γ is a new parameter, with dimension of (mass)^4. Its value is not free, but is fixed, $\gamma = \gamma(g)$, by requiring that the expectation value of H have a known value F, determined by the location of the horizon,

$$g^2\langle H\rangle = F. \tag{1.1}$$

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One may invert this relation and obtain $g = g(\gamma)$, which is a form of the familiar dimensional transmutation whereby the running coupling constant is substituted for the expansion parameter g after a perturbative calculation.

A close analogy exists here between lattice gauge theory and a classical statistical mechanical system with hamiltonian H and energy E, which is equivalent to a Boltzmann distribution at a temperature T determined by $\langle H \rangle = E$.

It was observed in refs. [2,3] that the perturbative expansion of the critical or continuum limit of lattice gauge theory, defined in eq. (2.1) below, in powers of the coupling constant g at fixed γ was renormalizable by power counting. In the present work, we shall show that this theory is indeed perturbatively renormalizable, and that the "horizon condition", eq. (1.1), also renormalizes, in the sense that it gives a finite relation between renormalized quantities, which is moreover compatible with the perturbative renormalization group. Renormalizability should not be a surprise, because the property of being renormalizable is a consequence of the insensitivity of the critical limit to short distance structure. On the other hand, it is a valuable consistency check on the hypotheses of ref. [3], by which the critical limit of lattice gauge theory was derived, that renormalizability does hold. Also in the present work, some specific consequences of these hypotheses are verified, as is explained in the concluding section, where various physical implications are also discussed.

In zeroth order perturbation theory, the gluon propagator is given by $k^2[(k^2) + N\gamma^4]^{-1}$, for SU($N$) gauge theory. There is no pole at $k = 0$, so the gluon is destabilized by the horizon. This propagator was originally found by Gribov [1], and was also obtained in ref. [4] as a non-perturbative solution of the Schwinger–Dyson equations without a horizon. The relation of this type of propagator to confinement is discussed [3–5]. We refer to ref. [3] for a detailed discussion of the horizon in lattice gauge theory and for further references.

In sect. 2 the non-local Boltzmann factor is expressed as an integral over auxiliary or ghost fields with a local action. In sect. 3 the BRS symmetry of the dimension-4 part of the local action is exhibited. The remaining, lower-dimensional, pieces of the action are treated in sect. 4 by the method of local sources, with local sources also introduced for the BRS transforms of these remaining pieces. In sect. 5 a theorem is derived which gives the explicit dependence of the effective action on the ghost fields. In sect. 7 the theory with γ arbitrary is shown to be renormalizable by solving the standard cohomology problem, and in sect. 8 the horizon condition is shown to renormalize. In sect. 9 the energy–momentum tensor $T_{\lambda\mu}$ is derived, and it is explained why the gluon condensate $\langle T_{\lambda\lambda} \rangle$ may be perturbatively calculable in the present scheme. In sect. 10 we shall show that the exact propagator of the fermi ghost field has a $1/(q^2)^2$ singularity at $k = 0$. This comes from an exact cancellation of the tree-level contribution to the inverse propagator by the quantum corrections, when the horizon condition is satisfied. In the concluding sect. 11, we briefly discuss some physical implications of this result for confinement, what it tells us about which configurations dominate the functional integral, and to what extent the hypotheses of ref. [3] are verified.

2. Local action

It was found in ref. [2] that the partition function of a continuum non-abelian gauge theory may be written in the form

$$Z = \int \mathrm{d}A \, \exp(-S_{\mathrm{YM}} - \gamma H)\delta(\partial \cdot A) \, \det(\mathbf{M}), \tag{2.1}$$

and in ref. [3] a lattice-regularized analogy of this expression was obtained. Here S_{YM} is the Yang–Mills action, and $\mathbf{M} = \mathbf{M}(A)$ is the Faddeev–Popov operator, defined by

$$\mathbf{M}^{ac}\varphi^c = -\partial \cdot (D^{ac}\varphi^c) = -\partial_\mu\left[\left(\delta^{ac}\partial_\mu + f^{abc}gA_\mu{}^b\right)\varphi^c\right], \tag{2.2}$$

where μ is a Lorentz index, φ^a is any field that transforms according to the adjoint representation of the structure group, and f^{abc} are the structure constants of a semi-simple Lie group which will be taken to be SU(N). Apart from the term γH in the action, formula (2.1) is the familiar Faddeev–Popov partition function. The horizon function H is defined by

$$H \equiv (A, \mathbf{M}^{-1}A) \equiv \int \mathrm{d}^D x \, f^{abc}A_\mu{}^b(\mathbf{M}^{-1})^{ad} f^{dec}A_\mu{}^e \equiv \int \mathrm{d}^D x \, h(x), \tag{2.3}$$

where repeated indices are summed over, and $h(x)$ is the horizon function per unit volume. Because of translation invariance, the horizon condition (1.1) which determines γ reads

$$g^2\langle h \rangle = f \equiv (N^2 - 1)D. \tag{2.4}$$

Here $(N^2 - 1)$ is the dimension of the adjoint representation of SU(N), and D is the dimension of euclidean space-time, so f is the number components of $A_\mu{}^b$, which is also the number of degrees of freedom per lattice site. The coefficient g^2 occurs because the classical connection has been written gA. Formula (2.1) is understood to be defined by its power series in g, calculated by gaussian quadrature with dimensional regularization.

[We give a brief word about how this expression for the critical limit is derived. Although it was originally obtained in continuum quantum field theory [2], it must be said that quantization of a gauge field in the continuum is not really a well-defined problem mathematically. Wilson's lattice gauge theory provides a satisfactory quantization and gauge-invariant regularization. To obtain its continuum limit, which is its critical limit, a gauge must be chosen which makes all link variables as close to unity as possible. To do this in an optimal way, equitably over the whole lattice, in ref. [3], the quantity

$$I[U] \equiv \sum_L \left[1 - N^{-1}\ \mathrm{Tr}(U_L)\right],$$

was taken as a measure of the deviation of the link variables from unity in the configuration U. Here the sum extends over all links L of the lattice, and U_L is the variable (an element of the SU(N) group) associated to the link L in the configuration U. The continuum analog of this expression is the Hilbert norm of the connection A, $I[A] = \int \mathrm{d}^D x\, |A(x)|^2$. The gauge was chosen which makes this quantity an absolute minimum. In this gauge, the function

$$F_U[g] \equiv I[U^g],$$

where U^g is the gauge transform of the configuration U by the local gauge transformation g, is an absolute minimum at $g(x) = 1$ with respect to all local gauge transformations $g = g(x)$. At a minimum this function is stationary, and its second variation is positive. These properties imply that $\partial \cdot A = 0$, and that the Faddeev–Popov operator $\mathbf{M}(A)$ is positive $\mathbf{M}(A) > 0$. The first condition is known as the Landau gauge condition, and the second defines the Gribov region. We refer the reader to ref. [3] for details on how the Wilson action in this gauge leads to the partition function (2.1). Although, as mentioned, the derivation requires hypotheses, they may at least in principle be verified by numerical simulation and numerical gauge fixing on the lattice, and some of their consequences are verified in the present work, as is discussed in sect. 11.]

In order to prove renormalizability of the theory defined by the partition function (2.1), we rewrite it in terms of a local action by integrating over auxiliary fields. Because the indices μ and c are mute in the horizon function (2.3) it is convenient to introduce the notation

$$A_i^{\ a} \equiv A_{\mu,c}^{\ \ a} \equiv f^{abc} A_\mu^{\ b}. \tag{2.5a}$$

Here we have written the single index

$$i \equiv (\mu, c) \tag{2.5b}$$

for the pair of mute indices, and i takes on $f = (N^2 - 1)D$ values. For the Boltzmann factor, we have by gaussian quadrature,

$$\exp\left[-\gamma(A, \mathbf{M}^{-1}A)\right] = \left[\det(\mathbf{M})\right]^f \int \mathrm{d}\varphi\, \mathrm{d}\varphi^* \times \exp\left[-(\varphi^*, \mathbf{M}\varphi) - \gamma^{1/2}(A, \varphi - \varphi^*)\right]. \tag{2.6}$$

Here $\varphi \equiv (\varphi_1 + i\varphi_2)/\sqrt{2}$ and $\varphi^* \equiv (\varphi_1 - i\varphi_2)/\sqrt{2}$ are a pair complex fields with components $\varphi_i^a = \varphi_{\mu,c}{}^a(x)$ where μ is a Lorentz index and a and c are in the adjoint representation of the SU(N) group, and similarly for φ^*, and

$$(A, \varphi - \varphi^*) \equiv \int d^D x \, A_i^a (\varphi - \varphi^*)_i^a. \tag{2.7}$$

$$(\varphi^*, \mathbf{M}\varphi) \equiv \int d^D x \; \varphi *_i^a \mathbf{M}^{ab} \varphi_i^b \tag{2.8}$$

The coefficient $[\det(\mathbf{M})]^f$ in eq. (2.6) appears because the mute indices in the last expression take on f values. To obtain a local expression, we write $[\det[\mathbf{M})]^f$ by means of gaussian quadrature over pairs ω and ω^* of Grassmann fields

$$\exp[-\gamma(A, \mathbf{M}^{-1}A)]$$

$$= \int d\varphi \, d\varphi^* \, d\omega \, d\omega^* \exp[-(\varphi^*, \mathbf{M}\varphi) + (\omega^*, \mathbf{M}\omega) - \gamma^{1/2}(A, \varphi - \varphi^*)]. \tag{2.9}$$

Here ω and ω^* are independent Grassmann variables that have the same components as φ and φ^*, namely $\omega_i^a = \omega_{\mu,c}{}^a$ and $\omega^*{}_i^a = \omega^*{}_{\mu,c}{}^a$. Finally we use the standard representation for the Faddeev–Popov measure and obtain the desired local expression for the partition function (2.1), namely

$$Z = \int dA \, dC \, dC^* \, d\lambda \, d\varphi \, d\varphi^* \, d\omega \, d\omega^* \exp(-S_\gamma), \tag{2.10}$$

where

$$S_\gamma \equiv S_{\rm YM} - (\lambda, \partial \cdot A) - (C^*, \mathbf{M}C) + (\varphi^*, \mathbf{M}\varphi) - (\omega^*, \mathbf{M}\omega) + \gamma^{1/2}(A, \varphi - \varphi^*). \tag{2.11}$$

Here C^a and C^{*a} are the usual pair of Faddeev–Popov Grassmann ghosts, and λ is an imaginary Lagrange multiplier which enforces the constraint $\partial \cdot A = 0$, characteristic of the Landau gauge. (We generally follow the notation of ref. [6].) In terms of these variables, the horizon condition reads

$$g^2 \langle A_i^a(x) \varphi_i^a(x) \rangle = -g^2 \langle A_i^a(x) \varphi^*{}_i^a(x) \rangle = f\gamma^{1/2} = (N^2 - 1) D \gamma^{1/2}. \tag{2.12}$$

3. BRS invariance

If one sets $\gamma = 0$, one should obtain a theory which is equivalent to the Faddeev Popov theory. To verify this point, consider the local action (2.11), with γ set to 0,

$$S_1 \equiv S_{\rm YM} - (\lambda, \partial \cdot A) - (C^*, \mathbf{M}C) + (\varphi^*, \mathbf{M}\varphi) - (\omega^*, \mathbf{M}\omega). \tag{3.1}$$

This action has a pleasant super-symmetry between the f pairs of Bose ghosts and the $f+1$ pairs of Fermi ghosts which acts on their lower indices. It also enjoys BRS invariance. To see this, let a BRS transformation be defined by

$$\begin{aligned} sA &= DC, & sC &= -(g/2)C\times C, \\ sC^* &= \lambda, & s\lambda &= 0, \\ s\varphi &= \omega, & s\omega &= 0, \\ s\omega^* &= \varphi^*, & s\varphi^* &= 0, \end{aligned} \tag{3.2}$$

which is nilpotent, $s^2=0$. Let S_0 be the action defined by

$$S_0 \equiv S_{\mathrm{YM}} + s[-(C^*, \partial\cdot A) + (\omega^*, \mathbf{M}\varphi)], \tag{3.3}$$

which satisfies

$$sS_0 = 0. \tag{3.4}$$

Since S_0 differs from S_{YM} by an exact BRS transform, we know from general arguments that for gauge-invariant observables it gives the same expectation values as standard Faddeev–Popov theory, provided only that the theory defined by S_0 is well defined, as we shall demonstrate. Keeping in mind that s anti-commutes with Grassmann fields, and that A is buried in $\mathbf{M} = \mathbf{M}(A) = -\partial\cdot D(\mathbf{A})$, one finds

$$S_0 = S_{\mathrm{cl}} - (\lambda, \partial\cdot A) - (C^*, \mathbf{M}C) + (\varphi^*, \mathbf{M}\varphi) - (\omega^*, \mathbf{M}\omega) - g(\partial\omega^*, (DC)\times\varphi). \tag{3.5}$$

This expression differs from the action S_1 above by the presence of the last term. However, we may and shall transform the action S_1 into S_0 by a shift in the variable ω

$$\omega' \equiv \omega + \mathbf{M}^{-1} g\partial\cdot[(DC)\times\varphi], \tag{3.6}$$

while keeping ω^* fixed. Thus, after dropping the prime, the partition function reads

$$Z = \int \mathrm{d}\Phi \, \exp[-S_0 - \gamma^{1/2}(A, \varphi - \varphi^*)], \tag{3.7}$$

where $\mathrm{d}\Phi$ represents integration over all fields, as in eq. (2.10). The full action which appears here is not BRS invariant. However the term proportional to $\gamma^{1/2}$ which breaks it is only of dimension 2 instead of 4. We shall show, by introducing a local source for it, that correlation functions with this composite field are renormalizable.

It is sometimes thought, erroneously, that BRS invariance is a form of gauge invariance, so some readers may conclude that the term in the action which violates BRS invariance means that a horrible gauge-violating error was made. To avoid possible confusion on this point, we emphasize that gauge invariance was in fact lost (more precisely, it was fully exploited) when the gauge was fixed in an optimal way, as described in the sect. 2. Moreover the theory with the parameter γ, whose renormalizability we wish to establish, does not represent a gauge theory at all, except when γ is assigned the value determined by the horizon condition, and this value is known only after calculations in the more general theory. On the other hand, BRS invariance is a new and useful symmetry that arises whenever a δ-function and its accompanying jacobian determinant are represented by integrals over a larger set of Bose and Fermi variables. The BRS transformation increases the fermion number by one, so it is defined only in the larger space. The BRS transformation may be isomorphic to an infinitesimal gauge transformation in some cases. The relevant issue is not whether the full action is BRS invariant but whether it is renormalizable. We shall see that it is sufficient that the dimension-4 part of the action be BRS invariant. This allows unambiguous, renormalizable calculations with local sources for lower-dimensional BRS-violating fields that may be elementary or composite.

The action S_0 possesses a U(1) symmetry which corresponds to conservation of ghost-fermion number. We assign ghost-fermion number 1 to the fields C and ω, and ghost-fermion number -1 to the fields C^* and ω^*. All other fields are bose fields with ghost-fermion number zero. The BRS operator defined in eq. (3.2) increases the ghost-fermion number by unity, and is nilpotent. [The space of functions of the fields, which is the sum of all spaces with fixed integer ghost-fermion number, is isomorphic to the space of differential forms, where the degree of the form is the ghost-fermion number. A slight generalization is required, because the starred ghost-fermion fields are assigned degree-1. Just as Cartan's exterior derivative increases the degree of forms by unity, the BRS operator s increases the ghost fermion number by unity.] The action S_0 also possess a U(f) symmetry, where $f=(N^2-1)D$, by which the fields φ_i, $\varphi^*{}_i$, ω_i, and $\omega^*{}_i$ are transformed on their lower index $i=(\mu, a)$ in an obvious way.

The two actions S_0 and S_1 differ by the vertex $g(\partial\omega^*, (DC)\times\varphi)$. This vertex increases the C-number by unity and decreases the ω-number by unity, whereas the remainder of the action separately conserves the number of these Fermi ghost fields. The important point about this new vertex is that it appears in the action without its complex conjugate. Consequently, although the action does not conserve the C-number and ω-number separately, it has the property that the C-number is non-decreasing and the ω-number is non-increasing. It follows that the new vertex contributes precisely n times in matrix elements or correlation functions where the C-number increases by n and the ω^* number decreases by n, and not at all in matrix elements where n is zero. Thus, for example, this vertex does not contribute at all to the horizon condition (2.10) in which n is zero. Moreover for the matrix elements in which n is zero, the actions S_0 and S_1 give equal values. Matrix elements where n is negative vanish. We call this new vertex the "C-ghost increasing vertex".

Observe that the term (A, φ^*) which appears in the action may be written

$$(A, \varphi^*) = s(A, \omega^*) - (DC, \omega^*). \tag{3.8}$$

We eliminate the last term by a linear shift in the ω variable (while ω^* is kept constant), so that the partition function is given by

$$Z = \int \mathrm{d}\Phi \, \exp(-S_{\mathrm{ph}}) \tag{3.9a}$$

$$S_{\mathrm{ph}} = S_0 + \gamma^{1/2}[(A, \varphi) - s(A, \omega^*)], \tag{3.9b}$$

where S_0 is given in eq. (3.5). This shift in φ by a term proportional to $\omega^* C$ does not affect expectation values of matrix elements in which these ghost numbers do not change, as we have just discussed.

4. Sources for composite fields

The partition function (3.9) contains the BRS violating term of dimension 2. We would like to treat it as in operator insertion in the BRS conserving theory. However, a direct expansion in powers of this term would lead to infrared divergences. For this reason it is convenient to introduce a local source for this term, renormalize the ultraviolet divergences and resum. We follow here the method by which the $m^2\varphi^2$ term is treated as an insertion into the massless φ^4 theory, as described in section 8.10 of ref. [6].

There is considerable freedom in the choice of local sources. We shall introduce those local sources which will allow us to solve the equations of motion of the ghost fields. Note that the term

$$\gamma^{1/2}(A, \varphi) = \gamma^{1/2} \int \mathrm{d}^D x \, \left(f^{abc} A_\mu{}^b \varphi_{\mu,c}{}^a \right)(x)$$

which appears in the action may also be written

$$\gamma^{1/2}(A, \varphi) = -g^{-1}\gamma^{1/2} \int \mathrm{d}^D x \, D_\mu{}^{ac} \varphi_{\mu,a}{}^c(x), \tag{4.1}$$

where the covariant derivative is defined in eq. (2.2), because the integral of an ordinary derivative vanishes. We shall introduce a source $M_{\mu i}{}^a(x)$ for each component of $(D_\mu \varphi_i)^a(x) = D_\mu{}^{ac}\varphi_i{}^c(x)$, and similarly a source $N_{\mu i}{}^a(x)$ for $D_\mu \omega^*{}_i{}^a(x)$. We also introduce sources $U_{\mu i}{}^a(x)$ and $V_{\mu i}{}^a(x)$ for the BRS transforms of these quantities, $s(D_\mu \varphi_i)^a(x)$ and $s(D_\mu \omega^*{}_i)^a(x)$. Sources for $D\varphi^*$ and $D\omega$ will not needed because they are closely related to $sD\omega^*$ and $sD\varphi$. Note that M and V are Bose fields, where N and U are Fermi fields, which have ghost number 1 and -1 respectively.

As is customary, we also introduce sources K and L for the composite BRS transforms of the elementary fields A and C, namely for $DC = sA$ and for $-g/2C \times C = sC$. Thus we are led to consider an extended action S which depends on all these local sources,

$$-S \equiv -S_0 + (K, DC) + (L, -g/2C \times C)$$
$$+(M, D\varphi) + (D\omega^*, N) + (U, sD\varphi) + (sD\omega^*, V), \qquad (4.2)$$

where S_0 is the BRS-invariant action given in eq. (3.5). We shall show by the technique of local sources that correlation functions with insertions of such fields are renormalizable. The original action S_{ph} given in eq. (3.9) is regained when the local sources are assigned the physical values

$$M_{\text{ph},\mu\nu b}{}^a(x) \equiv -V_{\text{ph},\mu\nu b}{}^a(x) \equiv \gamma^{1/2} g^{-1} \delta_{\mu\nu} \delta_b{}^a, \qquad (4.3)$$

$$K_{\text{ph}} = L_{\text{ph}} = N_{\text{ph}} = U_{\text{ph}} = 0. \qquad (4.4)$$

Here we have restored the notation $(\nu, b) = i$, and written $M_{\text{ph},\mu\nu b}{}^a(x)$ instead of $M_{\text{ph},\mu i}{}^a(x)$, and similarly for V_{ph}.

5. Solution of the ghost field equations of motion

To obtain the generating functional of correlation functions, we also introduce local sources for all the elementary fields,

$$Z = \exp W = \int \mathrm{d}\Phi \, \exp(-\Sigma), \qquad (5.1)$$

where

$$-\Sigma \equiv -S + (J, A) + (\eta^*, C) + (C^*, \eta) + (l, \lambda)$$
$$+(\rho^*, \varphi) + (\varphi^*, \rho) + (\sigma^*, \omega) + (\omega^*, \sigma), \qquad (5.2)$$

and S is given in eq. (4.2). Renormalization is most simply described in terms of the effective action Γ which is the Legendre transform of the generating functional of connected correlation functions $W = \ln Z$,

$$\Gamma(A, C, C^*, \lambda, \varphi, \varphi^*, \omega, \omega^*) + W(J, \eta, \eta^*, l, \rho, \rho^*, \sigma, \sigma^*)$$
$$= (J, A) + (\eta^*, C) + (C^*, \eta) + (l, \lambda) + (\rho^*, \varphi) + (\varphi^*, \rho) + (\sigma^*, \omega) + (\omega^*, \sigma), \tag{5.3}$$

where

$$\begin{aligned} A &= \delta W/\delta J, & J &= \delta\Gamma/\delta A \\ C &= \delta W/\delta\eta^*, & \eta^* &= -\delta\Gamma/\delta C, \\ C^* &= -\delta W/\delta\eta, & \eta &= \delta\Gamma/\delta C^*, \\ \lambda &= \delta W/\delta l, & l &= \delta\Gamma/\delta\lambda, \\ \varphi &= \delta W/\delta\rho^*, & \rho^* &= \delta\Gamma/\delta\varphi, \\ \varphi^* &= \delta W/\delta\rho, & \rho &= \delta\Gamma/\delta\varphi^*, \\ \omega &= \delta W/\delta\sigma^*, & \sigma^* &= -\delta\Gamma/\delta\omega, \\ \omega^* &= -\delta W/\delta\sigma, & \sigma &= \delta\Gamma/\delta\omega^*. \end{aligned} \tag{5.4}$$

The minus signs appear because the derivatives of Grassmann variables are defined to be left derivatives. The sources of the composite fields are constant parameters under this Legendre transformation and we have

$$\begin{aligned} \delta\Gamma/\delta K &= -\delta W/\delta K, & \delta\Gamma/\delta L &= -\delta W/\delta L, \\ \delta\Gamma/\delta M &= -\delta W/\delta M, & \delta\Gamma/\delta N &= -\delta W/\delta N, \\ \delta\Gamma/\delta U &= -\delta W/\delta U, & \delta\Gamma/\delta V &= -\delta W/\delta V. \end{aligned} \tag{5.5}$$

Before using BRS invariance to prove renormalizability, we shall establish a property of the effective action which greatly simplifies this task. It is possible to solve the equations of motion and determine the complete dependence of the effective action Γ on the six fields λ, C^*, φ, φ^*, ω and ω^*.

Theorem 5.1. The effective action is of the form

$$\Gamma = S + \Gamma_{\mathrm{qu}}(A, C, L, K', M', N', U', V'), \tag{5.6}$$

where S is given in eq. (4.2), and the primed variables are defined by

$$K' \equiv K + \partial C^* - g(U + \partial\omega^*) \times \varphi - g\omega^* \times V. \tag{5.7}$$

$$\begin{aligned} M' &\equiv M - \partial\varphi^*, & N' &\equiv N + \partial\omega, \\ U' &\equiv U + \partial\omega^*, & V' &\equiv V - \partial\varphi. \end{aligned} \tag{5.8}$$

Remark 1. The solvability is a particular property of the dynamics of these fields, and does not follow simply because we have introduced a plethora of sources. Indeed, if a source is also introduced for DC^* and its BRS transform, the equations are no longer solvable. Theorem 5.1 shows that the introduction of the sources for the composite fields does not complicate the dynamics, but is a natural structure.

Remark 2. We have introduced a local source $M_{\mu i}$ for $D_\mu \varphi_i = \partial_\mu \varphi_i + gA_\mu \times \varphi_i$. However the only change in Γ, as compared to a source $M_{\mu i}$ for $gA_\mu \times \varphi_i$, is simply the additive term $(M, \partial\varphi)$, and similarly for the other ghost fields. For let t^* be an independent source for φ, so that, with suppression of other fields,

$$\exp W(\rho^*, t^*) = \int \mathrm{d}\varphi \exp[-S + (\rho^*, \varphi) + (t^*, \varphi)],$$

and we have $\partial W/\partial \rho^* = \partial W/\partial t^*$. Now make the Legendre transformation from ρ^* to φ at fixed t^*,

$$\Gamma(\varphi, t^*) = (\rho^*, \varphi) - W(\rho^*, t^*).$$

This gives $\partial \Gamma(\varphi, t^*)/\partial t^* = -\partial W(\rho^*, t^*)/\partial t^* = -\partial W(\rho^*, t^*)/\partial \rho^* = -\varphi$, and we have

$$\Gamma(\varphi, t^*) = \Gamma_1(\varphi) - (t^*, \varphi),$$

where Γ_1 is independent of t^*. Upon setting $t^* = \partial \cdot M$, we obtain the relation between the effective action $\Gamma(\varphi)$ with source term $(M, D\varphi)$, and the effective action $\Gamma_1(\varphi)$ with source term $(M, gA \times \varphi)$,

$$\Gamma(\varphi) = \Gamma_1(\varphi) + (M, \partial\varphi).$$

The term $(M, \partial\varphi)$ is contained in S. Therefore the derivatives of Γ_{qu} with respect to M give insertions of $gA \times \varphi$. Of course this is just an illustration of the fact that Γ is the generator of one-particle irreducible correlation functions.

Remark 3. The appearance of the derivative of the ghost fields in eqs. (5.8), but not the ghost fields themselves makes manifest the factorization of incoming and outgoing ghost momenta from all proper (C-ghost conserving) diagrams, which is a well-known property of the Landau gauge [7]. (The appearance in K' of the undifferentiated ghost fields comes entirely from the C-ghost-increasing vertex). This reduces the degree of divergence of diagrams, and is presumably the reward for the optimal gauge-fixing described in sect. 2, whereby the fluctuations of the A-field are minimized. In particular, the ghost-ghost-gluon vertex is finite (after insertion of divergent subdiagrams), and the corresponding renormalization constant may be set to unity $Z_1' = 1$. For by virtue of the theorem we have

$$\delta^3 \Gamma/\delta\varphi^*(x)\delta\varphi(y)\delta A(z) = \partial^x \cdot \partial^y \cdot \delta\Gamma/\delta M(x)\delta V(y)\delta A(z).$$

By remark 2, the last expression represents insertions of $gA \times \varphi(x)$, $gA \times \varphi^*(y)$ and $A(z)$.

The remainder of this section is devoted to the proof of theorem 5.1. For the field λ we have

$$0 = \int \mathrm{d}\Phi\ \delta/\delta\lambda\ \exp(-\Sigma), \qquad 0 = \int \mathrm{d}\Phi(\partial\cdot A + l)\ \exp(-\Sigma),$$

$$0 = (\partial\cdot\delta/\delta J + l)Z, \qquad 0 = \partial\cdot\delta W/\delta J + l.$$

This gives

$$\delta\Gamma/\delta\lambda = -\partial\cdot A, \tag{5.9}$$

which has the solution

$$\Gamma = -(\lambda, \partial\cdot A) + \Gamma_1, \tag{5.10}$$

where Γ_1 is independent of λ.

For the field C^*, we have

$$0 = \int \mathrm{d}\Phi\ \delta/\delta C^*\ \exp(-\Sigma), \qquad 0 = \int \mathrm{d}\Phi\ (-\partial\cdot DC + \eta)\ \exp(-\Sigma),$$

$$0 = (-\partial\cdot\delta/\delta K + \eta)Z, \qquad 0 = -\partial\cdot\delta W/\delta K + \eta.$$

This gives

$$\delta\Gamma/\delta C^* = -\partial\cdot\delta\Gamma/\delta K, \tag{5.11}$$

which has the solution

$$\Gamma = \Gamma(K + \partial C^*), \tag{5.12}$$

giving the complete C^* dependence. From eq. (5.10), we get

$$\Gamma = -(\lambda, \partial\cdot A) + \Gamma_1(K + \partial C^*). \tag{5.13}$$

For the field φ^*, we have

$$0 = \int \mathrm{d}\Phi\ \delta/\delta\varphi^*\ \exp(-\Sigma), \qquad 0 = \int \mathrm{d}\Phi(\partial\cdot D\varphi + \rho - D(A)\cdot V)\ \exp(-\Sigma),$$

$$0 = (\partial\cdot\delta/\delta M + \rho - D(\delta/\delta J)\cdot V)Z, \qquad 0 = \partial\cdot\delta W/\delta M + \rho - D(\delta W/\delta J)\cdot V.$$

This gives

$$\delta\Gamma/\delta\varphi^* = \partial\cdot\delta\Gamma/\delta M + D(A)\cdot V, \tag{5.14}$$

which has the solution

$$\Gamma = -(D\varphi^*, V) + \Gamma_2(M - \partial\varphi^*), \tag{5.15}$$

giving the complete dependence on φ^*. From eq. (5.13), we get

$$\Gamma = -(\lambda, \partial\cdot A) - (D\varphi^*, V) + \Gamma_3(M - \partial\varphi^*, K + \partial C^*). \tag{5.16}$$

For the φ-field, we have

$$\begin{aligned}
0 &= \int \mathrm{d}\Phi\ \delta/\delta\varphi \exp(-\Sigma) \\
&= \int \mathrm{d}\Phi (D\cdot\partial\varphi^* + \rho^* + g\partial\omega^* \times (DC) - D(A)\cdot M + gU \times DC) \exp(-\Sigma) \\
&= \int \mathrm{d}\Phi [\partial\cdot(D\varphi^* + gDC\times\omega^*) - (\partial\cdot gA)\times\varphi^* + \rho^* - g\omega^*\times\partial\cdot(DC) \\
&\quad - D(A)\cdot M + gU\times DC] \exp(-\Sigma) \\
&= \int \mathrm{d}\Phi [\partial\cdot(sD\omega^*) - g(\delta/\delta\lambda - l)\times\varphi^* + \rho^* + g\omega^*\times(\delta/\delta C^* - \eta) - D(A)\cdot M \\
&\quad + gU\times\delta/\delta K] \exp(-\Sigma).
\end{aligned}$$

The exact derivatives integrate to zero, and we obtain

$$\begin{aligned}
0 &= (\partial\cdot\delta/\delta V + gl\times\delta/\delta\rho + \rho^* + g\eta\times\delta/\delta\sigma - D(\delta/\delta J)\cdot M \\
&\quad + gU\times\delta/\delta K) Z \\
0 &= \partial\cdot\delta W/\delta V + gl\times\delta W/\delta\rho + \rho^* + g\eta\times\delta W/\delta\sigma - D(\delta W/\delta J)\cdot M \\
&\quad + gU\times\delta W/\delta K
\end{aligned}$$

$$\delta\Gamma/\delta\varphi = \partial\cdot\delta\Gamma/\delta V + g\varphi^*\times\delta\Gamma/\delta\lambda + g\delta\Gamma/\delta C^*\times\omega^* + gU\times\delta\Gamma/\delta K + D(A)\cdot M. \tag{5.17}$$

From $\delta\Gamma/\delta\lambda = -\partial\cdot A$, this gives, for the complete φ-dependence,

$$\Gamma = -(\varphi^*, (\partial\cdot gA)\times\varphi) - (M, D\varphi) + \Gamma_4(V - \partial\varphi, K - gU\times\varphi, C^* - g\omega^*\times\varphi). \tag{5.18}$$

We set

$$\Gamma_4(V-\partial\varphi,\ K-gU\times\varphi,\ C^*-g\omega^*\times\varphi)$$
$$\equiv (D\varphi^*,\ \partial\varphi-V)+\Gamma_5(V-\partial\varphi,\ K-gU\times\varphi,\ C^*-g\omega^*\times\varphi),$$

where Γ_5 is a new arbitrary function, and obtain the alternate expression for the complete φ-dependence

$$\Gamma=(\partial\varphi^*-M,\ D\varphi)-(D\varphi^*,\ V)+\Gamma_5(V-\partial\varphi,\ K-gU\times\varphi,\ C^*-g\omega^*\times\varphi). \tag{5.19}$$

Upon comparison with eq. (5.16), we obtain

$$\Gamma=-(\lambda,\partial\cdot A)+(\partial\varphi^*-M,\ D\varphi)-(D\varphi^*,\ V)$$
$$+\Gamma_6(V-\partial\varphi,\ M-\partial\varphi^*,\ K-gU\times\varphi+\partial(C^*-g\omega^*\times\varphi)), \tag{5.20}$$

which gives the complete λ, C^*, φ and φ^* dependence. Because Γ_6 is a generic function of its arguments, this may also be expressed as

$$\Gamma=-(\lambda,\partial\cdot A)+(\partial\varphi^*-M,\ D\varphi)-(D\varphi^*,\ V)$$
$$+\Gamma_7(V-\partial\varphi,\ M-\partial\varphi^*,\ K-g(U+\partial\omega^*)\times\varphi+\partial C^*-g\omega^*\times V), \tag{5.21}$$

where Γ_7 is another generic function of its arguments.

For the ω^* field, we have

$$0=\int d\Phi\ \delta/\delta\omega^*\ \exp(-\Sigma),$$

$$0=\int d\Phi[-\partial\cdot(sD\varphi)+\sigma-D\cdot N+gDC\times V]\ \exp(-\Sigma),$$

$$0=[-\partial\cdot\delta/\delta U+\sigma-D(\delta/\delta J)\cdot N-gV\times\delta/\delta K]Z,$$

$$0=-\partial\cdot\delta W/\delta U+\sigma-D(\delta W/\delta J)\cdot N-gV\times\delta W/\delta K.$$

This gives

$$\delta\Gamma/\delta\omega^*=-\partial\cdot\delta\Gamma/\delta U-gV\times\delta\Gamma/\delta K+D(A)\cdot N, \tag{5.22}$$

which has the solution for the complete ω^* dependence

$$\Gamma=-(D\omega^*,\ N)+\Gamma_8(U+\partial\omega^*,\ K-g\omega^*\times V). \tag{5.23}$$

By comparison with eq. (5.21), we get

$$\Gamma = -(\lambda, \partial\cdot A) + (\partial\varphi^* - M, D\varphi) - (D\varphi^*, V) - (D\omega^*, N) + \Gamma_9(V-\partial\varphi, M-\partial\varphi^*, U+\partial\omega^*, K'). \tag{5.24}$$

where K' is defined in eq. (5.7). We set

$$\Gamma_9(V-\partial\varphi, M-\partial\varphi^*, U+\partial\omega^*, K') \equiv -(U+\partial\omega^*, D\omega) + \Gamma_{10}(V-\partial\varphi, M-\partial\varphi^*, U+\partial\omega^*, K'),$$

where Γ_{10} is a new generic function, and obtain

$$\Gamma = -(\lambda, \partial\cdot A) + (\partial\varphi^* - M, D\varphi) - (D\varphi^*, V) - (D\omega^*, N) - (U+\partial\omega^*, D\omega) + \Gamma_{10}(V-\partial\varphi, M-\partial\varphi^*, U+\partial\omega^*, K'). \tag{5.25}$$

For the ω-field, we have

$$0 = \int \mathrm{d}\Phi\ \delta/\delta\omega\ \exp(-\Sigma)$$

$$0 = \int \mathrm{d}\Phi (D\cdot\partial\omega^* - \sigma^* + D(A)\cdot U)\ \exp(-\Sigma)$$

$$0 = \int \mathrm{d}\Phi (\partial\cdot D\omega^* - g\partial\cdot A\times\omega^* - \sigma^* + D(A)\cdot U)\ \exp(-\Sigma)$$

$$0 = \int \mathrm{d}\Phi [\partial\cdot D\omega^* - g(\delta/\delta\lambda - l)\times\omega^* - \sigma^* + D(A)\cdot U]\ \exp(-\Sigma)$$

$$0 = [-\partial\cdot\delta/\delta N - gl\times\delta/\delta\sigma - \sigma^* + D(\delta/\delta J)\cdot U]Z$$

$$0 = -\partial\cdot\delta W/\delta N - gl\times\delta W/\delta\sigma - \sigma^* + D(\delta W/\delta J)\cdot U,$$

where the integral of an exact derivative was set to zero. This gives

$$\delta\Gamma/\delta\omega = -\partial\cdot\delta\Gamma/\delta N - g\delta\Gamma/\delta\lambda\times\omega^* - D(A)\cdot U, \tag{5.26}$$

which, with $\delta\Gamma/\delta\lambda = -\partial\cdot A$, has the solution

$$\Gamma = (\omega^*, g\partial\cdot A\times\omega) - (U, D\omega) + \Gamma_{11}(N+\partial\omega), \tag{5.27}$$

giving the complete ω-dependence. We set

$$\Gamma_{11}(N+\partial\omega) \equiv -(D\omega^*, N+\partial\omega) + \Gamma_{12}(N+\partial\omega),$$

where Γ_{12} is a new generic function, and obtain the alternate expression for the complete ω dependence,

$$\Gamma = -(\partial\omega^* + U,\, D\omega) - (D\omega^*,\, N) + \Gamma_{12}(N + \partial\omega). \tag{5.28}$$

By comparison with eq. (5.25), we get

$$\Gamma = -(\lambda,\, \partial\cdot A) + (\partial\varphi^* - M,\, D\varphi) - (\partial\omega^* + U,\, D\omega) - (D\varphi^*,\, V) - (D\omega^*,\, N)$$
$$+ \Gamma_{12}(V - \partial\varphi,\, M - \partial\varphi^*,\, N + \partial\omega,\, U + \partial\omega^*,\, K'). \tag{5.29}$$

Finally, we set

$$\Gamma_{12}(V - \partial\varphi,\, M - \partial\varphi^*,\, N + \partial\omega,\, U + \partial\omega^*,\, K')$$
$$\equiv S_{\mathrm{YM}} - (K',\, DC) - (L,\, -g/2C \times C)$$
$$+ \Gamma_{\mathrm{qu}}(V - \partial\varphi,\, M - \partial\varphi^*,\, N + \partial\omega,\, U + \partial\omega^*,\, K'), \tag{5.30}$$

and the theorem follows.

6. BRS identities for the effective action

We next derive the BRS identity for the effective action by standard procedures. We have from eqs. (4.2) and (5.2)

$$0 = \int \mathrm{d}\Phi\; s \exp(-\Sigma)$$
$$= \int \mathrm{d}\Phi[(M,\, sD\varphi) + (sD\omega^*,\, N) + (J,\, sA) - (\eta^*,\, sC) + (\lambda,\, \eta) + (\rho^*,\, \omega)$$
$$+ (\varphi^*,\, \sigma)] \exp(-\Sigma)$$
$$= [(M,\, \delta/\delta U) + (\delta/\delta V,\, N) + (J,\, \delta/\delta K) - (\eta^*,\, \delta/\delta L) + (\delta/\delta l,\, \eta)$$
$$+ (\rho^*,\, \delta/\delta\sigma^*) + (\delta/\delta\rho,\, \sigma)]Z$$
$$0 = (M,\, \delta W/\delta U) + (\delta W/\delta V,\, N) + (J,\, \delta W/\delta K) - (\eta^*,\, \delta W/\delta L) + (\delta W/\delta l,\, \eta)$$
$$+ (\rho^*,\, \delta W/\delta\sigma^*) + (\delta W/\delta\rho,\, \sigma).$$

This gives the desired identity satisfied by Γ,

$$(\delta\Gamma/\delta A,\, \delta\Gamma/\delta K) + (\delta\Gamma/\delta C,\, \delta\Gamma/\delta L) - (\lambda,\, \delta\Gamma/\delta C^*) - (\delta\Gamma/\delta\varphi,\, \omega)$$

$$-(\varphi^*,\, \delta\Gamma/\delta\omega^*) + (M,\, \delta\Gamma/\delta U) + (\delta\Gamma/\delta V,\, N) = 0. \tag{6.1}$$

This identity is geometrical in nature in the sense that the coupling constant g nowhere appears in it. Moreover it is satisfied order by order as a power series in g. In particular it is satisfied at order g^0 for which $\Gamma = S$, so the identity is satisfied by S itself, as may be verified by explicit calculation. We write $\Gamma = S + \Gamma_{\text{qu}}$, and it follows that Γ_{qu} satisfies

$$\delta\Gamma_{\text{qu}}/\delta A,\, \delta\Gamma_{\text{qu}}/\delta K + (\delta\Gamma_{\text{qu}}/\delta C,\, \delta\Gamma_{\text{qu}}/\delta L) + \sigma\Gamma_{\text{qu}} = 0, \tag{6.2}$$

where

$$\sigma \equiv (\delta S/\delta K,\, \delta/\delta A) + (\delta S/\delta A,\, \delta/\delta K) + (\delta S/\delta L,\, \delta/\delta C) + (\delta S/\delta C,\, \delta/\delta L)$$

$$-(\lambda,\, \delta/\delta C^*) - (\omega,\, \delta/\delta\varphi) - (\varphi^*,\, \delta/\delta\omega^*) + (M,\, \delta/\delta U) + (N,\, \delta/\delta V). \tag{6.3}$$

The fact that identity (6.1) is satisfied by S implies that σ is a nilpotent operator, $\sigma^2 = 0$. (To see this, note that identity (6.1) which is satisfied by S is of the form

$$\partial S/\partial x_i \partial S/\partial \xi_i + \eta_i \partial S/\partial y_i + z_i \partial S/\partial \zeta_i = 0, \tag{6.4}$$

where x, y and z and ξ, η, and ζ are three sets of Bose and Fermi variables, and that σ is the differential operator

$$\sigma = \partial S/\partial \xi_i \partial/\partial x_i + \partial S/\partial x_i \partial/\partial \xi_i + \eta_i \partial/\partial y_i + z_i \partial/\partial \zeta_i. \tag{6.5}$$

It is easy to verify by standard arguments [6,7], that eq. (6.4) is a sufficient condition for σ to be nilpotent.)

In sect. 5 we showed that Γ_{qu} depends on the six fields λ, C^*, φ, φ^*, ω, and ω^* only through the dependence of the primed variables, defined in eqs. (5.7) and (5.8), on these fields. It is natural to express the BRS identity in terms of the primed or "reduced" set of variables. For this purpose it is helpful to introduce the action

$$-S' \equiv -S_{\text{YM}} + (K',\, DC) + (L,\, -g/2C \times C) - (M',\, V') + (U',\, N'), \tag{6.6}$$

which depends on the same variables as Γ_{qu} namely, $S' = S'(A, C, K', L, M', N',$

U', V'). We also define a BRS operator σ' in terms of the reduced set of variables by analogy with eq. (6.3),

$$\sigma' \equiv (\delta S'/\delta K', \delta/\delta A) + (\delta S'/\delta A, \delta/\delta K') + (\delta S'/\delta L, \delta/\delta C) + (\delta S'/\delta C, \delta/\delta L) + (M', \delta/\delta U') + (N', \delta/\delta V'). \tag{6.7}$$

One may verify that the BRS identity, eq. (6.2), satisfied by $\Gamma_{\rm qu}$ may be written

$$(\delta\Gamma_{\rm qu}/\delta A, \delta\Gamma_{\rm qu}/\delta K') + (\delta\Gamma_{\rm qu}/\delta C, \delta\Gamma_{\rm qu}/\delta L) + \sigma'\Gamma_{\rm qu} = 0. \tag{6.8}$$

Because there is no coupling in the action S' between the sources M', N', U' and V' and the other variables, the solution to this equation, which determines possible divergences, has a trivial dependence on these variables.

7. Renormalizability

Feynman rules for calculation with the action (3.9) or (5.2) are easily derived, and we shall not trouble to give them explicitly. They are slightly different from the ones given explicitly in refs. [2,3], where f was assumed even, and f real Bose ghosts and $f/2$ pairs of Fermi ghosts were introduced. In particular, for the action (3.9), in addition to the A–φ and A–φ^* propagators, there are non-zero φ–φ and φ^*–φ^* propagators. Nevertheless power counting gives the same primitive divergences as found in refs. [2,3].

Power counting in Feynman integrals fixes the dimensions to be assigned to the fields. These remain to some extent arbitrary because of conservation laws which follow from the U(1) × U(f) symmetry of S_0, and the arbitrariness may be fixed by convention. The dimension of the propagators gives the following conditions on the dimensions of the fields

$$[A] = [\varphi] = [\varphi^*] = 1, \qquad [\lambda] = 2 \tag{7.1}$$

$$[\omega] + [\omega^*] = [C] + [C^*] = 2. \tag{7.2}$$

A symmetric and traditional assignment, that would give dimension 1 to all ghost fields, would give dimension 5 to the C-ghost increasing vertex, the last term of S_0, eq. (3.5), which would suggest that this vertex is not renormalizable. However, such an assignment ignores the reduction in the degree of divergence implied by theorem 5.1. The C-ghost increasing term is in fact safely contained in the term (K', DC) of S', eq. (6.6), and its renormalization is assured. As a result of this reduction of divergence, dimension 4 is also assigned to the C-ghost increasing vertex, and this provides the condition

$$[\omega^*] + [C] = 1. \tag{7.3}$$

It is convenient to fix the remaining arbitrariness in dimensions of the elementary fields by requiring that the BRS operator s does not change dimension. [This implies incidentally that s commutes with the generator of dilatations (see sect. 9).] Then from eqs. (3.2) and (7.1) we obtain

$$[\omega] = [\omega^*] = 1,$$

$$[C] = 0, \qquad [C^*] = 2. \tag{7.4}$$

These dimensions also imply conditions (7.2) and (7.3). The dimensions of the composite fields are then fixed,

$$[K] = 3, \qquad [M] = [N] = [U] = [V] = 2. \tag{7.5}$$

Feynman integrals may be evaluated by dimensional regularization. As usual the coupling constant g is replaced by $g\mu^{(D-4)/2}$, where μ plays the role of renormalization mass.

We now discuss the recursive construction of counter terms which cancel the divergences, following the approach of ref. [7], pp. 599 to 604. This will be done in two steps. In the first step we shall establish renormalizability in terms of the reduced set of fields introduced in sect. 5. In the second step we shall establish renormalizability in terms of the original set of fields.

At loop order l, the divergent piece, Γ^l_{div}, is a local function of the reduced set of fields and sources of dimension 4 that satisfies

$$\sigma' \Gamma^l_{\text{div}} = 0. \tag{7.6}$$

By conservation of ghost number and U(f) invariance, the dependence of Γ^l_{div} on the sources M', N', U' and V' is of the form $c_4(M', V') + c_5(U', N')$, where c_4 and c_5 are constants. The last equation implies that Γ^l_{div} is of the form

$$\Gamma^l_{\text{div}} = \Gamma^l_{\text{div,FP}} + c_4[(M', V') - (U', N')] \tag{7.7}$$

$$= \Gamma^l_{\text{div,FP}} + \sigma' c_4(U', V'), \tag{7.8}$$

where $\Gamma^l_{\text{div,FP}}$ satisfies eq. (7.6), but depends only on the fields that appear in Faddeev–Popov theory. According to standard arguments [7], it follows from eq. (7.6) that Γ^l_{div} is of the form

$$\Gamma^l_{\text{div}} = c_1 S_{\text{YM}} + c_2[(\delta S_{\text{YM}}/\delta A, A) + (K', \partial C)]$$
$$+ c_3[(L, -\tfrac{1}{2}gC \times C) + (K', DC)] + c_4[(M', V') - (U', N')],$$

which may be written

$$\Gamma^l_{\text{div}} = c_1 S_{\text{YM}} + \sigma'[c_2(K', A) + c_3(L, C) + c_4(U', V')]. \tag{7.9}$$

The ghost–ghost–gluon vertex is finite in the Landau gauge [7], as is shown explicitly by remark 3 of sect. 5, so

$$c_3 = 0. \tag{7.10}$$

This is equivalent to the well-known result in the Landau gauge $Z_1' = 1$. Moreover, as was noted above, apart from correlation functions which contain the C-ghost increasing vertex, the ghost diagrams for the various ghost fields both bose and fermi are equal. It follows that

$$c_4 = -c_2. \tag{7.11}$$

We thus obtain

$$\begin{aligned} \Gamma_{\text{div}}^l &= c_1 S_{\text{YM}} + c_2[(\delta S_{\text{YM}}/\delta A, A) + (K', \partial C)] - [(M', V') - (U', N')] \\ &= c_1 S_{\text{YM}} + c_2 \sigma'[(K', A) - (U', V')]. \end{aligned} \tag{7.12}$$

Here the standard solution to the cohomology problem is exhibited, namely a multiple of S_{YM} plus an exact form.

We must show that these divergent terms may be cancelled by a renormalization of the charge and the reduced set of fields that appear in S', eq. (6.6),

$$\begin{aligned} &A = Z_A A_r, \qquad K' = Z_{K'} K'_r, \qquad C = Z_C C_r, \qquad L = Z_L L_r, \\ &M' = Z_{M'} M'_r, \quad V' = Z_{V'} V'_r, \quad N' = Z_{N'} N'_r, \quad U' = Z_{U'} U'_r, \\ &g = Z_g g_r. \end{aligned} \tag{7.13}$$

The renormalization constants Z_i are determined recursively, and in loop order l are of the form $Z_i^l = 1 + \delta Z_i^l$. By comparison with eq. (6.6), we obtain for the cancellation of the divergent terms at loop order l, the conditions

$$\delta Z_M^l + \delta Z_{V'}^l = \delta Z_{N'}^l + \delta Z_{U'}^l = \delta Z_{K'}^l + \delta Z_C^l = c_2, \tag{7.14}$$

$$\delta Z_{K'}^l + \delta Z_g^l + \delta Z_A^l + \delta Z_C^l = \delta Z_L^l + \delta Z_g^l + 2\delta Z_C^l = 0. \tag{7.15}$$

To obtain the remaining conditions, we write $S_{\text{YM}} = S_{\text{YM}}(A, g)$, which gives to first order

$$S_{\text{YM}}[(1 + \delta Z_A^l)A, (1 + \delta Z_g^l)g] = S_{\text{YM}}(A, g) + \delta Z_A^l(A, \delta S_{\text{YM}}/\delta A) + \delta Z_g^l g \partial S_{\text{YM}}/\partial g.$$

Moreover, $S_{\text{YM}}(A, g)$ is of degree 2 in A and g^{-1} which means

$$(A, \delta S_{\text{YM}}/\delta A) - g\partial S_{\text{YM}}/\partial g = 2S_{\text{YM}}.$$

This gives

$$\delta S_{\rm YM} = -2\delta Z_g^l S_{\rm YM} + \left(\delta Z_A^l + \delta Z_g^l\right)(A,\, \delta S_{\rm YM}/\delta A),$$

and we obtain the additional conditions for cancellation of the divergences at loop order l,

$$2\delta Z_g^l = c_1, \qquad \delta Z_A^l + \delta Z_g^l = -c_2, \tag{7.16}$$

These equations imply the exact relations among the renormalization constants,

$$Z_{M'} Z_{V'} = Z_{N'} Z_{U'} = Z_{K'} Z_C = \left(Z_A Z_g\right)^{-1}, \tag{7.17}$$

$$Z_{K'} Z_g Z_A Z_C = Z_L Z_g Z_C^2 = 1, \tag{7.18}$$

which are consistent with each other.

This establishes renormalizability of the effective action Γ' defined by

$$\Gamma' \equiv S' + \Gamma_{\rm qu}, \tag{7.19}$$

and which depends on the reduced set of variables. To be explicit, we have shown that with the renormalization constants chosen recursively as described above, $\Gamma_{\rm r}'(X_{\rm r})$, which is defined by

$$\Gamma_{\rm r}'(X_{\rm r}) = \Gamma'(X) \tag{7.20}$$

is a finite function of its arguments. Here X represents the reduced set of unrenormalized fields and the coupling constant, and similarly for $X_{\rm r}$. This completes step one.

We next verify that relations (5.7) and (5.8) among the original unreduced set of variables may be maintained for the renormalized variables. They are satisfied if we define the renormalization constants

$$Z_M = Z_{\varphi*} = Z_{M'}, \quad Z_V = Z_\varphi = Z_{V'}, \quad Z_N = Z_\omega = Z_{N'}, \quad Z_U = Z_{\omega*} = Z_{U'},$$
$$Z_{C*} = Z_K = Z_{K'}, \tag{7.21}$$

and provided that the new condition

$$Z_g Z_{\omega*} Z_\varphi = Z_K \tag{7.22}$$

holds. These equations imply the relation between the renormalization constants of the elementary fields,

$$Z_{\varphi*} Z_\varphi = Z_{\omega*} Z_\omega = Z_{C*} Z_C = \left(Z_A Z_g\right)^{-1}, \tag{7.23}$$

$$Z_g Z_{\omega*} Z_\varphi = Z_{C*}. \tag{7.24}$$

Because of the arbitrariness in the renormalization due to conservation laws, we are free to impose

$$Z_{\varphi^*} = Z_{\varphi}, \qquad Z_{\omega^*} = Z_{\omega}, \tag{7.25}$$

which gives

$$Z_{\varphi^*} = Z_{\varphi} = Z_{\omega^*} = Z_{\omega} \equiv Z_3'^{1/2}, \tag{7.26}$$

$$Z_{C^*} Z_C = Z_3', \tag{7.27}$$

where we have introduced the standard notation for the ghost field renormalization constant. Upon multiplication of eq. (7.24) by Z_C, we obtain

$$Z_g Z_C = 1, \tag{7.28}$$

or

$$gC = g_r C_r. \tag{7.29}$$

Geometrically this means that the infinitesimal local gauge transformations generated by gC are not renormalized. From eq. (7.18) we obtain also

$$Z_K Z_A = Z_L Z_C = 1. \tag{7.30}$$

And finally, from eq. (7.23), we obtain

$$Z_g = \left(Z_3^{1/2} Z_3'\right)^{-1}, \tag{7.31}$$

where we have introduced the conventional notation

$$Z_3^{1/2} \equiv Z_A. \tag{7.32}$$

Eq. (7.31) is standard in the Landau gauge where $Z_1' = 1$.

To complete the proof we shall establish renormalizability of the effective action Γ, expressed as a function of the original variables. The original action S, defined in eq. (4.2), may be written

$$S = S' + S_{\text{in}} - [(M, V) - (U, N)], \tag{7.33}$$

where S', is defined in eq. (6.6) and depends only on the reduced set of variables, and S_{in} is defined by

$$-S_{\text{in}} \equiv (M', gA \times \varphi) + (gA \times \varphi^*, V) + (U', gA \times \omega) + (gA \times \omega^*, N). \tag{7.34}$$

Observe that S_{in} is invariant under renormalization,

$$S_{\text{in}}(X_r) = S_{\text{in}}(X), \tag{7.35}$$

which is a consequence of the Landau gauge formula $Z_g = (Z_3^{1/2} Z_3')^{-1}$. It follows that

$$\Gamma_{\mathrm{r}}(X_{\mathrm{r}}) \equiv \Gamma'(X) + S_{\mathrm{in}}(X) = \Gamma(X) + [(M, V) - (U, N)] \qquad (7.36)$$

is a finite function of its arguments. The term $[(M, V) - (U, N)]$, which depends only on the sources, must be added to the original effective action Γ to obtain a quantity which is made finite by the multiplicative renormalization of charge and fields. This is a typical occurrence, when sources for composite fields are present [6]. This completes the proof of renormalizability of the theory with local sources for composite fields of lower dimension.

8. Renormalization of the horizon condition

In this section we shall show that the horizon condition gives a finite equation when expressed in terms of renormalized quantities. In a translation-invariant theory the expectation value of the derivative of a field vanishes. Consequently the horizon condition, eq. (2.12), may be written in terms of the physical sources, eqs. (4.3) and (4.4), as

$$\left\langle D_\mu{}^{ac} \varphi_{\mu a}{}^{c} \right\rangle = \delta W / \delta M_{\mu\mu a}{}^{a} \big|_{\mathrm{ph}} = -\delta \Gamma / \delta M_{\mu\mu a}{}^{a} \big|_{\mathrm{ph}} = V_{\mathrm{ph},\mu\mu a}{}^{a}, \qquad (8.1)$$

$$\left\langle s D_\mu{}^{ac} \omega^*{}_{\mu a}{}^{c} \right\rangle = \delta W / \delta V_{\mu\mu a}{}^{a} \big|_{\mathrm{ph}} = -\delta \Gamma / \delta V_{\mu\mu a}{}^{a} \big|_{\mathrm{ph}} = M_{\mathrm{ph},\mu\mu a}{}^{a}. \qquad (8.2)$$

This may be written,

$$\delta[\Gamma + (M, V) - (U, N)] / \delta M_{\mu\mu a}{}^{a} \big|_{\mathrm{ph}} = 0,$$

$$\delta[\Gamma + (M, V) - (U, N)] / \delta V_{\mu\mu a}{}^{a} \big|_{\mathrm{ph}} = 0.$$

A striking simplicity now appears. For upon comparison with eq. (7.36), one recognizes that the quantity in square brackets equals the renormalized effective action. So the horizon condition in terms of renormalized quantities is not only finite, but is given by the simple homogeneous equations

$$\delta \Gamma_{\mathrm{r}} / \delta M_{\mathrm{r}\mu\mu a}{}^{a} \big|_{\mathrm{ph}} = \delta \Gamma_{\mathrm{r}} / \delta V_{\mathrm{r}\mu\mu a}{}^{a} \big|_{\mathrm{ph}} = 0. \qquad (8.3)$$

Remarkably, any other constant on the right hand side of the horizon condition (1.1) would not give a finite renormalized horizon condition, and thus would not correspond to a critical point.

We next give the physical values of the renormalized sources. From eq. (4.3) we have

$$M_{\mathrm{r,ph},\mu\nu b}{}^{a}(x) = -V_{\mathrm{r,ph},\mu\nu b}{}^{a}(x) = Z_3'^{-1/2} M_{\mathrm{ph},\mu\nu b}{}^{a}(x) = -Z_3'^{-1/2} V_{\mathrm{ph},\mu\nu b}{}^{a}(x)$$

$$= Z_3'^{-1/2} \gamma^{1/2} g^{-1} \delta_{\mu\nu} \delta_b{}^{a} = Z_3{}^{1/2} Z_3'^{1/2} \gamma^{1/2} g_r{}^{-1} \delta_{\mu\nu} \delta_b{}^{a}. \qquad (8.4)$$

Let a renormalized Boltzmann constant be defined by

$$\gamma_{\rm r}^{1/2} = Z_3^{1/2} Z_3'^{1/2} \gamma^{1/2}. \tag{8.5}$$

Then the physical values of the renormalized sources are

$$M_{{\rm r,ph},\mu\nu b}{}^{a}(x) = -V_{{\rm r,ph},\mu\nu b}{}^{a}(x) = \gamma_{\rm r}^{1/2} g_r^{-1} \delta_{\mu\nu} \delta_b{}^a. \tag{8.6}$$

The physical value of all other sources is zero. Thus if $\gamma_{\rm r}$, which has dimensions of $(\text{mass})^4$, is given a finite value in physical units, then the renormalized horizon condition, which is a finite equation in terms of the renormalized quantities, determines a finite value for $g_{\rm r}$.

In the method of local sources, the renormalization constants have the same value as in standard Faddeev–Popov theory. (Different normalization conventions were used in [3].) It follows that $g_{\rm r}$ satisfies the standard renormalization group equation,

$$\mu\partial g_{\rm r}/\partial\mu = \beta(g_{\rm r}) = b_0 g_{\rm r}^3 + b_1 g_{\rm r}^5 + \dots \tag{8.7}$$

From eq. (8.5) we obtain the corresponding renormalization group equation for $\gamma_{\rm r}^{1/2}$,

$$\mu\partial\gamma_{\rm r}^{1/2}/\partial\mu = \alpha(g_{\rm r})\gamma_{\rm r}^{1/2}, \tag{8.8}$$

where

$$\alpha(g_{\rm r}) \equiv (\tfrac{1}{2})\mu\partial[\ln(Z_3 Z_3')]/\partial\mu = a_0 g_{\rm r}^2 + a_1 g_{\rm r}^4 + \dots \tag{8.9}$$

and the coefficients are finite. The renormalization group equation for $g_{\rm r}$ has the familiar solution

$$g_{\rm r} = g_{\rm r}(\Lambda_{\rm QCD}/\mu), \tag{8.10}$$

where $\Lambda_{\rm QCD}$ is a constant of integration. This equation may be inverted to give

$$\Lambda_{\rm QCD} = \mu f(g_{\rm r}). \tag{8.11}$$

If we change independent variable from μ to $g_{\rm r}$, the renormalization-group equation for $\gamma_{\rm r}$ takes the form

$$\beta(g_{\rm r})\partial\gamma_{\rm r}^{1/2}/\partial g_{\rm r} = \alpha(g_{\rm r})\gamma_{\rm r}^{1/2}. \tag{8.12}$$

This gives

$$\partial(\ln\gamma_{\rm r}^{1/2})/\partial g_{\rm r} = [\beta(g_{\rm r})]^{-1}\alpha(g_{\rm r}) = b_0^{-1} a_0 g_{\rm r}^{-1} + {\rm O}(g_{\rm r}), \tag{8.13}$$

which we write in the form

$$\partial\left[\ln\left(\gamma_{\mathrm{r}}^{1/2}/g_{\mathrm{r}}^{c}\right)\right]/\partial g_{\mathrm{r}}=h(g_{\mathrm{r}})\equiv\left[\beta(g_{\mathrm{r}})\right]^{-1}\alpha(g_{\mathrm{r}})-b_0^{-1}a_0 g_{\mathrm{r}}^{-1}]=\mathrm{O}(g_{\mathrm{r}}),\quad(8.14)$$

where

$$c\equiv a_0/b_0.\quad(8.15)$$

This has the solution

$$\gamma_{\mathrm{r}}^{1/2}=(\Lambda'_{\mathrm{QCD}})^2 g_{\mathrm{r}}^{c}\exp\left[\int_0^{g_{\mathrm{r}}}\mathrm{d}g\,h(g)\right],\quad(8.16)$$

where Λ'_{QCD} is a new constant of integration with dimensions of mass, which is moreover a renormalization-group invariant.

The horizon condition determines $g_{\mathrm{r}}=g_{\mathrm{r}}(\gamma_{\mathrm{r}})$. Moreover it is compatible with the renormalization-group equations because it is obtained, as we have shown above, by renormalization of the unrenormalized horizon condition which is renormalization-group invariant. This means that we may replace g_{r} and γ_{r} in the equation $g_{\mathrm{r}}=g_{\mathrm{r}}(\gamma_{\mathrm{r}})$ by their expressions in eq. (8.10) and (8.16) and obtain a consistent equation for Λ_{QCD} in terms of Λ'_{QCD}. On dimensional grounds it can only be of the form

$$\Lambda_{\mathrm{QCD}}=c\Lambda'_{\mathrm{QCD}},\quad(8.17)$$

where c is a pure number which is determined by the horizon condition.

We shall not trouble to write the homogeneous renormalization-group equations for the correlation functions which are the exact analog of eq. (8.73) of sect. 8.10 of ref. [6].

9. Energy–momentum tensor and gluon condensate

The dimensionful parameter $\gamma^{1/2}$ in the action breaks dilatation invariance at the tree level. We shall derive the energy–momentum tensor and identify the gluon condensate.

We use dimensional regularization, and consequently we require the dimensions of the fields in generic euclidean dimension D. The formulas in sect. 7 for $D=4$ generalize to

$$[A]=[\varphi]=[\varphi^*]=(D-2)/2,\quad[\lambda]=D/2,\quad(9.1)$$

$$[\omega]+[\omega^*]=[C]+[C^*]=(D-2).\quad(9.2)$$

The dimension of g is fixed by the condition that the classical connection has dimensions of inverse length, so $[gA]=1$, which gives

$$[g]=(4-D)/2\equiv\epsilon/2. \tag{9.3}$$

From the C-ghost increasing vertex, we obtain

$$[\omega^*]=[C]=D-3. \tag{9.4}$$

We again fix the remaining arbitrariness in dimensions by imposing that the BRS operator leaves dimensions invariant. This gives

$$[\omega]=[\omega^*]=(D-2)/2, \tag{9.5}$$

$$[C]=(D-4)/2=-\epsilon/2, \quad [C^*]=D/2. \tag{9.6}$$

These dimensions also imply conditions (9.2) and (9.4), and give $[gC]=0$ in all dimensions. The dimension of γ is unchanged,

$$[\gamma^{1/2}]=2. \tag{9.7}$$

We shall derive a Ward identity by the change of variable [8]

$$\Phi_i\to\Phi_i'=\Phi_i+w[\eta]\Phi_i \tag{9.8}$$

in the partition function

$$Z=\int d\Phi\,\exp[-S+(J_i,\Phi_i)]. \tag{9.9}$$

Here the field Φ_i represents all the elementary fields, with sources J_i, and we collectively denote the dimensions of the elementary fields by

$$D_i\equiv[\Phi_i]. \tag{9.10}$$

The action S is defined by

$$S=S_{\text{ph}}-\int d^Dx\,f\gamma/g^2, \tag{9.11}$$

where S_{ph} is given in eq. (3.9b). The last term here is the last term of eq. (7.36) with the external sources set equal to their physical value. Finally $w[\eta]$ in eq. (9.8) represents the infinitesimal generator

$$w[\eta]\equiv\int d^Dx\;\eta^\mu(x)\{\partial_\mu\Phi_i\delta/\delta\Phi_i-D^{-1}D_i\partial_\mu(\Phi_i\delta/\delta\Phi_i)$$

$$+\tfrac{1}{2}\partial_\nu(\Sigma_{i,\mu\nu}\Phi_i\delta/\delta\Phi_i)\}, \tag{9.12}$$

where $\eta^{\mu}(x)$ is an arbitrary infinitesimal vector function, $\frac{1}{2}\Sigma_{i,\mu\nu} = -\frac{1}{2}\Sigma_{i,\nu\mu}$ is the generator of spin transformations for the field Φ_i, and a sum over i is implicit. The generator $w[\eta]$ has the property that for $\eta^{\mu}(x)$ of the special form

$$\eta^{\mu}(x) = \rho^{\mu} + \sigma x^{\mu} + \tau^{\mu\lambda}x^{\lambda}, \tag{9.13a}$$

where ρ^{μ}, σ and $\tau^{\mu\lambda}$ are infinitesimal and x-independent, with $\tau^{\lambda\mu} = -\tau^{\mu\lambda}$, $w[\eta]$ generates an infinitesimal translation, dilatation and (euclidean) Lorentz transformation, namely

$$w[\eta]\Phi_i = \rho^{\mu}\partial_{\mu}\Phi_i + \sigma(x^{\mu}\partial_{\mu} + D_i)\Phi_i + \tfrac{1}{2}\tau^{\mu\lambda}(x_{\lambda}\partial_{\mu} - x_{\mu}\partial_{\lambda} + \Sigma_{i\lambda\mu})\Phi_i, \tag{9.13b}$$

as is easily verified.

The change of variable (9.8) is linear in Φ so the jacobian is a constant, and we have

$$0 = \int \mathrm{d}\Phi[-wS + (w\Phi_i, J_i)]\ \exp[-S + (\Phi_i, J_i)]. \tag{9.14}$$

To obtain the Ward identity we evaluate

$$wS = \int \mathrm{d}^D x\ w\Phi_i(x)\delta S/\delta\Phi_i(x)$$

$$= \int \mathrm{d}^D x\ \eta^{\mu}\{\partial_{\mu}\Phi_i\delta S/\delta\Phi_i - \partial_{\mu}(D^{-1}D_i\Phi_i\delta S/\delta\Phi_i) + \tfrac{1}{2}\partial_{\lambda}(\Sigma_{i,\mu\lambda}\Phi_i\delta S/\delta\Phi_i)\}.$$

Because S is the integral of a local density L, we have

$$\delta S/\delta\Phi_i = \partial L/\partial\Phi_i - \partial_{\lambda}(\partial L/\partial\partial_{\lambda}\Phi_i),$$

which gives

$$\begin{aligned} wS = \int \mathrm{d}^D x\ \eta^{\mu}\{ & -\partial_{\lambda}(\partial_{\mu}\Phi_i\partial L/\partial\partial_{\lambda}\Phi_i) + \partial_{\mu}L \\ & -\partial_{\mu}[-\partial_{\lambda}(D^{-1}D_i\Phi_i\partial L/\partial\partial_{\lambda}\Phi_i) + L - D^{-1}\alpha L - D^{-1}\partial_{\lambda}\Phi_i\partial L/\partial\partial_{\lambda}\Phi_i] \\ & + \tfrac{1}{2}\partial_{\lambda}[-\partial_{\kappa}(\Sigma_{i,\mu\lambda}\Phi_i\partial L/\partial\partial_{\kappa}\Phi_i) - (\delta_{\kappa\mu}\partial_{\lambda}\Phi_i - \delta_{\kappa\lambda}\partial_{\mu}\Phi_i)\partial L/\partial\partial_{\kappa}\Phi_i]\}. \end{aligned} \tag{9.15}$$

Here we have used the fact that L has no explicit x-dependence, so

$$\partial_{\mu}L = \partial_{\mu}\Phi_i\partial L/\partial\Phi_i + \partial_{\mu}\partial_{\lambda}\Phi_i\partial L/\partial\partial_{\lambda}\Phi_i,$$

that L is a Lorentz scalar, so

$$0 = \Sigma_{i,\mu\lambda}\Phi_i\partial L/\partial\Phi_i + (\Sigma_{i,\mu\lambda}\partial_{\kappa}\Phi_i + \delta_{\kappa\mu}\partial_{\lambda}\Phi_i - \delta_{\kappa\lambda}\partial_{\mu}\Phi_i)\partial L/\partial\partial_{\kappa}\Phi_i,$$

and that L has engineering dimension D, so

$$DL = D_i\Phi_i\partial L/\partial\Phi_i + (D_i + 1)\partial_\lambda\Phi_i\partial L/\partial\partial_\lambda\Phi_i + \alpha L. \tag{9.16}$$

Here α is the operator,

$$\alpha \equiv 2\gamma^{1/2}\partial/\partial\gamma^{1/2} + (\epsilon/2) g\partial/\partial g, \tag{9.17a}$$

so αL is the contribution to the engineering dimension of L that comes from the dimensionful coupling constants. In euclidean dimension D, it is customary to make the substitution

$$g \to \mu^{\epsilon/2} g,$$

where g is now a dimensionless coupling constant, so α takes the form

$$\alpha = 2\gamma^{1/2}\partial/\partial\gamma^{1/2} + \mu\partial/\partial\mu \equiv \alpha_\gamma + \alpha_\mu. \tag{9.17b}$$

The term $\mu\partial/\partial\mu$ gives an anomalous contribution to the generator of dilatations, in the sense that it does not appear at tree level for $D = 4$, but the term $2\gamma^{1/2}\partial/\partial\gamma^{1/2}$ does. From eq. (9.15) we obtain

$$wS = \int \mathrm{d}^D x\ \eta^\mu(-\partial_\lambda T_{\lambda\mu} - \partial_\lambda\partial_\kappa R_{\kappa\lambda\mu}), \tag{9.18}$$

where

$$T_{\lambda\mu} \equiv \tfrac{1}{2}(\partial_\mu\Phi_i\partial L/\partial\partial_\lambda\Phi_i + \partial_\lambda\Phi_i\partial L/\partial\partial_\mu\Phi_i) - \delta_{\lambda\mu}D^{-1}(\partial_\kappa\Phi_i\partial L/\partial\partial_\kappa\Phi_i + \alpha L), \tag{9.19}$$

and

$$R_{\kappa\lambda\mu} \equiv \tfrac{1}{2}\Sigma_{i,\mu\lambda}\Phi_i\partial L/\partial\partial_\kappa\Phi_i - \delta_{\kappa\mu}D^{-1}D_i\Phi_i\partial L/\partial\partial_\lambda\Phi_i. \tag{9.20}$$

The energy–momentum tensor $T_{\lambda\mu}$, obtained here without the use of the equations of motion, is symmetric, and its trace

$$T_{\lambda\lambda} = -\alpha L \tag{9.21}$$

would vanish in the absence of dimensionful coupling constants.

Because $\eta^\mu(x)$ is an arbitrary infinitesimal function, eq. (9.14) gives the local Ward identity,

$$\begin{aligned} &-Z(\partial_\lambda T_{\lambda\mu} + \partial_\lambda\partial_\kappa R_{\kappa\lambda\mu}) \\ &\quad = \partial_\mu(\delta Z/\delta J_i)J_i - D^{-1}D_i\partial_\mu(\delta Z/\delta J_i J_i) + \tfrac{1}{2}\partial_\nu(\Sigma_{i,\mu\nu}\partial Z/\delta J_i J_i). \end{aligned} \tag{9.22}$$

The right-hand side renormalizes since it merely effects the same linear transformation on the renormalized and unrenormalized fields, which shows that insertion of the field $\partial_\lambda T_{\lambda\mu} + \partial_\lambda \partial_\kappa R_{\kappa\lambda\mu}$ gives finite correlation functions.

We shall not attempt to "improve" the energy–momentum tensor by adding an exact derivative, nor to obtain explicit renormalized formulas for the energy–momentum tensor such as is available for $\lambda\varphi^4$ theory [8]. Instead we shall exhibit the integrated Ward identity for dilatations which is insensative to such a change. We contract eq. (9.22) with x^μ, and integrate. The term with $\partial_\lambda \partial_\kappa R_{\kappa\lambda\mu}$ gives no contribution, and we obtain

$$\int \mathrm{d}^D x\, Z(T_{\lambda\lambda}) = -\int \mathrm{d}^D x\, Z(\alpha L) = \int \mathrm{d}^D x (x^\mu \partial_\mu + D_i)(\delta Z/\delta J_i) J_i. \quad (9.23)$$

Thus a global dilatation is obtained by insertion of $-\alpha S$, which is minus the contribution to the dimension of the action which comes from the dimensionful coupling constants.

The expectation value

$$\langle T_{\lambda\lambda}\rangle = \langle \alpha_\gamma L\rangle + \langle \alpha_\mu L\rangle \quad (9.24)$$

is also insensitive to the addition of an exact derivative to $T_{\lambda\mu}$. This quantity is a natural candidate for the gluon condensate which has been introduced with success in hadron phenomenology [9]. From the action (9.11), we obtain

$$\alpha_\gamma L = 2\gamma^{1/2} A_i^a (\varphi - \varphi^*)_i^a - 4f\gamma/g^2, \quad (9.25)$$

and the horizon condition (2.12) gives

$$\langle \alpha_\gamma L\rangle = 0. \quad (9.26)$$

So this term does not contribute to the gluon condensate nor to the cosmological constant, even though $\alpha_\gamma L$ is expected to be important in generating dilatations. We are left with

$$\langle T_{\lambda\lambda}\rangle = \langle \alpha_\mu L\rangle. \quad (9.27)$$

This quantity contains an explicit factor of $\epsilon = 4 - D$, as one sees from eq. (9.17). Therefore it is quite likely to be finite, when evaluated by dimensional regularization and continued analytically to $D = 4$, and expressed in terms of the mass scale Λ'_{QCD} which appears in eq. (8.16). (A similar proposal appears in ref. [4].) If so, a direct link will have been established between hadron phenomenology [9] and the global properties of the fundamental modular region.

10. Dipole ghost

In this section we will show that the propagator of the Fermi ghosts has a $1/(q^2)^2$ singularity at $q=0$.

The horizon condition, eq. (8.2), reads

$$Z^{-1}\int \mathrm{d}\Phi\ \exp(-S) sD_\mu^{ac}\omega^*{}_{\mu a}{}^c = M_{\mu\mu a}{}^a, \qquad (10.1)$$

where $\mathrm{d}\Phi$ represents integration over all fields, and the sources are assigned their physical values. Because the BRS operator s is a derivative, we have by partial integration

$$Z^{-1}\int \mathrm{d}\Phi\ (sS)\ \exp(-S) D_\mu^{ac}\omega^*{}_{\mu a}{}^c = M_{\mu\mu a}{}^a. \qquad (10.2)$$

With $sS = -(M, sD\varphi)$, this gives

$$\langle (M, sD\varphi) D_\mu^{ac}\omega^*{}_{\mu a}{}^c(x)\rangle + M_{\mu\mu a}{}^a(x) = 0. \qquad (10.3)$$

We evaluate this quantity by choosing for the local source

$$M_{\mu\nu b}{}^a(x) = \gamma^{1/2} g^{-1}\delta_{\mu\nu}\delta_b{}^a \exp(iq\cdot x). \qquad (10.4)$$

The limit $q\to 0$, by which M approaches its physical value, will be taken at the end. It is convenient to define the propagator of composite fields

$$G^{UN}{}_{\mu i}{}^a{}_{\kappa j}{}^c(q) \equiv \int \mathrm{d}^D x\ \exp(-iq\cdot x)\left\langle (D_\mu\omega^*{}_i)^a(x) s(D_\kappa\varphi_j)^c(0)\right\rangle$$

$$= \int d^D x\ \exp(-iq\cdot x)\delta^2 W/\delta U_{\kappa j}{}^c(0)\delta N_{\mu i}{}^a(x), \qquad (10.5)$$

where, as we recall from sect. 2, the index i represents the pair $i=(\nu, b)$, and similarly for j. In terms of G^{UN}, the horizon condition takes the form

$$-G^{UN}{}_{\mu\mu a}{}^a{}_{\kappa\kappa c}{}^c(q) + f = 0. \qquad (10.6)$$

We next derive some properties of G^{UN}. We have

$$\left\langle (D_\mu\omega^*{}_i)^a(x) s(D_\kappa\varphi_j)^c(0)\right\rangle = \left\langle (D_\mu\omega^*{}_i)^a(x)(D_\kappa\omega_j)^c(0)\right\rangle. \qquad (10.7)$$

Moreover, at every vertex on the ω line which is continuous across every graph, there is a δ-function on the $i-j$ indices, which comes from U(f) invariance. Thus G^{UN} is of the form

$$G^{UN}{}_{\mu i}{}^{a}{}_{\kappa j}{}^{c}(q) = \delta_{ij}\delta^{ac}A_{\mu\kappa}(q), \tag{10.8}$$

where $A_{\mu\kappa}(q)$ is an invariant Lorentz tensor.

We next use the equations of motion to determine the longitudinal part of G^{UN}. We have

$$q_\mu G^{UN}{}_{\mu i}{}^{a}{}_{\kappa j}{}^{c}(q) = -i\int \mathrm{d}^D x \; \exp(-iq\cdot x)\left\langle (\partial\cdot D\omega^*{}_i)^a(x)(D_\kappa\omega_j)^c(0)\right\rangle$$

$$= -i\int \mathrm{d}^D x \; \exp(-iq\cdot x) Z^{-1}\int \mathrm{d}\Phi$$

$$\times \delta[\exp(-S)]/\delta\omega_i^a(x)(D_\kappa\omega_j)^c(0)$$

$$= i\int \mathrm{d}^D x \; \exp(-iq\cdot x) Z^{-1}\int \mathrm{d}\Phi \; \exp(-S)\delta\left[(D_\kappa\omega_j)^c(0)\right]/\delta\omega_i^a(x),$$

which gives

$$q_\mu G^{UN}{}_{\mu i}{}^{a}{}_{\kappa j}{}^{c}(q) = \delta_{ij}\delta^{ac}q_\mu,$$

and so

$$G^{UN}{}_{\mu i}{}^{a}{}_{\kappa j}{}^{c}(q) = G^{UN,\mathrm{T}}{}_{\mu i}{}^{a}{}_{\kappa j}{}^{c}(q) + \delta_{ij}\delta^{ac}q_\mu q_\nu/q^2, \tag{10.9}$$

where $G^{UN,\mathrm{T}}{}_{\mu i}{}^{a}{}_{\kappa j}{}^{c}(q)$ is the part of G^{UN} which is transverse on the μ and κ indices. With $f=(N^2-1)D$, the horizon condition, eq. (10.6) reads

$$-G^{UN,\mathrm{T}}{}_{\mu\mu a}{}^{a}{}_{\kappa\kappa c}{}^{c}(q) + (N^2-1)(D-1) = 0. \tag{10.10}$$

The crucial point in the evaluation of $G^{UN,\mathrm{T}}$ is that only irreducible diagrams contribute to it. For the only one-particle intermediate state is the ω-line, and the index i on the ω-field is mute, so ω is effectively a Lorentz scalar particle. Therefore any reducible contribution to $G_{\mu i}{}^{a}{}_{\kappa j}{}^{c}(q)$ is, for example, of the form

$$\delta_{ij}\delta^{ab}B_\mu(q)(q^2)^{-1}C_\nu(q) = \delta_{ij}\delta^{ab}q_\mu b(q^2)(q^2)^{-1}q_\nu c(q^2)$$

which is purely longitudinal on μ and ν. Call Γ^{UN} the one-particle irreducible part of the G^{UN} propagator. It may be derived from the generating functional for one-particle irreducible correlation functions, namely the effective action

$$\Gamma^{UN}{}_{\mu i}{}^{a}{}_{\kappa j}{}^{c}(q) = -\int \mathrm{d}^D x \; \exp(-iq\cdot x)\delta^2\Gamma/\delta U_{\kappa j}{}^{c}(0)\delta N_{\mu i}{}^{a}(x), \tag{10.11}$$

(The minus sign is characteristic for sources of composite fields.) Eq. (10.8) gives

$$\Gamma^{UN}{}_{\mu i}{}^{a}{}_{\kappa j}{}^{c}(q) = \delta_{ij}\delta^{ac}\left[f(q^2)\delta_{\mu\kappa} + g(q^2)q_\mu q_\kappa\right]. \tag{10.12}$$

From the horizon condition (10.10), we conclude

$$f(0) = 1. \tag{10.13}$$

We now use the equations of motion of ω and ω^* to translate this into a condition on the one-particle irreducible ω–ω^* propagator defined by

$$\Gamma^{\omega^*\omega}{}_{i}{}^{a}{}_{j}{}^{c}(q) = \int \mathrm{d}^D x \, \exp[-iq\cdot(x-y)]\,\delta^2\Gamma/\delta\omega_i^a(x)\delta\omega^*{}_j^c(y). \tag{10.14}$$

From eq. (5.22), we have, with suppression of indices,

$$\delta^2\Gamma/\delta\omega_x\delta\omega^*{}_y = -\partial^y\cdot\delta^2\Gamma/\delta\omega_x\delta U_y - gV_y\times\delta^2\Gamma/\delta\omega_x\delta K_y.$$

The second term vanishes for physical values of the sources, and we have

$$\delta^2\Gamma/\delta\omega_x\delta\omega^*{}_y = \partial^y\cdot\delta^2\Gamma/\delta U_y\delta\omega_x$$

$$= \partial^y\cdot\delta/\delta U_y\left[-\partial^x\cdot\delta\Gamma/\delta N_x + g(\partial\cdot A_x\times\omega_x) - D^x(A)U_x\right],$$

where we have used the equation of motion (5.26). For physical values of the sources, this gives

$$\delta^2\Gamma/\delta\omega_x\delta\omega^*{}_y = -\partial^x\cdot\partial^y\delta(x-y) + \partial^x\cdot\partial^y\cdot\delta\Gamma^2/\delta N_x\delta U_y,$$

$$\Gamma^{\omega^*\omega}{}_{i}{}^{a}{}_{j}{}^{c}(q) = q^2\delta_{ij}\delta^{ac} - q_\mu q_\kappa\Gamma^{UN}{}_{\mu i}{}^{a}{}_{\kappa j}{}^{c}(q), \tag{10.15}$$

whereby the factorization of both external ghost momenta is manifest. This gives

$$\Gamma^{\omega^*\omega}{}_{i}{}^{a}{}_{j}{}^{c}(q) = q^2\delta_{ij}\delta^{ac}\left[1 - f(q^2) - q^2 g(q^2)\right].$$

Form the horizon condition $f(0) = 1$, just derived, and under the assumption that $f(q^2)$ and $g(q^2)$ are regular functions of q^2, we conclude that at $q = 0$, the inverse propagator of the Fermi ghost is of order $(q^2)^2$,

$$\Gamma^{\omega*\omega}{}_{i}{}^{a}{}_{j}{}^{c}(q) = \delta_{ij}\delta^{ac}\mathrm{O}(q^2)^2. \tag{10.16}$$

Remarkably, at $q = 0$, the quantum corrections precisely cancel the tree-level contribution to the inverse propagator of the Fermi ghost!

The $1/(q^2)^2$ singularity of the Fermi ghost propagator at $q=0$ shows that the cluster property does not hold. This may mean that the vacuum is degenerate or that the effective action Γ_r has a stationary point $\delta\Gamma_r/\delta\Phi_r = 0$, at $\Phi_r \neq 0$, where Φ_r represents the set of renormalized elementary fields.

11. Conclusion

The renormalizability which is established here argues strongly for the consistency of the critical limit of lattice gauge theory given in eqs. (1.1) and (2.1). It is particularly striking that renormalizability of the horizon condition holds only for the particular value $\langle h \rangle = f$. Moreover, the dipole singularity of the Fermi ghost propagator found in sect. 10 is a verification in detail of the hypotheses of ref. [3] which lead to the critical limit, as we now explain.

The vanishing of the leading term in $\Gamma_i^{\omega^*\omega a}{}_j^c(q)$ at $q=0$, found in sect. 10, means that if the value f which appears the horizon condition $\langle h \rangle = f$ were any larger, then this propagator would go negative. If that happened, it would indicate that contributions from configurations outside the Gribov horizon dominate this quantity. The fact that it is on the verge of going negative suggests that configurations just at the horizon dominate the functional integral. This might seem surprising. For recall that the propagator of Fermi ghosts is the inverse of the Faddeev–Popov operator **M**, which is positive inside the Gribov horizon, so the Boltzmann factor $\exp(-\gamma H)$ which appears in the partition function vanishes exponentially as the horizon is approached. (In a perturbative expansion, the horizon is always approached from the interior.) Thus one might expect the system to be strongly contained within the horizon. However, the possibility that configurations on the horizon dominate the functional integral is consistent with the hypothesis of ref. [3] that at large euclidean volume V, the probability distribution $P(e)$ of the horizon function per unit volume $h(x)$ is in fact concentrated just at the horizon. More precisely, it was proposed that at large V, $P(e)$ is of the form

$$P(e) = \exp[Vs(e)], \quad 0 < e < f, \tag{11.1}$$

where $s(e)$ has the properties of entropy in classical statistical mechanics, namely $s'(e) > 0$ and $s''(e) < 0$, which express positivity of the temperature and heat capacity respectively. In this case $s(e)$ has its maximum at the end point of the interval, namely at $e=f$, and as the volume V grows without limit, the support of the probability distribution $P(e)$ approaches the horizon. The Boltzmann factor $\exp(-\gamma H)$ modifies the distribution to

$$P(e) = \exp\{V[s(e) - \gamma e]\}, \quad 0 < e < f. \tag{11.2}$$

If γ is slightly larger than $s'(f)$, then, at large V, the new distribution peaks sharply just inside the horizon, because $s(e)$ is monotonically increasing and $s'(e)$

is monotonically decreasing. This condition is assured when γ is determined by the horizon condition written in the form $\langle h \rangle = f - \epsilon$, where ϵ approaches zero. Thus the singular behavior of the Fermi ghost propagator is consistent with the probability distribution being located precisely at the horizon in the infinite-volume limit.

In fact, the dipole singularity of the Fermi ghost propagator suggests that a particular part of the horizon dominates the functional integral. Consider the expectation value

$$\langle \omega^*(x)\omega(x) \rangle = \langle \mathbf{M}^{-1}(x, x; A) \rangle = \left\langle \sum_n \psi_n{}^*(x)\psi_n(x)/\lambda_n \right\rangle,$$

where the eigenfunction expansion is at fixed A, and the average is the ensemble average over A. By translation invariance we have

$$\langle \omega^*(x)\omega(x) \rangle = V^{-1} \int d^4x \langle (\omega^*(x)\omega(x)) \rangle = V^{-1} \left\langle \sum_n 1/\lambda_n \right\rangle.$$

With

$$\langle \omega^*(x)\omega(y) \rangle = (2\pi)^{-4} \int d^4q \, \exp[iq\cdot(x-y)] D(q),$$

this gives, in the infinite-volume limit

$$(2\pi)^{-4} \int d^4q \, D(q) = \int_0^\infty d\lambda \langle \rho(\lambda; A) \rangle / \lambda = \int_0^\infty d\lambda \, \rho(\lambda)/\lambda = \infty,$$

where $\rho(\lambda; A)$ is the density of levels per unit volume of $\mathbf{M}(A)$, in the infinite-volume limit, and $\rho(\lambda)$ is its ensemble average. The integral diverges because $D(q)$ has a $1/(q^2)^2$ singularity. Consequently the average density of levels $\rho(\lambda)$ cannot vanish as fast as any positive power of λ and we conclude that $\rho(0) \approx 1$ (or greater), to within logarithmic factors. This is in marked contrast to minus the Laplace operator, for which the density of levels for positive λ is given by $\rho(\lambda) = \text{const.} \times \lambda$. Thus the configurations A which dominate the functional integral, not only lie on the Gribov horizon where an eigenvalue is about to go negative, but are those with the property that there is a very strong accumulation of levels at $\lambda = 0$. This is consistent with the result in ref. [3]: "all horizons are one horizon" by which is meant that for the relevant configurations, an infinite number of eigenvalues go negative together. To be more explicit, in ref. [3], individual eigenvalues $\lambda_n(A)$ of $\mathbf{M}(A)$ were tracked as A approaches the Gribov horizon. The horizon is defined by $\lambda_1(A)/\lambda_1(0) = 0$. [The rescaling by $\lambda_1(0)$ is necessary to compensate the trivial vanishing of any finite number of eigenvalues which is present also for the Laplace operator in the limit $V \to \infty$. There is also a trivial and irrelevant eigenvalue $\lambda_0(A) = 0$ for all A, which corresponds to global gauge invariance.] It was found that in the limit $V \to \infty$, and for the relevant configurations A, the condition $\lambda_n(A)/\lambda_1(0) = 0$ is satisfied simultaneously for all finite n when it is satisfied for $n = 1$. It was hypothesized in ref. [3] that these special points on the Gribov horizon, where "all horizons are one horizon" also lie on the boundary of the fundamental modular region, and moreover that the measure is concentrated on this special part of the boundary. [The fundamental modular region is the set of absolute minima of the minimizing function defined in sect. 2. It is identified with the physical configuration space which is the quotient space U/G. The fundamental modular region is known to be smaller than the Gribov regions which is the set of relative minima. However, as shown by van Baal [10], their boundaries have n-dimensional manifolds in common, for all integer n, which lie on a single gauge orbit, and where the Faddeev–Popov operator has n vanishing eigenvalues. We refer to refs. [3,10] for a discussion of this interesting geometrical topic.] Thus the dipole singularity of the fermi ghost propagator is consistent with the hypothesis that the measure is concentrated on that part of the horizon where "all horizons are one horizon".

[It is interesting to compare the result $\rho(0) \approx 1$ for the average density of levels of the Faddeev–Popov operator-$\partial \cdot D$ with the analogous property of the Dirac operator $\gamma \cdot D$, when chiral symmetry is spontaneously broken, as indicated by a non-zero value of the order parameter $\langle \psi^* \psi \rangle$. We have

$$\begin{aligned}\langle \psi^*(x)\psi(x)\rangle &= V^{-1}\int \mathrm{d}^4x \langle \mathrm{Tr}(m+\gamma\cdot D)^{-1}(x,x)\rangle \\ &= V^{-1}\int \mathrm{d}^4x \langle m \ \mathrm{tr}\left[m^2-(\gamma\cdot D)^2\right]^{-1}(x,x)\rangle,\end{aligned}$$

where the trace is over spinor indices. Upon expanding in terms of the eigenfunctions of $\gamma \cdot D$ with eigenvalue $i\lambda_n$, we obtain

$$\begin{aligned}\langle \psi^*(x)\psi(x)\rangle &= mV^{-1}\int \mathrm{d}^4x \left\langle \sum_n \psi_n^*(x)\psi_n(x)\left(m^2+\lambda_n^2\right)^{-1}\right\rangle \\ &= mV^{-1}\left\langle \sum_n \left(m^2+\lambda_n^2\right)^{-1}\right\rangle = m\int \mathrm{d}\lambda \langle \sigma(\lambda;\ A)\rangle\left(m^2+\lambda m^2\right)^{-1} \\ &= m\int \mathrm{d}\lambda\ \sigma(\lambda)\left(m^2+\lambda^2\right)^{-1} = \int \mathrm{d}\alpha\ \sigma(m\alpha)\left(1+\alpha^2\right)^{-1},\end{aligned}$$

where $\sigma(\lambda;\ A)$ is the average density of levels of the Dirac operator $\gamma \cdot D$, and $\sigma(\lambda)$ is its ensemble average. In the chiral-invariant limit $m \to 0$, this gives

$$\langle \psi^*(x)\psi(x)\rangle = \pi\sigma(0) \neq 0.$$

Thus a non-zero value for the chiral symmetry breaking parameter means that the average density of levels at $\lambda = 0$ of the Dirac operator $\sigma(0)$ is of order unity like $\rho(0)$.]

In a second-order calculation, Gribov [1] also found a $1/(q^2)^2$ singularity for the Fermi ghost propagator. He pointed out that the corresponding constraints in the Coulomb gauge would imply that the three-dimensional Faddeev–Popov propagator behaves at small q like $1/(q^2)^2$ which corresponds to a linear increase at large distances in position space. Moreover, in a non-abelian gauge theory, the Coulomb potential is replaced by the Faddeev–Popov propagator, and Gribov proposed this as a possible confinement mechanism. (More recently he has considered alternative mechanisms [11].) This is not implausible. However, the Coulomb gauge is not renormalizable, and it remains a challenge to demonstrate that a confinement mechanism operates in the renormalizable gauge presented here. The elements of a theory of confinement appear to be at hand, because the gluon pole at $k = 0$ is eliminated by the proximity of the Gribov horizon in infrared directions [12], and because long-range forces are present, as indicated by the $1/(q^2)^2$ singularity.

A step in this direction would be to verify that the gluon condensate [9], identified as the trace of the energy–momentum tensor, is likely to be finite and calculable in the present scheme, as explained at the end of sect. 9. This would directly relate hadron phenomenology to the horizon of the fundamental modular region. Finally, we remark that the SU(2) gauge field of the electro-weak interactions is also restricted by the horizon that bounds the fundamental modular region, so the results obtained here are also relevant in that theory.

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Note added in proof

The BRS invariance of the Faddeev–Popov action may be used to prove not only renormalizability, but also unitarity of the S-matrix. By contrast, in the present article, BRS invariance of the modified action has been used to prove only renormalizability. This is because the zero-order gluon propagator $k^2[(k^2)^2 + N\gamma]^{-1}$ has unphysical poles at $k^2 = \pm i(N\gamma)^{1/2}$, corresponding to imaginary $(\text{mass})^2$, so there are no consistent physical poles in any finite order of perturbation theory. Moreover the exact asymptotic states of the theory are unknown. Consequently nothing can be said at present about unitarity of the S-matrix. The resolution of this issue may have to wait for a solution of the confinement problem, and the construction of physical hadronic states, both of which are beyond the scope of the present article.

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